Does the market provide sufficient employment protection?*

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Abstract

This paper examines the role of employment protection when firms learn over time about the value of the match. Positive severance payments are useful instruments to achieve efficient layoff decisions and smooth labor income profiles, if firms and workers could commit to future wages. However, when future bargaining over wages cannot be prevented, layoffs are too frequent as a consequence of the asymmetry of learning between firms and their employees. Mandatory severance payments are not a remedy for this inefficiency. Instead, a Pigouvian tax/subsidy scheme will correct the inefficiency by enhancing employment protection.

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1 Introduction

The desirability of existing employment protection legislation (EPL) is still subject to debate among economists. Some blame the poor performance
(at least relative to the US) of most European labor markets on EPL and other labor market institutions. In particular, the low level of labor flows observed in many European economies is often interpreted as the outcome of EPL interfering with the efficient functioning of markets.\footnote{See, for instance, Hopenhayn and Rogerson (1993) and OECD (1994).} However, others have a much more favorable view of EPL. In particular, recent literature has suggested that the presence of various types of market frictions lead to excessive layoffs, which provides an efficiency rationale for employment protection policies. Several types of frictions have been considered, including the absence of private insurance against idiosyncratic labor income risk (Pissarides, 2001, and Bertola, 2004)\footnote{These papers are reminiscent of the old literature on implicit contracts. See Rosen (1985) for a survey. In contrast, Alvarez and Veracierto (2001) by calibrating a general equilibrium model with costly search and risk-averse workers conclude that most of the efficiency gains from severance payments are associated to the reduction in search costs rather than to the smoothing of consumption flows.} and firms’ idiosyncratic shocks in efficiency wage models (Saint Paul, 1995, and Fella, 2000)\footnote{A key assumption is that disciplinary dismissals and those caused by shocks can be perfectly distinguished. In the real world disciplinary dismissals are declared unfair quite frequently. Thus, severance payments may distort incentives, raise wages, and reduce employment (Galdón-Sánchez and Güell, 2003)}. In our reading of this second strand of the literature, we find a failure to offer a rationale for public employment protection. Indeed, in the models analyzed in all these articles, firms and workers would find it in their interest to include severance payments in their private contracts. Thus, public intervention would be redundant, in the sense that market transactions would incorporate the right amount of employment protection, if agents could use severance clauses. In most of these papers, this possibility is simply ruled out.\footnote{For instance, Pissarides (2001), page 156, claims that inclusion of severance payments in private contracts would not be enforceable.} Then, mandatory severance payments are motivated not only by the presence of market frictions but also (and crucially) by the existence of a “contracting failure”: for some unspecified reason, the set of feasible contracts is severely restricted.

We find this approach unsatisfactory. It is by no means rare to find in the real world employment protection clauses in private contracts. For instance, workers not covered by common labor market regulations (such as managers or professional sports people) are often protected against firm-initiated separations by contracts that include large severance payments.
There is substantial empirical evidence suggesting a negative relationship between the hazard rate of employment separations and job tenure. Two learning processes can explain such a phenomenon: Learning-by-doing and learning about match quality, since in both cases the average productivity of workers increases with tenure. Nagypál (2000) uses a French dataset to distinguish between these two possible explanations and finds that learning about match quality clearly dominates.

The asymmetry of information on the realization of the match value is crucial, since it causes inefficient separations. Nagypál (2002) studies a model where both the firm and the worker learn over time about the quality of the match and bargain efficiently about wages (efficient separations). A tax on layoffs reduces productivity and welfare, since it distorts separation decisions.

Burguet, Caminal, and Matutes (2002) studied the optimal contracting arrangements...
other forms in a model where workers learn about workers' quality. The aim of that paper was to analyze the effect of the information structure on the optimal combination of different types of switching costs (layoff costs versus penalties on quits). In contrast, the goal of the current paper is to understand the effect of the contract length on the efficiency of layoffs and the role of public intervention. Although the two models differ substantially, they share the insight that, under some circumstances, a positive severance payment may be needed to guarantee ex-post efficiency.

Our model is highly stylized but provides some basic insights that we consider robust, in particular with respect to public employment protection, our second question of interest. If privately agreed severance payments are enforceable then it is hard to make a case in favor of mandatory severance payments. In the absence of non-pecuniary externalities, exogenously setting a floor to the level of transfers between firms and workers cannot help. Such a policy simply imposes a restriction on the set of feasible contracts. This does not imply that there is no room for other forms of public intervention. Indeed, what is needed is the direct intervention of a third party that receives or makes payments, and not only a regulation of private relations. In particular, we show that a tax-subsidy scheme can restore full efficiency. A tax on layoffs reduces firms' dismissal incentives without raising workers' outside options.

We undertake the analysis in an overlapping generations model with infinitely lived, competitive firms that use labor in a decreasing returns to scale technology. Workers are untested when young, but their match-specific productivity is known by their employers by the time they get old. Workers can borrow and save, but capital markets are imperfect. Workers prefer smooth consumption patterns, although we do not need to assume risk aversion.

The remainder of the paper is organized as follows. The next section presents the model. As a benchmark, in Section 3 we analyze in the solution to this model if parties could commit to future wages. Firms and workers would then be able to sign contracts that would guarantee an efficient
equilibrium. Section 4 contains the main results of the paper. There we analyze the model with all the assumptions, including the crucial one: parties cannot commit to future wages. Equilibrium is characterized by excessive layoffs, but efficiency can be restored by a tax/subsidy scheme and not by mandated severance payments. Finally, Section 5 contains some concluding remarks. A number of formal proofs and extensions are contained in the Appendix.

2 The set up

The purpose of this paper is to study private and public employment protection in a parsimonious model that emphasizes firms’ learning about match quality. Thus, the model abstracts from all other interesting aspects of the labor market, including mobility costs, market power, demand shocks, and labor market institutions other than EPL. We consider a partial equilibrium model with infinitely-lived firms and overlapping generations of workers who live for two periods. The size (mass) of generations is constant over time, and denoted by $N$, and the mass of firms is assumed to be 1.

Firms in each period produce output using labor as the only input, according to the production function $Y_t = f(L_t)$, where $f$ is a twice differentiable function, with $f' > 0, f'' < 0$, and $L_t$ is the total mass of labor employed, measured in efficiency units, in period $t$. Firms are small with respect to both the labor and the output market (whose price is normalized to one), although each firm hires a large number of workers, so that the realized efficiency units of employed workers (see below how uncertainty is introduced) coincide ex-post with the expected number of efficiency units with probability one. Firms maximize the expected present value of profits, and their discount factor is $\gamma$.

Worker $i$ (of any one generation) is able to supply $q_{ij}$ efficiency units of labor to firm $j$. All $q_{ij}$ are realizations of independent and identically distributed random variables over the interval $[\underline{q}, \overline{q}]$ according to the density function $h(q_{ij})$ with c.d.f. $H(q_{ij})$. The value of the match, $q_{ij}$, is constant over time. The independence assumption allows us to dispose of the subscripts $ij$ when we analyze decisions concerning a single firm-worker pair. For economy of notation, we assume $E(q) = 1$. Let $\Psi(q)$ be the inverse of the hazard rate, i.e., $\Psi(q) = \frac{1 - H(q)}{h(q)}$. We assume that $\Psi'(q) < 0$, which is a standard assumption in the literature on optimal contracting under asym-
metric information and implies that $h'$ is not excessively negative. The realization of $q$ is only observed by the incumbent firm after employing the worker for one period. Ex-post efficiency requires that a worker that has worked for a firm when young is retained if and only if $q \geq 1$. Indeed, old workers stay in the market for only one more period, and their expected supply of labor, measured in efficiency units, in any firm other than their former employers equals 1.

