Collective Action and the Group Size Paradox

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According to the Olson paradox, larger groups may be less successful than smaller groups in furthering their interests. We address the issue in a model with three distinctive features: explicit intergroup interaction, collective prizes with a varying mix of public and private characteristics, and nonlinear lobbying costs. The interplay of these features leads to new results. When the cost of lobbying has the elasticity of a quadratic function, or higher, larger groups are more effective no matter how private the prize. With smaller elasticities, a threshold degree of publicness is enough to overturn the Olson argument, and this threshold tends to zero as the elasticity approaches the value for a quadratic function. We also demonstrate that these results are true, irrespective of whether we examine group sizes over the cross-section in some given equilibrium or changes in the size of a given group over different equilibria.

Most political, social, or economic activities are carried out by groups or organizations, not individuals. Presumably, this is because the outcome of pooled efforts usually is larger than the sum of individual efforts. A critical factor, however, is whether the benefits from cooperation are distributed in ways that pay all potential partners to cooperate. Individual rewards depend on the contributions of other group members as well as one's own. Because of the free-rider problem, individuals bear only partially the adverse consequences of reducing their effort. Consequently, collective effort typically falls below the optimal level.

The free-rider or collective action problem is extremely pervasive and includes a wide variety of situations in which cooperation is necessary. Trade unions, lobbies, and the provision of public goods are standard examples. The problem also appears in the case of collusive behavior between organizations, and it can occur even within organizations when the outcome depends on combined efforts by different individuals at the management or production level.

Olson's (1965) celebrated thesis and the earlier insightful work of Pareto (1927) argue that the free-rider problem makes smaller groups more effective. For instance, Olson writes:

> The most important single point about small groups . . . is that they may very well be able to provide themselves with a collective good simply because of the attraction of the collective good to the individual members. In this, smaller groups differ from larger ones. The larger a group is, the farther it will fall short of obtaining an optimal supply of any collective good, and the less likely that it will act to obtain even a minimal amount of such a good. In short, the larger the group, the less it will further its common interests (1965, 36).

Pareto makes a very similar argument in the context of protectionist measures.

> In order to explain how those who champion protection make themselves heard so easily, it is necessary to add a consideration that applies to social movements generally . . . If a certain measure *A* is the case of the loss of one franc to each of a thousand persons, and of a thousand franc gain to one individual, the latter will expend a great deal of energy, whereas the former will resist weakly; and it is likely that, in the end, the person who is attempting to secure the thousand francs via *A* will be successful. A protectionist measure provides large benefits to a small number of people, and causes a very great number of consumers a slight loss. This circumstance makes it easier to put a protection measure into practice (1927, 379).

Individuals always have an incentive to shirk, but the effect is more pervasive when group size is large. There are two reasons for this. First, the larger the group, the smaller is the perceived effect of an individual defection. Second, if the prize has any element of privateness, then the larger the group, the smaller is the individual prize. Hence, larger groups will be less effective in pursuing their targets, which in essence is the group-size paradox.[1]

There is general agreement that, because of the free-rider problem, individuals will tend to contribute lower levels of "action" (money, effort, time, and so on) the larger the group to which they belong. The key question, however, is the aggregate potency of the group, which is what determines effectiveness in the sense of success probabilities. Decreasing personal contributions are not necessarily incompatible with increasing aggregate effectiveness. Among others, Chamberlin (1974), Marwell and Oliver (1993), McGuire (1974), Oliver and Marwell (1988), Sandler (1992), and Taylor (1987) point out that Olson's proposition of an inverse relationship between effective collective action and group size hinges on the assumption that the collective good is purely private, so that it must be divided among group members. They argue

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[1] For an extended discussion of the different factors relevant to the group size paradox, see Olson 1965, chap. 1.

(but do not formally demonstrate) that when the collective good is public—so that rewards to individual group members are fully nonexcludable—Olson's result is reversed: The larger the group, the higher is the level of the collective good it will be able to produce. The view that the Olson thesis holds when the prize is private but may be reversed when the prize is purely public will be termed the "common wisdom" for the rest of this article.

Two points are to be noted and will be reiterated below. First, the words *public* and *private* are relative to the group in question. A public prize means that a group member's payoff is unaffected by the number of members, but nonmembers may be partially or completely excluded. Likewise, a private prize is fully divided up, but only among group members. Second, the common wisdom applies to the extreme cases of "purely private" and "purely public" goods and is silent on the intermediate case.

Olson's thesis—or even the common wisdom version of it—is not necessarily consistent with some informal observations. For instance, there is a sense in which the received theory runs counter to the old maxim: Divide and conquer. Political entities have applied this rule with surprising universality, but if smaller groups are more potent, the division of one's opponent into a number of smaller units would entail more effective opposition. This universality can be reconciled with the traditional argument only if all potential gains are fully public to group members. But that is hardly the case. For example, groups seeking political autonomy may be just as motivated by private economic considerations (e.g., access to natural resources) as by public gains (e.g., feelings of independence). The universality of "divide-and-conquer" coupled with the mixed nature of group rewards demands an explanation.

