Transitory saturation and the frequency of new product introductions

Ramon Caminal*
Institut d'Anàlisi Ecònomica, CSIC, and CEPR
Campus UAB, 08193 Bellaterra, Barcelona, Spain
(Preliminary draft)

March 17, 2010

Abstract

The goal of this paper is to analyze the effect of transitory saturation on the frequency of new product introductions. We focus on those product categories in which a specific product is purchased only once but repeat purchases are made in the same product category: like movies, books, concerts, computer games, etc. The model considers infinitely-lived, forward-looking consumers and firms. We show that a monopolist may introduce new product too frequently with respect to both the first and the second best. We also show that, provided firms share enough information about their production plans, competition exacerbates the tendency towards excessively frequent introduction of new products.

JEL Classification numbers: L12, L13

key words: transitory saturation, repeat purchases, monopoly, competition

1 Introduction

This paper analyzes the performance of markets for non-durable goods in which consumer preferences are subject to transitory saturation. For many repeat-purchase, consumption goods, and specially for leisure goods, the utility derived from a consumption episode tends to increase with the time elapsed since the last purchase. The paper focuses on how consumers' transitory saturation affects the frequency of new product introductions.

*I would like to thank Martin Perry and Roberto Burguet for their useful comments, and the Spanish Ministry of Science and Innovation (ECO2008-01850), Barcelona GSE and Generalitat de Catalunya for their support.

Consider the example of the motion picture industry. Major Hollywood studios typically spend large amounts in advertising around the release date. Concentration of advertising efforts seems essential to the profitability of the movie since approximately 40% of box office revenues are obtained during the first week and very few movies generate significant revenue beyond the sixth week. Studios choose very carefully the timing of movie releases, trying to avoid as much as possible any overlapping with other movies that appeal to similar audiences. It has been reported that studios are constantly rescheduling their opening dates and that they often pre announce their plans in order to avoid head-to-head competition.[1] However, shifting opening dates probably involves significant costs and, hence, independent suppliers are likely to have a hard time trying to coordinate their release dates, which suggests that the degree of information sharing is critical. In fact, it is often suggested that sharing information about movie releases tends to relax competition and reduce welfare.[2]

Consumers' transitory saturation is likely to be an important ingredient of the timing game played in the motion picture industry. Moviegoers need some time to "recover" from a previous experience before they are ready to see another movie of a similar type. Nevertheless, the speed of recovery varies a lot across consumers. Transitory saturation has been ignored in previous analysis, which in contrast often assumed that demand at a point in time is exogenous. In this context, the timing of releases matters because studios trade off the high revenues of the holiday seasons with the intensity of competition (Krieder and Weinberg, 1998). However, Einav (2009) has shown that box office revenues would increase if distributors did not cluster their releases so much around big holiday weekends, which clearly indicates that total demand does depend on the timing of releases.[3] In a similar vein, Corts (2001) has shown that market

[1] For a useful discussion of common practices and stylized facts in the motion picture industry, see Corts (2001), Krider and Weinberg (1998), and Einav (2007).

[2] In 2006 major film distributors (members of the Spanish Federation of Film Distributors) were sanctioned by Spanish competition authorities. One of the reasons was that through the Federation film distributors exchanged information regarding the planned dates for releases.

[3] Movie theaters employ uniform prices. See Orbach and Einav (2007) for a discussion of potential reasons. Hence, changes in revenues are exclusively the result of changes in ticket sales.

structure affects the timing of movie releases. When two similar films are both jointly produced and jointly distributed then they are released further apart than in the case they have neither a producer nor a distributor in common. This suggests that integrated structures internalize negative externalities and set schedules in order to maximize joint profits. By spreading opening times more evenly they are able to increase total demand and profits.

Transitory saturation is likely to affect the timing of new product introductions not only in the motion picture industry, but also in a much broader set of goods. In particular, it may be important in those product categories in which a specific good is typically purchased only once but repeat purchases are made in the same product category. Examples include concerts, books, music recordings, computer games, etc.

In this paper we present a dynamic (discrete time) model with infinitely-lived consumers and firms in order to analyze the effect of transitory saturation on the timing of new product introductions. In our stylized model production, commercialization, and consumption occur simultaneously. Thus, the model literally describes the market for highly perishable goods, like concerts. Even though soem goods, like movies and computer games, may be available to the consumer for long periods of time, strong preference for purchasing the good as soon as it is available, reinforced by heavy advertising campaigns at the time of release, render the gap between commercialization and consumption negligible.[4]

Somewhat paradoxically, we show that monopoly power need not reduce the frequency of new product introductions below the level that maximizes social welfare. In fact, if the impact of transitory saturation is sufficiently strong then a monopolist may introduce new products too frequently with respect to both the first and the second best. Competition, at least under some conditions, is shown to exacerbate the tendency towards excessively frequent new product introductions.

Under transitory saturation consumers are willing to purchase the good only if the current net surplus (their current valuation minus the price) is higher than the opportunity cost of waiting (if they abstain from current consumption

[4] We do not distinguish either between production and commercialization.

their future valuation of the good will in average be higher). Thus, if new products are introduced with high frequency, consumers are choosier because the opportunity cost of waiting is larger. However, the opportunity cost of waiting decreases with the level of prices. In other words, consumers' incentives to speculate and delay the purchase increase with the frequency of new product introcutions and decrease with prices. Firms also have incentives to speculate. In the case new products are introduced relatively often, a monopolist finds it optimal to charge a price above the level that maximizes current profits. The reason is that abstaining consumers will have a higher (in average) willingness to pay for the next product.

When the monopolist introduces a new product then the optimal price leaves some consumers out of the market and profits are lower than total surplus (the standard apropriability problem). If the impact of transitory saturation is small then the appropriability problem implies that the frequency of new product introduction is inefficiently low: the monopolist only captures a fraction of the surplus but must pay the entire cost. In contrast, if the impact of transitory saturation is sufficiently strong, the monopolist is able to capture a higher proportion of total surplus under high frequency. As a result, a monopolist may introduce new products more frequently than a utilitarian social planner. In this case, we get the surprising result that monopoly power may imply both overproduction (new products are introduced too quickly) and underconsumption (new products are purchased by too few consumers).

It is also shown that in the monopoly case multiple (Markov) equilibria may exist and that the high frequency may generate lower surplus than the low frequency equilibrium. The economic intuition is discussed in Section 4, right after the analysis. Also in Section 5 we discuss under what conditions competition is likely to exacerbate the tendency towards excessively frequent new product introductions. It is also argued that information sharing among independent suppliers and coordination of production plans need not reduce social welfare.

