Speckle statistics of electromagnetic waves scattered from perfectly conducting random rough surfaces

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By means of the extinction-theorem method, previously reported by J. M. Soto-Crespo and M. Nieto-Vesperinas [J. Opt. Soc. Am. A 6, 367 (1989)], the intensity statistics of the s-polarized electromagnetic field scattered from randomly rough, perfectly conducting, one-dimensional surfaces are studied in the case of surface transverse correlation lengths larger than the wavelength. In particular, the speckle contrast, the intensity probability-density function, and the mean scattered intensity are calculated for both moderately rough, single-scattering surfaces and very rough, multiple-scattering surfaces, for which the rms height is comparable with the transverse correlation length. Two different geometries for which non-Gaussian, fully developed speckle patterns may be produced are considered. On the one hand, we study the scattered field in the Fresnel region. We find that within this region multiple-scattering surfaces may lead to negative-exponential intensity statistics even at distances from the interface for which single-scattering surfaces produce remarkable non-Gaussian effects. On the other hand, non-Gaussian effects arising as a consequence of the reduction of the illuminated area on the interface are encountered in the far zone either for moderately rough, single-scattering surfaces or for very rough, multiple-scattering surfaces. In addition, with respect to multiple-scattering surfaces, it is shown that a minimum distance from the interface and a minimum incident-beam width are required for the backscattering peak to appear in the angular distribution of the averaged speckle intensity, as follows from simple arguments based on the coherent interference among multiple scattered paths.

1. INTRODUCTION

The study of the statistics of the electromagnetic field scattered from random media has become a problem of continuing and widespread interest in the past two decades. It is well known that, as a result of the interference among the distinct random contributions from the scattering centers, the scattered-field pattern exhibits a random pattern consisting of bright and dark regions, usually referred to as speckle. By means of the central-limit theorem,1 the speckle statistics is demonstrated to be Gaussian distributed, regardless of the particular statistical properties of the object, provided that the number of contributing scatterers is large enough. As a matter of fact, earlier theoretical works based either on phenomenological statistical models2,3 or on the Kirchhoff approximation4 predicted Gaussian speckle patterns. In order to avoid misunderstandings over what concerns the notation and terminology from now on, we note that Gaussian speckle statistics means a circular Gaussian joint probability density for the real and the imaginary parts of the scattered field, which in turn leads to a Rayleigh distribution for the scattered field's amplitude (namely, its modulus) and to a negative-exponential distribution for its intensity.1 In addition, the statistical moments of the intensity for speckle patterns obeying Gaussian statistics are well defined: \( \langle I^m \rangle / \langle I \rangle^m = m! \). Thus the speckle contrast \( \sigma_I / \langle I \rangle \) should be 1 in this case \( \sigma_I^2 = \langle I^2 \rangle - \langle I \rangle^2 \).

However, when the number of scattering centers is not sufficiently large, the speckle may be non-Gaussian.5 One can easily show this by probing that the speckle contrast departs from 1. Indeed, there exist situations of practical interest that give rise to non-Gaussian speckle not only for optical waves6 but also for other electromagnetic waves such as non-Rayleigh sea echo encountered when a microwave radar illuminates the sea surface (see, e.g., Ref. 7 and references therein). Several mathematical approaches have been developed for describing non-Gaussian speckle statistics and for explaining existing experiments (for an excellent review, see Jakeman6 and Jakeman and Tough8 by the use of analytical and numerical models that exploit the Kirchhoff approximation8-13 and perturbative methods14 or by the use of discrete-scatterer models, which are easier to handle but less rigorous.2,5,15 These studies have improved our understanding of the non-Gaussian effects but are generally restricted to random surfaces (or surfacelike systems such as phase screens) whose roughness statistical parameters are not exceedingly high with respect to the wavelength and to the surface correlation length.4,15,17

A few years ago the observation of enhanced backscat-
tering from both dense media and rough surfaces, that resulted from multiple scattering by strongly disordered media was reported. Concerning random rough surfaces, multiple-scattering theories have undergone a notable advance thanks to the development of numerical Monte Carlo methods (MCM’s) that solve the exact rigorous scattering integral equations (see a review, e.g., in Ref. 22). These methods were first put forward for perfect conductors, and, later on, were extended to real metals and dielectrics as well as being developed in acoustics.