The quality of the match is only observed by the firm after one period of employment. Thus, the firm can experiment with young workers and dismiss those who do not perform satisfactorily.

Since below we focus on stationary equilibria, then the only reference to the time dimension that matters is the worker’s age. Thus, a subscript 1 will refer to young workers (first period of their life) and a subscript 2 will refer to old workers (second period of their life).

Workers have identical preferences. The utility of a representative worker is given by:

$$U = u (c_1) + \gamma u (c_2)$$

with $u' > 0$, $u'' < 0$, and $c_t$ denotes the expected value of consumption in period $t$. Thus, the concave utility function $u()$ captures the consumption smoothing motive, which will play some role in the model in combination with capital market imperfections. However, this smoothing motive does not imply here that workers are risk averse. In other words, workers wish to smooth their consumption across periods but not necessarily across states of nature.$^8$ Implicit in this utility function is the assumption that workers’ labor supply is inelastic. Finally, we assume that workers hold zero financial wealth at the beginning of their lifetimes.

Capital markets are not perfect. Let us denote the discount factor associated with workers’ lending and borrowing as $\beta$ and $\bar{\beta}$, respectively. We assume that $\beta < \gamma < \bar{\beta}$. In other words, workers face a lower lending rate and a higher borrowing rate than firms.$^9$

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$^8$Hence, our utility specification is in the spirit of Epstein and Zin (1989), in the sense of disentangling intertemporal substitution and risk aversion.

$^9$Very often intertemporal models of the labor market assume that workers are excluded from financial markets ($\beta = 0, \bar{\beta} = \infty$). See Bertola (2004) for a recent example. In our case we simply require a small interest rate differential in order to break the multiplicity of optimal contracts, as discussed below.
Finally, we assume that both output and labor markets are perfectly competitive and assume away any costs of reallocating workers.

3 The benchmark case: full commitment to future wages

As mentioned in the Introduction, we are interested in understanding the consequences of asymmetric learning on the outcomes of labor markets and on public employment protection when workers can try to renegotiate their wages throughout their employment relationship with firms. However, it will be useful to consider first the benchmark case where parties have no restriction on committing to the entire sequence of wages. Thus, the only restriction on contracting that we assume for the moment is the consequence of asymmetric learning: payments cannot depend (explicitly) on performance.

3.1 Market efficiency and severance payments

In each period a typical firm retains some of the young workers hired in the previous period and dismisses some others. It also hires displaced old workers and young workers. The labor inputs of all these workers are perfect substitutes, from the firm’s point of view. Nevertheless, firms take into account that they can experiment with young workers: learn their types and reallocate them later if necessary.

Old workers that change employer are hired at a wage, $w^*$ for their only remaining period of activity. In contrast, in this section contracts with young workers may stipulate a sequence of wages $\{w_1, w_2\}$. We also allow the contract to include a severance pay, $s$, to be paid in case of a layoff.\textsuperscript{10} Workers are allowed to quit and quitting penalties are ruled out for the usual reasons. An old worker can obtain a wage $w^*$ in the market, and therefore the no quitting restriction on contracts is $w_2 \geq w^*$, which in equilibrium is not binding.

\textsuperscript{10}As we have already mentioned wages and payments in general cannot explicitly be a function of $q$. The information structure also implies that the separation decision must be taken by the firm. Since payments can only depend on observables, and the only observable between period 1 and 2 is the separation decision, we are considering a sufficient set of instruments to span the range of all possible contracts.
In each period the sequence of events is the following:

a) **Stage 1: Labor market.** Firms take hiring and firing decisions, and workers decide whether to quit. Fired workers receive \( s \), and hired workers are offered different types of contracts depending on age: \( w^* \) to old workers and \( \{w_1, w_2, s\} \) to young workers.

b) **Stage 2: Production.** All workers supply their efficiency units, and firms obtain the output and learn the value of \( q \) of their workers.

c) **Stage 3: Payments and consumption.** Wages are paid according to their contracts, workers visit financial markets and finally they consume.

Note that the timing of the game implies that laid off workers receive the severance payment in their second period. It is easy to check that if \( s \) is collected in the first period, results would be qualitatively identical.\(^{11}\) Moreover, letting workers collect their severance pay in the second period is probably a more natural assumption. In the continuous time version of the model, the contract would specify wages and severance payments as a function of time: \( \{w(t), s(t)\} \). Wages are continuously paid, but the severance payment is only collected at the time of a layoff. Hence, wages can be enjoyed throughout the employment relationship, but the severance payment will be enjoyed only after the relationship is over. Our formulation is analogous.

We focus on stationary and symmetric competitive equilibria. Thus, prices and allocations are constant over time and all firms employ a pool of workers with identical age and productivity composition. Moreover, firms and workers take market prices and utility levels associated to contracts as given. Firms take their hiring and firing decisions in order to maximize the present value of profits, and workers choose their employer as well as their savings/borrowing decisions in order to maximize their expected utility. Finally, markets clear.

We first analyze firms’ dismissal and hiring decisions. In each period, when the labor market opens, firms know the realizations of \( q \) of the young employees they hired in the previous period and decide who stays and who leaves. Workers with a particular level of \( q \) will be laid off if replacing them is cheaper. More specifically, workers with \( q \) will be laid off if:

\[ qw^* + s \leq w_2 \]

\(^{11}\)In fact, in the latter case the equilibrium value of \( s \) would be higher. One of the advantages of the formulation that we adopted is that layoff decisions involve only contemporaneous variables, so that we do not need to play around with discount factors.
The right hand side is the cost of keeping the worker. The left hand side is the cost of replacing that worker with old workers, whose market price per efficiency unit is \( w^* \). Therefore, ex-post a worker is laid off if \( q \leq q^c \), where the cut off \( q^c \) depends on the terms of the worker’s contract and the market wage for old workers:

\[
q^c \equiv \frac{w_2 - s}{w^*}.
\]

The condition \( q \leq q^c \) is not only necessary but also sufficient if the following conditions also hold: (i) firms hire new workers every period, and (ii) firms are indifferent between hiring young and old workers. Since we only consider equilibria where all firms’ labor force have the same age composition, then (i) is clearly satisfied (generational turnover plus reallocation of old workers across firms). Condition (ii) must also hold if markets are to clear. Indeed, if firms strictly prefer to hire workers of a particular age then the more expensive generation will be in excess supply. Therefore, in equilibrium a worker is laid off if and only if \( q \leq q^c \), where \( q^c \) is given by equation 2.