The role of transitory saturation in repeat purchase, perishable goods is somewhat analogous to the effect of depreciation or quality improvements in

durable goods; in the sense that they both induce repeat purchases and generate a link between current demand and past purchases. Like transitory saturation, depreciation of durable goods also generates an endogenous pattern of repeat purchases. However, if there are no quality improvements then the good is continuously available; i.e., the timing of production is not an issue.[5]

The analogies between our framework and the study of quality improvements in durable goods are clearer. In the latter case, the pattern of repeat purchases is also endogenous. Moreover, the timing of product innovation is also a crucial variable. There is an extensive literature analyzing product innovation (quality upgrading) in durable goods. In line with the results of our paper, it has been shown that a monopoly supplier may introduce more upgrades than socially optimal (See, for instance, Waldman, 1993; Choi, 1994; Ellison and Fudenberg, 1998). These papers present two-period models and focus on network externalities and compatibility between old and new models. Perhaps, the model closest to ours is Fishman and Rob (2000), in the sense that they also consider an infinite horizon framework and study how efficient is the frequency of innovations generated under monopoly. A crucial assumption of their model is that innovations are cumulative and show that a monopolist introduces new products too slowly with respect to the social optimum (at least, in case of no price discrimination and no planned obsolescence). The reason is that current innovation efforts have a positive effect on all subsequent models, but consumers are only willing to pay for the incremental flow of services the current model provides.

Despite of certain similarities with the case of durable goods, our model focuses on non-durable goods for which consumer preferences are subject to transitory saturation. Whether or not the current framework can be adapted to deal with durable goods is left for future research.

[5] The literature on depreciating durable goods has focused on very different issues; for instance, on the role of replacement sales in preventing the Coase conjecture (Bond and Samuelson, 1984; Driskill, 1997), or the effect of scrapping subsidies (Adda and Cooper, 2000).

2 The model

This is a dynamic, infinite horizon, model of a market for a homogeneous, perishable good. For convenience, time is a discrete variable, denoted by $t, t = 0, 1, 2, ...$ The good can be produced in period t at a fixed cost, F, and constant marginal cost, which is normalized to 0. Thus, literally the model describes a perishable good that it may not be continously available (think, for instance, of concerts). However, it is straightforward to interpret it also as a model of new product introductions, in which consumers purchase a specific product only once but they make repeat purchases of a product category (movies).

The least conventional aspect of the model are consumer preferences, which are history dependent. In particular, the market is populated by a continuum of infinitely-lived consumers of mass one. In each period a consumer can purchase at most one unit of the good. Individual consumer preferences are stochastic and depend on the recent history of purchases. More specifically, consumer $i's$ instantaneous utility in period t, r_{it}, is a stochastic variable whose density function depends on the consumption behavior in the previous period. If consumer i did not consume in period $t-1$ then r_{it} is a realization of distribution NC, which is uniform on the interval $[0, 1]$. In contrast, if consumer i did consume in period $t-1$, then r_{it} is a realization of distribution C, such that:

$$r_{it} = \begin{cases} 0, \text{ with probability } 1-\mu \\ \sim U[0,1], \text{ with probability } \mu \end{cases}$$

where μ is a fixed parameter, $0 \leq \mu < 1$. Thus, conditional on not having consumed in period $t-1$ a consumer's expected utility from and additional consumption episode in period t is equal to $\frac{1}{2}$. However, if a consumer did consume in the previous period, her expected utility from additional consumption is equal to $\frac{\mu}{2} < \frac{1}{2}$. The parameter μ measures the extent of transitory saturation, with $\mu = 0$ describing the maximum saturation. Note that, for tractability, the preference cycle lasts for only two periods.

We denote by α_t the fraction of consumers that get a positive realization of r_{it}. Clearly, α_t will depend on α_{t-1} and on the consumption behavior in period $t-1$, in a way that will be specified below. Thus, α_0 is one of the exogenous parameters of the model, but α_t, for all $t > 0$, are endogenous variables. We

restrict ourselves to the particular case of $\alpha_0 = 1$. The reader will soon realize that this is equivalent to focusing on the medium and long-run performance of the industry. It turns out that studying the short-fun behavior for an arbitrary value of $\alpha_0 \in [0,1]$, is quite demanding technically (specially when discussing the efficiency of various market structures) and brings about little additional insights.

A consumer obtains a net surplus of $r_{it} - p_t$ if she chooses to consume in period t at a price p_t. Otherwise she gets 0.Consumers are forward looking and maximize the expected discount value of their net surplus, using a discount factor, $\delta, 0 \leq \delta < 1$. Firms are also infinitely-lived and maximize their expected discounted value of profits, using the same discount factor, δ.

3 The first best

An allocation can be described as a pair (γ_t, I_t) for each period $t = 0, 1, 2, ..$, where $\gamma = 1$ indicates that production takes place (and the fixed cost is paid) and $\gamma = 0$ indicates no production (and hence no consumption). The set $I \subseteq [0,1]$ describes the set of values of r_i for which agents consume in case production has taken place. Thus, if $\gamma_t = 0$, then $I_t = \varnothing$. In this section we characterize the allocation that maximizes the expected discounted value of total surplus (profits plus consumer surplus). We start by characterizing the optimal consumption pattern conditional on $\gamma_t = 1$ for all t. Next, we consider alternative production paths and their associated consumption patterns.

Suppose production takes place every period. Since marginal costs are constant the optimal choice of each consumer only depends on her own realization, r_{it}, and not on the aggregate consumption level. If consumer i acquires the good then her expected level of utility is $r_{it} + \delta U^C_{t+1}$, where U^C_{t+1} is the continuation value at the beginning of period $t+1$, before she gets a draw from distribution C. If she waits (does not consume) then she gets δU^{NC}_{t+1}, where U^{NC}_{t+1} is the continuation value at the beginning of period $t+1$, before she gets a draw from distribution NC. Note that neither U^C_{t+1} nor U^{NC}_{t+1} depend on r_{it}. Thus, from an efficiency point of view, consumer i should consume if and only if $r_{it} \geq \overline{r}_t$, where

$$\overline{r}_t = \delta \left(U_{t+1}^{NC} - U_{t+1}^{C} \right) \tag{1}$$

In other words, $I_t = [\overline{r}_t, 1]$. Thus, an efficient allocation consists of a sequence of threshold values, $\{\overline{r}_t\}_{t=0}^{\infty}$, that maximizes the present value of total surplus:

$$W_0 = \sum_{t=0}^{\infty} \delta^t \alpha_t R(\overline{r}_t) \tag{2}$$

where $R(\overline{r}_t)$ is the total surplus generated in period t if all consumers with $r_{it} \in [\overline{r}_t, 1]$ acquire the good. That is, $R(\overline{r}_t) = \int_{\overline{r}_t}^{1} r dr = \frac{1}{2}\left(1 - \overline{r}_t^2\right)$. Note that α_t is determined by past consumption behavior. More specifically, in period $t-1$ the fraction of agents who consumed was $(1 - \overline{r}_{t-1})\alpha_{t-1}$. A fraction μ of these consumers, plus all those who did not consume, $1 - (1 - \overline{r}_{t-1})\alpha_{t-1}$, will be able to draw in period t a positive r_{it}. Therefore, the law of motion of α_t is:

$$\alpha_t = 1 - (1 - \mu)(1 - \overline{r}_{t-1})\alpha_{t-1} \tag{3}$$

The solution of this optimization problem is characterized in the following lemma. If we let $z = \delta(1 - \mu)$ then

Lemma 1 *If production takes places every period, it is efficient that consumers purchase the good if and only if $r_{it} \geq \overline{r}^*$, which is given by:*

$$\overline{r}^* = \frac{1 + z - \sqrt{1 + 2z}}{z}$$

The proof can be found in the Appendix.