Consider how individuals organize themselves in order to influence government decisions. We can think (simplistically but usefully, we hope) of the activity of the government as obtaining revenue and then allocating it to a number of activities, following the pressures perceived from the public. To be sure, a number of government activities have the character of public goods. But many are very close to private goods. Think of transfers either in money or in kind made on the basis of various individual characteristics. These include pensions, subsidies, education, and health benefits, for which there may be a fixed budget. Here, Olson's thesis would suggest that we should observe citizens organized in extremely narrow interest groups, to cut down on the scope of such transfers. Thus, lobbies would be organized around a very exhaustive (and possibly artificial) list of "deserving characteristics," in an attempt to ward off the free-rider problem. For instance, we should see the very poorest organizing their own lobby to become the sole beneficiaries of public support, arguing that those immediately above them are not really deserving. In general, there should be extreme splintering of lobby groups, not to mention an artificial narrowing of the issues. We do not seem to observe anything close to this. Whenever an interest group tries to argue in its favor, this is usually (although not always) done by appealing to some general and inclusive characteristic. By and large, the tendency is to see individuals organized in quite broad platforms. This sort of evidence does not fully conform to the common wisdom that when goods are largely private, smaller groups will tend to be more effective.

Or view firms as organizations, to which the free-rider problem should be readily applicable, as Olson (1965, 54–5) himself argues. If the common wisdom is to be accepted, we should observe that larger firms switch from incentive schemes (such as stock ownership for high-level employees) to direct monitoring of employee actions. This is because—applying Olson—in large organizations the link between stock returns and individual performance is tenuous, so that stock-based incentive schemes should have no effect. But we do not observe a progressive transition to direct monitoring among large corporations. If anything (and especially if we neglect the recent dot com experience), we observe the opposite.

The discussion so far is meant to challenge the common wisdom, at least in some cases where it may apply. But one can go farther. In many cases, the common wisdom—with its reliance on the polar distinction of private versus public—fails to apply in the first place. This is because most situations contain public as well as private components. Indeed, one purpose of this article is to throw light on the mixed case.

In this article we develop a formal model of collective action that has three features. (1) Collective action is undertaken in order to counter similar action by competing groups. (2) Marginal individual effort is increasingly costly (as total individual effort rises). (3) The collective prize is permitted to have mixed public-private characteristics, within a range between pure publicness and pure privateness of the good.

The first feature is common to a large and growing literature,[2] but the second gives rise to new results of substantial empirical relevance. Our main point is that, if the marginal cost of effort rises sufficiently quickly with respect to resources contributed, then larger groups will have a higher win probability, even if the prize is purely private. We shall argue that this overturns Olson's hypothesis in circumstances in which marginal costs increase with effort supply.

The phrase "larger groups have a higher win probability" is ambiguous.[3] One interpretation is that it applies over the cross-section of groups, that is, larger groups have a higher win probability evaluated at a single equilibrium. A second interpretation is more in the nature of comparative statics: At some equilibrium,

[2] Chamberlin (1974) emphasizes that most collective goods theory deals with actions of a single group, and the next step should be to consider the behavior of several groups in competition. As Hardin (1995) points out, successful collective action often entails suppression of another group's interest. A sizable literature has emerged on this subject (e.g., Katz, Nitzan, and Rosenberg 1990; Nitzan 1991; Riaz, Shogren, and Johnson 1995; and Tullock 1980) and formalizes the problem of collective action with multiple groups as a contest game.

[3] We are indebted to an anonymous referee for bringing this point to our attention.

if the membership of a certain group exogenously rises, then the winning probability of that group rises. Under the second interpretation it is necessary, at least in the general equilibrium formulation that we adopt, to track the possible changes in equilibrium magnitudes before we deliver an answer. That is, other groups must be permitted to react to this change in group size. Yet, remarkably enough, we find that both interpretations yield the same results. Larger groups have higher win probabilities under precisely the same (sufficient) restrictions on marginal cost. Thus, without taking sides as to which interpretation of the Olson argument should be adopted, we find that the same critique uniformly applies.

In the formal analysis to follow, we make precise the crucial condition that "marginal cost of effort rises sufficiently quickly." Here we limit ourselves to some remarks of interpretation. Observe that group effort may be supplied in one (or both) of two ways: funds and time. Consider the former. The marginal cost of "effort" is then simply the opportunity cost of devoting additional funds to a collective enterprise. If credit markets were perfect, this opportunity cost would have a simple proxy, which is the rate of return on alternative economic activities or just the rate of interest. In other words, if funds can be borrowed (without limit) for lobbying purposes, there would no reason to expect that the marginal cost of those funds will rise with the amount borrowed. This picture is inaccurate, however, because credit markets are rarely, if ever, perfect; even less if it is known that borrowed funds may be diverted to lobbying. In this case, the marginal cost of funds is no longer the rate of interest, but the rate of return on economic alternatives (including consumption) to which an individual must apply his own funds. It is natural to suppose that the larger the funds devoted to collective action, the higher will be the marginal opportunity cost of those funds. Our restriction becomes focal.

Observe, moreover, that this interpretation clarifies an important conceptual issue: Our restriction on marginal costs is not just an ad hoc restriction on individual tastes and preferences. It may be a statement about the institutional context, in this example, about the nature of capital markets.

Next, consider time as an input into the lobbying process. Then the marginal cost of effort is just the marginal cost of time. It is obvious that as more and more time is devoted to collective action, the marginal value of what remains must certainly rise, leading once again to the centrality of the increasing marginal cost restriction. Notice, moreover, that under this view credit markets play no role. Therefore our conditions admit more than one contextual interpretation.

