Endogenous mergers and bargaining failures

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Abstract

In this paper we study endogenous mergers in a model of strategic bargaining. We allow for firm asymmetries and, in particular, we emphasize the fact that potential synergies generated by a merger vary depending on the identity of the participating firms. We make two main contributions. The first is to show that relatively inefficient mergers may take place. That is, a particular merger may materialize despite the existence of an alternative merger capable of generating larger synergies and hence higher profits and higher social surplus. Our second contribution is a methodological one. We use a bargaining model that is flexible, in the sense that its strategic structure does not place any exogenous restriction on the endogenous likelihood of feasible mergers.

1 Introduction

In this paper we study endogenous mergers in a model of strategic bargaining. We allow for firm asymmetries so that the identity of the merging partners affects the distribution of profits. In particular, we emphasize the fact that potential synergies generated by a merger do vary depending on the identity of the participating firms.

We make two main contributions. The first is to show that relatively inefficient mergers may take place. That is, a particular merger may materialize despite the existence of an alternative merger capable of generating larger synergies and hence higher profits and higher social surplus. Our second contribution is a methodological one. We use a bargaining model that is flexible, in the sense that its strategic structure does not place any exogenous restriction on the endogenous likelihood of feasible mergers.

It is well known that for strategic reasons the occurrence of a merger does not only depend on its relative profitability with respect to the status quo. Unprofitable mergers may take place and, on the contrary, profitable mergers may be delayed. The first outcome may occur when mergers are unprofitable (with respect to the status quo) but also attractive: firms prefer to be part of the merger rather than competing with merged rivals. The literature refers to them as preemptive mergers. The second outcome may take place when mergers are profitable but unattractive: firms prefer to stay out of the merger if one is going to occur. In this case firms are engaged in a war of attrition. Such "anomalies" are likely to have a positive effect on consumer surplus. For instance, preemptive mergers emerge when they generate sufficiently negative external effects on firms that do not participate in the merger. In this case mergers are likely to benefit consumers since the negative externality is probably associated to a substantial reduction in the marginal cost of the merged firm, which more than compensates the increase in market concentration. Similarly, whenever mergers are profitable but unattractive (war of attrition) this is probably because the dominant effect is the increase in market concentration. In this case, if a merger is delayed then this benefits consumers.

These anomalies occur in our model as well. However, our model also predicts a different kind of anomaly that we can term as an identity failure: the group of firms that agrees to merge need not be the one that maximizes neither industry profits nor total surplus.

This new kind of anomaly is of a different nature of the ones already identified in the literature. Indeed, the ones discussed in previous papers can be interpreted as instances of coordination failure. Equilibria where a profitable merger is delayed always coexist with other equilibria where the profitable merger takes place without delay. Similarly, equilibria where an unprofitable merger takes place always coexist with no-merger equilibria. Therefore, if players could coordinate their expectations they would be willing to do so. In contrast, the type of inefficiency we have found would survive such coordination of expectations. When mergers are profitable and attractive then the equilibrium is unique, and provided synergies generated by alternative mergers are not too different, then an inefficient mergers takes place with positive probability. Thus, such inefficiency is not caused by any coordination failure, but by a more generic bargaining failure. Unless players are sufficiently heterogeneous, the relatively inefficient firm still enjoys a strong bargaining position, which interferes with the implementation of the efficient merger. Fixing this type of failure would require a binding agreement with side payments among all relevant players.

Our paper is closely related to two different strands of the literature: on endogenous mergers and on non-cooperative bargaining.

There is a large literature that endogenizes the set of mergers that will occur in a market in the absence of merger control. Some authors have approached the problem using cooperative solution concepts for games in partition function form (since a merger creates externalities on non-merging firms). See, for instance, Barros (1998) and Horn and Persson (2001). Other authors have set up non-cooperative games where both the market structure and the division of surplus are determined simultaneously. See, for instance, Kamien and Zang (1990), or Gowrisankaran (1999). The two papers closest to ours are Inderst and Wey (2004) and Fridolffson and Stennek (2005). Our goal is similar to theirs, in the sense that we also predict the market structure that results from a particular non-cooperative bargaining game. However, these papers focus exclusively on the symmetric case and hence the identity of the merging firms is not an issue in their analysis. Moreover, there are important methodological differences. Inderst and Wey (2004) place symmetric firms in an asymmetric bargaining position. In particular, nature chooses one of the firms as the target and the rest place their bids in an auction where the target firm sets a reservation price. Some of their specific results hinges precisely on the extra market power of the target firm. In contrast, Fridolffson and Stennek (2005) propose a bargaining game that treats all firms symmetrically. However, the structure of the game imposes certain restrictions on (probability distribution over) the set of feasible mergers that tend to enhance the bargaining power of the weakest player. (We will comment on this below.) As a consequence, their game generates multiple equilibria in cases in which ours generates a unique one.

The merger problem we discuss in this paper is similar (and equivalent, for some parameter values) to what has been termed the three-person/three-cake problem (see, for instance, Binmore, 1985), or in general a (restricted) game of coalition formation. Non cooperative analysis of this sort of problems abound. Most of them use one version or another of a dynamic proponent-respondent game in the Rubinstein-Stahl tradition. (See Ray, 2007, for a general discussion including games with externalities, and Compte and Jehiel, 2010, for a recent example.) The particular timing and order of moves in such models differ and, as a consequence, the outcomes of these games also differ. In fact, as Ray (2007, page 140) puts it, "a theory that purports to yield solutions that are independent of proposer ordering is suspect". We agree with this assessment if the ordering of proposers and movements, whether random or deterministic, is exogenous. Our contribution in this line is to propose a game designed so that, in a precise way that will be discussed below, this ordering of proposers is endogenous.

2 The model

2.1 Mergers and the distribution of profits

We consider an industry that is initially populated by three firms: $1, 2, 3$. In the absence of mergers, the equilibrium level of profits per period is given by $\left(\pi_1^T, \pi_2^T, \pi_3^T\right)$. We assume that competition authorities rule out a merger that leads to full monopolization but are willing to consider mergers that lead to a duopoly. In case firms (i, j) merge and firm k stands alone then equilibrium profits per period are given by π_{ij}^D and π_k^D, respectively. Thus, in most of the paper we will only be concerned with these nine numbers and not with the fundamentals that determine these numbers through a particular form of competition. Firms discount the future using the same discount factor $\delta \in (0, 1)$.

Particular market structures and parameter values give rise to alternative relationships between these numbers. In order to simplify the presentation we focus attention on a representative case. First, if no merger takes place, then all firms make the same level of profits, which we normalize to zero: $\pi_i^T = 0$. Second, mergers between firms 1 and 3 or between firms 2 and 3 give rise to the same profit distribution: the merged firm makes π^{**} and the firm that stands alone makes π^*. However, if firms 1 and 2 merge then they make a level of profits equal to $\pi^{**} + \Delta$, $\Delta \geq 0$, while the stand alone firm (firm 3) makes $\pi^* - \gamma\Delta$,

$0 \leq \gamma < \frac{1}{2}$. Thus, in the case $\Delta = 0$ we are in the perfectly symmetric case. The parameter Δ measures the degree of asymmetry across different mergers. Note that if 1 and 2 merge, total profits, $(\pi^{**} + \Delta) + (\pi^* - \gamma\Delta)$, increase with Δ. in fact Δ can be interpreted as the amount of extra synergies created by the merger between 1 and 2, as compared with the other two possible mergers. Thus, the merger between these two firms is not only the most profitable but also is likely to be the most efficient (from the total surplus point of view). Assuming that mergers $\{1, 3\}$ and $\{2, 3\}$ are symmetric allows a drastic reduction in the number of cases that need to be considered with little loss of economic insights.

