Mobile Termination and Consumer Expectations under the Receiver-Pays Regime

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Abstract

We analyze how termination charges affect retail prices when taking into account that receivers derive some utility from a call and when firms may charge consumers for receiving calls. A novel feature of our paper is that we consider passive self-fulfilling expectations and do not allow for negative reception charges. Firms only charge for receiving calls when the termination charge is below cost. We reconfirm the finding of profit neutrality when firms cannot use termination-based price discrimination. When firms can use termination-based price discrimination profits do depend on the termination charge. When the call externality is strong, firms prefer a below cost termination charge and will use RPP. When the call externality is weak, firms prefer a termination charge above cost. The termination charge that maximizes total welfare is below cost and would induce an RPP regime.

Keywords: Bill and Keep; Call externality; Access Pricing; Interconnection; Receiver pays; Consumer Expectations

JEL classification: D43; K23; L51; L96

1 Introduction

Although the telecommunications sector has been liberalized in most industrialized countries, some regulation remains. A clear example is call termination on mobile telephone networks. Mobile operators must interconnect their networks so that their customers can communicate

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with the customers of other networks. This requires mobile operators to provide a wholesale service called ‘call termination’, whereby each completes a call made to one of its subscribers by a caller on another network. Call termination is provided in exchange for a fee.\textsuperscript{1} This fee, also called mobile termination rate, is paid by the originating operator to the terminating operator. Since the market for termination is monopolistic, one cannot rely on competition to get termination rates at the efficient level. Excessive termination rates are believed to inflate retail prices so that usage is inefficiently low. The European Commission has urged national regulators to step in and regulate these termination rates towards the true cost. Most countries around the world have followed suit and do regulate termination rates. Regulators often use so called gliding paths which reduce termination rates gradually over a period of several years. At present termination rates in most countries are still believed to be above the cost of termination and regulators intend to reduce them further over the next years. In May 2009, the European Commission recommended national regulatory authorities to set termination rates based on the costs (i.e., the actual incremental cost of providing call termination – without allowing for common costs) incurred by an efficient operator.\textsuperscript{2} The European Commission’s view was also supported by the European regulators group, who in the Common Position adopted on February 2008\textsuperscript{3} decided to take a position in favor of setting a unique and uniform termination rate for all network operators at the cost incurred by an hypothetical efficient operator. As a result, the average MTR in Europe could drop from about 8.55 euro cents per minute at the end of 2009, to approximately 2.5 euro cents per minute by 2012 [see Harbord and Pagnozzi, 2010]. In light of these announcements, Vodafone and other large European mobile operators warned the European Commission that cutting termination rates could mean the end of handset subsidies for consumers and lead to a price increase. Furthermore, Vodafone claimed that cutting termination rates could result in a US style business model, where users pay for both placing and receiving calls.

The burden of regulation of termination rates is quite high. Any attempt by a national regulator to lower termination charges has to be preceded by a formal investigation of the relevant market and a round of public consultations. Operators oppose cuts in termination rates and challenge any argument made by regulators. Often there are disputes about what the true costs are, how they should be calculated and also about what the real effect of lower termination charges is. In countries as the US and Canada, however, there seems to be no

\textsuperscript{1}The fee is sometimes equal to zero. This arrangement is known as Bill and Keep.


\textsuperscript{3}See “ERG’s Common Position on symmetry of fixed call termination rates and symmetry of mobile call termination rates”, adopted by the ERG-Plenary on 28th February 2008, p. 4-5. Available at http://www.erg.eu.int.
need for regulators to set termination rates. In these countries the operators must negotiate reciprocal termination rates between themselves and voluntarily agree to set them very low, sometimes even at zero (that is, a Bill and Keep regime is chosen). Another major difference between these countries and the European markets is that a so called Receiving Party Pays (RPP) regime is used, while in Europe a Calling Party Pays (CPP) regime is in place. RPP means that operators charge a price to their customers not only for placing calls but also one for receiving calls. In this way operators can recover the cost of termination from their own customers. Littlechild (2006) argues that RPP countries have lower usage prices and higher usage than CPP countries, but higher fixed fees (or lower hand-set subsidies) and perhaps lower penetration rates or at least slower growth in penetration rates. Dewenter and Kruse (2010) argue that penetration rates in CPP and RPP countries are not significantly different once one controls for endogenous regulation.

The seeming superiority of the RPP regime has lead some economists to call upon regulators in CPP countries to impose an RPP regime. However, the statistical evidence of correlations between the payment regimes, termination charges, penetration, and retail prices does not imply there is a specific causal relationship. In particular, it is not clear that using an RPP regime will bring all the benefits that seem to be correlated with RPP regimes. In fact, the CPP regime can be considered as a special case of RPP where consumers happen to be charged a zero price for reception. Moreover, nothing prevents operators in RPP countries without regulated termination rates to agree upon high termination charges. And even if RPP were superior from a social welfare point of view, it would be difficult to imagine that regulators could actually force firms to use such specific pricing structure. Only if it is in the firms’ interest in terms of profitability, RPP regimes will be used. In our view, regulators can at most influence the choice of firms between CPP and RPP by setting adequate termination rates. However, inducing RPP regimes should not be the objective of regulators per se. Regulators should set termination rates such that the resulting outcome in terms of retail prices is socially efficient.

The objective of this paper is to explore the implications of termination rates for retail pricing (including the choice of CPP or RPP), profits and efficiency when firms may charge both outgoing and incoming calls. We consider a duopolistic framework in which firms, located at the ends of a Hotelling line compete in non-linear prices. That is, firms set a fixed fee, together with charges for placing and receiving calls. We do not allow firms to set negative prices, although they are allowed to price below cost. This assumption alone explains why CPP regimes are to be found in countries with high termination rates while RPP regimes are only found in countries with low termination rates. Namely, if termination

\[4\text{See for example. De Bijl et al. (2005).}\]
rates are above the cost of termination, firms would like to subsidize the reception of calls (from rival networks) by setting negative prices. Once this possibility is excluded by our assumption, the optimal thing to do is to charge the minimal price of zero for receiving calls. That is, CPP is an endogenous outcome when termination rates are above cost. On the other hand, when termination rates are below cost, firms do want to charge a strictly positive price for the reception of calls. Hence, RPP will arise endogenously when termination rates are set very low. This does not yet explain why operators in countries with high termination rates consistently oppose reductions in these rates. Presumably they do this as they fear their profits being reduced. Nevertheless, in countries with termination charges below cost and RPP regimes, operators seem to be doing quite well. In fact, Average Revenue Per User (ARPU) is usually higher in RPP countries (see Marcus (2004). The fact that operators in some of these countries voluntarily agree on such arrangements should indicate that firms can make reasonable returns on investment. In order to address the issue of what firms would prefer and how regulators should intervene one needs to examine how profits and welfare vary with different termination charges.

The impact of termination rates on competition, profit and welfare has been extensively studied. This burgeoning literature starts with the seminal works of Armstrong (1998) and Laffont, Rey and Tirole (1998a, b) (henceforth ALRT). Most of the theoretical work on mobile network interconnection typically assumes that consumers derive utility only from making calls, ignoring the existence of call externalities — that is, the fact that not only callers but also receivers of a call enjoy a positive benefit. Clearly, if there is no utility at all for receiving calls, consumers would refuse to answer the phone if they have to pay a positive price for it, so that only the CPP regime makes sense. If termination charges are above cost, firms do use CPP so that the results obtained in the literature assuming CPP may be relevant also when call externalities exist and RPP is allowed. For example, Laffont et al. (1998b) consider the case when networks compete in nonlinear prices and can charge different price for on- and off-net calls. They show that profit is strictly decreasing in termination charge. Building on their analysis, Gans and King (2001) show that firms using a CPP regime strictly prefer below cost termination charges. The intuition for their results is that if termination charge is above cost, off-net calls will be more expensive than on-net calls.

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5For a complete review of the literature on access charges see Armstrong (2002), Vogelsang (2003) and Peitz et al. (2004).

6One assumption that is invoked to justify the absence of call externalities in models of network competition is that call externalities could be largely internalized by the parties (see Competition Commission [2003, paras 8.257 to 8.260]). However, as argued by Hermalin and Katz (2004, p. 424), "this assumption is applicable only to a limited set of situations in which either the communicating parties behave altruistically or have a repeated relationship". Additionally, Harbord and Pagnozzi [2010] argue that the empirical basis for the internalization of call externalities is unclear.
As there is a price differential between on- and off-net calls, consumers care about the size of each network (the so-called ‘tariff-mediated network externalities’). In particular, they will be more eager to join the larger network. Consequently, acquisition costs are reduced, which in turn intensifies competition for subscribers and results in lower subscription fees and profits. Firms would thus prefer termination charges below cost. As total welfare would be maximized by termination charges equal to cost, this implies that consumers are better off when termination charges are strictly above cost. Berger (2005) considers the same setting where firms again use CPP but where call externalities do exist. In this case the social welfare maximizing termination charge is shown to be below cost. Berger (2005) argues that regulation is not necessary as the preferences of firms and regulators are aligned. As a matter of fact, these results are at odds with real world observations since regulators around the world, and especially in the European Union, are concerned about too high termination charges and operators consistently oppose cutting termination rates.  