A simple application of these remarks further illustrates the situation. First, the less rich the groups in question, the more significant should be the marginal cost effect. This is so for two reasons: (1) If credit markets are imperfect, they are likely to be more so for the poor, and therefore for such individuals the marginal cost of funds expended will rise more steeply with expenditure. (2) The poor are more likely to use time rather than funds in the lobbying process. As already noted, the use of time has a built-in propensity to exhibit increasing marginal cost—there are only 24 hours in the day. It follows that lobby battles involving relatively low-income groups (e.g., public job training, Medicare, or welfare assistance) will entail the formation of large groups. In contrast, battles fought by high-income groups (e.g., tax reduction or tariff protection) often substitute financial contributions for personal effort, and the marginal cost restriction is less likely to apply. Such lobby groups tend to be narrow and small.

The third feature of our model is new, that is, we allow for intermediate outcomes that are partly public and partly private. The result is new insights about the interplay between publicness of the prize and the increasing marginal cost effect. Given some cost function, a certain critical degree of publicness is enough to overcome the group-size paradox, which only confirms earlier work in more general form. More significantly, we prove that the threshold degree of publicness of the outcome is decreasing in the elasticity of marginal cost of effort supply. Indeed, if the elasticity is at least as large as 1, then the probability of success increases with group size, irrespective of the degree of privateness of the prize.

It should be noted that our—and Olson's—notion of group effectiveness may not be the only variable that influences group formation. We push this qualification a step farther by noting that per-capita payoffs to group members are just as important. This motivates a second notion of effectiveness, which we briefly explore at the end of the article.

We also note that our framework is a highly abstract model of pluralism, not a model of lobbying in the way most formal political theorists understand it. Nevertheless, we believe it is the simplest structure within which to analyze group effectiveness, for it captures the idea that government policy is sensitive to group contributions (whether in units of money or effort). Indeed, as in Olson's original argument, the focus is on the free-rider problem. We achieve this focus by abstracting from other (possibly more realistic) aspects, but there is no reason to believe that a consideration of those aspects would change the results significantly.

A MODEL OF COLLECTIVE ACTION WITH FREE-RIDING

The Effort Cost of Collective Action

Suppose that several options are available to society. Think of these as different locations of a public facility, competing public projects (hospital, library, or museum), or different political parties in office. Only one of the available alternatives can come about. Individuals differ in their valuations of these options. All those who rank a certain one first form an "interest group." We assume that all individuals with the same favorite are identical.[4]

[4] We thus exclude the case of group members who might be prepared

Let G denote the number of options as well as interest groups. Let N be the total population and N_1, $N_2, \ldots, N_G$ be the membership of the G groups.

In every collective action problem at least two types of goods are involved: the various prizes from collective action and the efforts contributed by individuals to realize their favored ends. The structure of preferences over the various prizes may have complex implications. For instance, individuals may free-ride not only on the effort made by fellow group members but also on the effort contributed by other groups, as long as the options have some degree of publicness.[5] In order to isolate and examine the within-group component of free-riding, we remove these aspects of the problem by assumption.[6] In other words, we assume that individuals in any group care most about their favorite option and are indifferent to all others. The utility differential between the favorite and the remainder, denoted by w, will drive the arguments below. Note well that this description is compatible with a scenario in which each option has a universally public component that is enjoyed symmetrically by all groups.

We shall denote by a the level of effort contributed by each individual. These units, which can be added across group members to yield group effort, may represent dollars or hours contributed to the collective cause. This particular interpretation of a will be maintained in the discussion that follows.

Assume that individual preferences are represented by the (additively separable) utility function

$$u(w,a) = w - v(a), \quad \mathbf{(1)}$$

where v is an increasing, smooth, convex function with $v'(0) = 0$, and w is the per-capita benefit from the favored option. This is equivalent to measuring utility in units of the collective good: From the benefit w we subtract the cost of the effort contributed, translated into the equivalent units of the collective good.[7]

In order to examine the economic meaning of alternative shapes of the v function, consider the individual preferences over (w, a) pairs implied by equation 1. If v is linear, we have the case in which effort is directly subtracted from benefits, as in Olson. There are instances in which this might be a plausible assumption. Think, for instance, of the cost to a lobbying firm of borrowing an extra dollar from a frictionless credit market, so that the rate of interest i is insensitive to the amount borrowed. In this case $v(a)$ is just $(1 + r)a$. Yet, in many interesting instances this assumption does not appear appropriate. That is clearly the case when the collective action is contributed by individuals and consists of personal effort, time, or income. The class of nonlinearities broadens even more if capital markets are imperfect. In these situations it may be more appropriate to assume that additional units of effort are increasingly costly.

Put another way, the marginal rate of substitution between reward and effort—the amount of extra benefit that will just compensate an individual for contributing an extra unit of effort—increases as total effort increases. As it turns out, the rate of increase, that is, the elasticity of the marginal rate of substitution with respect to effort, is the key variable that determines the effect of group size in collective action problems.

To formalize this concept quickly, we note that the marginal rate of substitution, r, can be simply written as

$$r = v'(a), \quad \mathbf{(2)}$$

and its elasticity $\alpha(a)$ at any effort level as

$$\alpha(a) = \frac{\frac{dr}{r}}{\frac{da}{a}} = \frac{av''(a)}{v'(a)}. \quad \mathbf{(3)}$$

$\alpha(a)$ will play a central role in our main result.

The Benefits of Collective Action

We turn now to an exploration of the benefits of group action and to the way in which efforts influence these benefits. We allow for options that have both public and private components because we wish to examine a view that appears to be common wisdom (at least since Chamberlin [1974]): Individual effort always decreases as group size increases, but aggregate collective action increases when the good is purely public and decreases when the good is purely private.[8] To this end, each option will have a public component, to be denoted by P, and a private component, denoted by M.

to supply by themselves the necessary contributions. That issue was raised by Olson in classifying different types of groups and has been extensively analyzed by Marwell and Oliver (1993). We do not deny the potential empirical relevance of this issue but want to focus on group numbers alone.

