Bribery and Favoritism by Auctioneers in Sealed-Bid Auctions*

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Abstract

We consider a model of bribery in an asymmetric procurement auction. In return for a bribe from the dishonest supplier, the auctioneer has the discretion to allow this supplier to revise his bid downward to match the low bid of the honest supplier. The dishonest supplier can also win the contract outright without paying a bribe by bidding below the honest supplier. We investigate the effect of the bribe share and the cost distributions on the bidding functions, the allocative distortion, and the expected price paid by the buyer. The dishonest supplier bids more aggressively to win the contract outright when the auctioneer takes a larger bribe share. Bribery and the implied right of first refusal introduce a new allocative distortion in favor of the dishonest supplier. Finally, we use the power family of cost distributions to examine the expected price paid by the buyer. When the dishonest supplier has a more favorable cost distribution, there exist bribe shares sufficiently large such that the expected price paid by the buyer can actually decline as a result of bribery.

KEYWORDS: bribery, favoritism, auctions

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1. Introduction

In this paper we analyze bribery as a monetary side-payment by a supplier to an auctioneer in order to alter the award of a procurement contract in favor of the supplier. The auctioneer represents a buyer in the procurement of some good. The buyer could be a government or a corporation, and the auctioneer would be a procurement official or employee.

The award of the contract in return for a payment could take different forms in different contexts. In this paper, we will analyze the exchange between a corrupt auctioneer and a dishonest supplier whereby the auctioneer allows the supplier to revise his bid, when necessary to win the contract. That is, the auctioneer cannot award the contract at a price above the lowest bid from the auction, but he can favor the dishonest supplier by awarding him the contract at that price. In effect, when the dishonest supplier does not submit the lowest bid, he still has a right of first refusal to accept the contract at the lowest bid of the other suppliers. We use the term “favoritism” to describe the special case in which the auctioneer does not require a bribe from the dishonest supplier in exchange for awarding the contract under this right of first refusal.

This specification of the auctioneer’s discretion in awarding the contract resembles some documented examples of bribery. For instance, Ingraham (2005) examines bribery in contracts awarded by the New York City School Construction Authority from 1990 - 1997. When the bids were to be opened publicly to identify the winner of the contract, the auctioneer saved the bid from the bribing supplier to open last and then submitted a new, winning bid for this supplier. Since the bribing supplier could withdraw from the contract if the new winning bid underestimated his costs, the auctioneer secretly created a right of first refusal during the auction process. Lengwiler and Wolfstetter (2000) cite two major international construction projects (an airport in Berlin and a power station in Singapore) in which bribes were paid to obtain the bids that were submitted by the rivals. More generally, this paper provides some insights into favoritism with or without bribes. Government procurement officers are known to have accepted bribes from suppliers and corporations are known to favor certain suppliers of various inputs.

The goal of this paper is to examine the effects of this form of bribery or favoritism on the allocation of contracts and prices. We first examine how bribery or favoritism affects the bidding strategies of suppliers competing in a

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1 See Noonan (1984) for a history of bribery. See Rose-Ackerman (1999) for a general survey of corruption and potential solutions with specific examples of bribery to government officials for procurement contracts in Chapter 3.
procurement auction. We then identify the effects of bribery on the allocation of contracts and the expected price paid by the buyer.

In Section 2, we discuss the literature most related to our paper. This discussion explains the key distinctions between the differing models and results in the literature. In doing so, we also explain how our approach to bribery differs from the existing literature.

In Section 3, we present the model where two suppliers compete in a first-price auction (FPA) to win a procurement contract, but one supplier is favored by the auctioneer. The supplier who is favored will be called the “dishonest supplier” (DS), and the other supplier called the “honest supplier” (HS). The auctioneer demands a share of the post-auction “surplus” defined as the difference between the low bid of the HS and the cost of the DS, whenever this difference is positive. We call this a “bribe share”.

Bribery or favoritism alters the bidding strategies of both suppliers. The bidding strategy of the HS will account for the fact that the DS will be favored by the auctioneer. The bidding strategy of the DS will account for both the opportunity to bribe the auctioneer and the cost of doing so. In Section 4, we characterize these equilibrium bidding strategies and analyze how the size of the bribe share affects this behavior. Although one might conjecture that bribery would induce the HS to bid more aggressively and the DS to bid less aggressively, we explain why this need not be the case. In particular, when the bribe share paid to the auctioneer is large, the DS may bid more aggressively than the HS, as well as more aggressively than he would have in the absence of the opportunity to bribe the auctioneer.