Note that $\overline{r}^*$ is an increasing function of z and can take values in the interval $\left[0, 2 - \sqrt{3}\right]$. Consumers' history-dependent preferences matter to the extent that they value the future. Thus, the impact of transitory saturation is captured by this composite parameter z. If either $\mu = 1$ (in which case distributions NC and C coincide) or $\delta = 0$, then $z = 0$ and $\overline{r}^* = 0$, since there is no relevant intertemporal preference effect. In the other extreme, if consumers do not discount the future ($\delta = 1$) and transitory saturation is most intense ($\mu = 0$) then $z = 1$, and $\overline{r}^*$ reaches its highest possible value $(2 - \sqrt{3})$. In general, it is efficient

to consume if instantaneous utility is sufficiently higher than the opportunity cost of waiting, $\delta\left(U^{NC}-U^{C}\right)$, which is constant over time.

Since the optimal $\overline{r}_t$ is constant over time, then the law of motion becomes:

$$\alpha_t = 1-(1-\mu)(1-\overline{r}^*)\,\alpha_{t-1}. \tag{4}$$

Since $0<(1-\mu)(1-\overline{r})<1$, then α_t converges to the steady state value $\alpha^* = \frac{1}{1+(1-\mu)(1-\overline{r}^*)}$, following an oscillating trajectory.

Let us now turn to the endogenous determination of the frequency of production. All possible optimal trajectories can be identified with the number of production periods, N, that precede a period without production. In other words, for an arbitrary N, $\gamma_0=\gamma_1=...=\gamma_{N-1}=1$ and $\gamma_N=0$. As a result, $\alpha_{N+1}=1$ and a new cycle of N consecutive production periods starts. In particular, one of the possible optimal paths consists of producing every period $(N=\infty)$, which can be called option (∞), and whose associated payoff can be computed rewriting equation (2) using equation (4):

$$W^{\infty} = \frac{\alpha_0+\frac{\delta}{1-\delta}}{A^*} - \frac{F}{1-\delta} = \frac{1}{1-\delta}\left(\frac{1}{A^*}-F\right) \tag{5}$$

where $A^* = \frac{1+z(1-\overline{r}^*)}{R(\overline{r}^*)}$. Note that A^* increases with z.

A second possible trajectory consists of producing every other period $(N=1)$, starting in period 0,i.e., $\gamma_t=1$ if and only if $t=0,2,4,...$ In this case, in all subsequent production periods $\alpha_t=1$, independently of current consumption decisions. Hence, the value of waiting is 0 and consequently it is efficient to let agents consume provided $r_i\geq 0$. The expected payoff of option (1) is:

$$W^1 = \alpha_0 R(0) - F + \frac{\delta^2}{1-\delta^2}\left(R(0)-F\right) = \frac{1-2F}{2\left(1-\delta^2\right)} \tag{6}$$

In the Appendix we prove the following Lemma.

Lemma 2 *If $F<\frac{1}{2}$, an efficient allocation implies either production takes place every period $(N=\infty)$ or every other period $(N=1)$.*

Thus, the optimal policy can be found by simply comparing equations (5) and (6). If we let $F^* = \frac{1}{\delta}\left(\frac{1+\delta}{A^*}-\frac{1}{2}\right)$ we can state the following result:

Proposition 3 *The first best allocation consists of: (i) Producing every period ($\gamma_t = 1$ for all $t \geq 0$) and letting all agents with $r_{it} \geq \overline{r}^*$ consume, if $F \in [0, F^*]$, (ii) Producing every other period ($\gamma_t = 1$ if and only if $t = 0, 2, 4, ...$) and letting all agents with consume if $r_{it} \geq 0$, if $F \in \left[F^*, \frac{1}{2}\right]$.*

4 Monopoly

Consider the case in which there is a single supplier. In each period the monopolist can choose whether or not to produce, which is indicated by the variable $\gamma_t \in \{0, 1\}$.$\gamma_t = 1$ means that the firm produces in period t (and pays the fixed cost). In this case it must also announce a price p_t. After observing the price and their own realizations, r_{it}, consumers decide whether or not to consume. Since all consumers face the same continuation value, then consumers' decisions can be described by a threshold value, $\overline{r}_t$, i.e., consumers purchase the good if and only if $r_{it} \geq \overline{r}_t$. We restrict ourselves to Markov strategies; that is, γ_t, p_t, depend exclusively on the state variable, α_t, and $\overline{r}_t$ depend on α_t and p_t. Moreover, we focus on equilibria where transaction prices are constant over time. That is, along the equilibrium path $p_t = p$ whenever $\gamma_t = 1$.

It will be useful to start analyzing the case in which production takes place every period.

4.1 Preliminaries: production every period

Suppose that the fixed cost is sufficiently low so that the monopolists finds it optimal to produce every period, $\gamma_t = 1$ for all t. In this case, the strategies of the firm and consumers can be written as $p(\alpha_t) = p^m$ and $\overline{r}(p_t, \alpha_t) = f(p_t)$, respectively. We will denote by $\overline{r}^m$ the threshold value along the equilibrium path, i.e., , $\overline{r}^m = f(p^m)$. In equilibrium, p^m is the price that maximizes the expected value of profits, under the beliefs that consumers behave according to $\overline{r}(p_t, \alpha_t) = f(p_t)$ and that $(p^m, \overline{r}^m)$ will prevail in the future, and $\overline{r}(p_t, \alpha_t) = f(p_t)$ describes consumers' optimal behavior under the beliefs that $(p^m, \overline{r}^m)$ will prevail in the future.