[5] Consider the case of three groups and options, A, B, and C. Suppose that group A (whose first preference is A) strongly prefers option B over C if choice is restricted to this pair. Suppose, moreover, that group B is pushing very strongly for its most preferred outcome. When deciding how much effort to contribute, individuals in group A would take into account not only what their fellow members are contributing but also the fact that option B is not that bad. Therefore, the free-riding problem would not be confined to one's own group, but would also be affected by the amount of effort made by "nearby" groups and the distance of their preferences from that of one's own group.

[6] More complex preference structures among options are explored in detail in Esteban and Ray 1999. There, we examine the case in which free-riding occurs not within groups but across groups. We show that, in general, the structure of preferences over the entire set of options has implications for the magnitude and pattern of conflict. Here we avoid the general case in order to focus narrowly on the Olson argument.

[7] Recall that a is the variable directly added over all members to arrive at win probabilities for a group, which means that the shape of v—in particular, the fact that we allow v to be nonlinear—becomes crucial in what follows. Note that a could as well be contributions in money. In that case, $v(a)$ is the utility cost of the contributed amount of money.

Taylor (1987, chap. 2) points out that Olson's argument critically depends on the assumption that costs can simply be subtracted from the benefits and mentions that Olson (1965, 29, 46) was aware of this analytical weakness. We shall show that this important observation has implications far stronger than the ones derived by Taylor through his diagrammatic analysis.

[8] See the summary in Taylor 1987, chap. 2, or Sandler 1992, chap. 2.

By a public component P, we refer to a component that all group members enjoy equally, irrespective of group size. By assumption, outsiders have no access to the public benefits of a group and may even be hurt by such a policy. Various causes come under this label: "better trade terms for developing countries," "abortion rights," "establishment of a dominant Hindu state," "a shorter working day," "saving the dolphins," and so on. Observe that a public component, while often nonmonetary, may well be monetary: A 10% rate of tariff protection to a group of domestic producers will offer the same benefit to each producer regardless of the number of producers. In contrast, a private component M is typically, although not always, monetary; the important characteristic being that it is dissipated with group size. As examples, consider "an additional \$1 billion for Medicare," "an additional increase in the U.S. immigration quota," or "allocation of water rights." Note that the last two examples are not directly monetary. The point is that a public component is unaffected by group size, while a private component is affected.

We assume that any private, divisible part of the collective good is shared by the N_i members of group i on an equal division basis. Let $\lambda \in [0, 1]$ be the share of publicness in any alternative. Then, the per-capita benefit to each member of group i, provided that option i is chosen by society, is

$$w_i = w(\lambda, N_i) = \lambda P + (1 - \lambda)\frac{M}{N_i}, \qquad (4)$$

and the benefit is normalized to be zero if any other option is chosen.

Observe that this formulation posits perfect symmetry among all options, except for the possible dilution of per-capita rewards because of varying group size. Once again, this assumption is a deliberate attempt to focus on the effects of group size alone.

Notice that $\lambda = 0$ corresponds to the case of a purely private, distributable good, whereas $\lambda = 1$ corresponds to a pure public good with perfect nonexcludability within the group. Thus, λ serves as a useful parameterization of the degree of publicness in a collective good.

We emphasize that the perceived share of publicness in a given collective good depends on group size. For given λ, P, and M, the larger the group, the larger is the perceived share of the public component. We shall denote by θ_i the share of publicness as perceived by an individual member of group i. That is,

$$\theta_i \equiv \theta(\lambda, N_i) = \frac{\lambda P}{\lambda P + (1 - \lambda)(M/N_i)}. \qquad (5)$$

We now discuss how the choice of options depends on effort. Denote by A_i the total effort contributed by group i. (Recall that this is just the sum of individual efforts within the group.) We assume that the probability of success for group i, which is just the probability π_i that option i will be chosen, equals the effort level of group i relative to the aggregate amount of effort A exerted by all groups.[9] That is,

$$\pi_i = \frac{A_i}{A}. \qquad (6)$$

Therefore, $\pi_i w_i$ is the expected value of the collective good to each member of group i. Under this specification, the marginal return to an additional unit of individual effort (as well as to an additional unit of group effort) is positive but decreasing in effort.[10]

Equilibrium

Expected utility per capita is given by

$$\frac{A_i}{A} w(\lambda, N_i) - v(a_i).$$

Each individual in each group takes as given the efforts contributed by everyone else in society (including fellow group members) and chooses a_i to maximize expected utility. Our end-point and curvature conditions guarantee that this choice is interior and is completely described by the first-order condition

$$\left[\frac{1}{A} - \frac{A_i}{A^2}\right] w(\lambda, N_i) - v'(a_i)$$
$$= \frac{1}{A}(1 - \pi_i) w(\lambda, N_i) - v'(a_i) = 0 \qquad (7)$$

An *equilibrium* is a vector of individual contributions such that equation 7 is satisfied for every individual in every group. From equation 7, it is clear that in any equilibrium the choice made by each member of any given group will be identical. That is, $A_i = N_i a_i$, so that

$$a_i = \frac{A_i}{N_i} = A\frac{\pi_i}{N_i}. \qquad (8)$$

Using equation 8, we can rewrite equation 7 as

$$\phi(\pi_i, A, N_i) \equiv \frac{1}{A}(1 - \pi_i)\left[\lambda P + (1 - \lambda)\frac{M}{N_i}\right]$$
$$- v'\left(A\frac{\pi_i}{N_i}\right) = 0. \qquad (9)$$

An equilibrium can now be reinterpreted as a vector of success probabilities—therefore adding up to unity—and a positive number A, such that equation 9 is satisfied for all groups. A is an indicator of scale: It tells us the aggregate amount of collective action that is created in equilibrium.