Depending on industry characteristics and the impact of mergers on costs, (i) π^{**} may be higher of lower than 0, and (ii) π^{**} may be higher of lower than $2\pi^*$. Thus, we can distinguish four different regions (See Figure 1). In Region 1 mergers are *profitable* with respect to the status quo ($\pi^{**} > 0$) and *attractive*; that is firms prefer to be part of the merger rather than being left outside $\left(\frac{\pi^{**}}{2} > \pi^*\right)$. In Region 2 mergers are still *profitable* but relatively *unattractive*; that is firms prefer to stay out of the merger rather than being part of it $\left(\frac{\pi^{**}}{2} < \pi^*\right)$. In Region 3 mergers are *unprofitable* with respect to the status quo ($\pi^{**} < 0$) but they are *attractive* because, conditional on the occurrence of a merger, firms prefer to be part of the merger rather standing alone $\left(0 > \frac{\pi^{**}}{2} > \pi^*\right)$. Finally, in Region 4 mergers are both *unprofitable* and *unattractive*. In this case, it is trivial that the only equilibrium involves no merger and hence we will ignore it.[1]

The goal of our analysis is to predict which merger, if any, will arise, when, and how firms will share the proceeds. For this purpose, we consider a non-cooperative bargaining game that, we argue, is a natural and appropriate model for these negotiations. We will relate this non-cooperative game and its outcomes to a cooperative solution concept that we have developed in a separate

[1] See also Inderst and Wey (2004) and specially Fridolfsson and Stennek (2005)

paper.

2.2 The bargaining game

We propose a discrete time, infinite horizon game. Players discount the future using the same discount factor, $\delta \in (0,1)$. As long as no agreement has been reached in the past, in any particular period the three firms play the following sequential game, which consists of two stages: matching (selecting negotiation partners) and actual negotiation between two players. More specifically, this is the timing of the perfect information game in any one period See Figure 2:

Matching

(1) Nature selects one of the three players, each with probability $\frac{1}{3}$. Let that player be A.

(2) Player A invites one of the other two players to become her negotiation partner. Let us call her player B.

(3) Player B accepts or rejects. If she accepts then players (A, B) enter into the negotiation stage. If player B rejects then players (B, C) enter into the negotiation stage.

Negotiation between F and E.

(4) Nature selects one of the two players, each with probability $\frac{1}{2}$. Let that player be F.

(5) Player F makes an offer to player E: θ_F^E, understood as the per-period profits that E keeps if merged with F.

(6) Player E accepts or rejects F's offer. If E accepts then she gets θ_F^E per period ($\frac{\theta_F^E}{1-\delta}$ discounted total payoff), player F gets $\pi_{FE}^D - \theta_F^E$ ($\frac{\pi_{FE}^D - \theta_F^E}{1-\delta}$ discounted total payoff) and the game ends. If E rejects the offer then everyone obtains 0 in that period and the game moves to the next period.

What is important about the structure of the matching game is that nature's choice in the matching stage does not impose upper or lower bounds on the probability of any given match in the period. This would not be the case if, for instance, we assumed instead that in the third stage of the matching game when player B rejects the offer then player A (and only her) can still ask player C. In that case, a negotiation between players B and C would be impossible in that period. We will comment more on alternative specifications later.

We focus on subgame perfect Nash equilibria (SPE) in stationary strategies. Also, we are interested in situations where the bargaining friction is negligible. Thus, we will pay particular attention to the limit of equilibria as $\delta \to 1$.

Let i, j, k represent generic, different players. A strategy for player i consists of $\left(\mu_i^j, \lambda_i^j, \lambda_i^k\right)$ for the matching game and $\left(\theta_i^j, \rho_i^j, \theta_i^k, \rho_i^k\right)$ for the negotiation stage. μ_i^j is the probability that player i proposes player j to be her negotiation partner in node (2), if i is chosen by nature in node (1). Given the definition of the game, the probability that i proposes k is $\mu_i^k = 1 - \mu_i^j$. λ_i^j is the probability that player i accepts player j's invitation to become a negotiation partner in node (3), and λ_i^k is the probability that i accepts player k's invitation. In line with the restriction to stationary strategies, we will assume that $\lambda_i^j = 1 - \lambda_i^k$. We will comment on the effect of this assumption later. Thus, in case nature chooses player i, then the probability that players (i, j) negotiate in nodes (5) and (6) is $\mu_i^j \lambda_j^i$, the probability that (i, k) negotiate is $\mu_i^k \lambda_k^i = \left(1 - \mu_i^j\right) \lambda_k^i$, and the probability that (j, k) negotiate is $\mu_i^j \lambda_j^k + \mu_i^k \lambda_k^j = \mu_i^j \left(1 - \lambda_j^i\right) + \left(1 - \mu_i^j\right) \left(1 - \lambda_k^i\right)$.

In the negotiation game, θ_i^j is the (per period) offer that player i makes to player j with probability ρ_i^j in node (5) if the former is chosen by nature in node (4) as the proponent. θ_i^k and ρ_i^k are the corresponding values in a negotiation with k. In order to avoid open-set technical problems, and also to save in notation, we assume that in node (6) the respondent accepts with probability one any offer above or equal to the value of continuation. That is why we do not include these decisions in the definition of a strategy. As we will see in the analysis below, this is innocuous and in particular does not rule out the possibility of delay in case of indifference.[2] Again, note that in line with the restriction to stationary strategies, we are implicitly assuming that the answer to invitations to negotiate in node (3) and the offer in node (5) does not depend on who made the invitation to meet or who answered to that invitation, but only on the identity of the partner. Again, we will comment on this assumption later.

Let us denote by u_i player i's expected payoff per period in a particular equilibrium. Thus, the total payoff is $\frac{u_i}{1-\delta}$. The analysis of the negotiation stage is straightforward, given these values. If $\delta u_i + \delta u_j < \pi_{ij}$ then $\theta_i^j = \delta u_j$ and $\rho_i^j = \rho_j^i = 1$. If $\delta u_i + \delta u_j > \pi_{ij}$ then no acceptable offer is made with positive probability in the negotiation between i and j. Finally, if $\delta u_i + \delta u_j = \pi_{ij}$ then $\theta_i^j = \delta u_j$ and this is compatible with any $\rho_i^j, \rho_j^i \in [0,1]$. Also, if $\delta u_i + \delta u_j < \pi_{ij}$, if player i is the proponent then she gets $\pi_{ij} - \delta u_j$ and if she is the respondent she gets δu_i. If $\delta u_i + \delta u_j \geq \pi_{ij}$ then she gets δu_i. As a result, player i's expected payoff of reaching the negotiation stage with player j is $u_i^{ij} = \max\left\{\frac{1}{2}\left(\pi_{ij} - \delta u_j + \delta u_i\right), \delta u_i\right\}$.