In a stationary equilibrium, the indifference between hiring old and new workers (arbitrage condition), taking 2 into account, is given by:

\[
w^* = \frac{w_1 + \gamma \{ H(q^c) s + [1 - H(q^c)] w_2 \}}{1 + \gamma \int_{q^c} q dH(q)}.
\]

The left hand side is the wage of old workers per efficiency unit. The numerator of the right hand side is the expected present value of payments to a young worker and the denominator is the expected present value of the efficiency units to be supplied by a young worker. That is, the right hand side is the expected payment per unit of efficiency supplied by a young worker.

One fundamental element of our model is the existence and management of experimentation. Firms experiment with young workers and, when the outcome of the experiment is not satisfactory, workers are dismissed and find new jobs. This means that, measured in efficiency units, the supply of labor that is expected from a young worker is larger than 1 per period. Note that since markets are perfectly competitive, workers appropriate the returns of this experimentation. Indeed, according to equation 3 firms pay \( w^* \) for each efficiency unit (in expectation) provided by both old and young workers.
Given $w^*$, an equilibrium contract for young workers $(w_1, w_2, s)$ should maximize the (life-cycle, expected) utility of workers subject to 3 given 2, and given the optimal private choice of savings resulting from the contract and the realization of the random variable. If, alternatively, workers were offered contracts that satisfy 3 but do not maximize their utility, then firms could find a contract that offers workers the same level of utility to young workers and reduce the cost of each efficiency unit below $w^*$. Hence, this cannot be part of an equilibrium. Thus, the objective of an equilibrium contract is to achieve the best combination of (a) an efficient allocation of workers and (b) smooth labor income (smooth worker’s consumption, using the most efficient borrowing and saving instrument). In fact, in this case those two goals can be achieved simultaneously. Indeed, ex-post efficiency implies that $q^c = 1$, which is equivalent to:\[12]

$$w_2 = w^* + s$$

Labor income smoothing implies that:

$$w_1 = H(q^c)(w^* + s) + [1 - H(q^c)]w_2 = w_2$$

If we plug these two conditions into the arbitrage condition 3 then we find that these two goals are achieved if and only if:

$$s = \frac{\gamma}{(1 + \gamma)^2} \left[ E(q|q \geq 1) - 1 \right] w^*.$$  

All this discussion is summarized in the following proposition (the proof can be found in the Appendix):

**Proposition 1** An equilibrium contract for young workers involves full wage smoothing and efficient layoffs, i.e., $w_1 = w_2$, $q^c = 1$. Moreover, laid off workers receive a positive severance pay, $s > 0$.

These results can also be explained in terms of how experimentation rents are transferred to workers. Given $w^*$, competition implies that young workers are offered contracts that remunerate their expected supply of labor.

\[12\] Note that ex-post efficiency implies that the equilibrium contract is renegotiation-proof.
at this rate. Thus, young workers receive the expected value of experimentation rents, which evaluated in second period terms equals:

\[ [1 - H(q^c)] [E(q| q \geq q^c) - 1] w^*. \]

Given the imperfections in capital markets, workers strictly prefer contracts with smooth labor income, which implies that expected income in both periods must be above \( w^* \). In particular, expected second period labor income, \( H(q^c) (w^* + s) + [1 - H(q^c)] w_2 \), must be above \( w^* \). If \( s = 0 \), then this would require \( w_2 > w^* \), which would imply inefficient layoff decisions (equation 2). Therefore, efficient separations require a positive severance payment.

A consequence of efficiency and equation 2 is the following result.

Remark 1: The equilibrium contract provides full insurance (in spite of universal risk neutrality) in the sense that the workers’ second period income is constant across states of nature: \( w^* + s = w_2 \).

In this model, ex-post efficiency and insurance go hand in hand when future wages can be contracted.

Workers’ consumption smoothing is attained in this model through the most efficient channel, which is the contract with the employer. If capital markets were perfect, this would not be the only way to attain the same goal.

Remark 2: Under perfect capital markets \( (\gamma = \beta = \overline{\beta}) \) the contract characterized in Proposition 1 is still optimal given \( w^* \). However, it is not the only one. In particular, there is a contract with zero severance payments and a decreasing wage sequence that is also optimal. If \( \overline{\beta} < \gamma = \overline{\beta} \), there are also multiple solutions but in all of them \( s > 0 \).

See Appendix. Note that the result in Proposition 1 does not hinge on ruling out bonding (increasing wage sequences). Contracts with positive severance payments are strictly better than contracts with zero severance payments if workers dislike decreasing wages. In other words, it is sufficient that workers face a small transaction cost when lending.

If the legal environment was hostile to the presence of severance payments clauses in private contracts, then smooth labor income would be incompatible with efficient layoffs. More specifically, in our framework if severance payments are forbidden then under the equilibrium contract there are excessive layoffs and the wage sequence is increasing:

Remark 3: Under the constraint that \( s = 0 \), the equilibrium contract is characterized by \( w_1 < w_2, q^c > 1 \).
The rest of the equilibrium equations, which are needed to endogenize \( w^* \), are labor demand (profit maximization), and labor market clearing. The demand for labor is the result of static profit maximization and is characterized by the familiar condition:

\[
f' \left( L^d \right) = w^*. \tag{4}
\]

Market clearing in any period is given by:

\[
L^d = N \left\{ 1 + \int_{q^c} q dH(q) + H(q^c) \right\}. \tag{5}
\]

The right hand side, the supply of labor, is the sum of three terms. The first is the amount of labor in efficiency units supplied by young workers (each supplying one unit on average) in the period. A proportion \( 1 - H(q^c) \) of the young workers hired in the previous period are retained by their employers, and each one supplies \( E(q|q \geq q^c) \) units of labor in expectation. Thus, the second term is the amount of labor in efficiency units supplied by retained old workers. The rest of the young workers of the previous period, that is, a proportion \( H(q^c) \) of the old workers of today, are dismissed and then hired by new employers. On average, they supply 1 efficiency unit of labor. That is the third term.

Combining 4 and 5 we have that \( w^* \) is a non monotonic function of \( q^c \), decreasing if \( q^c < 1 \) and increasing otherwise (\( w^* \) reaches a minimum at \( q^c = 1 \)).

Summarizing, under long-run commitment the market delivers full efficiency, both ex-ante (full employment of young workers) and ex-post (the reallocation of workers is also efficient). Moreover, workers enjoy the right temporal consumption profile. As a result, there is no room for efficiency-motivated public intervention.

### 3.2 A few remarks on robustness

The model we have just discussed is a very stylized one. We next comment on how some of the assumptions can be relaxed without major consequences. First, the assumption that workers live for two periods is convenient but by no means essential.

Remark 4: All the main insights of Proposition 1 extend very easily to the case of workers who live for \( T \) periods, \( T \geq 2 \), and there is uncertainty
about the timing of the realization of \( q \). In such a case, severance payments increase with seniority and are always positive.

See Appendix. The value of a worker increases with the number of remaining periods of her active life. Since the value of experimentation is transferred to the worker, the utility of a new employment decreases with age. As a result the efficient cut-off point, \( q^c \), decreases over time. A contract can still achieve both allocation efficiency and wage smoothing, by stipulating a sequence of severance payments that increases over time.