It is very easy to check that an equilibrium always exists and is unique. Provisionally view π_i as a parameter in equation 9, and observe that

[9] The model can easily be rewritten with success probabilities depending nonlinearly on individual effort (see Skaperdas 1996), but in that case the interpretation of the nonlinearity of v becomes more complex.

[10] Marwell and Oliver (1993) use the term "decelerating" to describe this phenomenon.

$$\phi \rightarrow (1/A[\lambda P + (1 - \lambda)(M/N_i)] > 0 \; as \; \pi_i \downarrow 0,$$

whereas $\phi \rightarrow -v'(A/N_i) < 0$ *as* $\pi_i \uparrow 1$. Furthermore, it is obvious that ϕ is strictly decreasing in π_i. Therefore, for each given A and N_i there is a unique value of π_i that satisfies equation 9. In other words, equation 9 implicitly defines π_i as a function of A and N_i: $\pi_i = \pi(A, N_i)$. The equilibrium value of A is then determined by the condition that

$$\sum_{i=1}^{G} \pi(A, N_i) = 1. \quad \textbf{(10)}$$

It is easily seen that $\pi(A, N_i)$ is strictly decreasing in A and varies between 0 and 1. This completes the demonstration of existence and uniqueness of equilibrium.

GROUP SIZE AND COLLECTIVE ACTION

Winning Probabilities

We now analyze the effect of group size on the provision of collective goods and present a pair of propositions. Both are in sharp contrast to the common wisdom that Olson's result hinges on whether the collective good is private or public. We show that whenever preferences are nonlinear, that is, $\alpha(a) > 0$, Olson's tenet that the level of collective action diminishes with group size does not generally hold, irrespective of the degree of privateness of the collective good. If preferences are "sufficiently" nonlinear (in a sense to be made precise below), Olson's result is not true and, indeed, is exactly reversed.

As noted earlier, the phrase "larger groups have higher win probabilities" is ambiguous. It might refer to an examination of large versus small groups at some given equilibrium or to the implications of increasing the size of some group, in which case the comparison is across equilibria.

Proposition 1 adopts the first of these interpretations.

PROPOSITION 1. *Consider the equilibrium of the game described above.*

a. Whenever $\inf_a \alpha(a) > 1$, *the level of collective action (and therefore winning probabilities) is strictly increasing in group size for all* $\lambda \in [0, 1]$, *that is, irrespective of the degree of public/privateness of the collective good.*

b. More generally, winning probabilities are increasing over a pair of group sizes n and n', where n < n', if

$$\theta(\lambda, n) \geq 1 - \inf_a \alpha(a). \quad \textbf{(11)}$$

Provided $\inf_a \alpha(a) > 0$ *and the good is not fully private, this condition is automatically satisfied for large enough group sizes. Alternatively, under the same provisions and for any pair of group sizes, it is satisfied for* λ *close enough to unity.*

Proof. We prove the proposition by examining the behavior of π_i over the cross-section of groups, keeping A unchanged at its equilibrium value. The easiest way to do this is to pretend that N_i is a continuous variable in equation 7 and to differentiate π with respect to N_i. Some tedious calculations reveal that this derivative is given by

$$\frac{d\pi_i}{dN_i} = \frac{\pi_i}{N_i} \frac{\alpha(a_i) - (1 - \theta(\lambda, N_i))}{\alpha(a_i) + \dfrac{\pi_i}{1 - \pi_i}}. \quad \textbf{(12)}$$

Notice that this derivative is guaranteed to be positive when $\inf_a \alpha(a) > 1$, so that part a is immediately established.

Condition 11 of part b is also a nearly immediate consequence of equation 12. We only need to observe that $\theta(\lambda, n)$ is a nondecreasing function of n, so that the nonegativity of the derivative evaluated at $N_i = n$ is sufficient. The fact that condition 11 is satisfied for large group sizes or for λ close to unity (provided that $\inf_a \alpha(a) > 0$) follows simply from the fact that $\theta(\lambda, n) \rightarrow 1$ either as $\lambda \rightarrow 1$ or as $n \rightarrow \infty$. *Q.E.D.*

In the next proposition, we investigate what happens when the size of a particular group increases. This involves a comparison across equilibria. Remarkably enough, the results tightly parallel those of proposition 1.