The possibility that the receiving party enjoys benefits from a call is clearly important for the manner in which firms compete in the retail market. Once it is recognized that consumers enjoy benefits from receiving a call, it follows that they are prepared to pay for this. Indeed, in some countries (e.g. Canada, Singapore, Hong Kong and the United States) mobile operators charge their subscribers for the calls they receive. An incipient literature has started to examine the relationship between termination rates and equilibrium prices in an environment with call externalities and RPP regimes. Laffont et al. (2003; LMRT hereafter), Jeon et al. (2004; JLT hereafter), Hermolin and Katz (2006), Cambini and Valletti (2008) and López (2010) are the papers closest to ours. LMRT analyze Internet backbone competition and assume that there exist two types of users: websites (senders) and consumers (receivers). Hermolin and Katz study whether termination charges can induce carriers to internalize the externalities that arise when both senders and receivers of telecommunications messages enjoy benefits. But in contrast to the framework of LMRT, in which there are two different types of users, they consider that any given user has a one-half chance of being a sender and a one-half chance of being a receiver. In JLT, López (2010), and this paper, however, every

7 Nevertheless, this result has been shown to be very robust. For example, it holds for any number of networks [Calzada and Valletti, 2008] and when networks are asymmetric [López and Rey, 2009]. Also, Hurkens and Jeon (2009) show that this result holds when there are both network externalities (i.e., elastic subscription demand as in Dessein, 2003) and network-based price discrimination.

8 According to Dewenter and Kruse (2010) 14 countries used RPP from the beginning at least until 2003. Another 31 countries started with RPP but switched at some point to CPP.

9 Other related papers in this literature include Kim and Lim (2001); DeGraba (2003); Hahn (2003); Berger (2004, 2005); Hermolin and Katz (2001, 2004)).

10 López (2010) generalizes the framework of JLT by allowing a random noise in both the callers’ and receivers’ utilities, by removing (at some stages) the assumption of a given proportionality between the utility functions, and by allowing asymmetry between mobile operators with respect to the number of locked-in
consumer both sends and receives traffic, and moreover obtains surplus from and is charged for placing and receiving calls. JLT and López (2010) obtain the following results. On the one hand, in the absence of network-based price discrimination, mobile operators charge calls and call receptions at their off-net cost.\textsuperscript{11} Hence operators charge a positive price for incoming calls only when the termination charge is below cost (so as to recover the cost of providing the service of call termination).\textsuperscript{12} When termination charge is above cost operators will set negative prices and subsidize incoming calls in order to earn termination profits. On the other hand, when mobile operators can differentiate their calling and reception charges according to whether the communication is on- or off-net, connectivity is prone to breakdown. The reason is that off-net calling and reception charges allow network operators to create direct externalities on the customers of rival operators. If, for example, the callers obtain more utility than the receivers from a given call, the attractiveness of the offer of the network where the call is received will be reduced in comparison with the rival’s offer. Therefore, to avoid a loss of attractiveness, the terminating network will break connectivity by charging a prohibitively high reception price.

In the present paper we develop further the analysis of JLT and obtain new results that have implications for retail pricing. The main novelty of our analysis lies in studying how consumer expectations affect equilibrium end-user prices (and so equilibrium profit and welfare). We introduce this novelty because we have shown elsewhere that consumer expectations are crucial under CPP regimes. A further difference with the related literature mentioned is that we impose that prices cannot be negative. In particular, subsidizing the reception of calls is not allowed in our paper. Furthermore, acknowledging the existence of equilibria with connectivity breakdown, we focus on the equilibria in which there is no connectivity breakdown. In particular, we show how the termination charge affects the existence of such equilibria and the private and social benefits obtained in these equilibria.

In Hurkens and López (2010) we show that the way consumers form expectations about network sizes is crucial for the relationship between termination charges and equilibrium profit under the CPP regime. We observe that the intuition for the counter-intuitive results obtained by Laffont et al. (1998b), Gans and King (2001) and Berger (2005) relies on the assumption that consumers can correctly predict the size of each network, after any combination of prices. Consumers having such \textit{rationally responsive expectations} means that any change of a price, how tiny it may be, by one firm is assumed to lead to an instantaneous ra-

\textsuperscript{11}This so called “off-net-cost pricing principle” dates back to LMRT, who found this pricing rule in a framework for Internet backbone competition.

\textsuperscript{12}Cambini and Valletti (2008) obtain the same result in their framework of information exchange between calling parties with interdependency among outgoing and incoming calls.
tional change in expectations of all consumers, such that, given these changed expectations, optimal subscription decisions will lead realized and expected network sizes to coincide. So a unilateral change in price does not lead only to a change in market shares, but it also leads consumers to accurately predict how market shares will change. We propose to relax the assumption of rationally responsive expectations and to replace it by one of fulfilled equilibrium expectations. This concept was first introduced by Katz and Shapiro (1985). Basically, Katz and Shapiro (1985) assume that first consumers form expectations about network sizes, then firms compete, and finally consumers make optimal subscription or purchasing decisions, given the expectations. These decisions then lead to actual market shares and network sizes. Katz and Shapiro impose that, in equilibrium, realized and expected network sizes are the same. We will refer to such expectations throughout the paper as passive (self-fulfilled) expectations. They are passive as they do not respond to out of equilibrium deviations by firms. Hurkens and López (2010) show that this seemingly innocuous twist of the modeling of consumer expectations is able to reconcile the puzzle: When consumers have passive expectations, firms prefer termination charges above cost, and socially optimal termination charges are below or at cost (depending on the case that is under consideration).\textsuperscript{13}

It is worth mentioning that a few recent papers also attempt to reconcile the mentioned puzzle. Armstrong and Wright (2009)\textsuperscript{14}, Jullien, Rey and Sand-Zantman (2010)\textsuperscript{15}, and Hoernig, Inderst and Valletti (2009)\textsuperscript{16} have in common that they introduce additional realistic features of the telecommunication industry into the Laffont, Rey and Tirole (1998b) framework. They show that for some parameter range (and under rationally responsive expectations) joint profits increase as the termination charge increases above the cost. However, contrary to Hurkens and López (2010) these papers conclude that the need to regulate termination charges is reduced because the socially optimal termination charge would also be above cost.

Our paper proceeds as follows. Section 2 introduces the general model based on JLT and defines the concept of passive expectations. We assume that the utility of receiving calls is proportional but smaller than the utility from placing calls. In section 3 we examine

\textsuperscript{13}Moreover, this result is robust to the inclusion of call externalities, an arbitrary number of mobile operators, asymmetric networks and elastic subscription demand.

\textsuperscript{14}Armstrong and Wright (2009) argue that if MTM and FTM termination charges must be chosen uniformly, as is in fact the case in most European countries, firms will trade off desirable high FTM and desirable low MTM charges and arrive at some intermediate level, which may well be above cost.

\textsuperscript{15}Jullien, Rey and Sand-Zantman (2010) argue that the willingness to pay for subscription is related to the volume of calls. They introduce two types of users in the framework of ALRT: light users and heavy users. Light users only receive calls and are assumed to have an elastic subscription demand. Instead, full participation is assumed for heavy users, who can place calls and obtain a fixed utility from receiving calls.

\textsuperscript{16}Hoernig, Inderst and Valletti (2009) consider the existence of calling clubs so that the calling pattern is not uniform but skewed.
the case of no network-based price discrimination. We first describe the set of equilibria when the volume of calls is always determined by the same party (either caller or receiver). By introducing noise in the marginal utility of receivers both parties jointly determine the volume and a unique equilibrium exists. Focussing on this unique equilibrium, we observe that the off-net-cost pricing principle is robust to the way consumers form expectations about network sizes. The reason is that mobile operators set marginal prices at the opportunity cost of ‘stealing’ the customers away from the rival operators (this maximizes consumer surplus, which can then be extracted through the fixed fee). As marginal prices do not depend on market shares, consumer expectations do not alter them. This result has two implications. First, in equilibrium profit is neutral to the level of the termination charge. Second, mobile operators only charge for incoming calls when the termination charge is below cost. The analysis concludes by determining the socially optimal prices: As optimality requires a positive reception charge, optimal termination charge must be strictly below cost. We also consider an alternative way to select a unique equilibrium, without relying on noisy utilities. We simply assume that operators must set the same price for placing and receiving calls. This does not affect the profit neutrality result but does explain the widely used bucket plans of operators in the US where consumers pay a fixed fee and a low price per minute of use, independent on the direction of traffic.

Section 4 considers the case of network-based price discrimination. Again, we describe the set of equilibria when there is no noise in the utilities. After introducing vanishing noise, we focus on the possible existence of an equilibrium without connectivity breakdown. As long as the termination charge is not too low (such as Bill and Keep), such equilibrium exists. Profit is no longer neutral with respect to the termination charge. The termination rate that maximizes firms’ profits depends on the strength of the call externality. If the externality is strong, firms prefer a termination charge below cost and will set positive prices for receiving calls. On the other hand, if the externality is weak, firms prefer a termination charge above cost. The resulting off-net call price will be equal to the monopoly price in this case. Moreover, firms will not charge for receiving off-net calls. We also show that the lowest termination charge that allows the existence of an equilibrium without connectivity breakdown is the one that maximizes total welfare. For this socially optimal termination charge it happens to be that firms would earn higher profits if they would commit to a CPP regime. Section 5 concludes. The technical appendix presents some useful derivatives needed to follow the analysis of the paper.
2 The model

We consider the framework developed by JLT (2004), which extends the traditional framework of network competition by allowing receivers to obtain utility from receiving calls and firms to charge call receptions.

There are two network operators, $i = 1, 2$, each providing full coverage.

**Cost structure.** The fixed cost to serve each subscriber is $f$, whereas $c_O$ and $c_T$ denote the marginal cost of providing a telephone call borne by the originating and terminating networks. The marginal cost of an on-net call is then $c = c_O + c_T$. Network operators pay each other a reciprocal access charge $a$ when a call initiated on a network is terminated on a different network.\(^{17}\) The termination mark-up is equal to:

$$m \equiv a - c_T.$$

The perceived cost of calls is the true cost $c$ for on-net calls, augmented by the termination mark-up for the off-net calls $c_O + a = c + m$ for the caller’s network. The marginal cost of an off-net call is $c_T - a = -m$ for the receiver’s network.