Second, we have assumed that only the incumbent firm learns the quality of the match and that workers’ outside opportunities are deterministic. If we relax either of these features, and we do not expand contracting possibilities (in particular, if we do not allow penalties on quits) then it is not possible to achieve efficient separations. However, the qualitative characteristics of the equilibrium contract remain roughly unchanged, even under perfect capital markets.\(^{13}\) In order to illustrate this point in the Appendix we analyze the case where the outside wage that a particular worker can obtain after separation is \( \lambda u^* \), and \( \lambda \) is random. We show that in this case, the optimal contract involves a positive severance payment.\(^{14}\)

Remark 5: Under perfect capital markets and uncertainty concerning the outside wage that a particular individual can obtain in the second period the optimal contract involves \( s > 0 \).

Finally, we could also consider the case that \( q \)'s are positively correlated across firms, in which case layoffs convey a stigma for the worker. It turns out that in this case it is important whether contracts are or are not observable to outside firms. If they are observable then laid-off workers experience a utility loss (see Burguet, Caminal and Matutes, 2002), but the other features of the optimal contract remain unchanged. If contracts are unobservable then severance payments are higher and separations are excessively infrequent (\( q^c < 1 \)).\(^{15}\)

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\(^{13}\)The case of imperfect capital markets is slightly more complicated analytically, since in that case there is a trade-off between ex-post efficiency and consumption smoothing.

\(^{14}\)Booth and Chatterji (1989) present a model of relation-specific human capital investment where severance payments are part the equilibrium contract. Our remark points out that the key feature of their model is the uncertainty concerning the worker’s outside opportunity cost and not the sharing of firm specific investment.

\(^{15}\)On the other hand, if firms can compete for retained workers then second period wages must be very high in order to discourage quits, which would tend to raise \( q^c \) above one.
4 Wage bargaining

In the previous section we assumed that firms and workers could commit to future wages. Such commitment capacity could perhaps be achieved by writing long-run contracts, although it is not obvious how parties can commit in practice not to renegotiate the terms of the contract. Moreover, explicit contracts with a time horizon of more than two or three years are rarely observed in the real world. Alternatively, firms may attempt to develop commitment devices based essentially on reputation, like wage setting rules or promotion systems, to make their future wage offers credible. Firms’ concerns about their reputation may limit the ability of workers to pursue an improvement in their wages during their careers. However, whatever means firms have at their disposal, the assumption of full commitment to future wages in the benchmark model of the previous section seems very extreme. We turn to the analysis of the model incorporating the assumption that parties cannot ex-ante commit to future wages. In particular, workers cannot be prevented from trying their hand at improving their salaries during their employment relationship.

Formally, contracts for young workers do not include their second period wage with their current employer, \( w_2 \). Thus, at the beginning of their second period (when the labor market opens) workers bargain (under asymmetric information) with their employers. In particular, this bargaining is about how to share the possible difference between the value of the worker for the incumbent firm and the worker’s outside opportunity cost \((qw^* - w^*)\).\(^{16}\) Contracts can still include severance payments, \( s \). In fact, there is no room to renegotiate the transfer in case of separation, independently of the information available about the value of the match.\(^{17}\)

Different bargaining procedures result in different outcomes. Next we analyze a simple, extreme case, where the worker (the uninformed party) makes a take-it-or-leave-it offer to the firm (the informed party). This is to

\(^{16}\)Thus, our model has nothing to do with the existence of market power in the labor market, which is typically associated to unions and the structure of collective bargaining. Union-firm bargaining tends to create a gap between the marginal product of labor and the workers’ reservation wage. As a result, severance payments may also help to improve the efficiency of employment decisions. See Booth (1995).

\(^{17}\)In a continuous time version of the model, workers live for \( T \) units of time. Incomplete commitment implies that fully binding contracts have a shorter horizon, \( T' < T \). Thus, a contract would include \([w(t), s(t)]\) for \( t \in [0, T']\). Our formulation is the natural analog in a two-period framework.
keep the analysis in its simplest terms. Later in this section, we will discuss alternative procedures and show that the qualitative results we are about to present, in particular the existence of excessive layoffs, are generic. For the reader familiar with the literature on bargaining, and then with the work of Myerson and Satterthwaite (1983) this should be apparent.

4.1 Market solution with excessive layoffs

Thus, assume that firms and workers can contract on \((w_1, s)\), i.e., the wage for the current period and the severance payment that would be paid to the worker in case of separation.\(^{18}\) At the beginning of the second period the worker makes a take-it-or-leave-it, second-period wage demand, \(w_2\) to her fully informed incumbent firm. If the firm does not accept it, then the worker leaves the job and collects the severance payment, \(s\). Faced with a wage demand \(w_2\) of one of its employees, the firm’s acceptance rule is unchanged: it accepts if and only if \(q \geq q^c\), where \(q^c\) is given by equation 2. The only difference is that \(w_2\) is not part of a contract, but a decision of the worker. Taking this decision rule into account, the worker chooses \(w_2\) in order to maximize the expected second period consumption,

\[
c_2 = H(q^c)(s + w^*) + [1 - H(q^c)]w_2. \tag{6}
\]

The first order condition of this maximization problem characterizes the equilibrium value of \(q^c\), and can be written as

\[
q^c - \Psi(q^c) = 1. \tag{7}
\]

Since \(\Psi(q^c)\), the inverse of the hazard rate, is strictly positive, then the solution of equation 7, \(\overline{q}^c\), is higher than one; i.e., the cut-off point is above the efficient level (excessive layoffs). Also, notice that \(\overline{q}^c\) is independent of \(w^*\) and \(s\). Old workers will attempt to obtain from their employers a wage over and above their reservation value \((w^* + s)\). This reservation value coincides with the employer’s cost of replacing a worker whose \(q\) has turned out to be equal to 1. Thus, the firm strictly prefers to separate from such worker. That is, \(\overline{q}^c > 1\). Thus, from equation 2, the outcome of the bargaining process will be:

\(^{18}\)Under bargaining there is no distinction between quits and layoffs: separations occur when parties do not reach an agreement.
\[ w_2 = s + w^* \bar{q} \] (8)

Let us now turn to the beginning of the relationship, when firms and young workers sign contracts \((w_1, s)\). Since \(\bar{q}^c\) is independent of the terms of the contract, the optimal contract will still imply smooth labor income:

\[ w_1 = H(q^c) (s + w^*) + [1 - H(q^c)] w_2. \] (9)

Thus, taking \(w^*\) as given, in equilibrium parties sign contracts \((w_1, s)\) that lead to bargaining outcomes \((q^c, w_2)\), which are characterized by equations 3, 7, 8, and 9. As in the benchmark case, the model is closed with equations 4 and 5; that is firms’ demand for efficiency units of labor and market clearing. The difference here is that workers are inefficiently allocated when old, and hence the gains from experimentation are lower than in the benchmark model, \(L^d\) is lower and \(w^*\) is higher.

This discussion is summarized in the following proposition.

**Proposition 2** If contracts can only stipulate \((w_1, s)\) then separations occur with an inefficiently high probability, \(q^c > 1\), and as a result employment (in efficiency units) and output are lower than efficient.