PROPOSITION 2. *Suppose that the size of one group (of size n) is increased, but all other group sizes are kept unchanged. Then, under the new equilibrium, the winning probability of this group increases if condition 11 holds:*

$$\theta(\lambda, n) \geq 1 - \inf_a \alpha(a).$$

Just as in proposition 1, this condition is automatically satisfied for large enough group sizes or when λ *is sufficiently close to unity. In particular, when* $\inf_a \alpha(a) > 1$, *the winning probability increases irrespective of the degree of public/privateness of the collective good.*

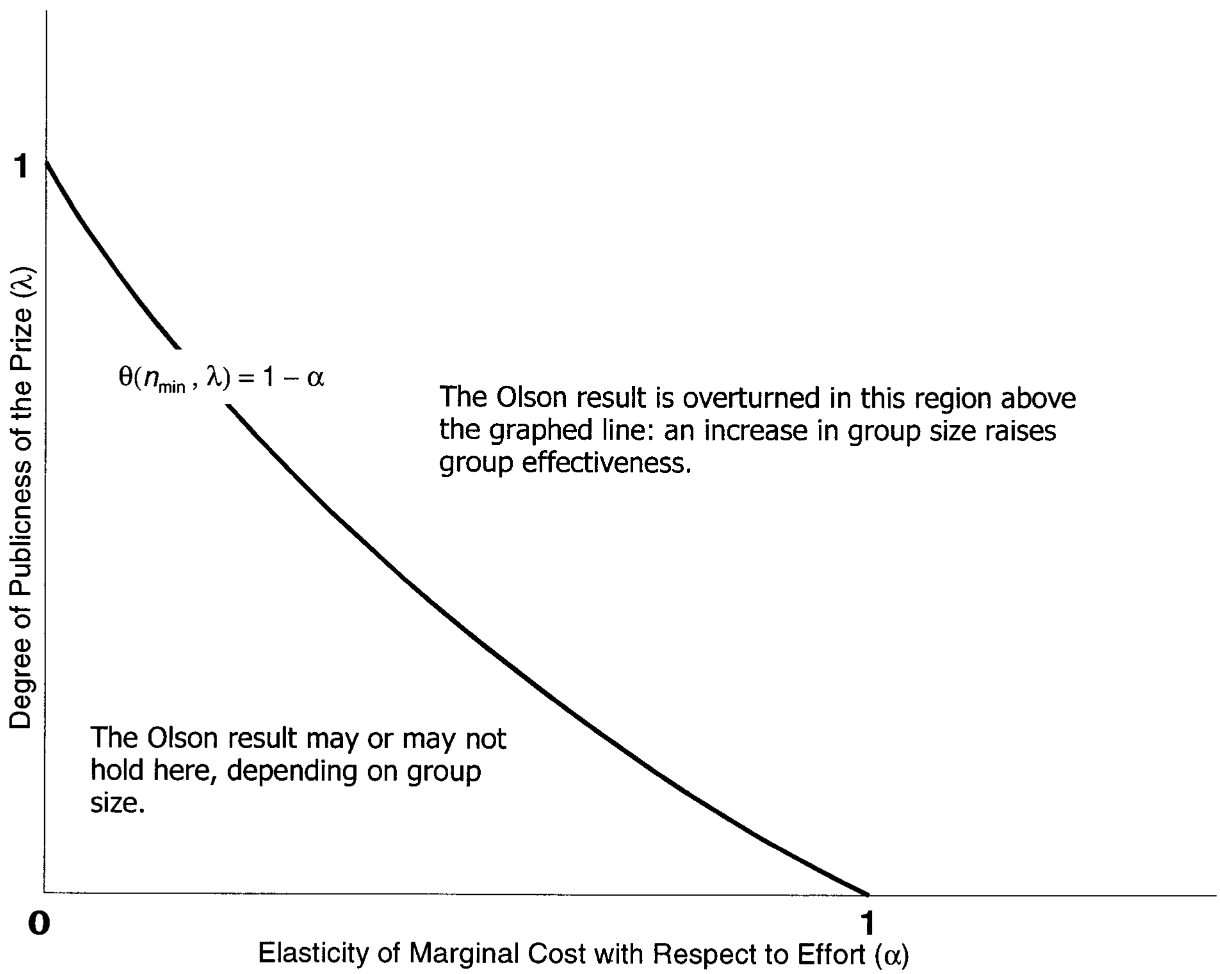
Proof. Let group i have size $N_i = n$. Just as in the proof of proposition 1, use equation 7 to differentiate π_i with respect to n, keeping A unchanged for the moment. We already know that this derivative is positive if equation 12 holds. Put another way, we have learned so far that $\pi(A, n)$ is increasing in n for *fixed A* (assuming equation 12 applies).

Now we have to allow for the change in equilibrium A. Notice that our exercise so far allows us to conclude that

$$\sum_{i=1}^{G} \pi(A, N_i)$$

has increased, evaluated at the earlier value of A and N_j, for $j \neq i$, and the new value of N_i. Therefore, using equation 10, the new equilibrium value of A must increase. But this proves that, for every group other than i, the equilibrium win probability must strictly fall.

FIGURE 1. Combinations of "Publicness" and Marginal Cost Elasticities that Overturn the Group Size Paradox



Because win probabilities sum to one, this completes the proof. (The other particular implications follow just as in proposition 1.) *Q.E.D.*

For expository convenience, assume for the moment that the elasticity of marginal cost of effort, α, is constant. If the cost function is itself of the constant-elasticity form $v(a) = a^{1+\alpha}$, it is easy to verify that the elasticity of marginal cost will also be constant and exactly equal to α. Then one can graphically view the region for which the Olson result is overturned (see Figure 1). The figure plots different elasticities of marginal cost (α) on the horizontal axis, and different degrees of publicness of the collective good (λ) on the vertical axis.

Let us suppose that we know that all group sizes are at least a certain minimum; call this $n_{\min}$. Figure 1 invokes condition 11 for this minimum group size, and this is used to divide the diagram into two regions. For any combination of parameters in the upper region, the Olson result must be overturned. That is, for any starting size (not lower than $n_{\min}$), an increase in group size must increase group effectiveness. In contrast, for parametric configurations in the lower region, the Olson result may or may not be valid, depending on how large the starting group size is (the larger is this starting size, the more likely it is that the result is invalid, even in this region). The only point that guarantees the Olson result is the origin of the diagrammatic axes (corresponding to both $\alpha = 0$ and $\lambda = 0$), which may be interpreted as a linear cost function together with a fully private prize. No other configuration of parameters can guarantee the Olson result for arbitrary group sizes.