For future reference it will be useful to note that in case $\rho_i^j = \rho_j^i = 1$ the probability that i and j merge is given by $p_{ij} = \frac{1}{3}\left[\mu_i^j \lambda_j^i + \mu_j^i \lambda_i^j + \mu_k^i \lambda_i^j + \left(1 - \mu_k^i\right)\lambda_j^i\right]$.

3 Profitable and attractive mergers (Region 1)

Consider the case in which all mergers increase the profits of all firms, and more so the joint profits of the firms that are parties to it: $\pi^{**} > 2\pi^* > 0$. Let us denote by Ψ the extra profits enjoyed by the merging firms, $\Psi \equiv \pi^{**} - 2\pi^* > 0$. In the region we are considering all incentives are favorable to the occurrence of a merger. However, what is not so clear is the identity of the merger.

[2] Indeed, apart from open-set issues, in a SPE there could be indifference between accepting and rejecting a partner's offer only if the sum of the continuation values for both partners is equal to what they have to share. In ths case, the fact that the proponent can choose any value ρ in $[0,1]$ already allows for any probability of delay.

The following proposition shows the unique equilibrium outcome, including the probability of each merger, in Region 1.

Proposition 1 *For δ sufficiently large, there exists a unique SPE outcome, both in payoffs and probability distribution over mergers. A merger occurs with probability* 1 *in the first period (no delay). There exists a threshold $\overline{\Delta}(\delta)$, with $\lim_{\delta \longrightarrow 1} = \frac{\Psi}{1-2\gamma}$, such that if $\Delta \geq \overline{\Delta}(\delta)$ then firms* 1 *and* 2 *merge with probability* 1*. If $\Delta < \overline{\Delta}(\delta)$ then all three potential mergers take place with positive probability.*

Proof. See Appendix. ■

If mergers are sufficiently heterogeneous, $\Delta \geq \frac{\Psi}{1-2\gamma}$, then the efficient merger is not challenged by the presence of alternative profitable mergers. Indeed, if such merger occurs with probability one, then in the limit the joint profits per period for players 1 and 3 (and, similarly, players 2 and 3) are $\frac{1}{2}(\pi^{**} + \Delta) + (\pi^{*} - \gamma\Delta)$. Therefore, if what players 1 and 3 can obtain by merging, π^{**}, is less than this amount then there are no profitable deviations. As $\delta \longrightarrow 1$, such a condition approaches precisely $\Delta \geq \frac{\Psi}{1-2\gamma}$. The proposition tells us that this is in fact the only equilibrium outcome.

The problem is much more interesting when $\Delta < \frac{\Psi}{1-2\gamma}$. It is clear that an equilibrium with $p = 1$ does not exist since firms can always have access to profitable deviations. An alternative way of putting it is that in this case the core of the cooperative game is empty. That is, player 3 can always offer either player 1 or player 2 a share of the gains from merging that renders the deal mutually beneficial. Hence, when the core is empty then a pure strategy equilibrium does not exist. In the unique equilibrium all three players are indifferent with respect to their merging partner and any of the three mergers can occur with positive probability. In other words, the implementation of the efficient merger is disturbed by the presence of player 3, since players 1 and 2 are actually indifferent between implementing the efficient merger or merging with player 3.

More specifically, in equilibrium $p_{13} = p_{23} \equiv q$, and $p_{12} + 2q = 1$. In the rest of the paper, and in order to simplify notation we use $p \equiv p_{12}$. If player 1 negotiates with 2, then she obtains $\frac{1}{2}(\pi^{**} + \Delta)$ per period; alternatively, if player 1 negotiates with 3, then she obtains $\frac{1}{2}(\pi^{**} + \delta u_1 - \delta u_3)$. Hence, player 1 is indifferent if and only if:

$$\delta u_1 - \delta u_3 = \Delta. \tag{1}$$

In equilibrium u_1 and u_3 are given by:

$$u_1 = (1-q)\frac{1}{2}(\pi^{**} + \Delta) + q\pi^*, \tag{2}$$

$$u_3 = (1-2q)(\pi^* - \gamma\Delta) + 2q\frac{1}{2}(\pi^{**} + \delta u_3 - \delta u_1). \tag{3}$$

If we solve equations (1),(2), and (3) for q and take the limit $\delta \to 1$, then we have:

$$q = \frac{\Psi - (1-2\gamma)\Delta}{3\Psi - (1-4\gamma)\Delta}. \tag{4}$$

Thus, q is a decreasing function of Δ, and it takes the value $q = \frac{1}{3}$ for $\Delta = 0$ and the value 0 for $\Delta = \frac{\Psi}{1-2\gamma}$. Consequently, p is an increasing function of Δ and it takes the value $\frac{1}{3}$ for $\Delta = 0$ and the value 1 for $\Delta = \frac{\Psi}{1-2\gamma}$.

As discussed in the introduction, the existing literature has focused mostly on symmetric market structures. It has pointed out two important phenomena: the possibility that profitable mergers might be delayed and the occurrence of unprofitable mergers, essentially in Regions 2 and 3. However, in the regions where the inefficient aggregate outcome may occur, there is always another equilibrium in which the efficient outcome takes place with probability one. Thus, these two phenomena can be thought of as the result of some kind of coordination failure: firms coordinate in the "wrong" equilibrium. If players could coordinate their expectations then they would be willing to do so. In line with the rest of the literature, we have found that in Region 1 a merger occurs immediately with probability one. However, we have found a different sort of phenomenon related to the identity of the merging partners: a relatively inefficient merger takes place with positive probability. Even more importantly, this equilibrium is unique. That is, it is not the result of any coordination failure, but is simply a consequence of the relative strength of player 3 that makes the possible occurrence of relatively inefficient mergers unavoidable. Fixing this type of bargaining failure would require more than coordinating expectations, and would require a binding agreement with side payments among the three players. That is, the formation of the grand coalition.

In addition to studying asymmetric mergers (which, of course, makes the identity of the merging partners a relevant issue) we view our bargaining game and some of its important properties as a methodological contribution. To illustrate this point, we first compare our game with some standard games that have been used to study endogenous mergers. Then, we relate the predictions of our non-cooperative game to a cooperative solution concept that we have developed somewhere else.

In the spirit of Stähl and Rubinstein, Fridolffson and Stennek (2005) propose the following bargaining game. In the first period, nature chooses as proponent one of the three players, each with equal probability. The proponent makes a specific offer to one of the other two players. The respondent accepts or rejects. If she accepts the merger takes place and the game is over. If she rejects then they move into the next period and the game starts again.[3] Using a notation similar to the one used above, let us denote by μ_i^j the probability that i makes an offer to j, and by λ_j^i the probability that j accepts i's offer. In

[3]In fact, they frame their game in continuous time and bidding rounds occur at random points in time.However, they also focus on the limit case that the expected difference between two bidding rounds goes to zero. This is equivalent to the deterministic version we discuss in the text.

an equilibrium where a merger occurs in the first period with probability one, then the probability of the efficient merger is given by $p = \frac{1}{3}\left(\mu_1^2\lambda_2^1 + \mu_2^1\lambda_1^2\right)$. That is, p is bounded above by $\frac{2}{3}$. This bound has nothing to do with decisions on the part of players, but it is an artifact (a rigidity) imposed by the design of the game on the outcomes of negotiation. Due to this rigidity, player 3 "must" be part of the merger in one out of three times unless there is delay, and as a consequence, her payoff is higher than the payoff in our game. Moreover, also as a consequence of this rigidity, in a subset of Region 1 the game studied by Fridolffson and Stennek (2005) has multiple equilibria in the asymmetric case. (Details are available upon request.) In particular, if player 1 expects that player 2 will accept player $3'$s offer, then u_1 will be relatively low and player 1 will also accept player 3's offer (the symmetric is true for player 2). However, if player 1 expects that player 2 will reject player $3'$s offer, then u_1 will be relatively higher, which will induce her to reject also player 3's offer and will generate some delay. Clearly, u_1 and u_2 are higher in the second equilibrium.