Some studies based on the MCM and related rigorous approaches focused on the speckle contrast and on angular correlations; these studies described the well-known memory effect and new long-range correlation effects. In this regard, note that, as mentioned in Refs. 37 and 38, the long-range correlations are due both to multiple scattering and to a low number of scattering centers. Under these circumstances, the joint probability density of the scattered field, at two pairs of angles of incidence and scattering, fails to obey a jointly Gaussian distribution, even though this number of scatterers may be large enough to produce negative-exponential first-order statistics for the intensity of the scattered field. It should also be mentioned that non-Rayleigh first-order statistics for the amplitude of the scattered field in highly disordered dense media has been reported.

Here we study the first-order statistics of speckle patterns in the regime of low distances to the rough surface, or of small illuminated areas, in which non-Gaussian statistics has previously been encountered within the range of validity of the Kirchhoff approximation. Hence we pay special attention to very rough, multiple-scattering, perfectly conducting surfaces, for which the rms height is comparable with the transverse correlation length, in contrast to the moderately rough, single-scattering surfaces previously studied. The scattering equations are numerically solved by means of the extinction-theorem method (see Ref. 16), thus allowing us to treat large roughness with a considerable amount of double- or triple-scattering contribution. In all cases analyzed here, the specular component is negligible, so that we avoid non-Gaussian effects that are due to partially developed speckle patterns. Two special geometries producing a small effective number of scattering centers are taken into account: first, effects appearing in the Fresnel region of the surface, and, second, effects in the far zone that are due to a notable reduction of the illuminated area in terms of the surface correlation length. We demonstrate that multiple scattering substantially modifies the statistical properties of the speckle pattern in both cases. The effects of these special situations on the enhanced backscattering phenomenon also are analyzed.

This paper is organized as follows. In Section 2 the scattering situation is described; in addition, a brief summary of some of the scattering equations and of the essentials pertaining to the numerical procedure that we used to solve them is included. The results concerning the speckle properties in the Fresnel and the Fraunhofer regions are shown and discussed in Sections 3 and 4, respectively; these results account for two different regimes of surface roughness. Finally, the main conclusions are outlined in Section 5.

2. SCATTERING GEOMETRY AND EQUATIONS

Let us consider a one-dimensional random rough surface that separates vacuum from a perfectly conductive semi-infinite medium. (The assumptions of one dimensionality and perfect conductivity arise for the sake of simplicity in the numerical treatment of the scattering equations.) The one-dimensional surface is assumed to be a stationary stochastic process with Gaussian statistics that has the following properties: zero mean, a rms deviation of the random height given by $\sigma$, and a Gaussian correlation function

$$c(\tau) = \frac{1}{\sigma^2} \langle D(x + \tau)D(x) \rangle = \exp\left(-\frac{\tau^2}{T^2}\right),$$

where $T$ is the correlation length. The parameters $\sigma$ and $T$ define the degree of roughness of the random interface: $\sigma$ is a measure of the mean deviation of the surface height from the mean plane $z = 0$, and $T$ approximately accounts for the mean distance between valleys and peaks in the surface.

A linearly polarized monochromatic incident wave of frequency $\omega$ impinging upon the random surface at an angle $\theta_0$ from the normal to the mean plane $z = 0$, i.e., the $z$ axis in our geometry. We restrict the analysis to TE polarization ($s$ waves). Therefore, inasmuch as the random interface is one dimensional, the three-dimensional electromagnetic problem becomes a two-dimensional scalar one, where only the $y$ component of the electric field is non-zero; moreover, this component depends only on the $x$ and $z$ coordinates. We choose as incident wave a beam of finite width whose electric field is given by

$$E^{(i)}(r) = \frac{W}{2\sqrt{\pi}c} \frac{\omega}{\epsilon_0} \int_{-\infty}^{\infty} d\beta \cos \beta \exp\left(-\frac{W^2 \omega^2}{4 \epsilon_0^2 \sin^2 \beta}\right) \times \exp\left(i \frac{\omega}{c} [x \sin(\beta + \theta_0) - z \cos(\beta + \theta_0)]\right),$$

where $r = (x, z)$ is the position vector in vacuum. It should be noticed that, in the limit $(\omega c/W) \gg 1$, Eq. (1) leads to a Gaussian beam of half-width $W = g \cos \theta_0$, $g$ being its half-intersect with the plane $z = 0$.