Notice that when the elasticity of marginal cost attains the value one (corresponding to a quadratic cost function for effort), the group size paradox is "fully" reversed, and remains so for all higher elasticities. Irrespective of the characteristics of the collective prize or the starting group size, larger groups are then more effective. Figure 1 graphically displays this by yielding all space to the upper region to the "right" of the point where $\alpha = 1$.

Discussion

Propositions 1 and 2 stand in sharp contrast to the common wisdom that collective action increases or decreases with group size, depending on whether the collective good is purely public or purely private, respectively. Instead, we prove that there is no inverse relation between collective action and group size except under either of two extreme assumptions: (1) individual indifference curves are straight lines (i.e, $\alpha(a) = 0$), or (2) $\alpha(a) \in [0,1]$ *and* the collective good is purely private (i.e., $\lambda = 0$).

Indeed, if $\alpha \geq 1$, the Olson assertion is fully overturned. Collective action increases with group size even when the prize is purely private, and this is true a fortiori when some degree of publicness is introduced. Moreover, even if the cost function exhibits lower elasticity (but nevertheless continues to be strictly convex), proposition 1 argues that there will always be some vector of group sizes for which the Olson thesis is false.[11] In summary, the private/public distinction matters, to be sure, but may be entirely swamped by considerations that involve the disutility cost of effort supply. This offers new insights regarding the Olson thesis,[12] especially in light of our argument that some curvature in the cost function is empirically likely to be the rule rather than the exception.

To obtain further intuition for the propositions, we drop the subscripts in operation 12 and rewrite it as

$$\frac{N}{\pi}\frac{d\pi}{dN} = \frac{1}{\alpha(a) + \dfrac{\pi}{1-\pi}}[\alpha(a) - (1 - \theta(\lambda, N))].$$

This expression states that the proportional change in the size of collective action induced by a given percentage increase in group size is proportional to the difference between the elasticity of the marginal rate of substitution and the elasticity of the value of the collective good with respect to group size.[13] Notice that the reciprocal expression $1/\alpha(a)$ is the elasticity of individual effort with respect to the marginal compensating reward. Thus, low values of α correspond to extremely elastic responses of individual effort supply to small variations in the marginal reward. Therefore, the propositions say that collective action will decrease or increase with group size depending on whether the proportional effect on the reward $(1 - \theta)$ does or does not induce a sufficiently elastic response in individual effort supply.

The sign and value of α play a critical role in our result. Which assumptions about α are most plausible? Chamberlin (1974) introduced the distinction among three categories of collective goods: inferior, normal, and superior. A collective good is inferior if the individual responds to an increase of one unit in effort by the rest of the group with a reduction in personal effort of no less than one unit; it is normal if the personal reduction is strictly less than one unit; it is superior when the personal response is increased effort.

In order to see which assumptions would give rise to each category of goods, we differentiate a_i with respect to A_i in the best response function, equation 7, where $A_{-i} \equiv A_i - a_i$. After performing this differentiation and dividing each side by the corresponding side of equation 7, we find that

$$\frac{da_i}{dA_{i-}} = -\frac{1}{1 + \alpha(a_i)\dfrac{A}{2a_i}}.$$

It follows that the collective good is normal if and only if $\alpha(a) > 0$. Therefore, our main result simply requires the collective good to be normal in the sense of Chamberlin.[14]

Another Notion of Group Effectiveness

A second definition of group effectiveness relates group size to per-capita payoffs. This relationship cannot, in general, be predicted by the change in winning probabilities (as described in propositions 1 and 2). For instance, when the good is purely private, it may be true that larger groups have a higher win probability (if $\alpha > 1$). Moreover, as is well known (Chamberlin 1974), they also put in lower effort per capita. Yet, larger numbers do diminish the per-capita value of the prize. Therefore, holding constant the overall value of the prize, large groups are at some intrinsic disadvantage in terms of payoffs.

If the good is purely public, then this disadvantage vanishes altogether, and we are simply left with the two positive effects for large groups. This informal discussion suggests that our second notion of effectiveness may be more closely tied to the private/public distinction.

PROPOSITION 3. *In equilibrium, the expected payoff to a player increases with group size when the collective good is purely public* ($\lambda = 1$) *and decreases when it is purely private* ($\lambda = 0$).

Proof. Write individual expected utility in an equilibrium as

$$\pi(A, N_i)w(\lambda, N_i) - v\left(\frac{\pi_i A}{N_i}\right),$$

[11] Indeed, if $\alpha(a) \geq (1 - \lambda)M/(\lambda P + (1 - \lambda)M)$, the Olson thesis is false regardless of the configuration of group sizes in society.

[12] To be sure, the amount of individual effort contributed diminishes with group size for all $\alpha(a) \geq 0$ and $\lambda \in [0, 1]$, a result known since Chamberlin (1974); see also Riaz, Shogren and Johnson 1995.

[13] To see why $1 - \theta(\lambda, N)$ serves as a measure of the latter elasticity, consider the percentage variation in per-capita reward $w(\lambda, N)$ that corresponds to a given percentage increase in population size. It is easy to see that

$$\frac{\partial w}{w} / \frac{\partial N}{N} = -\frac{N}{w}(1-\lambda)\frac{M}{N^2} = -\frac{(1-\lambda)(M/N)}{w} = -[1 - \theta(\lambda, N)].$$

Therefore, the degree of perceived privateness $(1 - \theta)$ can also be interpreted as the elasticity of w—the reward—with respect to group size.