In contrast, in our game if a merger occurs immediately with probability one, then $p = \frac{1}{3}\left[\mu_1^2\lambda_2^1 + \mu_2^1\lambda_1^2 + \frac{1}{2}\left(\lambda_1^2 + \lambda_2^1\right)\right]$. That is, p can take any value between 0 and 1, and whether this happens or not will depend on the decisions of players. In other words, as discussed in the introduction our bargaining game is flexible in the sense that nature does not impose any restriction on the likelihood of any match. This has as one of its consequences sharper predictions in Region 1 (unique equilibrium).

Our non-cooperative game in Region 1 implements the PSBN (Prediction for Simultaneous Bilateral Negotiations). In Burguet and Caminal (2010) we define PSBN as the generalization of Nash Bargaining Solution to simultaneous bargaining negotiations. The underlying assumption is that all pairs simultaneously bargain à la Nash, and in each of these negotiations fallback options are endogenous and determined by (consistent) expectations on the consequences of failing to reach an agreement in that particular negotiation. Players are assumed to share the same beliefs on the probability distribution over the success of different coalitions. In our restricted model this is equivalent to sharing beliefs on (p, q) such that $p + 2q = 1$. Therefore, if negotiations between players 1 and 3 break down then they will expect that coalition $(1, 2)$ will succeed with (conditional) probability $\frac{p}{1-q}$, and coalition $(1, 3)$ with probability $\frac{q}{1-q}$. Other than that, the PSBN only imposes a weak restriction on the set of admissible beliefs: a coalition cannot have positive probability of success if both players prefer to reach an agreement with the third player, with one of the preferences being strict.

4 Profitable but unattractive mergers (Region 2)

Consider the case in which a merger benefits all firms, including the one that is left standing alone, but in fact the latter benefits more on average: $2\pi^* > \pi^{**} > 0$. That is, $\Psi < 0$. In this case, firms may engage in a war of attrition, and as in any such situation, even though forming a merger is profitable this may be delayed by the even greater gain of been left out of the merger if one occurs. See again Inderst and Wey (2004) and Fridolfsson and Stennek (2005). It is also well known that in this region (as well as in Region 3) there are very strong reasons for the existence of multiple equilibria, and our game will not be an exception. Thus, for simplicity, we restrict ourselves to equilibria where strategies are partially symmetric. More specifically $\lambda_1^2 = \lambda_2^1$, $\mu_1^2 = \mu_2^1$, $\mu_3^1 = \lambda_3^1 = \frac{1}{2}$. As a result $p_{13} = p_{23} = q$, and $u_1 = u_2$.

Like in Region 1, in the limit as $\delta \longrightarrow 1$ an equilibrium with $p = 1$ exists if and only if $\Psi \leq (1 - 2\gamma)\,\Delta$. Moreover, since $\Psi < 0$ this condition is satisfied for all $\Delta \geq 0$.

The polar case, that is, an equilibrium with $q = \frac{1}{2}$ exists if and only if player 1 (player 2) prefers negotiating with player 3 to negotiating with player 2 (player 1):

$$\pi^{**} - \delta u_3 \geq \pi^{**} + \Delta - \delta u_1. \tag{5}$$

But now $\widetilde{u}_1$ and $\widetilde{u}_3$ are given respectively by:

$$u_1 = \frac{1}{2}\left[\frac{1}{2}\left(\pi^{**} - \delta u_3 + \delta u_1\right) + \pi^*\right], \tag{6}$$

$$u_3 = \frac{1}{2}\left(\pi^{**} - \delta u_1 + \delta u_3\right). \tag{7}$$

If we solve equations (6) and (7) for the limiting case of $\delta = 1$ and we plug the solution in equation (5) then we get that an equilibrium with $q = \frac{1}{2}$ exists if and only if $\Delta \leq -\Psi$. That is, provided that players 1 and 2 are worse off than player 3 if they merge, then there is an equilibrium where one of the two inefficient mergers occurs in the first period with probability one.

It turns out that an equilibrium where a merger occurs with probability one in the first period but $p > 0$ and $q > 0$ does not generically exist in this region. Indeed, when we solve the system of equations (1) to (3) in this region, we obtain $u_1 > \frac{1}{2}\left(\pi^{**} + \Delta\right)$ and therefore players 1 and 2 should not be willing to merge. The intuition is relatively simple. If all meetings lead to mergers with probability 1, then refusing to merge will lead with positive probability to be left out of the merger, and with the complement probability it will lead to be part of a meeting again in the next period.[4] For δ close to 1, some firm should find such deviation profitable unless all of them make at least (almost) π^* by merging, which is impossible in this region.

There exists a third type of equilibrium, as in any war of attrition, where the probability of a merger in any given period is positive but lower than one.

[4] Moreover, if all mergers have positive probability that implies that all firms are indifferent as to which of the other two firms it merges, just as in the similar equilibrium in Region 1.

More specifically, suppose that in any negotiation players are indifferent between reaching an agreement or moving to the next period. In this case:

$$(\pi^{**}+\Delta)-\delta u_1=\delta u_1, \tag{8}$$

$$\pi^{**}-\delta u_1=\delta u_3. \tag{9}$$

Also u_1 and u_3 are given respectively by:

$$u_1=p\frac{1}{2}\left(\pi^{**}+\Delta\right)+q\delta u_1+q\pi^{*}+\left(1-p-2q\right)\delta u_1, \tag{10}$$

$$u_3=p\left(\pi^{*}-\gamma\Delta\right)+2q\delta u_3+\left(1-p-2q\right)\delta u_3. \tag{11}$$

Clearly, as δ goes to 1, p and q go to zero. In other words, as rounds are closer in time, the probability that a merger materializes in any one round goes to zero. In order to study the expected delay as $\delta \longrightarrow 1$, we can solve the system of equations (8) to (11), for $\frac{p}{1-\delta}$ and $\frac{q}{1-\delta}$ and study the behavior of those solutions as $\delta \longrightarrow 1$. Solving that system we get

$$\frac{p}{1-\delta}=\frac{\pi^{**}-\Delta}{\Psi+\left(1-2\gamma\right)\Delta}, \tag{12}$$

$$\frac{q}{1-\delta}=\frac{\pi^{**}+\Delta}{\Psi-\Delta}. \tag{13}$$

An equilibrium in mixed strategies exists if and only if $p,q\in[0,1]$, and $\frac{p}{1-\delta}+2\frac{q}{1-\delta}\leq 1$. These conditions are satisfied if and only if $\Delta \leq \min\left\{-\Psi,\pi^{**}\right\}$. Note that p is decreasing in Δ while q is increasing in Δ. Also, note that delay is decreasing in Δ: as Δ increases $(p+2q)$ increases.