Under the assumptions stated above, and with use of the rigorous formalism of integral equations developed earlier (see Ref. 22), the scattered field in the vacuum region above the rough surface can be expressed as follows:

$$E^{(s)}(r) = \frac{\pi k}{c} \int_{-\infty}^{\infty} dx F(x') H_0^{(1)}(k|r - r'|),$$

where $x' = [x', z'] = D(x')]$ is the position vector on the interface and $k = \omega c/2 \pi \lambda$, $\lambda$ being the wavelength. $H_0^{(1)}(k|r - r'|)$ is the zeroth-order Hankel function of the first kind, which indeed constitutes the two-dimensional Green's function of the present problem. The source function $F(x')$ proportional to the induced electric current density, is obtained from a nonlocal boundary condition (see, for instance, Eq. (A15) of Ref. 16), which in turn comes from the extinction theorem evaluated on the surface.
We are interested in calculating the scattered field at any point in the vacuum half-space, so that both Fresnel and Fraunhofer regions are studied. In the latter case, the asymptotic expression as kr $\rightarrow \infty$ (r = |r|) for the far field is used instead of Eq. (2) [see Eq. (3) of Ref. 16]. In order to characterize the statistics of the scattered field, we focus on the following quantities:

- The normalized mean scattered intensity (MSI):

$$\langle I \rangle = \frac{r}{L} \langle |E^0(x, z)|^2 \rangle,$$

where $r = (x^2 + z^2)^{1/2}$ and $L$ is the incident power flow;

- The speckle contrast (SC):

$$SC = \left( \frac{\langle I^2 \rangle}{\langle I \rangle^2} - 1 \right)^{1/2};$$

- The probability distribution function (PDF) of the scattered intensity $p(i)$, where $i = I/\langle I \rangle$.

We use the same numerical procedure as in Refs. 16, 28, and 30 to solve the scattering integral equations. First, the surface samples are extracted out of a sequence of random numbers (typically 10$^6$), numerically generated by means of the MCM according to the desired surface statistics. Then for each surface profile the scattered field is computed for several incident angles; afterward, it is averaged over a certain number of surface samples. It should be mentioned that, as in Refs. 16, 24, and 30, the number of surface samples is effectively doubled by inclusion of the averaging beams incident at both $\theta_0$ and $-\theta_0$ for each profile.

The calculations were carried out on a VAX 9210 computer. The effective number of surface profiles taken for averages to be smooth enough (namely, with minimum speckle noise) has proven to be 8000 for both the SC and the PDF, 28 while only 200 surface samples suffice for $\langle I \rangle$. 16,24,26,30,31 Each surface profile has a length $L = 30\lambda$ and consists of $N = 300$ sampling points. This means 10 sampling points per wavelength, which is indeed an accurate-enough sampling interval. The results are checked by a probe of the following restrictive conditions: energy conservation, reciprocity theorem, and numerical convergence with $N$.

It should be pointed out that, within the regime of surface correlation lengths studied in this work ($T \equiv \lambda$), the choice of a perfect-conductor scatterer instead of a metallic scatterer is not relevant, as shown in Ref. 28; the advantage of this choice lies in that the scattering equations do become notably simplified, and thus the computer time required for solving them is considerably reduced. On the other hand, we have not studied $p$-polarized (TM) incident waves, because the behavior of the field scattered from perfect conductors in this case for $T \equiv \lambda$ does not differ appreciably from that corresponding to $s$ polarization.

3. SPECKLE IN THE FRESNEL REGION

Throughout this section we study the speckle statistics as a function of the distance z from the random rough surface. The scattered field is calculated on several planes

$z = \text{const.}$ (strictly speaking, horizontal lines in our two-dimensional geometry) through the Fresnel region, from the top of the random surface ($z = 4\sigma$) to a distance of $z = 10\lambda$ (which actually lies in the Fraunhofer region for the scattering parameters used below). The $z = \text{const.}$ lines are sampled with the scattering angle $\theta$ kept fixed, so that $x = z \tan \theta$.