**Retail pricing.** We consider competition in nonlinear pricing under two different frameworks. First, we consider competition in the absence of network-based (i.e., on-net/off-net) price discrimination (Section 3). Here we examine two scenarios: when network $i$ offers three-part tariffs $\{F_i, p_i, r_i\}$, where $F_i$ is the monthly subscriber charge, $p_i$ is the per-unit calling price and $r_i$ is the per-unit reception charge, and when network $i$ sets the same price for outgoing and incoming calls $\{F_i, p_i, p_i\}$, where $p_i$ is the per-unit calling and reception charge. Secondly, we consider competition in the presence of network-based discrimination (Section 4): Network $i$ offers five-part tariffs of the form: $\{F_i, p_i, \hat{p}_i, r_i, \hat{r}_i\}$, where $\hat{p}_i$ and $\hat{r}_i$ denote the off-net calling and reception charges.

**Market shares.** The networks (i.e., firms) sell a differentiated but substitutable product. Consumers are uniformly distributed on the segment $[0, 1]$ and the two networks are located at the two extremities of the segment ($x_1 = 0$, $x_2 = 1$). Given income $y$, a consumer located at $x$ and joining network $i$ has utility

$$y + v_0 - t|x - x_i| + w_i,$$

where $v_0$ represents a fixed surplus from being connected to either network (it is assumed to be large enough so that all consumers want to subscribe to one network), $t|x - x_i|$ is the

\(^{17}\)Reciprocity means that a network pays as much for termination of a call on the rival network as it receives for completing a call originated on the rival network.
cost of subscribing to a network with "address" $x_i$, and $w_i$ is the net surplus of a network-$i$ consumer from making and receiving calls on that network. Network 1’s market share is given by

$$\alpha_1 = \frac{1}{2} + \sigma (w_1 - w_2),$$

(1)

where $\sigma \equiv 1/2t$ measures the degree of substitutability between the two networks. As there is full participation, 2’s market share is $\alpha_2 = 1 - \alpha_1$.

**Individual demand.** Subscribers obtain positive utility from making and receiving calls. The caller’s utility from making a call of length $q$ minutes is $u(q)$, whereas the receiver’s is $\tilde{u}(q)$ from receiving a call of that length. $u(\cdot)$ and $\tilde{u}(\cdot)$ are twice continuously differentiable, and concave. For tractability, we assume that

$$\tilde{u}(q) = \beta u(q) \quad \text{with } 0 < \beta < 1.$$  

We consider the case in which callers and receivers can hang up.

**Volume of calls without noise.** In equilibrium two cases may arise depending on whether the caller or the receiver determines the volume. When the caller hangs up first, the length of a call from network $i$ to network $j$ is given by $D(p_i, r_j) = q(p_i)$ for $i, j = 1, 2$. Conversely, when the receiver determines the volume: $D(p_i, r_j) = q(r_j/\beta)$ for $i, j = 1, 2$. As we argue below, under this specification there is a range of equilibria. By letting the marginal utilities of communications be random we can single out one equilibrium. Thus, we will also examine the case with noise, where individual demand is defined as follows.

**Volume of calls with noise.** The utility that a receiver derives from receiving a call is subject to a noise $\varepsilon$\textsuperscript{18}:

$$\tilde{u}(q) + \varepsilon q.$$  

$\varepsilon$ follows the distribution function $F(\cdot)$, with wide enough support $[\xi, \Xi]$, zero mean, and density function $f(\cdot)$, which is strictly positive for all $\varepsilon$ in the support. Additionally, $\varepsilon$ is identically and independently distributed for each caller-receiver pair.

As receivers are allowed to hang up, for a given pair of prices $(p_i, r_j)$ the length of a call from a caller of network $i$ to a receiver of network $j$ is given by $q(\max(p_i, (r_j - \varepsilon)/\beta))$. Therefore, the volume of calls from network $i$ to network $j$ is $\alpha_i \alpha_j D(p_i, r_j)$ with

$$D(p_i, r_j) \equiv [1 - F(r_j - \beta p_i)]q(p_i) + \int_\xi^{r_j - \beta p_i} q \left( \frac{r_j - \varepsilon}{\beta} \right) f(\varepsilon) d\varepsilon.$$  

Similarly, the utility that a network-$i$ consumer obtains from placing calls to network-$j$ \textsuperscript{18}Introducing also noise in the marginal utility of the caller will not change the results.
consumers is \( \alpha_j U(p_i, r_j) \) with

\[
U(p_i, r_j) \equiv [1 - F(r_j - \beta p_i)]u(q(p_i)) + \int_{\xi}^{r_j-\beta p_i} u \left( q \left( \frac{r_j - \varepsilon}{\beta} \right) \right) f(\varepsilon) d\varepsilon.
\]

Notice that

\[
\frac{\partial U(p_i, r_j)}{\partial p_i} = p_i \frac{\partial D(p_i, r_j)}{\partial p_i}. \quad (2)
\]

The utility that a network-\( j \) consumer obtains from receiving calls from network-\( i \) consumers is \( \alpha_j \tilde{U}(p_i, r_j) \) with

\[
\tilde{U}(p_i, r_j) \equiv \int_{\xi}^{r_j-\beta p_i} \left[ \tilde{u}(q(p_i)) + \varepsilon q(p_i) \right] f(\varepsilon) d\varepsilon
\]

\[
+ \int_{\xi}^{r_j-\beta p_i} \left[ \tilde{u} \left( q \left( \frac{r_j - \varepsilon}{\beta} \right) \right) + \varepsilon q \left( \frac{r_j - \varepsilon}{\beta} \right) \right] f(\varepsilon) d\varepsilon.
\]

And,

\[
\frac{\partial \tilde{U}(p_i, r_j)}{\partial r_j} = r_j \frac{\partial D(p_i, r_j)}{\partial r_j}. \quad (3)
\]

We make the standard assumption of a balanced calling pattern, which means that the percentage of calls originating on a given network and completed on another given (including the same) network is equal to the fraction of consumers subscribing to the terminating network.\(^{19}\)

**Timing.** We assume that the terms of interconnection are negotiated or established by a regulator first. Then, for a given access charge \( a \) (or equivalently, a given \( m \)) the timing of the game is the following:

1. Consumers form expectations about the number of subscribers of each network \( i \) (\( \beta_i \)) with \( \beta_1 \geq 0, \beta_2 \geq 0 \) and \( \beta_1 + \beta_2 = 1 \).

2. Firms take these expectations as given and choose simultaneously retail tariffs: i) in the absence of network-based price discrimination: \( T_i = (F_i, p_i, r_i) \) for \( i = 1, 2 \); ii) in the presence of network-based price discrimination: \( T_i = (F_i, p_i, \hat{p}_i, r_i, \hat{r}_i) \) for \( i = 1, 2 \).

3. Consumers make rational subscription and consumption decisions, given their expectations and given the networks’ tariffs.

Therefore, market share \( \alpha_i \) is a function of prices and consumer expectations. Self-fulfilling expectations imply that at equilibrium \( \beta_i = \alpha_i \).

\(^{19}\)Dessein (2003, 2004) examines how unbalanced calling patterns between different customer types affect retail competition when network operators compete in the presence of the caller-pays regime.
3 No Network-Based Price Discrimination

While in many European countries on-net/off-net price discrimination is a common practice, there are countries as for example the US where this is less common. In addition, in mobile telecommunications markets, it is not uncommon for firms to offer price plans in which both regimes (network-based and no network-based price discrimination) coexist. In this section, we consider for a given reciprocal access charge $a$ and consumer expectations $\beta_1$ and $\beta_2$, competition under the receiver-pays regime and no network-based price discrimination. We first analyze price competition when networks offer different prices for outgoing and incoming calls (what we term ”party-based price discrimination”). We then analyze the case in which networks set the same price for placing and receiving calls. The analysis of the latter case is partially motivated by the observation that in some countries where the receiver-pays regime is used, network operators offer a volume of minutes in a exchange for a fee independently of whether the user is placing or receiving an on- or off-net call. Notice that setting $p_i = r_i$ resembles a tariff whereby firms allow consumers to make and receive a volume of $q(p_i/\beta)$ minutes and charge them $p_i$ per minute of use.

Before turning to the analysis of these two cases, though, we examine price competition in the absence of noise. We show that there exists a range of equilibria, and argue that either the case of party-based price discrimination with noise or price competition with $p_i = r_i$, are only criteria to select one equilibrium in the range of the feasible equilibria.

3.1 Deterministic utility and party-based price discrimination

In the absence of noise in the marginal utilities, as Jeon et al. (2004) point out, there exists a potential indeterminacy of equilibria. If the caller determines the volume, then as the reception charge has no impact on volume, from the viewpoint of firms and subscribers only the sum $\{F_i + r_i q\}$ matters, not its composition. Similarly, when the volume is determined by the receiver, only the sum $\{F_i + p_i q\}$, and not its composition, matters. However, both call and reception charges matter in both cases. In what follows we construct the range of equilibria for both cases. The idea is to show that the assumptions we make next are only criteria that allow us to select one of these equilibria. We will also show that in any symmetric equilibrium, profit is neutral to the access charge.

First, assume that callers determine the volume, in equilibrium it must hold that $r \leq \beta p$. Given the balanced calling pattern assumption and consumer expectations $\beta_1$ and $\beta_2$, the surplus from subscribing to network $i$ (gross of transportation costs) is given by (for $i \neq j = $}

20In the case of the US there is a technical reason: as the prefixes of mobile and fixed line numbers are not different, it is difficult for users to identify to which network the person being called belongs to.
1, 2):

\[ w_i = \varphi_i (\beta_i, p_i, p_j) - F_i \]

with

\[ \varphi_i (\beta_i, p_i, p_j) = u(q(p_i)) - p_i q(p_i) + \beta_i \tilde{u}(q(p_i)) + \beta_j \tilde{u}(q(p_j)) - r_i (\beta_i q(p_i) + \beta_j q(p_j)) \].