In the Appendix we also show that no other type of equilibrium with partially symmetric strategies exists. All this information can be summarized in the following proposition:

Proposition 2 *In the limit as δ goes to 1 and: (i) if $\Delta\leq\min\left\{-\Psi,\pi^{**}\right\}$ then there are three equilibrium outcomes: (a) $p=1$, (b) $q=\frac{1}{2}$, (c) $p,q>0$, $p+2q<1$; (ii) if $\Delta\in\left[\min\left\{-\Psi,\pi^{**}\right\},-\Psi\right]$ there are two equilibrium outcomes: (a) $p=1$, (b) $q=\frac{1}{2}$; if $\Delta>-\Psi$ then there is a unique equilibrium with $p=1$.*

Provided that firms 1 and 2 are worse off than firm 3 if they merge, then the multiplicity of equilibria prevails in spite of the asymmetry. Moreover, in two of these equilibria a relatively inefficient merger may occur. Thus, in addition to the aggregate problem (a profitable merger is delayed), the identity of the merger may not be the most desirable.

5 Unprofitable but relatively attractive mergers (Region 3)

Suppose that mergers are not profitable, but staying out of a merger is even more damaging; i.e., $0 > \pi^{**} > 2\pi^{*}$. The literature has pointed out the possible occurrence of preemptive mergers in such case.

Let us first consider equilibria with $p = q = 0$. In this case, $u_1 = u_3 = 0$. Players 1 and 2 will find it profitable to deviate if and only if their merger is profitable. Thus, this is an equilibrium provided that $\Delta \leq -\pi^{**}$.

Also, $p = 1$ will be an equilibrium outcome if $\pi^{**} + \Delta \geq 2\delta u_1 = \delta\left(\pi^{**} + \Delta\right)$. Hence, as $\delta \longrightarrow 1$ this is equivalent to $\Delta \geq -\pi^{**}$. Thus, these two types of equilibria are mutually exclusive and efficient.

Like in Region 1 there is also an equilibrium where a merger takes place immediately with probability one: $p = 1 - 2q$ and $p, q > 0$. In this case q is given by equation (4). Thus, an equilibrium of this type exists if and only if $\Delta \leq \frac{\Psi}{1-2\gamma}$.

Finally, by looking at equation (12) it is immediate that an equilibrium with $p < 1 - 2q$ and $p, q > 0$ does not exist since in this region this would involve $p < 0$.

In the Appendix we also show that no other type of equilibrium with partially symmetric strategies exists. We summarize this discussion for the case $\frac{\Psi}{1-2\gamma} > -\pi^{**}$:

Proposition 3 *If* $\frac{\Psi}{1-2\gamma} > -\pi^{**}$*, in the limit as* δ *goes to* 1*: (i) if* $\Delta \leq -\pi^{**}$*, then there are two equilibrium outcomes: (a)* $p = q = 0$*, (b)* $p, q > 0$*,* $p+2q = 1$*; (ii) if* $\Delta \in \left[-\pi^{**}, \frac{\Psi}{1-2\gamma}\right]$*, then there are two equilibrium: (b)* $p, q > 0$*,* $p+2q = 1$*, and (c)* $p = 1$*, (iii) if* $\Delta > \frac{\Psi}{1-2\gamma}$ *then there is a unique equilibrium with* $p = 1$*. If* $\frac{\Psi}{1-2\gamma} < -\pi^{**}$*,in the limit as* δ *goes to* 1*: (i) if* $\Delta \leq \frac{\Psi}{1-2\gamma}$*, then there are two equilibrium outcomes: (a)* $p = q = 0$*, (b)* $p, q > 0$*,* $p + 2q = 1$*; (ii) if* $\Delta \in \left[\frac{\Psi}{1-2\gamma}, -\pi^{**}\right]$*, then there is a unique equilibrium outcome with* $p = q = 0$*, (iii) if* $\Delta > -\pi^{**}$ *then there is a unique equilibrium with* $p = 1$

Like in Region 2 in addition to the aggregate inefficiency (non profitable mergers may take place) there is an issue with the identity of the merger. Unless heterogeneity is sufficiently large, there is an equilibrium where a relatively inefficient merger may occur with positive probability.

6 Discussion (to be completed)

Bargaining failure in Region 1 is of a different nature than in the other two regions. In the former case there is a unique equilibrium in which there is a positive probability that the relatively inefficient merger occurs, while in the other two regions there is a coordination failure: whenever there is an equilibrium where a profitable (with respect to the status quo) merger is delayed or an unprofitable merger that takes place, then there is also another equilibrium where merger decisions are taken according to their profitability. However, in all these cases (including the inefficiency found in Region 1) the underlying problem is that the three firms cannot reach a binding agreement. This may have important policy implications. Suppose merger control is imperfect (makes type one and type two mistakes) and social and private incentives to merge are not always aligned. Consider two scenarios. In the first one, the three firms cannot get together and merger proposals arise as equilibrium outcomes of our game. In the second one, a merger is submitted only if it maximizes industry profits (firms 1 and 2 are able to make a payment to firm 3). Then, there is an interesting trade-off. If binding agreements among the three firms are allowed then, on the one hand, no merger proposal which is worse than another one is submitted; but, on the other hand, some socially inefficient but profitable mergers are immediately submitted (delay is eliminated) and, moreover, efficient but unprofitable mergers will no longer be submitted.

7 References

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8 Appendix

8.1 Proof of Proposition 1

As a preliminary stage we derive the following properties of any SPE.

8.1.1 Property 1: At least in one negotiation there is a strictly positive surplus; i.e., there exist a pair (i, j) such that $\pi_{ij} > \delta u_i + \delta u_j$.

Suppose not; i.e., for all (i, j) $\pi_{ij} \leq \delta u_i + \delta u_j$. Then, $u_i = p_{jk}\pi_i^D + (1 - p_{jk})\,\delta u_1$. Hence, $u_i \leq \max\left\{0, \pi_i^D\right\}$. Similarly, $u_j \leq \max\left\{0, \pi_j^D\right\}$. Therefore, $\delta u_i + \delta u_j \leq \max\left\{0, \pi_i^D, \pi_j^D, \pi_i^D + \pi_j^D\right\} < \pi_{ij}^D$. We have reached a contradiction.$\square$

8.1.2 Property 2: It cannot be the case that in two and only two negotiations there is a strictly positive surplus.

Suppose that in two and only two negotiations there is a strictly positive surplus, i.e.,

$$
\begin{aligned}
\delta u_i + \delta u_j &\geq \pi_{ij}^D \\
\delta u_i + \delta u_k &< \pi_{ik}^D \\
\delta u_j + \delta u_k &< \pi_{jk}^D
\end{aligned}
$$