Consumer decisions

If a consumer with r_{it} purchases the good then she gets $r_{it} - p_t + \delta U^C$, otherwise she gets δU^{NC}. Along a constant price equilibrium U^{NC} and U^C are independent of p_t and constant over time. Thus, for all t :

$$\overline{r}_t = f(p_t) = p_t + \delta \left(U^{NC} - U^C\right) \tag{7}$$

Note that $\frac{\partial \overline{r}_t}{\partial p_t} = 1$. Consumers compare the instantaneous utility from consumption, $\overline{r}_t - p_t$, with the option value of waiting, $\delta \left(U^{NC} - U^C\right)$, and hence purchase if and only if r_{it} is sufficiently higher than p_t. In fact, along a constant price equilibrium we have that:

$$U^{NC} = \int_{\overline{r}}^{1} (r - p)\, dr + (1 - \overline{r})\, \delta U^C + \overline{r} \delta U^{NC}$$

$$U^C = \mu \int_{\overline{r}}^{1} (r - p)\, dr + \mu (1 - \overline{r})\, \delta U^C + (\mu \overline{r} + 1 - \mu)\, \delta U^{NC}$$

and hence:

$$U^{NC} - U^C = \frac{(1 - \mu) \left[\frac{1}{2}\left(1 - \overline{r}^2\right) - (1 - \overline{r})\, p\right]}{1 + z(1 - \overline{r})}$$

Finally, if we plug the above equation into equation (7) and evaluate it at $\overline{r}_t = \overline{r}$, and $p_t = p$, then we have:

$$p = \overline{r}\left[1 + z(1 - \overline{r})\right] - \frac{z}{2}\left(1 - \overline{r}^2\right) \tag{8}$$

Note that if $p = 0$ then equation (7) implies that $\overline{r} = \overline{r}^*$. Unsurprisingly, what prevents efficiency is the market power that allows the monopolist to charge a price above marginal cost.

Firm's optimal pricing

In period t the monopolist's payoff is given by:

$$\Pi(\alpha_t) = \arg\max_{p_t} \alpha_t (1 - \overline{r}_t)\, p_t - F + \delta \Pi(\alpha_{t+1})$$

where $\overline{r}_t$ is given by equation (7), $\Pi(\alpha_{t+1})$ is the continuation value at the beginning of period $t + 1$, and α_{t+1} is given by equation (4). In particular,

$$\alpha_{t+1} = 1 - (1-\mu)(1-\overline{r}_t)\alpha_t$$

Note that $\frac{\partial \alpha_{t+1}}{\partial \overline{r}_t} = (1-\mu)\alpha_t$, and $\frac{\partial \alpha_{t+1}}{\partial \alpha_t} = -(1-\mu)(1-\overline{r})$.

Let us conjecture that $\Pi(\alpha)$ is a linear function of α, and in particular that $\frac{d\Pi(\alpha_{t+1})}{d\alpha_{t+1}} = k$, which is independent of α_t. Then, the first order condition of the firm's optimization problem (the second order condition is satisfied) is:

$$(1 - \overline{r}_t - p_t) + kz = 0 \tag{9}$$

The first important remark is that the optimal p_t does not depend on α_t, and therefore a constant price equilibrium may exist. The second remark is that the intertemporal effect on demand induces the firm to raise its price. In other words, if the firm increases its price slightly above the level that maximizes static profits, it causes a second order loss on current profits but it raises future profits (first order effect) by increasing demand in the next period.

By the envelop theorem if we evaluate $\Pi(\alpha_t)$ at the constant price equilibrium then:

$$k = \frac{d\Pi(\alpha_t)}{d\alpha_t} = (1-\overline{r})p - kz(1-\overline{r})$$

Hence, k is independent of α_t. Solving for k and plugging into equation (9) we obtain:

$$p = (1-\overline{r})[1 + z(1-\overline{r})] \tag{10}$$

Equilibrium

Equations (8) and (10) uniquely determine the equilibrium values of $(p^m, \overline{r}^m)$:

$$\overline{r}^m = \frac{2 + 3z - \sqrt{4+6z}}{3z} \tag{11}$$

$$p^m = \frac{2 + 6z - \sqrt{4+6z}}{9z} \tag{12}$$

The red lines in Figure 1 depict the locus describing optimal consumer choices (equation 8), on the one hand, and optimal monopoly prices (equation 10), on the other in the case $z = 0$. Similarly, the blue lines correspond to the case $z = 1$. Note that $\overline{r}^m$ is an increasing function of z, with $\overline{r}^m(z = 0) = \frac{1}{2}$, and $\overline{r}^m(z = 1) = \frac{5-\sqrt{10}}{3}$.Similarly, p^m is also an increasing function of z, with $p^m(z = 0) = \frac{1}{2}$ and $p^m(z = 1) = \frac{8-\sqrt{10}}{9}$. Finally, $\overline{r}^m(z) > \overline{r}^*(z)$.

For future reference we introduce additional notation: $\Delta \equiv \overline{r}^m - p^m = \frac{4+3z-\sqrt{4+6z}}{9z}$.

4.2 The frequency of production under commitment

Let us now study the frequency of production that arises in equilibrium. In this subsection we consider, as a benchmark, the case where the firm can choose the entire production path, $\{\gamma_t\}_{t=0}^{\infty}$, in period 0. Given the production path, in each period t the firm sets the price, p_t, and consumers decide whether or not to purchase the good. We compute the equilibrium behavior for alternative production paths, and then characterize the optimal production paths in different regions of the parameter space.

If production takes place every period, $N = \infty$, then equilibrium prices and consumer behavior are $(p^m, \overline{r}^m)$, which are given in equations (11) and (12). Hence, the present value of profits is given by:

$$\Pi^{\infty}(\alpha_0) = \frac{\alpha_0 + \frac{\delta}{1-\delta}}{A^m} - \frac{F}{1-\delta} = \frac{1}{1-\delta}\left(\frac{1}{A^m} - F\right) \tag{13}$$

where $A^m = \frac{1+z(1-\overline{r}^m)}{(1-\overline{r}^m)p^m}$.

As in the previous section, option (1) consists of producing in periods $t = 0, 2, 4, ..$ In this case, both firm's and consumers' optimization problems become static. Consumers anticipate that in the next production period they will draw their r_i from distribution NC independently of their current behavior, and hence purchase if and only if $r_{it} \geq p_t$. Similarly, the monopolists anticipates that demand in the next production period is independent of current prices and hence p maximizes static profits. As a result $\overline{r}_t = p_t = \frac{1}{2}$ (profits per production period are equal to $\alpha_t \frac{1}{4} - F$). In this case the firm's payoff is given by:

$$\Pi^1(\alpha_0) = \alpha_0 \frac{1}{4} - F + \frac{\delta^2}{1-\delta^2}\left(\frac{1}{4} - F\right) = \frac{1}{1-\delta^2}\left(\frac{1}{4} - F\right) \tag{14}$$

In the Appendix we prove the following Lemma:

Lemma 4 *If $F < \frac{1}{4}$, then the monopolists either commits to producing every period ($N = \infty$) or to producing every other period ($N = 1$).*