Excessive layoffs are a consequence of (asymmetric information) bargaining between firms and their employees. As we mentioned above, we have chosen a simple, yet extreme form of bargaining. It is extreme in the sense that it gives all the ex-post bargaining power to the worker (but all the information advantage to the firm). A first question we need to address is how robust our qualitative results are with respect to the bargaining procedure. In particular, how general is the existence of excessive layoffs (insufficiency of employment protection). We claim that indeed excessive layoffs are a quite robust consequence of wage bargaining. In a very influential paper, Myerson and Satterthwaite (1983) have shown that bargaining under two-way asymmetric information will result always, and independently of the bargaining mechanism, in insufficient trade. We are considering a one-way asymmetric information scenario: only the firm has private information. Yet, it is clear that the same result applies to this model unless the firm has all the bargaining power ex-post. That is, unless workers are never able to obtain a salary above \(w^* + s\), their fall back
option; put in other words, unless workers do not effectively have the possibility of bargaining over salaries with the firm. Indeed, if the firm has all the bargaining power ex-post, it can credibly offer to young workers what we found was the optimal contract under full commitment: \( w_2 = w^* + s \). Employees will not be able to obtain a wage above this level, and the firm would not be able to retain workers by offering a second period wage below \( w^* + s \) even if the contract did not include that \( w_2 \). In summary,

**Remark 6:** If future wages are subject to any meaningful type of negotiation, that is, if employees get a wage \( w_2 \) above \( w^* + s \) with positive probability, then the market outcome will be characterized by excessive layoffs independently of the bargaining procedure.

See Appendix.

With wage bargaining between employers and employees, retained workers obtain a wage above \( w^* \) even in the absence of positive severance payments, and then their second period expected revenue include rents from experimentation. That is, and as opposed to what happened in our benchmark model, severance payments are not the only channel through which workers receive experimentation rents when old. A positive severance payment, on the other hand, affects (amplifies) the wage that employees will demand in the second period. In fact, even with no severance payments these wage demands may be excessive as compared to the experimentation rents, and then equilibrium may call for negative severance payments.\(^{20}\)

The following remark is concerned with the feasibility of negative severance payments.

**Remark 7:** If only non negative severance payments are allowed, then for some distribution functions of \( q \) this restriction is binding and hence in

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\(^{19}\)For instance, this scenario would correspond to a situation where the firm can make a take-it-or-leave-it wage offer to its employees for their second period in the firm.

\(^{20}\)The non contractibility of \( w_2 \) affects the equilibrium value of \( s \) through different channels. On the one hand, for a given \( w^* \), \( s \) is lower than under full commitment. The reason is that \( s \) is both an instrument for transferring experimentation rents to the worker (part of her compensation package) and also part of the fall-back option in the second period bargaining. Experimentation rents are lower than under commitment, which calls for a reduction in the worker’s compensation. Also, consumption smoothing requires a moderate \( w_2 \), which can only be obtained by reducing the worker’s fall-back option (lower \( s \)). On the other hand, less efficient separations induce lower supply of labor in efficiency units, which raises the short term wage, \( w^* \), and increases the total compensation of a young worker (higher \( s \)). Obviously, if the demand for labor is sufficiently elastic the first effect dominates and the level of \( s \) is lower than under commitment.
equilibrium we have that \( s = 0 \).

See the Appendix. In these cases, consumption smoothing cannot be attained by adjusting the pattern of wages, and \( w_1 < w_2 \). Thus, workers would have to rely on borrowing in the less efficient capital market, if they want to flatten their consumption pattern.

Finally, if severance payments are not contractible \( (s = 0) \), then if our previous equilibrium implied positive severance payments, workers will have to resort to saving, \( w_1 > w_2 \), if they still want to flatten their consumption pattern.

Remark 8: If workers borrow or save through the capital market, either because severance payments are non contractible or because the non negativity constraint is binding, separations will still occur too often.

5 Optimal public intervention

Previous subsections have shown that layoffs are excessively frequent (there is too little private employment protection) due to the inability of parties to commit to future terms of trade in the presence of asymmetric learning. This raises the question of whether public intervention can improve market performance. Given the nature of the market failure, welfare enhancing policies require that firms and workers make payments to or receive them from a third party.\(^{21}\) This is where a public authority can make a difference by means of a Pigouvian tax/subsidy scheme.

Consider a combination of a tax on layoffs, \( \tau \), and an employment subsidy, \( \sigma \). We require that the scheme balances the budget. The firm’s firing decision must now consider the extra cost that the tax represents: besides paying the severance payment \( s \), a firm has to pay the tax if negotiations with the employee break down, i.e., if a separation occurs. Then, firms will not accept a wage demand \( w_2 \) from workers with \( q < q^c \), where this threshold is given by:

\[
q^c = \frac{w_2 - (\tau + s)}{w^*} \quad (10)
\]

\(^{21}\)Any measure that does not imply transfers from or to third parties would be covered by the theorem of Myerson and Satterthwaite (1983), and then we should expect excessive layoffs if the worker is to obtain any positive surplus from bargaining with positive probability.
The worker’s ex-post problem is not affected, although she now takes this new layoff decision rule into account. Thus, substituting this rule into the first order condition for the optimal offer, $w_2$, we have:

\[ q^c - \Psi(q^c) = 1 - \frac{\tau}{w^*} \]  

(11)

Note that, since we are assuming that the inverse of the hazard rate, $\Psi(q^c)$, is a monotonically decreasing function, then for given $w^*$, $q^c$ is decreasing in $\tau$. The arbitrage (cost minimization) condition 3 becomes:

\[
w^* = \frac{w_1 - \sigma + \gamma \{ H(q^c)(s + \tau) + [1 - H(q^c)] w_2 \}}{1 + \gamma \int_{q^c} qdH(q)}
\]

(12)

Finally, the balanced budget condition is $\sigma = -H(q^c) \gamma \tau$.

Suppose that $w^*$ takes the value that prevailed in the equilibrium of the benchmark model (Section 4), $w^* = \hat{w}^*$. Then, from equation 11 we can compute the value of $\tau$ that implies that $q^c = 1$, that is $\tau = \hat{w}^* \Psi(1)$. This determines (equation 10) the optimal wage demand: $w_2 = \hat{w}^* + s + \tau$. Anticipating this behavior, parties sign contracts $(w_1, s)$ that aim at labor income smoothing, which implies that $w_1 = w_2 - \frac{\tau}{2}$. Plugging the balanced budget condition, the efficiency condition, the optimal wage demand, and the first period wage that guarantees labor income smoothing, into arbitrage condition 12 gives the equilibrium value of $s$, which is $\frac{\tau}{2}$ units lower than in the benchmark model, $s = \hat{s} - \frac{\tau}{2}$. As a result, expected second period labor income, $\frac{1}{2} (\hat{w}^* + s + w_2) = \hat{w}^* + s + \frac{\tau}{2}$ is the same as first period labor income, and identical to the level obtained in the benchmark model. Finally, since the level of experimentation rents are the same as in the benchmark model, $(w^*, L^d)$ will also be the same. Hence, it is possible to implement an efficient allocation of workers without distorting workers’ consumption profile.

The next proposition summarizes this discussion:

**Proposition 3** If contracts can only stipulate $(w_1, s)$, then there exists a tax/subsidy scheme that implements the first best, i.e., eliminates inefficient layoffs, without distorting workers’ intertemporal allocation of consumption, and hence increasing welfare.