In Fig. 1 the SC at $\theta_0 = 0$ and $\theta = 12^\circ$ for two random rough surfaces is plotted versus $z$: one surface with $\sigma = 0.5\lambda$ and $T = 5\lambda$ and the other one with $\sigma = 1.9\lambda$ and $T = 3.16\lambda$. The surface statistical parameters are chosen so that, for the former surface, the Kirchhoff approximation is valid, 16,24 whereas the latter surface is known to produce a considerable contribution of double and even triple scattering. 26,28,30 The half-width of the incident beam guarantees that the speckle pattern in the far zone is Gaussian for both rough surfaces ( $g = 1.5T$ and $g = 3T$). First we discuss the result for the smoother surface ($\sigma = 0.5\lambda$), for which only single scattering occurs. The qualitative behavior resembles that obtained through both analytical models and experimental measures for random phase screens with a smaller value of the roughness parameter $\sigma$, but quantitative differences can be found. The SC exhibits a remarkable structure as it evolves through vacuum, which reflects the existence of different-length scales in the speckle pattern. 5 Near the interface, only a few small portions of the surface contribute to the formation of the speckle, which in this case looks rather uniform. As a consequence, the SC takes values less than unity; moreover, it tends to zero as $z$ further approaches the surface profile. On leaving the surface vicinity, the contributions from different individual caustics are combined in such a way that the speckle pattern gradually becomes less uniform. At a certain region near the geometrical focus $z_f \sim T^4/\sigma = 25\lambda$, the SC reaches its maximum above SC = 1. The corresponding non-Gaussian speckle pattern shows bright and dark structures. 5 Nevertheless, in light of Fig. 1, it is evident that, for the surface scattering parameters used here, this absolute maximum is not very remarkable and does not exceed the value SC = 1.1 for the surface with $\sigma = 0.5\lambda$. This means that the focusing phenomenon seems to be somewhat weak in the case studied here. If the distance is further increased, the SC decreases slightly toward SC = 1 (see Fig. 1 for $\sigma = 0.5\lambda$ at $z \geq T^4\sigma$). This can be interpreted
as being due to the increment in the number of scatterers contributing to the intensity pattern, which in turn leads to negative-exponential statistics.

In Fig. 2, \( p(i) \) is shown for the surface with \( \sigma = 0.5 \lambda \) and \( T = 5 \lambda \), considering normal incidence and scattering \((\theta_0 = \theta = 0)\) at three different distances: \( z/\lambda = 2, 9, \) and 40. The negative exponential function \( p(i) = \exp(-i) \) is included, too. For \( z = 2 \lambda \), the PDF is narrow and centered about a value close to \( i = 1 \). This reveals that the intensity pattern looks almost uniform, with a most probable value very close to the mean value \( I = \langle I \rangle \), which supports the interpretation based on geometrical optics given above for the SC in the vicinity of the rough surface. In fact, as mentioned in Ref. 28 for a similarly rough interface, the scattered field upon the surface profile has an almost constant amplitude and varies only in its phase. The PDF at \( z = 9 \lambda \) manifests a less marked structure than that encountered at \( z = 2 \lambda \), but it still is considerably different from negative-exponential intensity statistics. Therefore, it should be noted that, even though SC \( \geq 1 \) at \( z = 9 \lambda \) (see Fig. 1), the above-mentioned physical mechanisms involved in the scattering within this focusing region generate an intensity PDF that is clearly not a negative exponential. In this respect it should be mentioned that, among the analytical distribution functions proposed for this scattering situation, the better fit to the computed PDF is obtained for a generalized I-K distribution of order \( \alpha \), as those suggested by Andrews and Phillips for atmospheric scintillation [see Eq. (6) of Ref. 15]. These distributions were introduced on the basis of a weak-scattering model for which \( (I) \) should contain a coherent component and seem to generalize Jakeman’s homodyned distributions of a random walk process with a finite number of steps. In fact, although it is not shown here, the matching with the computed PDF is achieved when the coherent component of the intensity \( i_0 \) entering the I-K distribution is approximately equal to the most probable value of \( p(i) \), namely, \( i_0 = 0.85 (\alpha = 3) \) for \( z = 2 \lambda \) and \( i_0 = 0.4 (\alpha = 5) \) for \( z = 9 \lambda \) (see Fig. 2). This is in striking disagreement with the calculated \( (I) \), which in this case has no coherent component at all. Nevertheless, as mentioned by Jakeman and Tough (see Ref. 6, pp. 485 and 525), there is not yet enough justification of the occurrence of the I-K distributions on physical grounds.