Since consumers’ expectations are assumed passive we have that \( w_i \) is a function of expectations and prices, instead of market shares and prices as it is in the case of rationally responsive expectations. The profit of network \( i \) is

\[ \pi_i = \alpha_i [(p_i - c - \alpha_j m) q(p_i) + \alpha_j mq(p_j) + r_i (\alpha_i q(p_i) + \alpha_j q(p_j)) + F_i - f] . \]

Adjusting \( F_i \) so as to maintain net surpluses \( w_1 \) and \( w_2 \) and thus market shares constant, leads network \( i \) to set \( p_i \) so as to maximize (for \( i \neq j = 1, 2 \)):

\[ \pi_i = \alpha_i [(p_i - c - \alpha_j m) q(p_i) + \alpha_j mq(p_j) + r_i (\alpha_i q(p_i) + \alpha_j q(p_j))] + \frac{1}{\sigma} (\alpha_i - \frac{1}{2}) - f] . \]

The first-order condition yields

\[ p_i = c + \alpha_j m - \beta_i r_j . \] (4)

Observe that the optimal call price depends on the reception charge (of the rival). As the caller determines the volume, firms set the calling price at the perceived (or, as termed by JLT, ”strategic marginal”) cost of placing a call, which is given by: the average marginal cost of a call \( c + \alpha_j m \) minus the pecuniary externality imposed on the subscribers of the rival network \( \beta_i r_j \).\(^{21}\) In a symmetric equilibrium \( (\alpha_1 = \alpha_2 = 1/2) \) under symmetric reception charges \( r_i = r_j = r^* \), Eq. (4) boils down to

\[ p^* = c + \frac{m - r^*}{2} . \] (5)

\(^{21}\)A decrease in \( p_i \) will increase the volume of calls from network \( i \) to network \( j \). This in turn increases the surplus of network-\( j \) subscribers from the calls they receive from network \( i \) (direct externality), but also their payment as they have to pay for receiving these extra calls (pecuniary externality). Since the direct externality on \( i \) and \( j \)’s subscribers is the same (the volume of calls received by consumers increases by the same amount independently of the network they are attached), only the pecuniary externality matters.
Assume that \( p_i = p_j = p \), then \( i \)'s profit can be rewritten as follows

\[
\pi_i = \alpha_i [(p - c)q(p) + r_i q(p) + F_i - f] \tag{6}
\]

with \( \alpha_i = \frac{1}{2} + \sigma (\varphi(\beta_i,p,p) - F_i - \varphi(\beta_i,p,p) + F_j) \). Using \( \partial \alpha_i / \partial F_i = -\sigma \), we have

\[
\frac{d\pi_i}{dF_i} = -\sigma [(p - c)q(p) + r_i q(p) + F_i - f] + \alpha_i.
\]

At symmetric equilibrium

\[
F^* + r^* q(p^*) = f + \frac{1}{2\sigma} - (p^* - c)q(p^*). \tag{7}
\]

As hinted above, any combination \((F^*, p^*, r^*)\) satisfying Eqs. (5) and (7) is an equilibrium (provided that \( r^* \leq \beta p^* \) and that no firm wants to deviate and set the reception charge above \( \beta p^* \)). By replacing Eqs. (5) and (7) into (6), we have that at symmetric equilibrium: \( \pi = \frac{1}{2\sigma} \). That is, equilibrium profit is neutral to the access charge and equals the profit that firms would obtain under unit demand.

We now turn to the case in which receivers determine the volume. In equilibrium the condition \( r \geq \beta p \) must hold. The surplus from subscribing to network \( i \) (gross of transportation costs) is now given by (for \( i \neq j = 1,2 \)):

\[
w_i = \tilde{u}(q(r_i/\beta)) - r_i q(r_i/\beta) + \beta_i u(q(r_i/\beta)) + \beta_j u(q(r_j/\beta)) - p_i(\beta_i q(r_i/\beta) + \beta_j q(r_j/\beta)) - F_i.
\]

Maximizing \( \pi_i \) with respect to \( r_i \) while adjusting \( F_i \) to keep the relative attractiveness \((w_i - w_j)\) of the two networks, yields the following first-order condition:

\[
r_i = \alpha_i c - \alpha_j m - \beta_i p_j. \tag{8}
\]

Note that the optimal reception charge depends on the call price chosen by the rival. When the receiver determines the volume, firm \( i \) sets the reception charge at the perceived (or strategic marginal cost) of receiving a call, which is given by: the average unit cost of receiving calls on a given network \( \alpha_i c - \alpha_j m \) minus the pecuniary externality imposed on the subscribers of the rival network \( \alpha_i p \). At symmetric equilibrium, Eq. (8) reads as

\[
r^* = \frac{c - (m + p^*)}{2}.
\]

As above, it is straightforward to show that at the symmetric equilibrium the first-order
condition with respect to $F$ is given by
\[ F^\ast + p^\ast q(\frac{r^\ast}{\beta}) = f + \frac{1}{2\sigma} - (r^\ast - c) q(\frac{r^\ast}{\beta}). \] (9)

Any combination $(F^\ast, p^\ast, r^\ast)$ satisfying Eqs. (8) and (9) is an equilibrium (provided that $r^\ast \geq \beta p^\ast$ and that no firm has an incentive to deviate and set call price so high that all volume is determined by the caller). As in the previous case, equilibrium profit equals $\frac{1}{4\sigma}$.

### 3.2 Deterministic utility and no party-based price discrimination

Here we assume that networks charge a fixed fee and set the same per-unit price $p$ for outgoing and incoming calls. We also assume that there is no noise. Notice that $\beta < 1$ implies that at (symmetric) equilibrium the volume is always determined by the receiver. Therefore, we have that $D(p_i, p_j) = q\left(\frac{p_j}{\beta}\right)$ for $i, j = 1, 2$. The surplus from subscribing to network $i$ (gross of transportation costs) is then given by
\[ w_i = \beta_i u(q(\frac{p_i}{\beta})) + \beta_j u(q(\frac{p_j}{\beta})) + \tilde{u}(q(\frac{p_i}{\beta})) - p_i q(\frac{p_i}{\beta})\beta_i q(\frac{p_j}{\beta}) + \beta_j q(\frac{p_j}{\beta}) - p_i q(\frac{p_i}{\beta}). \]

The profit of network $i$ is (for $i \neq j = 1, 2$):
\[ \pi_i = \alpha_i[p_i - c]q(\frac{p_i}{\beta}) + \alpha_j(p_i - c - m)q(\frac{p_j}{\beta}) + \alpha_jmq(\frac{p_i}{\beta}) + p_i q(\frac{p_i}{\beta}) + F_i - f]. \] (10)

Let $p^\ast$ denote the symmetric equilibrium usage price. The next proposition characterizes the equilibrium.

**Proposition 1 (no party-based price discrimination).** When $m < c$, there exists a unique symmetric equilibrium, in which firms set $p^\ast = \frac{c-m}{3}$ and $F^\ast = f + \frac{1}{2\sigma} - (2p^\ast - c)D(p^\ast, p^\ast)$. The symmetric equilibrium profit is independent of the access charge and given by $\pi_1 = \pi_2 = \frac{1}{4\sigma}$.

To understand why firms charge $p^\ast = \frac{c-m}{3}$, note that when the receiver is who determines the volume, firm $i$ sets the usage price at the perceived or strategic marginal cost of receiving a call, which is given by: the average unit cost of receiving calls on a given network $(\alpha_i c - \alpha_j m)$ minus the pecuniary externality imposed on the subscribers of the rival network $\alpha_i p^\ast$ (i.e.,

\[ A \text{ decrease in } r_i \text{ will increase the volume of calls from network } j \text{ to network } i. \text{ This in turn increases the surplus of network-} j \text{ subscribers from the calls they place on network } i \text{ (direct externality), but also their payment as they have to pay for placing these extra calls (pecuniary externality). Since the direct externality on } i \text{ and } j \text{'s subscribers is the same (the volume of calls placed by consumers increases by the same amount independently of the network they are attached), only the pecuniary externality matters.} \]
Eq. (8)). Since we are imposing that \( p = r \), it follows that at the symmetric equilibrium \( p = p^* \).

We observe that profits are neutral to the access charge as in the case of competition in two-part tariffs under the caller-pays regime (Laffont et al. 1998a). In the latter case, an increase in the access charge boosts usage prices (as they are set equal to the average marginal cost of a call), which in turn increases the per-user profit or, in other words, reduces the opportunity cost of servicing a customer. This makes it more attractive to build market share and thus intensifies competition for subscribers, resulting in lower fixed fees. In our setting an analogous relationship exists, though in the opposite direction. Here an increase in the access charge, decreases the usage price. This reduces the per-user profit and makes it less attractive to attract subscribers, resulting in higher fixed fees.\(^{23}\) As there is full participation of subscribers, in both cases the pass-through rates of cost into prices is one-to-one\(^{24}\), thus the two opposite effects cancel and the impact of the access charge on profit is totally neutralized (100% waterbed effect).

Turning now to the socially efficient price, note that it must satisfy the condition \( u'(q) + \tilde{u}'(q) = c \). Hence the socially efficient price equals \( \frac{c}{1+\beta} \). We have the following,

**Proposition 2** (social optimum). (i) The socially efficient termination mark-up is given by \( m = m^* \equiv c(\frac{1-2\beta}{1+\beta}) \). Thus \( m^* > (<) 0 \) when call externalities are weak (strong) so that \( \beta < (>) \frac{1}{2} \). (ii) Under cost-based access charges (\( m = 0 \)), the volume of calls is socially excessive only if \( \beta > \frac{1}{2} \). (iii) Under bill-and-keep (\( m = -\frac{c}{2} \)), the volume of calls is socially insufficient.