The tax decreases the incentives of firms to break off negotiations without increasing the workers’ threat point. It does so by introducing a gap
between what a firm pays and what a worker receives. This sort of third party intervention is what is needed to guarantee efficiency under asymmetric information. Excessive layoffs call for public employment protection, but this employment protection is not guaranteed by only restricting the contracting possibilities of parties.

The result in Proposition 3 assumes no constraint on the sign of $s$. If negative severance payments are not allowed (and such a constraint is binding), and in equilibrium $w_1 < w_2$, a tax/subsidy scheme can still restore ex-post efficiency, but makes it more costly for workers to smooth their consumption. Thus, 

*Remark 9:* If $s \geq 0$ is binding and workers borrow in equilibrium without intervention, then the proposed tax/subsidy scheme cannot implement the first best, but a sufficiently small tax/subsidy still improves welfare.\(^{22}\)

See Appendix.\(^{23}\) A tax raises $w_2$, since the worker faces a lower probability of being laid off for a given wage demand, and a subsidy raises $w_1$. Consumption smoothing then calls for a lower $s$. If this is not possible then there is a trade-off between productive efficiency and consumption smoothing. If the tax is such that $q^c = 1$ then the expected consumption in the second period is too high and consumption in the first period is too low. However, if workers are smoothing their consumption (imperfectly) by borrowing in the capital market, a small tax/subsidy will have a second order effect on consumption smoothing and a first order effect on efficiency. This explains the unambiguous remark above.\(^ {24}\)

In most European countries the typical instrument for employment protection consists of mandating a minimum level of severance payments. However,

\(^{22}\)There is still a third possibility, which is that $s = 0$, but workers do not borrow in equilibrium. That is, workers would like to borrow *through the firm*, but are not willing to borrow in the more expensive capital market. In this case, even a low tax rate has first order effects both on efficiency and consumption smoothing, and then the net welfare effect of this intervention has an ambiguous sign.

\(^{23}\)If negative values of $s$ are not feasible, and such a constraint is binding, then in the optimal policy the worker must pay some share of the tax on separations. This may look implausible. In order to analyze this issue rigorously we need to explicitly model the reasons behind restricting to non-negative severance payments.

\(^{24}\)Again, more efficiency does not have to translate into a higher wage bill. If workers’ welfare is the goal, this labor market intervention may need a complementary, more standard redistributive public intervention.
Remark 10: In the model in this section mandatory severance payments cannot improve welfare.

In the absence of transaction costs, a binding, minimum level of severance payments cannot affect firms’ layoff decisions and hence can only make things worse, i.e., raise second period wage at the cost of a lower first period wage (distort the workers’ consumption profile). This results in lower utility for the worker, with no effect on the level of employment (measured in efficiency units).\textsuperscript{25}

6 Concluding remarks

When the quality of the matches between workers and their employers is uncertain and is only revealed through experience, labor contracts have to address the problem of how to share the resulting experimentation rents. This is the issue analyzed in this paper, under the assumption that the employer has an advantage in observing the quality of the match, but competes with other firms to attract new workers. In this context, contracts should pursue possibly conflicting goals. One of these goals is to provide the right incentives for workers’ reallocation (dismissals). The second one is to provide an adequate labor income profile, when workers have a preference for consumption smoothing and capital markets are less than perfect. We have shown how these two goals could be made compatible if contracts could stipulate positive severance payments and also the sequence of wages throughout the employment relationship.

When firms and workers cannot commit to future wages, the two goals are no longer compatible. In this case, negotiations under incomplete information will result in excessive dismissals (too little employment protection). Equilibrium contracts may still include positive severance payments, but nevertheless labor relations will break down too often.

This inefficiency is due to a friction, the information asymmetry, which does not cause any non-pecuniary externality. Therefore, a public intervention that limits itself to restricting the set of feasible contracts cannot improve efficiency. Thus, mandatory severance payments, which simply provide a floor for the terms of contracts that employers and employees can

\textsuperscript{25}In fact, if we assumed an elastic supply of labor, this lower utility of workers would translate into a lower level of employment.
sign, do not remedy this inefficiency.

Yet, a more active public policy may do the job. The ex-post allocation inefficiency in which the two parties are locked in as a consequence of bargaining under asymmetric information may be corrected only by the intervention of a third party. In our case, this intervention may take the form of taxes on dismissals, coupled with subsidies for new hires.

Our paper also contributes to the debate on the desirability of experience rated unemployment insurance.\(^{26}\) In the US, a firm’s contribution to the unemployment insurance fund depends on the number of workers who claimed unemployment benefits after being laid off by that firm. Thus, such a scheme works as a tax on layoffs. According to our theory, experience rated unemployment insurance is a more appropriate device for discouraging layoffs than the standard European EPL.\(^{27}\)

### 7 References


Blanchard, O. and J. Tirole (2004), The optimal design of unemployment insurance and employment protection, mimeo.


\(^{26}\)Fuest and Huber (2003) and Fath and Fuest (2002) provide different arguments in favor and against experience rated unemployment insurance and review the existing literature.

\(^{27}\)Very recently, Blanchard and Tirole (2004) have also advocated the use of layoff taxes as the optimal way to finance public unemployment insurance, since they induce firms to take efficient layoff decisions. In our model layoffs are socially excessive, and a layoff tax can improve efficiency, even in the absence of any public unemployment insurance mechanism. Obviously, these two approaches are complementary.


Fuest, C. and B. Huber (2003), Is experience rated unemployment insurance bad for employment in the presence of asymmetric shocks? the role of decentralisation, mimeo University of Cologne.


8 Appendix

8.1 Proof of Proposition 1

Given \((w_1, w_2, s)\) a young worker chooses the optimal level of savings, \(b\), in order to maximize:

\[
U = u(c_1) + \gamma u \{ H(q^c) \zeta_2 + [1 - H(q^c)] \bar{\zeta}_2 \}
\]
where:

\[ c_1 = w_1 - b \]
\[ c_2 = w^* + s + \frac{b}{\beta} \]
\[ \bar{c}_2 = w_2 + \frac{b}{\beta} \]

where \( \beta = \beta \) if \( b < 0 \), and \( \beta = \bar{\beta} \) if \( b > 0 \). From the first order condition, if \( b \neq 0 \) (\( c_2 \) stands for expected second period consumption):

\[ u'(c_1) = \frac{\gamma}{\beta} u'(c_2) \quad (13) \]

The optimal contract consists of choosing \((w_1, w_2, s)\) in order to maximize 1 subject to 3. Suppose then by the envelope theorem and if we denote the worker’s utility under optimal savings by \( U^* \), the first order conditions are (using 13):

\[ \frac{\partial U^*}{\partial w_2} = \gamma u'(c_2) \left\{ [1 - H(q^c)] \left( 1 - \frac{\gamma}{\beta} \right) + \frac{h(q^c)}{w^*} (w^* + s - w_2) \right\} = 0 \quad (14) \]

\[ \frac{\partial U^*}{\partial s} = \gamma u'(c_2) \left\{ H(q^c) \left( 1 - \frac{\gamma}{\beta} \right) - \frac{h(q^c)}{w^*} (w^* + s - w_2) \right\} = 0 \quad (15) \]

Note that \( 1 - \frac{\gamma}{\beta} \neq 0 \). Hence, these two first order conditions cannot hold simultaneously. It must be the case that under the optimal contract \( b = 0 \). In this case the optimal contract consists of choosing \((w_1, w_2, s)\) in order to maximize:

\[ U = u(w_1) + \gamma u \left\{ H(q^c)(w^* + s) + [1 - H(q^c)] w_2 \right\} \]

subject to 3. From the first order condition, we obtain that \( w_1 = w_2 = w^* + s \), which implies that \( q^c = 1 \) (Ex-post efficiency).