As seen in Fig. 2 at \( z = 40 \lambda \), which is beyond the focusing region, the effective number of scatterers becomes large enough to give negative-exponential intensity statistics. In addition, although it is not shown here, the PDF remains invariantly a negative exponential as \( z \) is further increased, thus reaching the far zone, as expected. It should be pointed out that a negative-exponential intensity statistic does not necessarily imply a circularly symmetric Gaussian PDF for the scattered field. Yet this condition strongly suggests that, for the simple physical system that we are dealing with, the scattered field most likely obeys Gaussian statistics.

So far we have been concerned only with single-scattering speckle patterns. To what extent is this scheme altered when multiple scattering occurs? To answer this question, we next study the SC for the rougher surface of Fig. 1 \((g = 3 T)\). The field scattered from this surface contains a considerable contribution from double- and triple-scattering processes and exhibits a large back-scattering peak in the far-zone mean intensity and enhanced long-range correlations, as shown in Refs. 26, 28, and 37 by means of rigorous and iterative MCM calculations. Note that the first point in the graph of SC in Fig. 1 for this surface corresponds to \( z = 7 \lambda \); as a matter of fact, inasmuch as the maximum of the rough surface lies approximately at \( z \approx 4 \lambda \) it does not make sense to consider points closer to the mean surface \( z = 0 \), since then some of them might be placed in the selvedge and even inside the perfect-conductor semi-infinite medium. The intensity PDF for this scatterer is shown in Fig. 3, at normal incidence and detection for \( z = 9 \lambda \).

The most striking feature of these results is the absence of non-Gaussian effects in the speckle pattern at any distance (calculations at other not-too-large scattering angles reveal identical features in both SC and PDF). The curves in Fig. 3 manifest that the intensity statistics is a negative exponential, and, consequently, Fig. 1 yields SC \( = 1 \) for this surface, apart from a certain amount of speckle noise. This behavior results from the extremely high roughness that characterizes the random surface under consideration. Even if the detector is placed as close to the top of the surface profile as possible, the addition of multiple-scattering contributions is equivalent in practice to this speckle pattern’s being regarded as a result of the contribution of a large effective number of individ-
ual scatterers (provided that the beamwidth is large enough); this large effective number of scatterers leads to a negative-exponential statistics of the intensity. There exists no geometrical focusing region such as that found near rough interfaces for which the Kirchhoff approximation holds (see Fig. 1 for the smoother surface). Furthermore, within the cavities in the selfedge of the random surface, the amplitude of the scattered field may exhibit a rather complex behavior with strong oscillations. These oscillations have been shown to be due to the interference effects between double-scattering paths, by a comparison of the results for the rigorous MCM with those for an iterative treatment up to the double-scattering contribution (see Fig. 9 of Ref. 28; as mentioned in Ref. 28, the single-scattering field on the surface yields only a constant factor of 2 for its amplitude). Thus we believe that the multiple-scattering effects in the amplitude prevent the SC from tending to zero when z tends to the top of the surface profile (see Fig. 1 for \( \sigma = 1.9 \lambda \) and \( T = 3.16 \lambda \)). Therefore, in order to achieve non-Gaussian speckle patterns within this regime of surface roughness, one should artificially reduce the number of scatterers by illuminating a small portion of the rough surface; this is done in Section 4 below.

In light of this study, it is interesting to analyze how the averaged speckle intensity evolves through propagation. For moderately rough surfaces, our calculations, not shown here, reveal that the MSI adopts its far-zone envelope when a relaxed condition for the lower bound of the Fraunhofer region is satisfied, namely, when \( z \geq k g^2 = 10^2 \lambda \). The occurrence of enhanced backscattering makes the case of very rough surfaces doubly interesting. In Fig. 4 we plot \( \langle I \rangle \) versus the scattering angle \( \theta \) for \( \theta_0 = 0 \) at different distances for a surface with \( \sigma = 1.9 \lambda \) and \( T = 3.16 \lambda \), within the \( \theta \) interval \( [0, 45^\circ] \) (the part of the graph for negative \( \theta \) is omitted because of its symmetry). The structure of the backscattering peak clearly appears at \( z = 10^2 \lambda \), which is less restrictive than the condition for the far zone (in this case the half-width of the incident beam is \( g = 10 \lambda \)). This can be explained as follows: The constructive interference between double- or higher-scattering paths, which, owing to time-reversal invariance, leads to enhanced backscattering, becomes effective only provided that its angular width \( (k l^*)^{-1} \) \((l^* \text{ being the transport mean free path; see Maradudin et al. in Ref. 23)} \) is larger than \( l^*/z \). Through this simple reasoning we obtain the approximate condition for the appearance of this peak \( z > k (l^*)^2 = 10^3 \lambda \), which actually is very similar to the value of \( z \) inferred from Fig. 4.