In Section 5 we first analyze the allocative effects of bribery. We compare the allocative distortion with bribery to that which arises in an asymmetric FPA without bribery and an optimal auction. Bribery favors allocation of the contract to the DS, whether he is ex ante stronger or weaker than the HS in the sense of first order stochastic dominance of the cost distribution. This contrasts with both the FPA and the optimal auction. Moreover, with bribery, the allocative distortion is more pronounced for low cost realizations of the HS. Indeed, the HS chooses his bid to compete against the cost of the DS. Thus, the allocative distortions will be solely determined by the bidding behavior of the DS. Under mild conditions, the lower the cost of the HS, the higher the margin of his bid above his cost.

In Section 6 we show that bribery may in fact result in a lower expected price paid by the buyer, even though the allocative distortions are not optimal and even though the bribe paid to the auctioneer is a pure loss for the buyer. We illustrate this result by analytically solving the model for a convenient family of cost distributions. The expected price will be lower than either the expected price in a FPA or in an efficient auction (such as a second-price auction (SPA)) when the bribe share is large and the HS is ex ante weaker than the DS.
Section 7 concludes the paper and briefly discusses some other interesting questions that can be addressed with this model.

2. Related Literature

There is a substantial and growing literature on bribery and favoritism in auctions. In this section, we briefly summarize the major issues and findings of that literature which are most closely related to our paper.²

Several papers examine models in which any bidder (or supplier) can bribe the auctioneer: Beck and Maher (1986) and Lien (1986), Lien (1990), Lengwiler and Wolfstetter (2000, 2005), Menezes and Monteiro (2001, 2006), Compte, Lambert-Mogiliansky, and Verdier (2005), and Koc and Neilson (2005). These papers differ in the specification of the bribe, but the common feature is that the auctioneer for a procurement auction has discretion to allow any supplier with the lowest bid to receive a price equal to the second lowest bid. Lengwiler and Wolfstetter (2005) describe this feature as “type I corruption”. Similarly, we will refer to this feature as type I discretion by the auctioneer. The primary finding of all these papers is that the introduction of a corrupt auctioneer will modify a standard auction held by a seller (or buyer) into a “bribery auction” held by the auctioneer. The term “bribery auction” is used to describe an auction in which the presence of a corrupt auctioneer has no effect on the allocation of the good (or the contract) and no effect on the expected profits of the bidders. In particular, the bribery auction remains efficient and the bidders are indifferent to an auction with or without bribery. Thus, bribery simply results in a transfer of rents from the buyer to the auctioneer.

In our paper this transformation into a bribery auction does not occur. There are two primary reasons for this. First, the model specifies that only some suppliers can bribe the auctioneer, but that the other suppliers cannot. Second, our model assumes a different specification for the auctioneer’s discretion. In particular, the auctioneer has the discretion to award the contract to a losing supplier at the price equal to the lowest bid by the winning supplier. Thus, the favored losing supplier will bribe the auctioneer to receive the contract whenever his cost is below the lowest bid of the other suppliers. Lengwiler and Wolfstetter (2005) describe this feature as “type II corruption”, so we will similarly refer to it as type II discretion by the auctioneer.

In this paper, we examine type II discretion by the auctioneer. There are several reasons why type II discretion is interesting. First, type II discretion does

² There is a related literature in which suppliers bribe a third party who provides an assessment of the quality in a multi-attribute procurement auction. For example, see Celentani and Gauza (2002), and Burguet and Che (2004). See also Laffont and Tirole (1991).
not result in a bribery auction that simply preserves efficiency while transferring
rents to the auctioneer.\footnote{Lengwiler and Wolfstetter (2005) allow the auctioneer to choose between type I corruption and type II corruption. Using numerical examples, they find that the equilibrium bidding functions do not always result in efficient allocations. Thus, despite symmetry, the auction does not degenerate into a bribery auction. These results suggest that type II corruption introduces inefficiencies even when type I corruption is present.} Second, we have shown in Burguet and Perry (1999) that a model with type II discretion can be easily extended to include type I discretion. Third, favoritism by the auctioneer (or the buyer) toward one supplier is a natural special case of type II discretion. Favoritism means that the favored supplier who loses the bidding (or does not bid) can still obtain the contract at a price equal to the lowest bid from the other suppliers, but without paying a bribe to the auctioneer (or the buyer). As such, the favored supplier effectively has a right of first refusal on the contract at a price equal to the lowest bid from the other suppliers, even if he never bids to win the contract.