**Proof.** For \( m = 0 \), users will place \( q(\frac{c}{3+\beta}) \) calls. The volume of calls is socially excessive if \( \frac{c}{3+\beta} > \frac{c}{1+\beta} \), which holds for \( \beta > \frac{1}{2} \). Under bill-and-keep, users will place \( q(\frac{c}{2+\beta}) \) calls. This volume of calls is socially insufficient since \( \frac{c}{2+\beta} > \frac{c}{1+\beta} \). Finally, \( p^* = \frac{c}{1+\beta} \) for \( m = m^* \). \( \blacksquare \)

### 3.3 Random utility and party-based price discrimination

Network \( i \) offers the three-part tariff \( \{F_i, p_i, r_i\} \). Given the balanced calling pattern assumption and consumer expectations \( \beta_1 \) and \( \beta_2 \), the surplus from subscribing to network \( i \) (gross of transportation costs) is given by (for \( i \neq j = 1, 2 \)):

\[ w_i = \phi_i(\beta_i, p_i, r_i, p_j, r_j) - F_i \]

\(^{23}\) This is true whenever \( m > -c \). If \( m \) is too negative, then increasing the access charge could increase the per-user profit, leading to lower fixed fees.

\(^{24}\) As the market is assumed to be covered, the equilibrium mark-up is independent of the cost per subscriber, implying that there is a 100% pass-through (see Jullien and Rey, 2008).
with

$$
\phi_i(\beta_i, p_i, r_i, p_j, r_j) = \beta_i U(p_i, r_i) + \beta_j U(p_i, r_j) + \beta_i \tilde{U}(p_i, r_i) + \beta_j \tilde{U}(p_j, r_i) - p_i \left[ \beta_i D(p_i, r_i) + \beta_j D(p_i, r_j) \right] - r_i \left[ \beta_i D(p_i, r_i) + \beta_j D(p_j, r_i) \right].
$$

The profit of network $i$ can be written as

$$
\pi_i = \alpha_i [\alpha_i (p_i - c) D(p_i, r_i) + \alpha_j (p_i - c - m) D(p_i, r_j) + \alpha_j m D(p_j, r_i)] + r_i (\alpha_i D(p_i, r_i) + \alpha_j D(p_j, r_i)) + F_i - f].
$$

Adjusting $F_i$ so as to maintain net surpluses $w_1$ and $w_2$ and thus market shares constant, leads network $i$ to set $p_i$ and $r_i$ so as to maximize

$$
\pi_i = \alpha_i [\alpha_i (p_i - c) D(p_i, r_i) + \alpha_j (p_i - c - m) D(p_i, r_j) + \alpha_j m D(p_j, r_i)] + r_i (\alpha_i D(p_i, r_i) + \alpha_j D(p_j, r_i)) + \phi_i(\beta_i, p_i, r_i, p_j, r_j) - \phi_j(\beta_j, p_j, r_j, p_i, r_i) + F_j - \frac{1}{\sigma} (\alpha_i - \frac{1}{2}) - f].
$$

Assume that $r_i = r_j = r$, by differentiating (12) with respect to $p_i$ and using (2), we obtain the following first-order condition:

$$
p_i = c + \alpha_j m - \alpha_i r.
$$

Similarly, assuming $p_i = p_j = p$, and by differentiating (12) with respect to $r_i$ and using (3), we obtain the following first-order condition:

$$
r_i = \alpha_i c - \alpha_j m - \alpha_i p.
$$

If $p_i = p$ and $r_i = r$, equations (13) and (14) simplify to

$$
p = c + m,
$$

$$
r = -m.
$$

In equilibrium $p$ and $r$ do not depend on market shares, and network operators charge calls and call receptions at their off-net cost. This is the so-called ‘off-net-cost pricing principle’:

Each network sets prices for a subscriber’s outgoing and incoming traffic at the marginal cost

---

25The off-net-cost pricing principle dates back to Laffont et al. (2003), who found this pricing rule in a framework for Internet backbone competition.
that it would incur if all other subscribers belonged to the rival network. To understand this result, notice that the off-net cost is also the opportunity cost of stealing the customers away from the rival network. As usual with two-part tariffs, firms set the marginal price(s) at marginal cost so as to maximize the consumer surplus, which can then be extracted through the fixed part. JLT and López (2010) also find this pricing rule under the assumption of rationally responsive expectations. Therefore the off-net-cost pricing principle is robust to the assumption of consumer expectations. The reason is that firms set marginal prices at the opportunity cost of stealing the customers away from the rivals, and so marginal prices do not depend on market shares. The way consumers form expectations is then irrelevant for the level of the equilibrium marginal prices.

By setting calling and reception charges at the off-net cost, we have that \( \alpha_i = \frac{1}{2} - \sigma (F_i - F_j) \) (for \( i \neq j = 1,2 \)). At equilibrium, market shares do not depend on expectations because there is full participation and, as commented above, usage prices are independent of market shares and symmetric. Thus, \( i \)'s profit can be rewritten as follows:

\[
\pi_i = \left(\frac{1}{2} - \sigma (F_i - F_j)\right) (F_i - f).
\]

(17)

Solving the first-order conditions, we obtain the equilibrium fixed fees \( F_i = f + \frac{1}{2\sigma} \). The equilibrium profit is therefore \( \pi_i = \frac{1}{4\sigma} \), which is the profit that each network would obtain under unit demands. We also have that at equilibrium, profits are independent of the level of the access charge. As López (2010) points out, the reason is that all call activities yield zero profit: on-net calls cost (per unit) \( c \) and yield revenue (per unit) \( p + r = c \), originating an off-net call costs \( c_O + a \) while it yields revenue \( p = c + m = c_O + a \), and the cost of terminating an off-net call is \( c_O \) while it yields revenue \( a + r = a - m = c_O \).

If reception charges are restricted to be non-negative, the above analysis is only correct for \( m \leq 0 \), that is, for termination charges below the cost of termination. Suppose \( m > 0 \) and reception charges cannot be negative. Then it will be optimal to set reception charges at the minimum, i.e., \( r_1 = r_2 = r = 0 \). Hence, if termination charges are above cost, firms will not charge consumers for the reception of calls, even if they are allowed to do so. And the optimal call price will then be \( p_1 = p_2 = p = c + \frac{m}{2} \). In this case, call charges are again set at average marginal cost, but reception is "charged" (at zero) above the true cost of termination \( -m < 0 \). Firms now do make profits from traffic, in particular from terminating calls. Given the symmetry in call and reception charges, market share is again given by

26The opportunity cost of stealing a caller away from the rival network is \( c_O + a = c + m \), whereas the opportunity cost of stealing a receiver away from the rival network is \( c_O - a = -m \). See López (2010) for a complete characterization of the off-net-cost pricing equilibrium.
\alpha_i = \frac{1}{2} - \sigma(F_i - F_j). \text{ Hence, firms choose the fixed fee so as to maximize}

\pi_i = \alpha_i(mq(p) + F_i - f) \tag{18}

The first-order condition for a symmetric equilibrium now reads

0 = -\sigma(mq(p) + F - f) + \frac{1}{2} \sigma

so that \( F = f + \frac{1}{2\sigma} - mq(p) \) and equilibrium profit equals, again, \( 1/(4\sigma) \).\footnote{This equilibrium exists and is unique when \( \sigma \) or \( m \) are not too high (see proof of Proposition 7 in Laffont et al. [1998a, Appendix B]).}

We have the following,

\textbf{Proposition 3 (Equilibrium)} (i) if \( -\left(\frac{\beta}{1+\beta}\right) c < m < 0 \), then as the noise vanishes there exists a unique symmetric equilibrium where marginal prices are set at the off-net cost \( p = c + m \) and reception charges cannot be negative, then (for \( \sigma \) and \( m \) not too high) there exists a unique equilibrium in which \( p = c + \frac{m}{2} \), \( r = 0 \), \( F = f + \frac{1}{2\sigma} - mq(c + \frac{m}{2}) \) and profit is neutral to the termination charge: \( \pi = \frac{1}{4\sigma} \).

The profit neutrality result is independent of the exact specification of the randomness in the marginal utility for receivers. In particular, it holds even if noise does not vanish. It is straightforward to show that the socially optimal call and reception prices converge to \((p^*, r^*) = (\frac{c}{1+\beta}, \frac{\beta c}{1+\beta})\) as the noise vanishes. The intuition is that efficiency requires that the volume of calls \( q \) satisfies \( u'(q) + \tilde{u}'(q) - c = 0 \). Notice that optimality requires a positive reception charge and therefore \( m \) must be strictly negative. In fact, the socially optimal reception charge will be

\[ m^* = \frac{-\beta c}{1+\beta} \]

Figure 1 illustrates the sets of equilibria when utility is deterministic for a specific value of \( m \in (-\beta c/(\beta + 1), 0) \). They are indicated by the two thick parts. Point \( X \) indicates the intersection of both first-order condition equations. This point is the equilibrium when utility is random, so that sometimes the caller and sometimes the receiver determines the volume of calls. Finally, the point \( Y \) indicates the possibility of an equilibrium where firms restrict themselves to charge the same price for placing and receiving calls. This practice may be chosen as it is perhaps easier to market to consumers. In any case, as we have shown above, at any equilibrium, profit is neutral to the access charge. This is caused by the full
coverage assumption of the Hotelling model and the impossibility to discriminate between on- and off-net calls.

![Equilibria with random or deterministic utility.](image)

Figure 1: Equilibria with random or deterministic utility.