Therefore, the production trajectories in the equilibrium with commitment can be characterized by comparing equations (13) and (14) .If we let $F^m = \frac{1}{\delta}\left(\frac{1+\delta}{A^m} - \frac{1}{4}\right)$ we can state the following result:

Proposition 5 *If the firm can commit to production trajectories, the (generically) unique constant price equilibria consists of: (i) producing every period, charging a price p^m, and selling to consumers with $r_i \geq \overline{r}^m$, if $F \in [0, F^m]$, and (ii) producing every other period, charging a price $\frac{1}{2}$ and selling to consumers with $r_i \geq \frac{1}{2}$, if $F \in \left[F^m, \frac{1}{4}\right]$.*

In order to asses the efficiency of the frequency of production selected by a monopolist (under commitment) we simply need to compare F^* and F^m. Suppose δ is close to zero. In this case, F^* is close to $\frac{1}{2}$ and F^m is close to $\frac{1}{4}$. Therefore, $F^* > F^m$. In this case, monopoly power tends to slow down production with respect to the first best. However, if $\mu = 0$, as δ goes to 1 then F^* goes to 0.0360 and F^m goes to 0.0502. That is, if δ is sufficiently close to 1 then $F^* < F^m$. In this case, a monopolist tends to produce too often with respect to the first best. Summarizing:

Remark 6 *Under commitment a monopolists may choose a frequency of production higher or lower than in the first best. In particular, excessively frequent production occurs when the impact of transitory saturation is sufficiently high.*

Monopoly power causes two types of inefficiencies. First, the static price distortion leads to underconsumption whenever production is available. Second, the frequency of production is typically inefficient, although the sign of the inefficiency is ambiguous. The reason behind this ambiguity can be described as follows. Suppose the monopolist is able to appropriate a fraction λ of total

surplus under all circumstances. With respect to the first best, the payoff differential associated to alternative frequencies (gross of fixed costs) is scaled down by λ, but nevertheless it has to incur the entire fixed cost. In this case, the firm's limited ability to appropriate surplus tends to slow down its frequency of production. However, under transitory saturation the fraction of the surplus appropriated by the monopolist varies with the frequency of production. If production takes place every other period (and $\overline{r} = p = \frac{1}{2}$) then the monopolist captures one half of the total surplus (gross of fixed costs). However, if production takes place every period then the monopolist captures more than one half of the total surplus (gross of fixed costs). The reason is that an increase in the price above marginal costs reduces consumers' value of waiting (reduces $U^{NC} - U^{C}$), which reduces the gap, $\overline{r} - p$. In other words, as consumers become less defensive the firm can capture a higher fraction of total surplus. Thus, if the impact of transitory saturation is sufficiently strong, the monopolist's ability to appropriate a higher proportion of total surplus under $N = \infty$ may induce the monopolist to overproduce. In such a case, and somewhat paradoxically, underconsumption coexists with overproduction. That is, the monopolists may produce too often, with respect to the efficient allocation, but when it does produce then it sells to too few consumers.

4.3 The frequency of production in the absence of commitment

In the absence of commitment, the firm chooses every period γ and p, and consumers select $\overline{r}$. A strategy for the firm is a pair $\{\gamma(\alpha_t), p(\alpha_t)\}$, and consumers' strategy can be written as $\overline{r}(p_t, \alpha_t)$. It turns out that, despite of the restrictions on the strategy space, for most parameter values multiple equilibria exist. In order to illustrate the multiplicity of equilibria, let us we consider two types of equilibria.

Equilibrium type I

Let us consider the strategies that support an equilibrium where production takes place every period ($N = \infty$):

If $\alpha_t \in [\overline{\alpha}, 1]$ then $\gamma_t(\alpha_t) = 1, p_t(\alpha_t) = p^m$, and $\overline{r}(p_t, \alpha_t) = p_t + \Delta$.

If $\alpha_t \in [0, \overline{\alpha})$ then $\gamma_t = 0, p_t(\alpha_t) = p^m$,and $\overline{r}(p_t, \alpha_t) = p_t + \Delta$.

It has been shown above that this reflects consumers' optimal behavior whenever they expect production in every period. Also, given such consumer behavior, this strategy is optimal for the firm provided it does not have incentives to deviate at $\alpha_t = 1$. The only potentially profitable deviation would consists of setting a price p^d such that in the following period $\alpha_{t+1} < \overline{\alpha}$ (and hence $\gamma_{t+1} = 0$). If the firm conjectures that $\gamma_{t+1} = 0$ then it will set the price that maximizes current profits, i.e. $p^d = \frac{1-\Delta}{2}$. Therefore, the expected value of profits obtained from such deviation is given by:

$$\Pi^d(1) = \frac{(1-\Delta)^2}{4} - F + \delta^2 \Pi^d(1)$$

Thus, an equilibrium of type I requires that $\Pi^d(1) \leq \Pi^\infty(1)$, which is equivalent to $F \leq F_1$, where F_1 is given by:

$$F_1 = \frac{1}{\delta}\left[\frac{1+\delta}{A^m} - \frac{(1-\Delta)^2}{4}\right]$$

If $\delta = 1, \mu = 0$, then $F_1 = 0.0863$.

Equilibrium type II

Let us now consider the strategies that support an equilibrium where production takes place every other period:

If $\alpha_t = 1$, $\gamma_t(\alpha_t) = 1, p_t(\alpha_t) = \frac{1}{2}$, and $\overline{r}(p_t, \alpha_t) = p_t$.

If $\alpha_t \in [\overline{\alpha}, 1)$ then $\gamma_t(\alpha_t) = 1, p_t(\alpha_t) = p^m$, and $\overline{r}(p_t, \alpha_t) = p_t + \Delta$.

If $\alpha_t \in [0, \overline{\alpha})$ then $\gamma_t = 0, p_t(\alpha_t) = p^m$, and $\overline{r}(p_t, \alpha_t) = p_t + \Delta$

We have shown above that consumers' strategy is the optimal response to the firm's strategy. In equilibrium the firm's expected profit is equal to $\Pi^1(1)$. The only potentially profitable deviation at $\alpha_t = 1$ is to select a price p^d under the expectation that in the next period $\alpha_{t+1} \geq \overline{\alpha}$ and hence $\gamma_{t+1} = 1$. In this case, p^d is chosen in order to maximize:

$$\Pi^d(p) = (1-p)p - F + \delta \Pi^\infty [1 - (1-\mu)(1-p)]$$

Thus,

$$p^d = \frac{A^m + z}{2A^m}$$

Finally, an equilibrium of type II requires that the optimal deviation is not profitable: $\Pi^d\left(\frac{A^m+z}{2A^m}\right) \leq \Pi^1(1)$, i.e., $F \geq F_0$, where F_0 is given by:

$$F_0 = \frac{1}{\delta(1+\delta)-1}\left[\frac{1-\delta^2}{4}\left(1+\frac{z}{A^m}\right)^2 + \frac{\delta(z+\mu)(1+\delta)}{A^m} - \frac{1}{4}\right]$$

It must also be the case that for values of α arbitrarily close to 1, the firm must find it optimal to set $p = p^m$. Hence, an equilibrium of type II will exist if and only if an equilibrium of type I also exists, i.e., if $F \leq F_1$.