4. SPECKLE IN THE FRAUNHOFER REGION

In this section we restrict the analysis to the far zone \( z \gg k W^2 \). In this region non-Gaussian speckle statistics can be found as a consequence of the reduction in the illuminated area of the rough surface\(^{3,11}\); the ratio \( g/T \) between the incident beam half-width and the correlation length constitutes the scaling parameter.

First, we consider a smooth surface whose roughness parameters \( (\sigma = \lambda \) and \( T = 5 \lambda \)) lie near the limit of validity of the Kirchhoff approximation for normal incidence.\(^{16}\) Figure 5 shows the SC for this random surface as a function of the scattering angle \( |\theta| \leq 45^\circ \) for beams incident at (a) \( \theta_0 = 0 \) and (b) \( \theta_0 = 15^\circ \). In each case, two values of the half-intersect of the incident beam are accounted for: \( g/T = 1 \) and \( g/T = 0.5 \). Several facts should be pointed out concerning the results shown in Fig. 5. First, it is clearly seen that, even for the largest \( g \), the SC takes on values substantially above \( SC = 1 \). These non-Gaussian effects in the far field are due entirely to the small value of \( g/T \), which can be understood as a limitation in the effective number of scatterers contributing to the speckle pattern.\(^9\) As a matter of fact, further decreasing \( g \), \( g/T = 0.5 \) produces an increase in the SC, namely, an enhancement of the non-Gaussian effects. Second, the SC exhibits a remarkable dependence on the scattering angle.\(^7\) The SC is almost constant within an

\[ \frac{(I^2)(\langle I \rangle^2 - 1)^{1/2}}{\langle I \rangle^2} \]

\[ \theta (\deg.) \]

\[ \frac{(I^2)(\langle I \rangle^2 - 1)^{1/2}}{\langle I \rangle^2} \]

\[ \theta (\deg.) \]

Fig. 5. Speckle contrast versus \( \theta \) for a perfectly conductive surface with \( \sigma = \lambda \) and \( T = 5 \lambda \). Stars, \( g/T = 1 \); circles, \( g/T = 0.5 \). (a) \( \theta_0 = 0 \), (b) \( \theta = 15^\circ \). Average over \( N = 8000 \) samples.
The width of the MSI angular distribution is similar to the negligible. In addition, our calculations reveal that the direction, with the coherent, specular component being that of the MSI. It is well understood if one compares the distribution of the SC with when moving out of such interval. This effect can be shown.

Fig. 6. Stars, intensity PDF for a perfectly conductive surface with $\sigma = \lambda$ and $T = 5\lambda$, with $g/T = 0.5$ and $\theta_0 = \theta = 0$. Average over $N = 8000$ samples. A negative-exponential function (solid curve) and a $K$ distribution $P_2(i)$ (dashed curve) are also shown.

interval of $\theta_0 = 30^\circ$ about the specular direction and rises when moving out of such interval. This effect can be understood if one compares the distribution of the SC with that of the MSI. It is well known\(^{16}\) that, for this surface, $(I)$ has an almost-Gaussian shape centered on the specular direction, with the coherent, specular component being negligible. In addition, our calculations reveal that the width of the MSI angular distribution is similar to the width of the SC angular distribution. Thus the increase in the SC for $|\theta - \theta_0| \approx \theta_0/2$ is most likely related to the decrease in the effective number of scatterers that radiate in directions contributing to the intensity pattern at such scattering angles.