4 Network-based price discrimination

In this section we allow the firms to set a fixed fee and (non-negative) prices for making and receiving calls that can depend on the network receiving and originating the call. That is, firm $i$ chooses $(F_i, p_i, r_i, \hat{p}_i, \hat{r}_i)$. We use the same set-up as in JLT (2004), except for the fact that we do not allow for negative reception charges and that we assume that consumers form expectations in a passive way. We will again assume that there is randomness in the marginal utility of receivers. For technical reasons we assume that noise vanishes in the following regular way (similar to the definition in JLT):

**Definition 4** A sequence of distributions $F_n(\varepsilon)$ with zero mean on domain $[\underline{\varepsilon}, \bar{\varepsilon}]$ is called
regular if for any continuous function \( h(\cdot) \) we have

\[
\lim_{n \to \infty} E[h(\varepsilon) | \varepsilon \geq \varepsilon_0] = h(\varepsilon_0) \quad \text{for all } \varepsilon_0 \geq 0
\]

and

\[
\lim_{n \to \infty} E[h(\varepsilon) | \varepsilon \leq \varepsilon_0] = h(\varepsilon_0) \quad \text{for all } \varepsilon_0 \leq 0.
\]

Since we assume that all consumers subscribe to one of the networks, we can use the method of maximizing profits with respect to usage prices for making and receiving calls, keeping market share constant, by adapting the fixed fee accordingly. It is not surprising that the usage prices in a symmetric equilibrium candidate we find are the same as the ones found by JLT (2004) under the assumption that consumer expectations vary with respect to prices. For completeness, we include the analysis.

We start the analysis with the market for on-net calls. It is optimal for network \( i \) to maximize the size of the pie for on-net calls. The first-order conditions with respect to \( p_i \) reads

\[
\frac{d[U(p_i, r_i) + \tilde{U}(p_i, r_i) - cD(p_i, r_i)]}{dp_i} = 0,
\]

while the one with respect to \( r_i \) reads

\[
\frac{d[U(p_i, r_i) + \tilde{U}(p_i, r_i) - cD(p_i, r_i)]}{dr_i} = 0.
\]

As the noise vanishes, these equations can be solved to yield \( p_i = c/(1 + \beta) \equiv p^* \) and \( r_i = c\beta/(\beta + 1) = \beta p^* \equiv r^* \). (This is the same exercise as determining the socially optimal call and reception prices, as we did in the previous section.)

It is clear that there always exists an equilibrium with both off-net call and reception charges equal to infinity, so that no off-net calls will be made. This is independent of the level of the termination mark-up. Both networks then just offer efficient levels of on-net traffic and compete for subscribers by means of the fixed fees. When consumers expect both networks to be of equal size, then the equilibrium fixed fees will be equal to \( f + 1/(2\sigma) \). Thus profits will be equal to \( 1/(4\sigma) \) for each firm. This type of equilibrium is pretty bad in generating consumer surplus as only on-net calls will be made. Clearly, if there are more than two networks this type of equilibrium is even worse.

We now solve for the optimal off-net call and reception charges in an equilibrium without connectivity breakdown. When consumers expect market shares to be equal, to keep market
shares constant at one half, network $i$ should adjust fixed fee as follows:

$$F_i = F_j + \frac{1}{2} \left\{ U(\hat{p}_i, \hat{r}_i) + \tilde{U}(\hat{p}_j, \hat{r}_i) - \hat{p}_i D(\hat{p}_i, \hat{r}_j) - \hat{r}_i D(\hat{p}_j, \hat{r}_i) - U(\hat{p}_j, \hat{r}_i) - \tilde{U}(\hat{p}_i, \hat{r}_j) + \hat{p}_j D(\hat{p}_j, \hat{r}_i) + \hat{r}_j D(\hat{p}_i, \hat{r}_j) \right\}.$$

The first-order derivative of profit with respect to $\hat{p}_i$ reads

$$\frac{1}{4} \left[ \frac{\partial U(\hat{p}_i, \hat{r}_j)}{\partial \hat{p}_i} - (c + m - \hat{r}_j) \frac{\partial D(\hat{p}_i, \hat{r}_j)}{\partial \hat{p}_i} - \frac{\partial U(\hat{p}_i, \hat{r}_j)}{\partial \hat{r}_i} \right],$$

which can be rewritten as

$$\frac{1}{4} \left[ 1 - F(\hat{r}_j - \beta \hat{p}_i) \right] q'(\hat{p}_i) \left[ (1 - \beta)\hat{p}_i - c - m + \hat{r}_j - \int_{\hat{r}_j - \beta \hat{p}_i}^{\infty} \varepsilon f(\varepsilon) \frac{1}{1 - F(\hat{r}_j - \beta \hat{p}_i)} d\varepsilon \right].$$

Hence, the first-order condition is satisfied when

$$(1 - \beta)\hat{p}_i - c - m + \hat{r}_j = E(\varepsilon \mid \varepsilon \geq \hat{r}_j - \beta \hat{p}_i).$$

Note that the second-order derivative of the profit function with respect to $\hat{p}_i$, evaluated at the solution of the first-order condition, equals

$$\frac{\partial^2 \pi_i}{\partial \hat{p}_i^2} = \frac{1}{4} \left[ 1 - F(\hat{r}_j - \beta \hat{p}_i) \right] q'(\hat{p}_i) (1 - \beta) - \frac{1}{4} q'(\hat{p}_i) \beta f(\hat{r}_j - \beta \hat{p}_i) \left( 1 - \int_{\hat{r}_j - \beta \hat{p}_i}^{\infty} \varepsilon f(\varepsilon) \frac{1}{1 - F(\hat{r}_j - \beta \hat{p}_i)} d\varepsilon \right).$$

The first-order derivative of $\pi$ with respect to $\hat{r}_i$, keeping market share constant at one half, reads

$$\frac{1}{4} \left[ \frac{\partial U(\hat{p}_j, \hat{r}_i)}{\partial \hat{r}_i} + (\hat{p}_j + m) \frac{\partial D(\hat{p}_j, \hat{r}_i)}{\partial \hat{r}_i} - \frac{\partial U(\hat{p}_j, \hat{r}_i)}{\partial \hat{r}_i} \right].$$

This can be rewritten as

$$\frac{1}{4} \int_{-\infty}^{\hat{r}_i - \beta \hat{p}_j} \frac{1}{\beta} [\hat{r}_i + \hat{p}_j + m - \hat{r}_i - \varepsilon] \frac{1}{\beta} q'(\frac{\hat{r}_i - \varepsilon}{\beta}) f(\varepsilon) d\varepsilon,$$

which in turn is equal to

$$\frac{1}{4 \beta} F(\hat{r}_i - \beta \hat{p}_j) E((\hat{r}_i + \hat{p}_j + m - \hat{r}_i - \varepsilon) q'(\frac{\hat{r}_i - \varepsilon}{\beta}) \mid \varepsilon \leq \hat{r}_i - \beta \hat{p}_j).$$
Note that the second-order derivative of the profit function with respect to $\hat{r}_i$, evaluated at the solution of the first-order condition, equals

$$\frac{\partial^2 \pi_i}{\partial \hat{r}_i^2} = \frac{1}{4\beta} (\hat{r}_i + m) q'(\hat{p}_j) f(\hat{r}_i - \beta \hat{p}_j) + \frac{1}{4\beta} \int_{\hat{r}_i - \beta \hat{p}_j}^{\hat{r}_i} \left[ (1 - 1/\beta) q'(\frac{\hat{r}_i - \varepsilon}{\beta}) + (\hat{r}_i + \hat{p}_j + m - \frac{\hat{r}_i - \varepsilon}{\beta}) q''(\frac{\hat{r}_i - \varepsilon}{\beta}) \frac{1}{\beta} \right] f(\varepsilon) d\varepsilon.$$ 

Let $F^{(n)}$ represent a series of noise distributions that is regular according our definition. Let $(\hat{p}^{(n)}, \hat{r}^{(n)})$ denote the corresponding symmetric equilibrium candidate usage prices. By taking a suitable subsequence one may assume that either $\hat{r}^n - \beta \hat{p}^n \leq 0$ for all $n$ or that $\hat{r}^n - \beta \hat{p}^n \geq 0$ for all $n$.

Consider the first case. Then in the limit, as noise vanishes, the limit point $(\hat{p}, \hat{r})$ must satisfy $\hat{r} - \beta \hat{p} \leq 0$ and

$$0 = (1 - \beta) \hat{p} - c - m + \hat{r} \quad (19)$$
$$0 = \hat{r} + m \quad (20)$$

so that $\hat{r} = -m$ and $\hat{p} = (c + 2m)/(1 - \beta)$. The condition $\hat{r} - \beta \hat{p} \leq 0$ is satisfied if and only if $m \geq -\beta c/(1 + \beta)$. Note that these symmetric candidate equilibrium usage prices were reported in JLT (2004). However, they did not verify whether the second-order condition is satisfied and whether the equilibrium candidate is indeed an equilibrium. We show that, under some additional assumptions, these equilibrium candidates are indeed equilibria.

**Lemma 5** Suppose that $\hat{r} - \beta \hat{p} < 0$ and that $q(p)$ is linear. Furthermore, assume that the noise distributions are normal with zero mean and variance $\sigma^2$. Then the second-order conditions are satisfied in the equilibrium candidate of the noisy game as $\sigma \downarrow 0$.