If $\delta = 1, \mu = 0$, then $F_0 = 0.0502$. Therefore, if $F \in [0.0502, 0.0863]$ then and equilibrium of type I and another equilibrium of type II both exist.

Remark 7 *There exists parameter values for which both type I and type II equilibria exist.*

The intuition of the multiplicity of equilibrium is the following. Suppose $\alpha_t = 1$. If both consumers and the firm expect that production will take place in every period, then consumers find it optimal to purchase only if their current net surplus is sufficiently high (higher than Δ), and firms prefer to set a relatively high price $\left(p^m > \frac{1}{2}\right)$. As a result, sales are relatively low, and hence α_{t+1} is relatively high. Consequently, the firm finds it optimal to produce in period $t+1$. In contrast, if consumers and the firm expect that $\gamma_{t+1} = 0$, then consumers purchase even if the current surplus is arbitrarily low, but positive, and the firm prefers to set a relatively lo price $\left(\frac{1}{2}\right)$. As a result, sales are relatively abundant and α_{t+1} is relatively low. Consequently, the firm prefers not to produce in period $t+1$.

When both types of equilibrium exist it may be interesting to learn which equilibrium is preferred. In the case $\mu = 0$ and $\delta = 1$ the payoffs per period in each type of equilibrium are given in the following table.

	Type I equilibrium	Type II equilibrium
Profits	$0.1501 - F$	$0.125 - \frac{F}{2}$
Consumer surplus	0.0750	0.0625
Total surplus	$0.2251 - F$	$0.1875 - \frac{F}{2}$

Thus, in spite of higher prices consumers always prefer the type I equilibrium, since the good is continuously available. In contrast, the firm prefers the type I equilibrium if and only if $F \leq 0.0502$. Finally, a utilitarian social planner prefers the type I equilibrium if and only if $F \leq 0.0752$. Thus, when both types of equilibrium exist there is a conflict of interest. Consumers and the firm will never agree about which equilibrium they should play. Also, a utilitarian social planner prefers the equilibrium with high frequency of production if $F \in [0.0502, 0.0752]$ but prefers the equilibrium with low frequency if $F \in [0.0752, 0.0863]$. Thus, the possibility of overproduction under monopoly arises not only when we compare the equilibrium with respect to the first best (which requires price controls), but also there may be overproduction in a different sense. Perhaps, agents play an equilibrium with a high frequency of production (type I), when there is another equilibrium with a lower frequency of production that generates higher social surplus.

5 Competition

The model can accommodate more than one firm provided firms are symmetric (same cost structure) and produce a homogeneous good.

We proceed in two steps. First, we discuss the effect on competition in a simple version of the model with rectangular demand functions and next we discuss pricing behavior in the original model.

5.1 Competition without the static price distortion

Consider the following consumer preferences. If agent i did not consume in period $t-1$ then $r_{it} = 1$ with probability 1, but if she did consume in period $t-1$ then $r_{it} = 1$ with probability μ and 0 with probability $1-\mu$.

Suppose there are $n \geq 2$ identical firms (with the cost structure considered above) that are able to produce a homogeneous good. At the beginning of each period, firms decide whether or not to produce (and pay the fixed cost). After observing which firms have decided to incur the fixed costs, then these firms simultaneously and independently quote a price. Those consumers that prefer to consume choose the firm with the lowest price. In case of a tie consumers are equally split between firms.

Analogously,we denote by α_t the mass of consumers with $r_{it} = 1$. Since in equilibrium all consumers with $r_{it} = 1$ purchase the good, then the law of motion is given by:

$$\alpha_t = 1 - (1 - \mu)\, \alpha_{t-1}$$

If production takes place every period then α_t converges to $\frac{1}{2-\mu}$.

5.1.1 Symmetric Markov equilibria

If in period t only one firm pays the fixed cost, then it quotes a price equal to one and sales are equal to α_t. If two or more firms pay the fixed costs, then there is Bertrand competition and prices are driven down to zero. Thus, firms always prefer not to produce if another firm is producing. Hence, a symmetric Markov equilibrium involve mixed strategies. In equilibrium each firm is indifferent between producing and not producing. In case of production, a favorable state is one where the firm is a transitory monopolist. However, if more than one firm produce the good then revenues are zero and profits are equal to $-F$. Thus, from a social point of view, competition may not be desirable for two reasons. First, with some probability the good may not be available as often as efficiency requires. Second, the good may be available but fixed costs may be paid by more than one firm at a time.

5.1.2 Coordination of production decisions

In order to avoid duplication of fixed costs and head-to-head competition, firms have incentives to coordinate their production decisions. If they also coordinate on prices then the market structure is equivalent to a monopoly. Suppose firms

do not coordinate on prices. We can think of a very simple and symmetric coordination method. At the beginning of each period one firm is randomly selected, with equal probability and independent of history. Formally, equilibrium behavior under such coordination mechanism form a correlated equilibrium. The selected firm can choose whether or not to produce. In case of production, the firm will be a transitory monopolist and charge a price equal to 1 and sell to α_t consumers.[6] The selected firm will choose to produce in period t if and only if:

$$\alpha_t - F + \delta\Pi(\alpha_{t+1}) \geq \delta\Pi(1) \tag{15}$$

where $\Pi(\alpha)$ is the continuation value of any firm at the beginning of the period before learning the outcome of the lottery. Thus, if production takes place in period $t+1$, then:

$$\Pi(\alpha_{t+1}) = \frac{1}{n}(\alpha_{t+1} - F) + \delta\Pi(\alpha_{t+2}) \tag{16}$$

We can compute the continuation value of a firm in an equilibrium where production takes place every period by solving equation (16) recursively:

$$\Pi(\alpha_{t+1}) = \frac{\delta - F(1+z) + (1-\delta)\alpha_{t+1}}{n(1+z)(1-\delta)} \tag{17}$$

Since $\alpha_0 = 1$, $\alpha_1 = \mu$. Hence, a necessary and sufficient condition for the existence of an equilibrium with production every period is equation (15), evaluated at $\alpha_t = \mu$, where the continuation values are given by equation (17), which is equivalent to $F \leq F^c$ where:[7]

$$F^c = \mu\left[1 - \frac{z}{n(1+z)}\right]$$

If $n = 1$the firm can appropriate the entire surplus and thus implement the first best solution. Production every period is efficient only if at $\alpha = \mu$ the current surplus is sufficiently high as to compensate for the future value of a higher consumer base: 1 instead of $1 - \mu(1-\mu)$. In this case the threshold

[6] In this context it is immediate to show that the unique equilibrium with constant prices involves $p = 1$.