The intensity PDF in the specular direction for the surface corresponding to Fig. 5 is shown in Fig. 6 in the case of the narrowest incident beam ($g/T = 0.5$). The negative exponential is included, too. The departure of the intensity statistics from the negative-exponential function is evident from these curves and is especially notable for small $i$. Furthermore, the computed $p(i)$ resembles a $K$ distribution of index $n$, namely, $P_n(i)$, which depends on the modified Bessel function of the second kind, $K_{n-1}$, as introduced by Jakeman and Pusey in the context of multi-scale surfaces and small illuminated areas by means of random-walk models for negative binomial distribution of scatterers\(^7\) (see also Ref. 5, p. 456). This distribution can be written in a normalized form as follows:

$$P_n(i) = \frac{2}{\Gamma(n)} \sqrt{n^{2i+1}K_{n-1}(2\sqrt{n}i)}.$$  \hspace{1cm} (5)

The index of the $P_n(i)$ distribution is proportional to the number of scatterers effectively participating in the scattering process\(^5\); in the limit $n \to \infty$, the negative exponential is retrieved. In our case, this index depends on the incident beam width, on the surface roughness, and, less strongly, on the scattering angle. In fact, if one of these parameters is changed, the statistics changes and hence so does the value of $n$ in the function $P_n(i)$. For the scattering parameters used in the calculation shown in Fig. 6, we have chosen as the distribution that better matches the computed PDF the function $P_2(i)$, which involves a $K_1$ function.

Let us turn once again to a very rough surface with a considerable contribution from multiple-scattering events ($\sigma = 2\lambda$ and $T = 5\lambda$). The angular dependence of the SC is shown in Fig. 7 for this surface for two illuminated widths $g/T = 1$ and $g/T = 0.25$: (a) $\theta_0 = 0$ and (b) $\theta_0 = 30^\circ$. Unlike the SC for the surface that produces single scattering (see Fig. 5), we perceive from Fig. 7 that the SC for this case does not so substantially increase from its minimum value around the specular direction as the scattering angle increases, at least within the interval $|\theta| \leq 45^\circ$. As we see below, for $\theta$ out of the backscattering peak, the angular distribution of the MSI is a rather constant distribution between $\theta = -45^\circ$ and $\theta = 45^\circ$, manifesting no particular dependence on the scattering direction. This suggests that, owing to the large roughness, the effective number of scatterers that radiate at a given angle hardly varies as this angle increases within the range shown in Fig. 7. Hence the SC should take an almost-constant value, as occurs in Fig. 7. Obviously, as grazing observation angles ($|\theta| \leq 90^\circ$) are approached, $(I) \to 0$, so that the SC should grow up (as obtained in the calculations, though not shown here).

In Fig. 8 we plot $p(i)$ for the rough surface with $\sigma = 2\lambda$ and $T = 5\lambda$, taking $g/T = 0.75$ and $\theta_0 = -\theta = 30^\circ$, along with a negative exponential and a $P_1(i)$ distribution. The departure from the negative-exponential statistics of the intensity is evident, in a way analogous to that encountered in Fig. 6. Moreover, the intensity statistics resembles a $P_1(i)$, as noted above. In this respect, these non-Gaussian effects appear to be very similar to those of Fig. 6 (for which $\sigma = \lambda$) around the specular direction, in spite of the fact that the surface roughness is two times larger ($\sigma = 2\lambda$).
direction; far from $\theta = \theta_0$, multiple scattering increases the number of scatterers, thus reducing non-Gaussian effects.

Figure 9 shows the dependence of the SC on the width of the incident beam for the two surfaces ($\sigma = \lambda$ and $\sigma = 2\lambda$, the correlation length being $T = 5\lambda$). To avoid the appearance of enhanced non-Gaussian effects as $|\theta_0 - \theta|$ becomes large, we chose $\theta_0 = \theta = 30^\circ$. Actually, we obtain each point of the curve by averaging the SC over a narrow interval of scattering angles ($\Delta \theta = 5^\circ$), centered about normal detection. The qualitative behavior looks similar to that accounted for by means of the Kirchhoff approximation by Jakeman and McWhirter\textsuperscript{14} and more accurately by Levine,\textsuperscript{13} although there exist quantitative differences. For $g/T \gg 1$, the speckle-intensity statistics tends to a negative-exponential PDF. As $g/T$ diminishes, the SC increases, owing to the reduction in the number of scatterers, so that non-Gaussian effects appear. At a certain value of $g/T$ (in this case $g/T = 0.25$), these non-Gaussian effects reach a maximum enhancement. On further decrease of $g/T$, the SC rapidly decreases toward zero. One can explain this by realizing that the limit $g/T \rightarrow 0$ corresponds to conversion of the illuminated area to a point on the surface; such geometry implies a uniform speckle pattern with constant intensity, for which $SC = 0$.