Several recent papers have discussed the implications of a right of first refusal in auctions. These papers include Arozamena and Weinschelbaum (2004), Porter and Shoham (2005), and Bikhchandani, Lippman, and Ryan (2005), Choi (2003) and Lee (2004). Applied to a procurement auction, these papers examine the implications of the right of first refusal granted to one supplier by a buyer (or auctioneer) on the bidding behavior of the other suppliers and the resulting price paid by the buyer.\footnote{Bikhchandani, Lippman, and Ryan (2005) examine a right of first refusal in a second-price auction, whereas Choi (2003) examines a right of first refusal in a first-price auction. With private values, the gains of the favored buyer exactly offset the seller’s loses in a second-price auction, whereas the joint expected profits of the seller and the favored buyer are higher in a first-price auction.} The papers by Arozamena and Weinschelbaum (2004) and Porter and Shoham (2005) introduce an auctioneer who grants the right of first refusal instead of the buyer. However neither paper has an explicit model of the bribery payments to the auctioneer. As a result, the supplier with the right of first refusal can be interpreted as bidding his cost, bidding any price above his cost including his highest possible cost, or not bidding at all. In contrast, our model explicitly defines the bribery payments as a share of the surplus generated by the right of first refusal. The supplier with this right of first refusal still has an incentive to bid against the other suppliers because he could win the contract outright and avoid paying a bribe when he has a low cost and submits the lowest bid. This will determine the bidding behavior of the supplier with the right of first refusal, alter the bribery payments to the auctioneer, and affect the expected price paid by the buyer in a variety of ways.
3. **The Model of Bribery**

The buyer has a value $v$ for a good with a fixed quantity and quality. The buyer employs an auctioneer who receives bids from suppliers and awards a contract for the buyer to purchase the good from one of the suppliers using a sealed-bid first-price auction (FPA). In a fair auction without bribery or favoritism, the contract would be awarded to the supplier with the lowest bid at a price equal to that bid. In contrast, we examine an auction in which the auctioneer can provide a *right of first refusal* to one of the suppliers. If no bribe is required from this supplier for the *right of first refusal*, then we will refer to him as the “favored” supplier. But more generally, if this supplier must pay a bribe for the *right of first refusal*, then we will refer to him as the “dishonest” supplier (DS). We assume that there is one other supplier called the “honest” supplier (HS). We do not allow the auctioneer to consider bribes from both suppliers. Moreover, we do not address which supplier is the DS and which is the HS.

An important feature of our model is that the two suppliers are asymmetric in that they have different distributions for their costs of producing the good. We assume that each supplier draws his cost of production $c_i$, where $i = d$ (DS) or $h$ (HS), from a distribution $G_i(c)$ with a common support $[0,1]$, and a positive density $g_i(c)$ over this support. The cost $c_i$ is private information for each supplier, but the distribution functions are common knowledge. For simplicity, we also assume that the value of the buyer exceeds the highest possible cost realization ($v > 1$). Finally, we assume that the costs of the suppliers are independently distributed. Thus, we will examine bribery and favoritism in an asymmetric independent private value (cost) FPA. The suppliers simultaneously bid for the contract, knowing their cost, knowing the cost distribution of the other supplier, and also knowing the form in which bribery or favoritism occurs.

The auctioneer runs a sealed-bid FPA and must award the contract at a price equal to the lowest bid. If the auctioneer has some discretion in awarding the contract and/or setting the price paid by the buyer, he can extract a bribe from the dishonest supplier. In this paper, we will focus on a specification of the auctioneer’s discretion in which he must set the price equal to the lowest bid, but that he need not award the contract to the honest supplier who makes the lowest

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5 The model with general cost distributions could be characterized in terms of multiple symmetric honest suppliers. However, the general insights are fully illustrated with one honest supplier. In Appendix 2 where Proposition 4 is proved, we discuss the extension to multiple honest suppliers.

6 This question obviously requires that the suppliers be asymmetric. This question has been partially addressed in a companion paper Burguet and Perry (2003, revised 2005) where we examined upfront payments directly to the buyer (or similarly, upfront bribes to the auctioneer) in return for acquiring a *right of first refusal* (called “preference”) during the subsequent auction.
bid. If the bid of the DS is higher than that of the HS, the auctioneer can award the contract to the DS at a price equal to the bid of the HS. Even though the DS submitted a higher bid, the DS has a right of first refusal at a price equal to the lower bid by the HS. Conversely, if the bid of the DS is below the bid of the HS, we assume that the DS is simply awarded the contract at a price equal to his bid.

Let \( b_d \) and \( b_h \) be the bids of the DS and the HS respectively. If \( b_h > b_d \), the contract is awarded to the DS at a price equal to his bid. However, if \( b_d > b_h \), bribery or favoritism may occur. In particular, when \( b_d > b_h > c_d \), there is “surplus” \( (b_h - c_d) \) which can be divided between the auctioneer and the DS. In these cases, the auctioneer awards the contract to the DS at a price \( b_h \), and the auctioneer receives a share \( \alpha \in [0,1] \) of the surplus \( (b_h - c_d) \). We call \( \alpha \) the “bribe share”. Finally, if \( b_d > c_d > b_h \), there is no surplus that the auctioneer and DS can share, and the HS is awarded the contract at a price equal to his bid.