[7] If $F > F^c$ then production takes place every other period and the expected payoff for each firm is $\frac{1-F}{n(1-\delta^2)}$.

value is $F^c = \frac{\mu}{1+z}$. As n increases, then each each firm discounts more heavily the future gains associated with a higher consumer base and as a result has incentives to produce even when current profits are relatively low (for relatively high values of the fixed cost). In other words, F^c increases n. In fact, if n is sufficiently large, then the firm which is selected to produce in the current period essentially disregards future profits and is willing to produce provided current profits are not negative. In other words, F^c goes to μ as n goes to infinity.

Summarizing:

Proposition 8 *Under rectangular demand, if firms coordinate their production decisions using a symmetric static mechanism, then competition generates overproduction (the frequency of production is inefficiently high). The range of parameter values under which there is overproduction expands with the number of firms.*

5.2 Coordination of production with elastic demand (to be completed)

It is important to understand how the results of the previous subsection change if we reintroduce the static price distortion. More specifically, we would be interested in the characteristics of equilibria in the framework of Section 4 but allowing for an arbitrary number of firms that coordinate their production decisions using the symmetric, static mechanism described above. In such a mixed model the results would also be a combination of those in Sections 4.3 and 5.2.

Let us start discussing pricing behavior. Even though firms behave as transitory monopolists there is intertemporal competition. If $n > 1$ each firm discounts future profits more heavily than the monopolist and therefore sets lower prices. In fact, prices decrease with n, and as n goes to infinity firms set the price that maximize current profits.

Competition also changes private incentives to produce. The most important change is that under competition firms discount more heavily the effect of current action future profits and hence they tend to produce more often. In fact, if n is arbitrarily large then firms produce as long as current profits are not negative. Also, since intertemporal effects are less important, total surplus

associated with production every period is higher than in the case of monopoly. In any case, the frequency of production under competition (and coordination of production) may be higher or lower than in the first best. Moreover, more competition (higher n) may involve lower total surplus (despite of lower prices) because of the incentives to produce too frequently.

In order to illustrate this discussion let us consider the case that n is arbitrarily large. Let us first consider equilibria where production takes place every period. In this case, consumer behavior is still given by equation (8). However, the behavior of firms is now different from the monopoly case. A firm which is called to produce if it chooses to do so then it will set a price that maximizes current profits $[1-\overline{r}(p)]\,p$, where $\overline{r}(p)=p+\Phi$. The firm takes Φ as given (it is the value of waiting, which is a function of expectations about future behavior) but in equilibrium $\Phi=\overline{r}-p$.

Thus, the profit maximizing price is $p=\frac{1-\Phi}{2}$.Combined with equation (8), we obtain the equilibrium values of $(\overline{r}^c.p^c)$:

$$\overline{r}^c=\frac{2+z-\sqrt{2(2+z)}}{z}$$

$$p^c=\frac{-2+\sqrt{2(2+z)}}{z}$$

Note that $p^c<\frac{1}{2}<\overline{r}^c<\overline{r}^m$. Since consumers are still forward looking $\Phi>0$. However, firms behave myopically. As a result prices are lower than $\frac{1}{2}$, which induces higher levels of consumption than under monopoly, although consumption is still lower than in the case consumers are myopic or they do not suffer from transitory saturation.

An equilibrium with production every period will exist if for the lowest possible value of α_t along the equilibrium path profits are non-negative. In other words, a necessary and sufficient condition for the existence of an equilibrium with production every period is that: $[1-(1-\mu)(1-\overline{r}^c)](1-\overline{r}^c)\,p^c-F\geq 0$. In the case $\delta=1,\mu=0$ such a condition is $F\leq 0.1110$.

It turns out that in this parameter range forcing firms to produce every other period might increase total welfare. If production takes place every period, and behavior is summarized by $(\overline{r}^c.p^c)$ given above, then total surplus per period is

equal to $0.2090 - F$. However, if firms are forced to produce every other period then $\overline{r} = p = \frac{1}{2}$, and total surplus per period is equal to $0.1875 - \frac{F}{2}$. Therefore, total surplus is higher under low frequency production if $F \geq 0.0430$.

6 Discussion

In this paper we have presented an infinite horizon model to analyze the effect of transitory saturation on market performance. If consumers' expected valuation for the good tends to increase with the time elapsed since the last consumption episode then consumers purchase the good only if their current valuation is sufficiently higher than the current price. The gap is explained by the option value of waiting (the consumer's expected future valuation of the good increases with abstinence). Also, sellers tend to set prices higher than those that maximize current profits. The loss in current profits is compensated by higher future profits associated to a customer base with higher willingness to pay.

These effects are most relevant in those markets where the timing of production (new product introductions) is crucial. We have shown that if the effect of transitory saturation is strong enough then monopoly power may have counterintuitive consequences: overproduction may coexist with underconsumption. In other words, firms may introduce new products too often but when they do they sell them to too few consumers. This is true if we compare market equilibrium with the first best (where the social planner controls both the frequency of production and prices) and also with the second best (where the social planner only controls the frequency of production.

Finally, competition tends to increase the frequency of production and hence to make overproduction more likely. In fact, information sharing among independent suppliers and coordination of production plans need not reduce social welfare.

7 References

Adda, J. and R. Cooper (2000), Balladurette and Juppette: A Discrete Analysis of Scapping Subsidies, Journal Political Economy 108(4), 778-806.

Bond, E. and L. Samuelson (1984), Durable good monopolies with rational expectations and replacement sales, Rand Journal of Economics 15 (3), 336-345.

Choi, J. (1993), Network Externality, Compatibility Choice, and Planned Obsolescence, Journal of Industrial Economics 42 (2), 167-182.

Corts, K. (2001), The strategic effects of vertical market structure: common agency and divisionalization in the US motion picture industry, Journal of Economics and Management Strategy 10 (4), 509-528.

Driskill, R. (1997), Durable-Goods Monopoly, Increasing Marginal Cost and Depreciation, Economica 64, 137-154.

Einav, L. (2009), Not all rivals look alike: estimating an equilibrium model of the release date timing game, Economic Inquiry , 1–22.

Einav, L. (2007), Seasonality in the US motion picture industry, Rand Journal of Economics 38(1), 745-788.

Ellison, G. and D. Fudenberg (2000), The neo-Luddite's lamment: excessive upgrades in the software industry, Rand Journal of Economics 31 (2), 253-272.

Fishman, A. and R. Rob (2000), Product innovation by a durable-good monopoly, Rand Journal of Economics 31 (2), 237-252.