Finally, using \( w_1 = w_2 = w^* + s \) in 3, it can be seen immediately that \( s > 0 \).
8.2 Proof of Remark 2

If $\beta_1 = \beta = \gamma$ then the first order conditions 14 and 15 hold and both imply that $w_2 = w^* + s$, but no further restriction is placed on the optimal contract. As a result there is a continuum of solutions. Using 3, it can be confirmed that these solutions include $w_1 = w_2$, $s > 0$, and $w_1 > w_2$, $s = 0$.

8.3 Proof of Remark 4

Let us prove Remark 4 for the case $T = 3$. The same method can be applied for any value of $T$.

Suppose that $q$ is revealed at the end of any period with probability $\mu$, $0 < \mu < 1$. If $q$ is revealed at the end of $t = 2$, then as in the text the ex-post efficient separation rule at the beginning of $t = 3$ is $q_3^e = 1$. Under ex-post efficiency, the expected present value of efficiency units that a worker delivers if she takes a new job at the beginning of $t = 2$ is given by:

$$Q_2 \equiv 1 + \gamma + \gamma \frac{\mu}{2} [E(q|q \geq 1) - 1]$$

Note that $Q_2 > 1 + \gamma$. The optimal separation rule at the beginning of $t = 2$ if $q$ has been revealed is:

$$q_2^e = \frac{Q_2}{1 + \gamma}$$

Hence, $q_2^e > 1$. Under ex-post efficiency, the expected present value of efficiency units that a worker delivers if she takes a new job at the beginning of $t = 1$ is given by:

$$Q_1 \equiv 1 + \gamma Q_2 + \gamma (1 + \gamma) \mu [1 - H(q_2^e)] [E(q|q \geq q_2^e) - q_2^e]$$

Note that $Q_1 > 1 + \gamma + \gamma^2$. As in Proposition 1 equilibrium contracts can achieve both ex-post efficiency and wage smoothing (and full insurance). By $w_n^*$ let us denote the (constant) wage in a contract for $n$ remaining periods, $n = 1, 2, 3$. Thus, a three-period contract specifies $(w_3^*, s_2, s_3)$, where $s_t$ is the severance payment that the worker obtains if laid off at the beginning of period $t, t = 2, 3$. In order to implement ex-post efficiency it must be the case that:
\[ q_3^c = \frac{w_3^* - s_3}{w_1^*} = 1 \]

\[ q_2^c = \frac{w_3^*}{w_1^*} - \frac{s_2}{(1 + \gamma) w_1^*} \]

In equilibrium, firms must be indifferent between hiring workers in the third period of their lives and those in their first period. This arbitrage condition can be written as:

\[ w_1^* = \frac{(1 + \gamma + \gamma^2) w_3^*}{Q_1} \]

Let us first show that \( s_3 > s_2 \). From the cut-off points, we have that:

\[ \frac{s_3 - s_2}{w_1^*} = Q_2 - \frac{\gamma Q_1}{1 + \gamma + \gamma^2} \]

Using the definitions of \( Q_1 \) and \( Q_2 \) and manipulating:

\[ sg \{ s_3 - s_2 \} = sg \{ [E(q|q \geq 1) - 1] - [E(q|q \geq q_2^c) - q_2^c] \} \]

Thus, the sign of \( s_3 - s_2 \) is positive if \( \Omega'(x) < 1 \), where

\[ \Omega(x) \equiv E(q|q \geq x) \]

Let us check the latter condition:

\[ \Omega'(x) = \frac{\Omega(x) - x}{\Psi(x)} > 0 \]

\[ \Omega''(x) = \frac{[\Omega'(x) - 1] \Psi(x) - \Psi'(x) [\Omega(x) - x]}{\Psi(x)^2} > 0 \]

Thus, the function \( \Omega(x) \) is convex because the inverse of the likelihood ratio has been assumed to be negative. As a result, \( \Omega'(x) \) will attain its maximum at \( x = 2 \). Applying l’Hôpital’s rule:
\[ \lim_{x \to 2} \Omega'(x) = \frac{1}{2} \]

Therefore, \( \Omega'(x) < 1 \). Next, let us show that \( s_2 > 0 \). From the equation associated with the implementation of the optimal \( q^*_2 \), we have that:

\[
\frac{s_2}{(1 + \gamma) w^*_1} = \frac{Q_1}{1 + \gamma + \gamma^2} - \frac{Q_2}{1 + \gamma}
\]

Note that \( Q_1 \) is higher than when learning takes place only at the end of the first period in a new job and that the separation rule when learning takes place is \( q^c = 1 \). Hence,

\[
\frac{Q_1}{1 + \gamma + \gamma^2} > 1 + \frac{\gamma (1 + \gamma) \mu}{2(1 + \gamma + \gamma^2)} [E(q|q \geq 1) - 1] >
\]

\[
1 + \frac{\gamma \mu}{2(1 + \gamma)} [E(q|q \geq 1) - 1] = \frac{Q_2}{1 + \gamma}
\]

### 8.4 Proof of Remark 5

Assume that \( \bar{\beta} = \beta = \gamma \) and that each individual worker in the second period will have access to a wage equal to \( \lambda w^* \), where \( \lambda \) is distributed over \([1 - \bar{X}, 1 + \bar{X}]\) according to \( G(\lambda) \), \( E(\lambda) = 1 \). Ex-post efficiency requires that the worker be retained if and only if \( q \geq \lambda \). The firm fires the worker if and only if \( q < q^c \), where:

\[
q^c = \frac{w_2 - \frac{\gamma}{G(\lambda^c)} w^*_1}{w^*}.
\]

The worker quits if and only if \( \lambda > \lambda^c \), where \( \lambda^c = \frac{w_2}{w^*} \).

The arbitrage condition can be written as:

\[
w^* \left[ 1 + \gamma G(\lambda^c) \int_{q^c} q dH(q) \right] = w_1 + \gamma G(\lambda^c) \left[ 1 - H(q^c) \right] w_2 + H(q^c) s
\]

Since capital markets are perfect the worker only cares about the expected present value of her income:
\[
U = w_1 + \gamma G(\lambda^c) [1 - H(q^c)] w_2 + \gamma H(q^c) (s + w^*) + \gamma [1 - H(q^c)] \int_{\lambda^c} \lambda w^* dG(\lambda)
\]

Using the arbitrage condition we can rewrite the worker’s utility:
\[
U = w^* \left[ 1 + \gamma G(\lambda^c) \int_{q^c} q dH(q) \right] + \gamma H(q^c) w^* + \gamma [1 - H(q^c)] \int_{\lambda^c} \lambda w^* dG(\lambda)
\]

The first order conditions fully characterize \((\lambda^c, q^c)\) from where \((w_2, s)\) can be recovered:
\[
\lambda^c = E(q|q \geq q^c)
\]

\[
G(\lambda^c) q^c + \int_{\lambda^c} \lambda dG(\lambda) = 1
\]

Condition 16 is satisfied if and only if \(\lambda^c > q^c\), which implies that \(s > 0\).