**Proof.** See Appendix. 

Consider the second case next. Then in the limit, as noise vanishes, the limit point $(\hat{p}, \hat{r})$ must satisfy $\hat{r} - \beta \hat{p} \geq 0$ and

$$0 = \hat{p} - c - m \quad (21)$$
$$0 = \hat{r} (1 - 1/\beta) + \hat{p} + m \quad (22)$$

so that $\hat{p} = c + m$ and $\hat{r} = \beta (c + 2m)/(1 - \beta)$. The condition $\hat{r} - \beta \hat{p} \geq 0$ is satisfied if and only if $m \geq -\beta c/(1 + \beta)$. JLT (2004) discard this candidate equilibrium on the ground of second-order considerations.
Proposition 6  As the noise vanishes, for \( m < -\beta c/(1 + \beta) \) the only symmetric equilibrium has connectivity breakdown. The unique symmetric equilibrium candidate without connectivity breakdown has

(i) If \( m \geq 0 \),
\[
p = \frac{c}{1 + \beta}, \quad r = \frac{\beta c}{1 + \beta}, \quad \hat{p} = \frac{c + m}{1 - \beta}, \quad \hat{r} = 0, \quad F = f + \frac{1}{2\sigma}.
\]

(ii) If \( -\beta c/(1 + \beta) \leq m < 0 \),
\[
p = \frac{c}{1 + \beta}, \quad r = \frac{\beta c}{1 + \beta}, \quad \hat{p} = \frac{c + 2m}{1 - \beta}, \quad \hat{r} = -m, \quad F = f + \frac{1}{2\sigma}.
\]

Equilibrium profit per firm is given by
\[
\pi^* = \frac{1}{4\sigma} + \frac{1}{4}(\hat{p} + \hat{r} - c)q(\hat{p}).
\]

Proof. We already showed the result for the variable prices in the text. We just need to calculate the equilibrium fixed fees. Given the variable prices, firms make no profits from on-net calls and each firm \( i \) thus solves
\[
\max_{F_i} \alpha_i [F_i - f + (1 - \alpha_i)(\hat{p} + \hat{r} - c)q(\hat{p})].
\]
Because expectations are passive, \( \partial \alpha_i / \partial F_i = -\sigma \) so that the first-order condition reads
\[
-\sigma [F_i - f + (1 - 2\alpha_i)(\hat{p} + \hat{r} - c)q(\hat{p})] + 1/2 = 0.
\]
At a symmetric equilibrium (with \( \alpha_i = 1/2 \)), we thus have
\[
-\sigma(F - f) + \frac{1}{2} = 0,
\]
from which the result follows. \( \square \)

Note that even though the termination charge affects the price for making and receiving off-net calls, the equilibrium fixed fee is independent of \( m \). The reason is that consumers with passive expectations expect each firm to obtain half of the market, even when firms would charge different fixed fees. As consumers expect to get the same surplus from each network, the equilibrium fixed fee is as in the standard Hotelling model without network effects. Thus, there exists no waterbed effect on subscription or, in other words, there is no pass-through of costs into retail prices. This is due to the fact that we consider a symmetric
duopoly. With asymmetric firms or symmetric oligopolies a partial waterbed effect can be shown to exist.

For a non-negative $m$, firms will not charge the reception of off-net calls, in which case the socially efficient termination mark-up would be $m = -2\beta c/(1 + \beta)$. Nevertheless, this level of termination mark-up would lead them to charge users for receiving off-net calls. So in terms of social welfare, the best $m$ with $\hat{r} = 0$ is $m = 0$. But, overall $m = -\beta c/(1 + \beta)$ is the socially efficient termination mark-up as it leads automatically to the RPP regime with the efficient volume. For this level of termination discount, however, firms will not make profit from off-net traffic: $\hat{p} + \hat{r} - c = 0$. This may induce them to agree to adopt a CPP regime in which case they will increase their profits.

It is worthwhile to remark that when termination charge is above cost ($m \geq 0$) it does not really matter whether firms use the RPP or the CPP regime. Prices for off-net calls are identical and profits and consumer surplus are exactly the same under both regimes. Only when termination charge is below cost there are differences. First, under RPP there exist equilibria with connectivity breakdown. Focussing on the equilibria without connectivity breakdown, we notice that not only $\hat{p}^{RPP} < \hat{p}^{CPP}$, but even $\hat{p}^{RPP} + \hat{r}^{RPP} < \hat{p}^{CPP}$. Hence when termination charge is below cost, consumers make more or longer calls and pay less, even when one accounts for the fact that then consumers pay for receiving calls as well. This is consistent with the finding of Littlechild (2006) that RPP regimes are correlated with lower average revenues (per minute) as well as higher average usage (measured by call minutes). This not so surprising if one takes into account that CPP regimes are observed in countries with high termination charges while RPP regimes are often observed in countries with low termination charges. However, our result suggests that this is even so when comparing RPP and CPP regimes with similar termination charges.

It is not clear whether firms will have more profit under the CPP or the RPP regime if allowed to set freely the termination mark-up. Next we address this issue under the assumption of constant elasticity call demand.

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28 For a non-negative $m$, firms will compete under the CPP regime, in which case the socially efficient termination mark-up would be $m = -2\beta c/(1 + \beta)$. However, this level of termination mark-up would lead them to use the RPP regime. So in terms of social welfare, the best $m$ under the CPP regime is $m = 0$. Nonetheless, overall $m = -\beta c/(1 + \beta)$ is the socially efficient termination mark-up as it leads automatically to the RPP regime with the efficient volume.

29 Under passive expectations, call externalities and the CPP regime, the equilibrium profit is $\pi^* = \frac{1}{4\tau} + \frac{1}{4} [q(\hat{p}^* - c)q(\hat{p}^*) - (p^* - c)q(p^*)]$ with $\hat{p}^* = \frac{c + m}{1 + \beta}$ and $p^* = \frac{c}{1 + \beta}$ (see Hurkens and López, 2010). Thus, $\pi^* > \frac{1}{4\tau}$.
4.1 Constant elasticity call demand

In order to get some insights about the termination charges that maximize industry profit, we will assume throughout the rest of the section that call demand has constant elasticity. That is, we will assume that $q(p) = p^{-\eta}$ where $\eta > 1$. The termination charge that maximizes industry profit in this case depends on the strength of the call externality $\beta$. When the call externality is relatively weak, firms prefer a termination charge above cost ($m > 0$) that will allow them to set the price for making an off-net call equal to the monopoly price $p^M = \arg\max(p - c)q(p)$. For strong call externalities, however, firms prefer to have a termination discount ($m < 0$) that will allow them to charge for receiving calls. The resulting off-net call price will be below the monopoly price but the firms will earn above monopoly profits (for off-net calls) because of the positive reception charge. The total price charged for a call will also be below the monopoly price.

**Proposition 7** Suppose that $q(p) = p^{-\eta}$ where $\eta > 1$. Then there exists a critical value $\bar{\beta} \in \left(\frac{1}{2\eta - 1}, \frac{1}{\eta}\right)$ such that

(i) for $\beta < \bar{\beta}$, the termination mark-up that maximizes industry profit equals

$$m = (1 - \beta)p^M - c > 0.$$  

Off-net calls are then charged at monopoly price $p^M$ and there is no reception charge for receiving off-net calls.

(ii) for $\beta > \bar{\beta}$, the termination mark-up that maximizes industry profit equals

$$m = \frac{c(2\eta\beta - \beta - 1)}{2(1 + \beta)(1 - \eta)}.$$  

The resulting prices then satisfy $\hat{p} < p^M$ and $\hat{p} + \hat{r} < p^M$.

**Proof.** If the optimal termination mark-up is positive, it must be equal to $m^+(\beta) = (1 - \beta)p^M - c$, as this makes the off-net call price equal to the monopoly price $p^M$. Of course, $m^+(\beta)$ is only positive if $\beta$ is small enough. In fact, $p^M = -\eta c/(1 - \eta)$ so that a necessary condition for $m^+(\beta) > 0$ is that $\beta < 1/\eta$.

If the optimal termination mark-up is negative, it must be that $m^-(\beta) = \arg\max(\hat{p} + \hat{r} - c)q(\hat{p})$. It is straightforward to show that

$$m^-(\beta) = \frac{c(2\eta\beta - \beta - 1)}{2(1 + \beta)(1 - \eta)}.$$  

26
A necessary condition for \( m^-(\beta) < 0 \) is that \( \beta > 1/(2\eta - 1) \).

The critical value \( \bar{\beta} \) is such that the profit obtained from termination mark-up equal to \( m^-(\bar{\beta}) \) is exactly equal to the one obtained from \( m^+(\bar{\beta}) \), that is, equal to monopoly profit. Straightforward calculations show that \( \bar{\beta} = 2^{1/\eta} - 1 \).

5 Concluding remarks

We have analyzed how termination charges affect retail price competition when firms can charge consumers for receiving calls. Compared to earlier literature on this topic we assume that consumers form expectations about network sizes in a passive, but ex-post rational way. Moreover, we restrict reception charges to be non-negative. When firms cannot set different prices for on-net and off-net traffic, expectations over network sizes do not matter and we obtain the standard profit neutrality result in this case. Firms will set a positive reception charge only if termination charge is below termination cost. In this sense we confirm European operators’ warnings that further reductions in termination charges may end the Calling Party Pays Regime. This is not necessarily a bad thing in terms of social welfare. In fact, when receivers’ utility is random and thus receivers sometimes determine the call volume, it is optimal to have strictly positive reception charges. This can only be achieved by setting termination charge below cost. If receiver and caller derive the same benefit, and if termination cost constitutes half of the total cost of a call, then Bill and Keep leads to the socially optimal outcome. (DeGraba (2003) makes this point without formal model.) In this case call and reception charges are the same. This may resemble the situation in the US market pretty well. We also saw that if firms are restricted to set the same price for incoming and outgoing traffic, an equilibrium exists with low usage prices. This may also resemble the case of the US and other RPP countries where where so called buckets of minutes of use are offered.

When firms are allowed to distinguish between on-net and off-net traffic, we already know from Hurkens and López (2010) that under the CPP regime the way expectations are formed are very important. Firms typically will prefer high termination charges while below or at cost termination charges are optimal from a social point of view. This is also true for relatively low levels of receiver utility. Only for very strong levels of call externalities firms would prefer below cost termination charges. The fact that most operators in Europe strongly oppose cuts in termination rates suggests that call externalities are not believed to be very strong.