These inequalities imply that:

$$
u_k < \frac{\pi_{ik}^D + \pi_{jk}^D - \pi_{ij}^D}{2\delta} \tag{14}
$$

Since $u_i^{ik} > \delta u_i = u_i^{ij}$ then $\lambda_i^k = 1$. Similarly, $\lambda_j^k = 1$. As a result, $p_{ij} = 0$

and $p_{ik}+p_{jk}=1$. Hence, we can write:

$$\begin{aligned}
u_i &= p_{ik}\frac{1}{2}\left(\pi_{ik}^D+\delta u_i-\delta u_k\right)+\left(1-p_{ik}\right)\pi_i^D \\
u_j &= p_{ik}\pi_j^D+\left(1-p_{ik}\right)\frac{1}{2}\left(\pi_{jk}^D+\delta u_j-\delta u_k\right) \\
u_k &= p_{ik}\frac{1}{2}\left(\pi_{ik}^D+\delta u_k-\delta u_i\right)+\left(1-p_{ik}\right)\frac{1}{2}\left(\pi_{jk}^D+\delta u_k-\delta u_j\right)
\end{aligned}$$

If we solve the system for u_k then it turns out that for any $p_{ik}\in[0,1]$, the solution violates inequality (14). We have reached a contradiction.□

8.1.3 Property 3: If player i strictly prefers to meet player j and viceversa, then $i=1$ and $j=2$.

Consider first the case where there is only one negotiation with a strictly positive surplus. Then it has to be the negotiation between players i and j. That is, we have that $u_i^{ij}>u_i^{ik}\geq\delta u_i$, and $u_j^{ij}>u_j^{jk}\geq\delta u_j$. Therefore, $\pi_{ij}^D=u_i^{ij}+u_j^{ij}>\delta u_i+\delta u_j$. Suppose that either palyer i or player j is player 3 (the reader should remember that players 1 and 2 are symmetric) i.e.,

$$\begin{aligned}
\delta u_1+\delta u_2 &\geq \pi^{**}+\Delta \\
\delta u_1+\delta u_3 &< \pi^{**} \\
\delta u_2+\delta u_3 &\geq \pi^{**}
\end{aligned}$$

From these inequalities we obtain that $\delta u_2>\frac{1}{2}\left(\pi^{**}+\Delta\right)$. But since player 2 is not able to reach an agreement, we have that $u_2=p_{13}\pi^*+\left(1-p_{13}\right)\delta u_2$, which implies that $u_2=\frac{p_{13}\pi^*}{1-\delta+\delta p_{13}}\leq\max\{0,\pi^*\}<\frac{1}{2\delta}\left(\pi^{**}+\Delta\right)$. We have reached a contradiction.

Alternatively, if all three negotiation involve a strictly positive surplus, then suppose that:

$$\begin{aligned}
u_1^{13} &> u_1^{12} \\
u_3^{13} &> u_3^{23}
\end{aligned}$$

These inequalities imply that:

$$\begin{aligned} u_2 - u_3 &> \frac{\Delta}{\delta} \\ u_1 &< u_2 \end{aligned}$$

In case $u_2^{12} > u_2^{23}$ then $\lambda_2^1 = \lambda_1^2 = 1$ and hence $p_{23} = 0$ and $p_{12} + p_{13} = 1$ (since all negotiations end up in agreement). Then, we can write:

$$u_1 = p_{12}u_1^{12} + p_{13}u_1^{13} \geq u_1^{12} \tag{15}$$

The last inequality holds because $u_1^{12} > u_1^{13}$. Similarly,

$$u_2 = p_{12}u_2^{12} + p_{13}\pi^* \leq u_2^{12} \tag{16}$$

The last inequality holds because $u_1 < u_2$ implies that $u_2^{12} > \frac{\pi^{**}+\Delta}{2} > \pi^*$. But (15) and (16) together contradict that $u_1 < u_2$.

In case $u_2^{12} < u_2^{23}$ then $p_{12} = 0$ and $p_{13} + p_{23} = 1$. Then, we can write:

$$u_2 = p_{13}\pi^* + p_{23}u_2^{23} \leq u_3^{23} \tag{17}$$

The last inequality holds because $u_2 > u_3$ implies that $u_2^{23} > \frac{\pi^{**}}{2} > \pi^*$. Similarly,

$$u_3 = p_{13}u_3^{13} + p_{23}u_3^{23} \geq u_3^{23} \tag{18}$$

The last inequality holds because $u_2^{12} < u_2^{23}$. But (17) and (18) together contradict that $u_3 < u_2$.

Finally, in case $u_2^{12} = u_2^{23}$ and if $p_{12} = 0$, then we are back to the previous case; but $p_{12} > 0$ if and only if either $u_1^{13} \leq \lambda_2^1 u_1^{12} + \left(1 - \lambda_2^1\right)\pi^*$ (that is, player 1 sets $\mu_1^2 > 0$), which implies that $\pi^* > u_1^{13}$, or $u_3^{13} \leq \lambda_2^1\left(\pi^* - \gamma\Delta\right) + \left(1 - \lambda_2^1\right)u_3^{23}$ (that is player 3 sets $\mu_3^2 > 0$), which implies that $\pi^* - \gamma\Delta > u_3^{13}$. Suppose that $\pi^* > u_1^{13}$ then:

$$\begin{aligned} u_1 &= p_{12}u_1^{12} + p_{13}u_1^{13} + p_{23}\pi^* \geq u_1^{12} \\ u_2 &= \left(p_{12} + p_{23}\right)u_2^{12} + p_{13}\pi^* \leq u_2^{12} \end{aligned}$$

and hence we reach a contradition. If $\pi^* - \gamma\Delta > u_3^{13}$ then:

$$\begin{aligned} u_2 &= (p_{12} + p_{23})\, u_2^{23} + p_{13}\pi^* \leq u_2^{23} \\ u_3 &= p_{12}\,(\pi^* - \gamma\Delta) + p_{13}u_3^{13} + p_{23}u_3^{23} \geq u_3^{23} \end{aligned}$$

and again we reach a contradiction.□

8.1.4 Property 4: Preference cycles cannot occur: If i weakly prefers to meet with j, j weakly prefers to meet with k, and k weakly prefers to meet with i, then they all must be indifferent.

Suppose not. If all three negotiations end up in agreement, $\delta u_i + \delta u_j \leq \pi_{ij}$, for all i, j, then

$$\pi_{ij} - \delta u_j \geq \pi_{ik} - \delta u_k \tag{19}$$

$$\pi_{jk} - \delta u_k \geq \pi_{ij} - \delta u_i \tag{20}$$

$$\pi_{ik} - \delta u_i \geq \pi_{jk} - \delta u_j \tag{21}$$

If we add up these three inequalities then this can only be satisfied if the three hold with equality.

Suppose instead that only players i and j are willing to reach an agreement. In this case, the system becomes:

$$\pi_{ij} - \delta u_j \geq \delta u_i \tag{22}$$

$$\delta u_j \geq \pi_{ij} - \delta u_i \tag{23}$$

$$\delta u_k \geq \delta u_k \tag{24}$$

If we add up equations (22) to (24) then again this can only be satisfied if the three hold with equality.□

8.1.5 Property 5: If player 1 strictly prefers to meet player 2, and viceversa, then $p_{12} = 1$.

If $u_1^{12} > u_1^{13}$ then $\lambda_1^2 = 1$. Similarly, if $u_2^{12} > u_2^{23}$ then $\lambda_2^1 = 1$. Since $u_1^{13} \geq \delta u_1$ and $u_2^{23} \geq \delta u_2$, then $\pi^{**} + \Delta = u_1^{12} + u_2^{12} > \delta u_1 + \delta u_2$. That is, $\rho_1^2 = \rho_2^1 = 1$ and $p_{12} = \frac{1}{3}\left(\mu_1^2 + \mu_2^1 + 1\right) > 0$.