[14] This result represents a substantial exploration of the observation made by Taylor (1987, chap. 2) regarding the critical role of flatness in the individual indifference curves between reward and effort.

where A satisfies the equilibrium condition 10. Again, treating N_i as a continuous variable, differentiate this expression with respect to N_i but keep A fixed. The idea is that we are moving over a cross-section of groups in a given equilibrium. If we carry out this exercise and manipulate the results a bit, we see that

$$\frac{\partial u_i}{\partial N_i} = \frac{\pi(A, N_i) w(\lambda, N_i)}{N_i} \cdot \left\{ \theta(\lambda, N_i) - \left[1 - \frac{1 - \pi(A, N_i)}{N_i} \right] \cdot \left[1 - \frac{N_i}{\pi(A, N_i)} \frac{\partial \pi(A, N_i)}{\partial N_i} \right] \right\} \quad \mathbf{(13)}$$

The sign of this derivative depends on whether the product of the two brackets is smaller or larger than $\theta(\lambda, N_i)$. Since $N_i \geq 1$, it is plain that the first bracket is positive and does not exceed 1. As for the second bracket, we know—paraphrasing Chamberlin (1974) (see note 12)—that it is always positive.

When the collective good is purely private, we have $\lambda = 0$ and $\theta(\lambda, N_i) = \theta(0, N_i) = 0$. Consequently, the derivative in equation 13 is strictly negative: In equilibrium, members of larger groups attain lower levels of per-capita utility than members of smaller groups.

When the collective good is purely public, we have $\lambda = 1$ and $\theta(\lambda, N_i) = \theta(1, N_i) = 1$. Moreover, by proposition 1, win probabilities rise with group size. It follows that the value of the second square bracket in equation 13 does not exceed 1. Consequently, when the good is purely public, the derivative in equation 13 is positive: Members of larger groups attain higher levels of per-capita utility than members of smaller groups. *Q.E.D.*

The argument behind this result is quite straightforward. Consider the case of a pure public good. By proposition 1 we know that, provided it is a normal good, the larger the group, the smaller is the individual effort, but the higher is the level of collective action. Since the size of the group does not reduce the availability of the collective good to individual members, membership in a larger group has the effect of increasing the benefit and reducing the costs. Larger groups are more desirable on all counts. When the good is purely private, an increase in group size still has the effect of reducing individual effort (but the effect on win probabilities is ambiguous). This effect possibly enhances individual utilities, but (as the proposition shows) it is never enough to counteract the fall in per-capita benefit created by larger group size.

Combining propositions 1 (or 2) and 3, we see that there are situations in which collective actions and utilities do not move in the same direction, so that our two notions of group effectiveness are really distinct. For instance, when $\inf_a \alpha(a) > 1$ and the collective good is purely private, larger groups contribute more resources and therefore enjoy larger win probabilities (proposition 1). By proposition 3, however, they must have lower payoffs at the individual level.

Notice that proposition 3 is not as comprehensive as proposition 1, in that it does not characterize group utilities in the intermediate cases. Observe, however, that whenever the good is not purely private (i.e., $\lambda \in (0,1]$), θ tends to 1 as N_i becomes large. It can be easily seen from equation 13 that, for sufficiently large N_i, this expression is strictly positive, so that further increases in group size will increase the equilibrium utility of members. It follows that whenever the collective good has some public content, large groups may do better (depending on the configuration of group sizes in society).

CONCLUSION

The common wisdom concerning Olson's thesis is that small groups are more effective, even when the collective prize is fully private, but that this relationship is overturned when the prize if fully public. By explicitly modeling the costs of effort, as well as situations in which the prize has mixed characteristics, we not only extend the common wisdom but also show that there are cases in which it is false.

In particular, if marginal costs rise sufficiently fast with contributions, even when the prize is purely private, large groups have higher win probabilities than small groups (propositions 1 and 2). The Olson result is critically dependent on the linearity of cost functions, an unrealistic assumption that we question here.

When a good is fully private, however, large groups have lower per-capita payoffs than smaller groups, if we control for the overall size of the prize (proposition 3). This payoff-based notion of effectiveness is different from the win-based notion that has received attention, but it certainly should be considered in theories of group formation. We do not pursue it here, but some remarks may be useful.

First, it is unclear whether group formation occurs exclusively on the basis of per-capita payoffs, or whether effectiveness in the sense of win probabilities (the perception of being successful) also is a factor. If we view firms as an instance of groups, we see that this ambiguity is closely related to the age-old question of what firms "maximize": profits, size and presence, other public perceptions of success, or some combination of these? To the extent that these other factors also matter, win-based notions of effectiveness enter into the theory of group formation, and there is no guarantee that society will be splintered into small Olson-style lobbies.

Second, even if per-capita payoffs are the sole criterion for group formation, proposition 3 (unfortunately) throws little light on the issue of small versus large group selection. The reason is that the proposition is unequivocal only in the extreme cases of pure privateness or publicness. Just how much privateness is required (under this criterion) for the small group effect to dominate remains an interesting and open question.

Finally, it is important to remember that many groups are defined by their ideal points, and there may be little or no room for group formation. So there is no necessary contradiction between the possibility that large groups have low per-capita payoffs yet still exist.

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