The decrease in the illuminated area has interesting consequences concerning the MSI. When $g/T \geq 1$, multiple scattering produces enhanced backscattering. By decreasing $g/T$, we cut off in practice the length of time-reversed double- or higher-scattering paths within the selvedge of the surface. Hence when the maximum length permitted becomes smaller than the transport mean free path $l^* \propto T$, the constructive interference that gives rise to enhanced backscattering no longer takes place. This is shown in Fig. 10, in which the angular distribution of $\langle I \rangle$ is plotted for different values of $g/T$. As predicted, the backscattering peak decays with $g/T$ and finally disappears for $g/T = 0.25$.

5. CONCLUSIONS

In what follows, we outline the main conclusions derived from this work with respect to the speckle properties in rough surface scattering.

Within the Fresnel region of a single-scattering rough surface ($\sigma = 0.5\lambda$ and $T = 5\lambda$), we retrieve the qualitative behavior for the SC and the intensity statistics previously predicted through approximate analytical treatments,\textsuperscript{5} although some quantitative differences appear: more precisely, the SC hardly exceeds the value $SC = 1$ about the focusing region. The situation changes drastically when multiple scattering takes place. For a very rough surface ($\sigma = 1.9\lambda$ and $T = 3.16\lambda$) presenting a great amount of double-scattering contribution, it is demonstrated that non-Gaussian effects considerably diminish in the Fresnel region (if the incident-beam half-width is large enough: $g = 3T$). We find that, for $x$ approaching the surface profile, the SC no longer tends to zero (see Fig. 1), and the PDF is practically a negative exponential (see Fig. 3). We argue that in this case multiple scattering implies that the effective number of scattering centers is large even if $x$ is close to the top of the surface profile. For this surface with $\sigma = 1.9\lambda$ and $T = 3.16\lambda$, the MSI has been studied as...
a function of the distance \( z \) within the Fresnel region (see Fig. 4). The backscattering peak is shown to appear beyond \( z \approx k^{(*)} = 100 \text{A} \), the minimum distance at which multiple-scattering constructive interference between time-reversed paths takes place.

In the Fraunhofer region, non-Gaussian effects arise as a consequence of the small illuminated area on the surface (in terms of the correlation length \( T \)), for rough surfaces both without and with relevant multiple-scattering contributions (their respective parameters being \( \sigma = \lambda \) and \( T = 5\lambda \), and \( \sigma = 2\lambda \) and \( T = 5\lambda \); see Figs. 5–9). These effects present certain features with respect to their dependence on the scattering angle \( \theta \), the illuminated area \( g/T \), or the surface roughness \( \sigma \). First, they are enhanced at directions of observation far away from the specular direction. In addition, the angular interval of constant SC is shown to be wider for the multiple-scattering surface than for the single-scattering surface. This bears a connection with the angular distribution of MSL, which turns out to be wider for the former surface than for the latter. On the contrary, we find that, within the angular interval of constant SC about the specular direction, the behavior of the SC versus \( g/T \) for the multiple-scattering, rougher surface is similar to that obtained for the single-scattering, moderately rough surface. Therefore we conclude that once a maximum roughness of only single-scattering contribution is reached (recall that, within the regime of validity of the Kirchhoff approximation, it is well known that the SC grows with \( \sigma = \lambda \) and \( T = 5\lambda \), fixed\(^1\)), a further increase in the surface roughness (so that multiple scattering begins to appear) does not result in an enhancement of the non-Gaussian effects about the specular direction (see Fig. 9), apart from a slight decrease in the value of the beamwidth required for the SC to be a maximum. The intensity PDF corresponding to this kind of non-Gaussian effects is shown to be fitted rather well by the \( K \) distributions, as discussed in Ref. 5 [see the \( P_n(i) \) functions in Eq. (5)]. The precise value of the index \( n \), which indicates the degree of departure from negative-exponential statistics and is related to the effective number of scatterers, varies depending on the scattering parameters. We note also that when the illuminated area is made smaller than the transport mean free path of the waves on the rough surface, the constructive interference between multiple-scattering reciprocal paths is eliminated, and hence the backscattering peak disappears (see Fig. 10).

Finally, we point out that it would be quite interesting to design and perform multiple-scattering experiments that might permit corroboration or extension of the results presented in this paper.

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