Suppose that $u_1 = u_2$. In this case, $u_1^{12} = u_2^{12} = \frac{\pi^{**}+\Delta}{2} > \pi^*$. Thus, if player 1 sets $\mu_1^2 = 1$ then se obtains u_1^{12}, which is higher than anything she might obtain in case $\mu_1^2 = 0$; namely, $u_1^{13}, \delta u_1, \pi^*$ (or a combination of these three elements). Hence, player 1 finds it optimal to set $\mu_1^2 = 1$. Similarly, $\mu_2^1 = 1$, and hence $p_{12} = 1$.

Suppose now (without loss of generality) that $u_1 > u_2$. In this case, $u_1^{12} > \frac{\pi^{**}+\Delta}{2} > \pi^*$. Hence, player 1 clearly finds it optimal to set $\mu_1^2 = 1$. The argument for player 2 is a bit more complicated. Suppose that the negotiation between 1 and 2 is the only one that generates a strictly positive surplus. In this case, if player 2 sets $\mu_2^1 = 1$ then she obtains u_1^{12}, and if she sets $\mu_2^1 = 0$, then she obtains δu_2. But note that $\delta u_2 = u_2^{23} < u_2^{12}$, and hence she chooses $\mu_2^1 = 1$. Because of Property 2 the only alternative is that all three negotiations generate a strictly positive surplus. In this case $u_3^{23} = \frac{1}{2}(\pi^{**} + \delta u_3 - \delta u_2)$ $> \frac{1}{2}(\pi^{**} + \delta u_3 - \delta u_1) = u_3^{13}$, and hence $\lambda_2^2 = 1$. Hence, if player 2 sets $\mu_2^1 = 0$ then she obtains u_2^{23}. Hence, in this case player 2 also finds it optimal to set $\mu_2^1 = 1$. Therefore, $p_{12} = 1$.$\square$

8.1.6 Property 6: Players 1 and 2 obtain the same expected payoff

Without loss of generality suppose $u_1 > u_2$. First, suppose that the negotiation between 1 and 2 is the only one that generates a strictly positive surplus. In this case, $u_1^{12} > \delta u_1 = u_1^{13}$ and $u_2^{12} > \delta u_2 = u_2^{23}$, and from Property 5, $p_{12} = 1$, which implies that $u_1 = u_2 = \frac{1}{2}(\pi^{**} + \Delta)$. Contradiction.

Suppose now that all three negotiations generate a strictly positive surplus.

In this case player 3 strictly prefers to meet player 2 rather than player 1: $u_3^{23} > u_3^{13}$. Then from Property 4 there are two possibilities; (a) $u_2^{23} > u_2^{12}$, and (b) $u_1^{12} > u_1^{13}$ and $u_2^{23} \leq u_2^{12}$. Case (a) is ruled out by Property 3. In case (b) Note that $u_1^{12} > u_1^{13}$ implies that $\lambda_1^2 = 1$, and $u_3^{23} > u_3^{13}$ implies that $\lambda_3^2 = 1$. Hence, $p_{13} = 0$ and $p_{12} + p_{23} = 1$. Thus,

$$u_1 = p_{12}u_1^{12} + p_{23}\pi^* \leq u_1^{12} \tag{25}$$

$$u_2 = p_{12}u_2^{12} + p_{23}u_3^{23} \tag{26}$$

If $u_2^{23} < u_2^{12}$ then from Property 5, $p_{12} = 1$, and equations (25) and (26) imply that $u_1 = u_2$. If $u_2^{23} = u_2^{12}$ then $u_2 = u_2^{12}$, which together with inequality (25) contradicts that $u_1 > u_2$.□

8.1.7 Property 7: There are two possible types of equilibria: (I) $u_1^{12} > u_1^{13}$ and $u_2^{12} > u_2^{23}$, (II) $u_i^{ij} = u_i^{ik}$ for all i, j, k.

Since $u_1 = u_2$ (Property 6) then $u_3^{23} = u_3^{13}$. Thus, both $u_2^{12} \leq u_2^{23}$, $u_1^{12} \geq u_1^{13}$, and $u_2^{12} \geq u_2^{23}$, $u_1^{12} \leq u_1^{13}$ would violate Property 4, unless both inequalities hold with equality. Thus, besides the case where all players are indifferent, there are two other cases to consider: (a) $u_2^{12} < u_2^{23}$, $u_1^{12} < u_1^{13}$ and (b) $u_2^{12} > u_2^{23}$, $u_1^{12} > u_1^{13}$. Case (a) cannot be part of an equilibrium, since in this case $\lambda_1^2 = \lambda_2^1 = 0$ and hence $p_{12} = 0$. Moreover, $\pi_{13} > \delta u_1 + \delta u_3$, and $\pi_{23} > \delta u_2 + \delta u_3$. Therefore, $p_{13} + p_{23} = 1$. In this case:

$$\begin{aligned}
u_1 &= p_{13}\frac{1}{2}(\pi^{**} - \delta u_3 + \delta u_1) + p_{23}\pi^* \\
u_2 &= p_{13}\pi^* + p_{23}\frac{1}{2}(\pi^{**} - \delta u_3 + \delta u_2) \\
u_3 &= \frac{1}{2}(\pi^{**} - \delta u_1 + \delta u_3)
\end{aligned}$$

Since $u_1 = u_2$ then $p_{13} = p_{23.}$ If we solve the above system we get that $u_3 = \frac{(2-\delta)\pi^{**} - \delta\pi^*}{4-3\delta} > \frac{\pi^{**}}{2}$. As a result $u_1^{13} < \frac{\pi^{**}}{2} < \frac{\pi^{**}+\Delta}{2} = u_1^{12}$ We have reached a contradiction.□

We can now proceede to characterize the two types of equilibria.

8.1.8 Equilbrium type I

Consider and equilibrium with $u_1^{12} > u_1^{13}$ and $u_2^{12} > u_2^{23}$. From Property 4, $p_{12} = 1$. Hence:

$$u_1 = u_2 = \frac{1}{2}\left(\pi^{**} + \Delta\right)$$
$$u_3 = \pi^* - \gamma\Delta$$

Thus, a profitable deviation for either player 1 or player 2 exists whenever $\frac{1}{2}\left(\pi^{**} + \Delta\right) < \frac{1}{2}\left[\pi^{**} - \delta\frac{1}{2}\left(\pi^{**} + \Delta\right) + \delta\left(\pi^* - \gamma\Delta\right)\right]$ .Therefore, $p_{12} = 1$ is an equilibrium if and only if:

$$\Delta \geq \frac{\delta\Psi}{2 - \delta - 2\delta\gamma} \equiv \overline{\Delta}\left(\delta\right) < \Psi$$