Under the RPP regime we have shown that with termination based-price discrimination firms will charge on-net reception (since this maximizes the surplus from on-net traffic when
there is vanishing noise in the receiver’s utility). There always exist equilibria in which off-net traffic is choked by high call and or reception charges. However, for a wide range of parameters there also exist equilibria in which there is no connectivity breakdown. Our analysis focuses on these equilibria. For positive termination mark-ups there is no reception charge, and our results on profits and welfare are as in the case where a CPP regime is used. That is, when the call externality is not too strong, firms see their profits reduced when termination charges are reduced toward cost. However, when termination charges are reduced below cost, things change. Namely, then firms start charging for reception (i.e., RPP is used). Profits first increase and then decrease as termination charge is further reduced. Depending on the strength of the call externality the profit of firms with below cost termination charges under an RPP regime may be higher than those obtained under a CPP regime with high termination charges.

Apart from the difference between Europe and the US in termination rates and the regimes used, there is the matter of penetration, which is much higher in Europe. In order to address how penetration rates are related to termination rates and pay regimes, we need to allow for elastic subscription demand. We plan to do so in the near future.

**APPENDIX**

We introduce some notation and derive some useful derivatives.

Define

\[ D_{ij} = D(p_i, r_j) = [1 - F(r_j - \beta p_i)]q(p_i) + \int_{\epsilon}^{r_j - \beta p_i} q(r_j - \epsilon) \frac{\epsilon}{\beta} f(\epsilon)d\epsilon, \]

\[ U_{ij} = U(p_i, r_j) = [1 - F(r_j - \beta p_i)]u(q(p_i)) + \int_{r_j - \beta p_i}^{r_j} u(q(r_j - \beta \epsilon)) f(\epsilon)d\epsilon, \]

and

\[ \tilde{U}_{ij} = \tilde{U}(p_i, r_j) = \int_{r_j - \beta p_i}^{\epsilon} [\beta u(q(p_i)) + \epsilon q(p_i)] f(\epsilon)d\epsilon \]

\[ + \int_{\epsilon}^{r_j - \beta p_i} [\beta u(q(r_j - \beta \epsilon)) + \epsilon q(r_j - \beta \epsilon)] f(\epsilon)d\epsilon. \]
Then,
\[
\frac{\partial D_{ij}}{\partial p_i} = [1 - F(r_j - \beta p_i)]q'(p_i)
\]
and
\[
\frac{\partial D_{ij}}{\partial r_j} = \int_{\epsilon}^{r_j - \beta p_i} \frac{1}{\beta} q'(\frac{r_j - \epsilon}{\beta}) f(\epsilon) d\epsilon.
\]
Further,
\[
\frac{\partial U_{ij}}{\partial p_i} = [1 - F(r_j - \beta p_i)]p_i q'(p_i) = p_i \frac{\partial D_{ij}}{\partial p_i}.
\]
and
\[
\frac{\partial U_{ij}}{\partial r_j} = \int_{\epsilon}^{r_j - \beta p_i} \frac{1}{\beta} q'(\frac{r_j - \epsilon}{\beta}) f(\epsilon) d\epsilon.
\]
Finally,
\[
\frac{\partial \tilde{U}_{ij}}{\partial p_i} = \int_{r_j - \beta p_i}^{\epsilon} (\beta p_i + \epsilon) q'(p_i) f(\epsilon) d\epsilon
\]
and
\[
\frac{\partial \tilde{U}_{ij}}{\partial r_j} = \int_{\epsilon}^{r_j - \beta p_i} r_j q'(\frac{r_j - \epsilon}{\beta}) \frac{1}{\beta} f(\epsilon) d\epsilon = r_j \frac{\partial D_{ij}}{\partial r_j}.
\]

**Proof of Proposition 1.** Maximizing i’s profit with respect to \(p_i\), while adjusting the fee \(F_i\) so as to maintain market share, yields the following first-order condition:

\[
\alpha_i [-\alpha_i c + \alpha_j m + (\alpha_i + \beta_j) p_i + \beta_i p_j] q'(\frac{p_i}{\beta}) \frac{1}{\beta} = 0.
\]

Since at a symmetric equilibrium \(\alpha_i = \beta_i = \frac{1}{2}\), the above expression leads to:

\[
p_1 = p_2 = p^* = \frac{c - m}{3}.
\]

Therefore, \(p^* > 0\) if \(c > m\). For given equilibrium prices \(p_i = p_j = p^*\), network i’s profit is equal to:

\[
\pi_i = \alpha_i [(2p^* - c) D(p^*, p^*) + F_i - f],
\]

with \(\alpha_i = \frac{1}{2} + \sigma (\varphi_i(\beta_i, p^*, p^*) - F_i - \varphi_j(\beta_j, p^*, p^*) + F_j)\). Differentiating with respect to the subscription fee \(F_i\) yields, at a symmetric equilibrium:

\[
\left. \frac{\partial \pi_i}{\partial F_i} \right|_{F_1 = F_2 = F, p_1 = p_2 = p^*} = -\sigma [(2p^* - c) D(p^*, p^*) + F - f] + \frac{1}{2}.
\]
Therefore, the equilibrium fixed fee $F^*$ is given by

$$F^* = f + \frac{1}{2\sigma} - (2p^* - c)D(p^*, p^*)$$

Second-order conditions are satisfied at the candidate equilibrium. Finally, substituting $F^*$ and $p^*$ into (10), we obtain $\pi_1 = \pi_2 = 1/4\sigma$.

**Proof of Lemma 5.** The second-order derivative of profits with respect to $\hat{p}_i$ converges to $\frac{1}{4}q'(\hat{p})(1-\beta) < 0$. Hence, for low levels of noise the second-order condition is satisfied. The second-order derivative of profits with respect to $\hat{r}_i$ reads, in this case

$$\frac{\partial^2 \pi_i}{\partial \hat{r}_i^2} = \frac{q'(\hat{p})F(\hat{r}_i - \beta \hat{p}_j)}{4\beta}(\hat{r}_i + m) \frac{f(\hat{r}_i - \beta \hat{p}_j)}{F(\hat{r}_i - \beta \hat{p}_j)} + \int_{\hat{r}_i - \beta \hat{p}_j}^{\hat{r}_i - \beta \hat{p}_j} (1 - 1/\beta) \frac{f(\varepsilon)}{F(\hat{r}_i - \beta \hat{p}_j)} d\varepsilon.$$

Hence the sign of the second-order derivative is the sign of

$$- \left\{ (\hat{r}_i + m) \frac{f(\hat{r}_i - \beta \hat{p}_j)}{F(\hat{r}_i - \beta \hat{p}_j)} + 1 - 1/\beta \right\}.$$

To show that this sign is negative, we now use that the noise distributions are normal with variance $\sigma^2$. We denote the CDF and PDF of these distributions by $F_\sigma$ and $f_\sigma$, respectively. Let $\hat{r}_i(\sigma)$ denote the solution of the first-order condition when the noise distribution has variance $\sigma^2$. Note that

$$r_i(\sigma) + m = -E(\hat{p}_j + \frac{\varepsilon - \hat{r}_i(\sigma)}{\beta} | \varepsilon \leq \hat{r}_i(\sigma) - \beta \hat{p}_j) > 0.$$

Hence, for $\sigma > 0$ small enough we have that $r'(\sigma) \geq 0$. Note that the first-order condition can be written as

$$(\hat{r}_i(\sigma) + m)F_\sigma(\hat{r}_i(\sigma) - \beta \hat{p}_j) = -\int_{\hat{r}_i(\sigma) - \beta \hat{p}_j}^{\hat{r}_i(\sigma) - \beta \hat{p}_j} (\hat{p}_j + \frac{\varepsilon - \hat{r}_i(\sigma)}{\beta}) f_\sigma(\varepsilon) d\varepsilon.$$

Taking the total derivative with respect to $\sigma$ of the above expression, we find

$$\hat{r}_i'(\sigma)F_\sigma(\hat{r}_i(\sigma) - \beta \hat{p}_j) + (\hat{r}_i(\sigma) + m) f_\sigma(\hat{r}_i(\sigma) - \beta \hat{p}_j) \hat{r}_i'(\sigma) =$$

$$-\int_{\hat{r}_i(\sigma) - \beta \hat{p}_j}^{\hat{r}_i(\sigma) - \beta \hat{p}_j} \left[ \frac{-\hat{r}_i'(\sigma)}{\beta} f_\sigma(\varepsilon) + (\hat{p}_j + \frac{\varepsilon - \hat{r}_i(\sigma)}{\beta}) \frac{\partial f_\sigma(\varepsilon)}{\partial \sigma} \right] d\varepsilon.$$
Dividing both sides by \( F_\sigma(\hat{r}_i(\sigma) - \beta \hat{p}_j) \), and using that

\[
\frac{\partial f_\sigma(\varepsilon)}{\partial \sigma} = f_\sigma(\varepsilon) \frac{\varepsilon^2 - \sigma^2}{\sigma^3},
\]

we find after rearranging that

\[
\hat{r}_i'(\sigma) \left\{ 1 - \frac{1}{\beta} + \frac{(\hat{r}_i(\sigma) + m) f_\sigma(\hat{r}_i(\sigma) - \beta \hat{p}_j)}{F_\sigma(\hat{r}_i(\sigma) - \beta \hat{p}_j)} \right\} = -E \left[ (\hat{p}_j + \frac{\varepsilon - \hat{r}_i(\sigma)}{\beta}) \frac{\varepsilon^2 - \sigma^2}{\sigma^3} | \varepsilon \leq \hat{r}_i(\sigma) - \beta \hat{p}_j \right],
\]

For \( \sigma \) small enough the right-hand side of this equation is strictly positive. Since \( \hat{r}_i'(\sigma) \geq 0 \), it must be that the term between curly brackets is strictly positive. This exactly implies that the sign of the second-order derivative is strictly negative.

References