### 8.5 Proof of Remark 6

We show that unless the worker obtains \(w_2 = s + w^*\) with probability one, the outcome of bargaining will be characterized by excessive layoffs. First, notice that workers will be dismissed whenever \(q < 1\). Indeed, if this is the case a firm would only retain the worker if \(w_2 < s + w^*\), but the worker would only accept \(w_2 \geq s + w^*\). Also, when the worker is dismissed then the firm will not pay more than \(s\) (individual rationality). We only need to show that if the probability of employment with the firm is 1 when \(q > 1\), then the worker obtains \(w_2 = s + w^*\) with probability 1 as well.

Thus, any bargaining mechanism will result in a probability of trade (employment) and (possibly) some transfers between parties, say from firm to worker, both (possibly) a function of the type \(q\). Let \(t(q)\) represent the expected transfer from firm to worker. Let \(x(q)\) represent the expected probability of employment. In this bargaining process, we can evaluate the value (surplus) of one efficiency unit of labor at its market value, \(w^*\). Finally, let \(U(q)\) represent the expected payoff for the firm for a given realization of \(q\). That is,
\[
U(q) = w^* x(q) q - t(q) + (1 - x(q))(w^* - s - w^*).
\]
Incentive compatibility of the firm requires that for all \( q, q' \)
\[
\begin{align*}
w^* x(q)q - t(q) - (1 - x(q))s & \geq \\
w^* x(q')q - t(q') - (1 - x(q'))s,
\end{align*}
\]
and
\[
\begin{align*}
w^* x(q)q' - t(q) - (1 - x(q))s & \leq \\
w^* x(q')q' - t(q') - (1 - x(q'))s.
\end{align*}
\]
Since efficiency implies that \( x(q) = 1 \) for all \( q \geq 1 \), these conditions translate into
\[
0 \geq t(q) - t(q') \geq 0,
\]
for all \( q, q' \geq 1 \). Therefore, the transfer from firm to worker must be constant for all \( q \geq 1 \). Finally, for \( q = 1 \), a firm should not prefer laying the worker off, and then \( t(q) = t(1) = w^* + s \).

### 8.6 Proof of Remark 7

Equations 2, 3 and 9 determine \((w_1, s, w_2)\) for given values of \((w^*, q^c)\). Manipulating these equations it can be shown that \( s < 0 \) if and only if \( \eta(q^c) < 0 \), where \( \eta(q^c) \) is given by:
\[
\eta(q^c) \equiv 1 + \gamma \int_{q^c} q dH(q) - H(q^c) - (1 + \gamma) \int_{1} H(q^c) q^c
\]
In the case that \( q \) is uniformly distributed, and \( \gamma = 1 \), then according to equation 7, we have that \( q^c = \frac{3}{2} \), and \( \eta = -\frac{3}{8} \).

### 8.7 Proof of Remark 8

Workers make their \( w_2 \) offers to maximize 6, whether \( s \) is positive or not, and whether they borrow or save. On the other hand, firms’ firing decisions still satisfy 2. Thus, 7 still describes the equilibrium firing decisions.

### 8.8 Proof of Remark 9

Assume that the solution to 2, 3, 4, 5, 7, and 9 gives \( s < 0 \), and this is not feasible. Also assume that when setting \( s = 0 \), equations 2, 3, 4, 5, 7 still hold but we have that \( b > 0 \), where \( b \) satisfies:
\[ u'(w_1 - b) = \frac{\gamma}{\beta} \left\{ [1 - H(q^c)] w_2 + H(q^c) w^* + \frac{b}{\beta} \right\} \]

That is, the non negativity condition on \( s \) is binding (workers would prefer negative severance payments) but imperfections in the capital market are not too great, so that workers use the less effective substitute which is borrowing (negative savings).

Note that from (10), \( \frac{d q^c}{d \tau} < 0 \). Also note that in the absence of public intervention \( \frac{d q^c}{d \tau} > 1 \). That is, an increase in \( \tau \) and \( \sigma \) from their zero levels will induce higher efficiency in the use of labor (higher output in equilibrium). Workers are smoothing their consumption (at the rate given by \( \frac{2}{\beta} \)), both before and after the policy change. Thus, total welfare unambiguously increases: there exists a monetary transfer from firms to workers that leaves both workers and firms better off.

As an example of such a transfer, consider the following. Each worker receives a subsidy of \( \alpha \left\{ 1 + \gamma \int_{q^c}^{q^c}(q - q^c)dH(q) \right\} \) when young and a subsidy of \( \alpha \left\{ H(q^c) + [1 - H(q^c)] q^c \right\} \) when old, where

\[ \alpha = -\frac{d w^*}{d q^c} \frac{d q^c}{d \tau} \bigg|_{\tau=0}. \]

That is, they are compensated in their wages for the loss they would have incurred from lower spot market wages, had the cut-off level not changed. With this complementary transfer, we compute the change in utility of the worker due to an increase in \( \tau \) (and, correspondingly, \( \sigma \)) when \( \tau = 0 \). Using the envelope theorem, so that \( \frac{\partial u}{\partial b} = 0^{28} \), this effect is:

\[ \frac{d U}{d \tau} \bigg|_{\tau=0} = \left( \frac{d q^c}{d \tau} \bigg|_{\tau=0} \right) \left[ u'(c_1) \frac{\partial c_1}{\partial q^c} + \gamma u'(c_2) \frac{\partial c_2}{\partial q^c} \right], \]

where \( c_1 = w^* \left( 1 + \gamma \int_{q^c}^{q^c}(q - q^c)dH(q) \right) - b, \) and \( c_2 = w^* \left( H(q^c) + [1 - H(q^c)] q^c \right) + \frac{b}{\beta}. \) Taking derivatives, we obtain

\[ \frac{\partial c_1}{\partial q^c} = -w^* \gamma \left[ 1 - H(q^c) \right] < 0, \]

\[ ^{28} \text{Here we use the fact that the savings/borrowings are non zero.} \]
and
\[
\frac{\partial c_2}{\partial q^c} = -w^* (h(q^c)(q^c - 1) + [1 - H(q^c)]) = 0,
\]

where this last equality follows from 11 at \( \tau = 0 \). Thus, since \( \frac{dq^c}{d\tau} < 0 \), we conclude that this transfer renders the effect on the utility of the worker positive. The transfer may be financed by a lump sum tax on firm’s profits. Indeed, the net wage bill per worker (including the lump sum tax and the hiring/firing tax/subsidies) then increases by

\[
\frac{\partial (c_1 + b)}{\partial q^c} + \gamma \frac{\partial}{\partial q^c} \left( \frac{dq^c}{d\tau} \bigg|_{\tau=0} \right) = - \left( \frac{dq^c}{d\tau} \bigg|_{\tau=0} \right) w^* \gamma [1 - H(q^c)],
\]

which equals the increase in the value of efficiency units per worker obtained:

\[
-w^* \left( \frac{dq^c}{d\tau} \bigg|_{\tau=0} \right) \gamma h(q^c)(q^c - 1),
\]

an equality that again follows from 11. QED