Krieder, R. and Ch. Weinberg (1998), Competitive Dynamics and the Introduction of New Products: The Motion Picture Timing Game, Journal of Marketing Research 35, 1-15.

Orbach, B. and L. Einav (2007), Uniform prices for differentiated goods: The case of the movie-theater industry, International Review of Law and Economics 27, 129-153.

Waldman, M. (1993), A new perspective on planned obsolescence, Quarterly Journal of Economics 108, 273-283.

8 Appendix

8.1 Proof of Lemma 1

We apply Bellman's principle: choose $\overline{r}_t$ in order to maximize:

$$W(\alpha_t) = \frac{\alpha_t}{2}\left(1 - \overline{r}_t^2\right) - F + \delta W(\alpha_{t+1})$$

The first order condition of an interior solution (second order condition holds) is:

$$\overline{r}_t = zW'(\alpha_{t+1}) \tag{18}$$

By the envelop theorem:

$$W'(\alpha_t) = \frac{1}{2}\left(1-\overline{r}_t^2\right) - zW'(\alpha_{t+1})(1-\overline{r}_t)$$

Combining the two previous equations we can write:

$$W'(\alpha_t) = \frac{1}{2}(1-\overline{r}_t)^2$$

Hence, the optimal policy must satisfy:

$$\overline{r}_{t-1} = \frac{z}{2}(1-\overline{r}_t)^2 \tag{19}$$

The stationary solution of this difference equation is:

$$\overline{r}^* = \frac{1+z-\sqrt{1+2z}}{z}$$

All the trajectories except $\overline{r}_0 = \overline{r}^*$ are explosive, and hence the unique solution to the optimization problem is $\overline{r}_t = \overline{r}^*$.

8.2 Proof of Lemma 2

Since $\alpha_0 = 1$ then it is obvious that $\gamma_0 = 1$. Let us now consider an arbitrary value of N, $\infty > N > 1$. Since, $\alpha_{N+1} = 1$ this implies (equation (18))that $\overline{r}_{N-1} = 0$ and $\overline{r}_t$ is obtained recursively from equation (19) for $0 \leq t \leq N-2$. Also, note that since $\alpha_0 = 1$,then $0 < \alpha_t < 1$ for $0 < t \leq N-1$.

First of all, we show that for any N, $\infty > N > 1$, $\frac{dW^N(\alpha_0)}{d\alpha_0} \in \left(0, \frac{1}{2}\right)$. In period $t, 0 \leq t \leq N-2$, we have:

$$W(\alpha_t) = \alpha_t R(\overline{r}_t) - F + \delta W(\alpha_{t+1})$$

where $\alpha_{t+1} = 1-(1-\mu)(1-\overline{r}_t)\alpha_t$. Hence,

$$\frac{dW(\alpha_t)}{d\alpha_t} = R(\overline{r}_t) - z(1-\overline{r}_t)\frac{dW(\alpha_{t+1})}{d\alpha_{t+1}}$$

Let us consider the case $t = N-1$. In this case $\frac{dW(\alpha_N)}{d\alpha_N} = 0$, since there is no production in period N. Moreover, $\overline{r}_{N-1} = 0$. As a result, $\frac{dW(\alpha_{N-1})}{d\alpha_{N-1}} = \frac{1}{2}$. For an arbitrary $t, 0 \leq t \leq N-2$, if $\frac{dW(\alpha_{t+1})}{d\alpha_{t+1}} \geq 0$, $\frac{dW(\alpha_t)}{d\alpha_t} < R(\overline{r}_t) \leq \frac{1}{2}$. Finally, if $\frac{dW(\alpha_{t+1})}{d\alpha_{t+1}} \leq \frac{1}{2}$, $\frac{dW(\alpha_t)}{d\alpha_t} > 0$.

Suppose that in period 0 it is optimal to follow option N, $\infty > N > 1$. This implies that in period $N-1$ option 1 is preferred to option N. That is,

$$W^1(\alpha_{N-1}) \geq W^N(\alpha_{N-1})$$

where $\alpha_{N-1} < 1$. Then since $\frac{1}{2} = \frac{dW^1(\alpha_{N-1})}{d\alpha_{N-1}} > \frac{dW^N(\alpha_{N-1})}{d\alpha_{N-1}}$, it must be the case that:

$$W^1(1) \geq W^N(1)$$

And we reach a contradiction. Therefore, at $\alpha_0 = 1$, the only optimal options are $N = 1$, and $N = \infty$.

8.3 Proof of Lemma 3

The proof is analogous to that of Lemma 2. Applying Bellman's principle:

$$\Pi(\alpha_t) = \arg\max_{p_t} \{\alpha_t(1-\overline{r}_t)p_t - F + \delta\Pi(\alpha_{t+1})\}$$

where $\alpha_{t+1} = 1-(1-\mu)(1-\overline{r}_t)\alpha_t$, and $\overline{r}_t$ is a linear function of p_t with slope equal to 1.

From the first order condition of this optimization problem, we obtain:

$$p_t = 1 - \overline{r}_t + z\Pi'(\alpha_{t+1})$$

Using the envelop theorem:

$$\Pi'(\alpha_t) = (1-\overline{r}_t)[p_t - z\Pi'(\alpha_{t+1})] = (1-\overline{r}_t)^2 > 0$$

Fix $N, 1 < N < \infty$. In period $t = N - 1$, which is the period that preceeds the no production period, $\overline{r}_{N-1} = p_{N-1}$ and $\Pi'(\alpha_N) = 0$. As a result, $p_{N-1} = \overline{r}_{N-1} = \frac{1}{2}$, and $\Pi'(\alpha_{N-1}) = \frac{1}{4}$.

For an arbitrary $t, 0 \leq t \leq N-2, \overline{r}_t > p_t = 1 - \overline{r}_t + z\Pi'(\alpha_{t+1})$, which implies that $\overline{r}_t > \frac{1}{2}$. Hence, $\Pi'(\alpha_t) < \frac{1}{4}$.

Suppose that in period 0 the firm finds it optimal to follow option N, $\infty > N > 1$. This implies that in period $N - 1$ option 1 is preferred to option N. That is,

$$\Pi^1(\alpha_{N-1}) \geq \Pi^N(\alpha_{N-1})$$

where $\alpha_{N-1} < 1$. Then since $\frac{1}{4} = \frac{d\Pi^1(\alpha_{N-1})}{d\alpha_{N-1}} > \frac{d\Pi^N(\alpha_{N-1})}{d\alpha_{N-1}}$, it must be the case that:

$$\Pi^1(1) \geq \Pi^N(1)$$

And we reach a contradiction. Therefore, at $\alpha_0 = 1$, the only optimal options for the firm are $N = 1$, and $N = \infty$.

Figure 1