8.1.9 Equilibrium type II

Consider and equilibrium with $u_i^{ij} = u_i^{ik}$ for all i, j, k. In this case:

$$\pi^{**} + \Delta - \delta u_3 = \pi^{**} - \delta u_1 \tag{27}$$

and

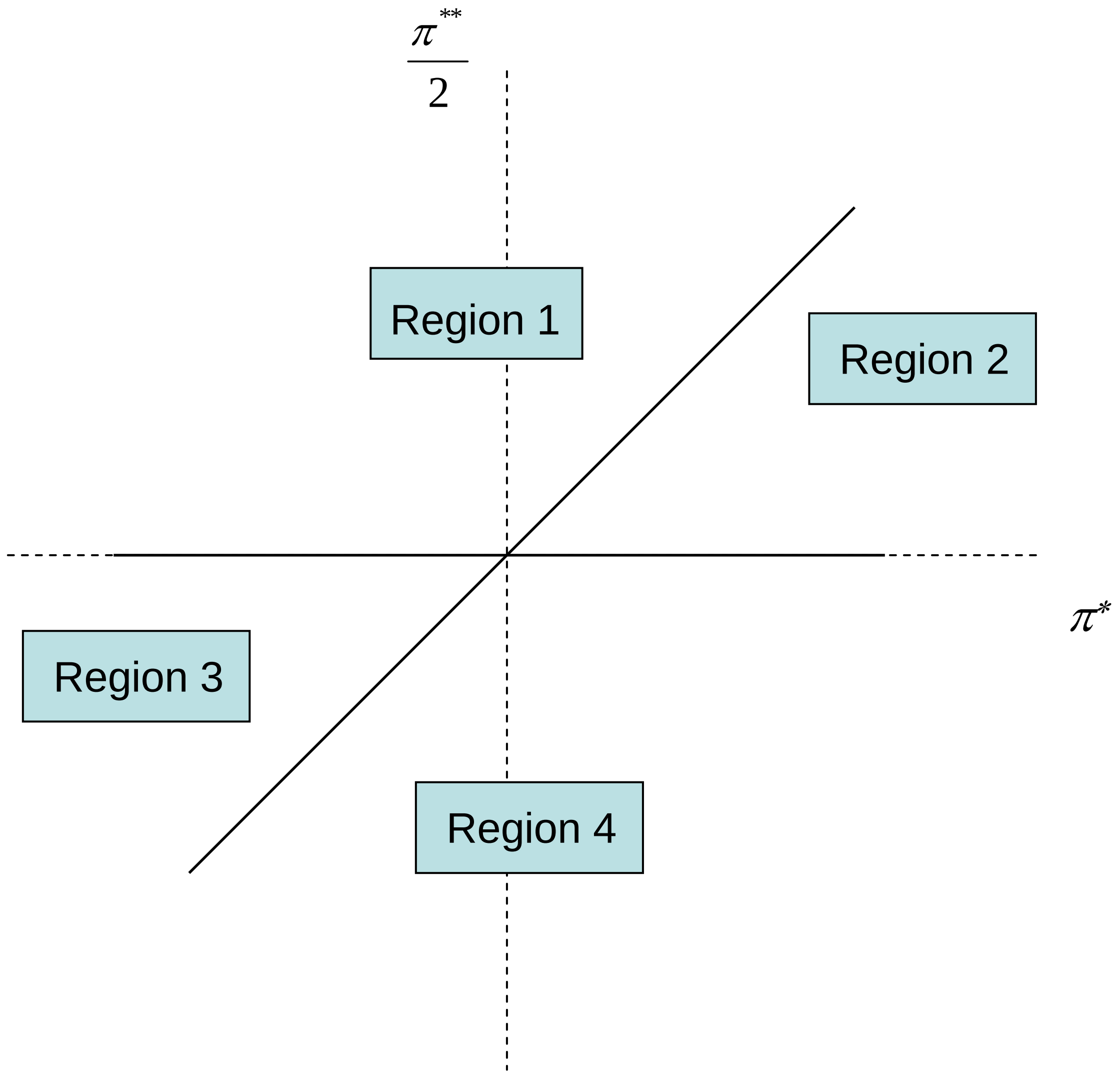
$$\begin{aligned} u_1 &= \left(p_{12} + p_{13}\right)\frac{1}{2}\left(\pi^{**} + \Delta\right) + p_{23}\pi^* \\ u_2 &= \left(p_{12} + p_{23}\right)\frac{1}{2}\left(\pi^{**} + \Delta\right) + p_{13}\pi^* \\ u_3 &= p_{12}\left(\pi^* - \gamma\Delta\right) + \left(p_{13} + p_{23}\right)\frac{1}{2}\left(\pi^{**} - \Delta\right) \end{aligned}$$

Since $u_1 = u_2$ then $p_{13} = p_{23} \equiv q$. Also note that if $\Delta \leq \frac{\Psi}{1-2\gamma}$ then $u_1 \leq \frac{1}{2}\left(\pi^{**} + \Delta\right)$ and $u_3 \leq \frac{1}{2}\left(\pi^{**} - \Delta\right)$. Hence, $\delta u_1 + \delta u_2 < \frac{1}{2}\left(\pi^{**} + \Delta\right)$ and $\delta u_1 + \delta u_3 < \frac{1}{2}\pi^{**}$. Therefore, any of the three negotiation will end up in agreement: $p_{12} = 1 - 2q$. If we plug u_1 and u_3 into equation (**??**) and solve for q :

$$q = \frac{\Psi - \Delta\left(\frac{2}{\delta} - 1 - 2\gamma\right)}{3\Psi - \Delta\left(1 - 2\gamma\right)}$$

Note that $q \geq 0$ if and only if $\Delta \leq \frac{\Psi}{\frac{2}{\delta} - 1 - 2\gamma} \equiv \overline{\Delta}$. Finally, if $\Delta > \frac{\Psi}{1-2\gamma}$ then for δ sufficiently close to 1 then $\delta u_1 + \delta u_3 > \frac{1}{2}\pi^{**}$ and players 1 and 2 cannot be indifferent between negotiating between them or with player 3.

Summarizing, for any δ sufficiently close to 1 the equilibrium exists and is unique.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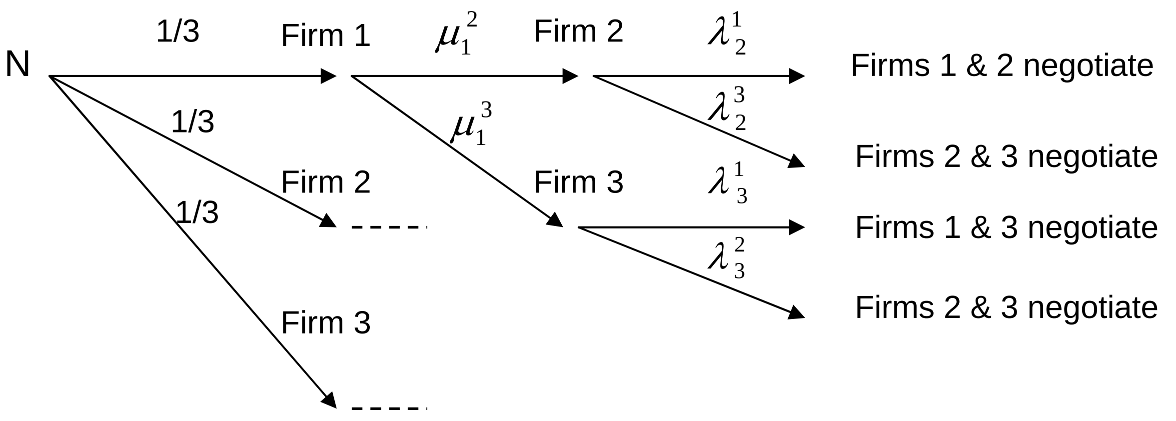
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