The bribe share \( \alpha \) is determined prior to the auction and thus is independent of the outcome of the auction. The DS knows the value of \( \alpha \) prior to submitting his bid. The HS need not know the specific value of the bribe share \( \alpha \). Rather, all the HS needs to know is that the DS has a right of first refusal at a price equal to his bid and will thereby obtain the contract whenever \( b_h > c_d \).

The bribe share could be interpreted as the relative bargaining power between the auctioneer and the DS. It may have arisen informally from past practice of the auctioneer. Also, it could be determined by giving the auctioneer a percentage of the stock in the subsidiary of the DS handling the contract. The auctioneer would then receive bribery payments in the form of dividends from that subsidiary after the contract is completed and the DS is paid by the buyer.

In order to calculate the bribery payments when the bribe share is strictly positive, the bid of the HS must be verifiable to the DS and the cost of the DS must be verifiable to the auctioneer. If the bids are submitted in writing, the auctioneer can verify the bid of the HS from the signed bidding materials submitted by the HS. However, it may be more difficult for the DS to verify his cost. The cost of the DS might be verifiable from the ex ante bid preparation documents of the DS. However, it seems more natural to assume that the cost of

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7 One justification for restricting the price to be equal or below the bid of the HS is that the price may become known at the end of the auction. The HS who was not awarded the contract could complain to the buyer if his bid were below the resulting price. In government procurement, the HS may be able to sue the government.

8 We assume that any compensation to the auctioneer from the buyer is independent of the auctioneer’s actions in awarding the contract, and that there are no punishments for accepting bribes. We are not attempting to model the agency problem between the buyer and the auctioneer. On this, see Krueger (1974), Rose-Ackerman (1975, 1978), Rasmusen and Ramseyer (1994), and Mookherjee and Png (1995).
the DS is verifiable from the ex post accounting records of expenditures by the DS. It should be noted that when the bribe share is zero, the costs of the DS need not be verifiable.9

4. Equilibrium Conditions for the Bidding Functions

In this section, we characterize the equilibria in a sealed-bid FPA for the basic model of bribery assuming general distribution functions for the costs of the two suppliers. Bribery will generally alter the equilibrium bidding functions of both suppliers. However, we find that bribery does not make the equilibrium bidding functions uniformly more or less aggressive for either supplier.

In general, it is difficult to solve for the equilibrium bidding functions in an asymmetric FPA. The asymmetric FPA becomes more tractable with bribery. In particular, the game is dominance solvable. Once we exclude bids by the DS below his cost and exclude the rejection of contracts by the DS at a price above his cost, the HS has a dominant strategy against which the DS can then choose his bidding strategy. Indeed, the HS is effectively bidding against cost of the DS. As a result, the bidding strategy of the HS is independent of both the bidding strategy of the DS and the value of the bribe share \( \alpha \). The HS calculates his bidding strategy \( b_h(c) \) by solving the following problem:

\[
\max_b \Pi_h[b; c] = (b - c) \cdot [1 - G_d(b)].
\]

The first-order condition of this problem is

\[
[1 - G_d(b)] - (b - c) \cdot g_d(b) = 0. \tag{2}
\]

The best response of the DS against the bidding strategy \( b_h(c) \) of the HS is obtained by solving the following problem:

\[
\max_b \Pi_d[b; c, b_h(\cdot)]=
\]

\[
(b - c) \cdot \left[1 - G_h(b_h^{-1}(b))\right] + \int_{b_h^{-1}(c)}^{b_h^{-1}(b)} (1 - \alpha) \cdot (b_h(x) - c) \cdot dG_h(x).
\]

9 If the DS has no credible way to inform the auctioneer about its true cost, then bargaining would occur under asymmetric information. In this case, efficient bargaining is possible only in the special case in which \( \alpha = 0 \) (favoritism). See Myerson and Satterthwaite (1983).

10 Note that this bidding function is equivalent to the best take-it-or-leave-it offer that a supplier with cost \( c \) can make to a buyer with a random valuation in the interval \([0,1]\) given by the distribution function \( G_d \).
The first term is the expected profit when the DS wins the auction outright without bribery or favoritism. The second term is the expected profit when the DS loses the auction \((b > b_h)\), but bribes the auctioneer because his costs are below the bid of the HS \((b_h > c)\). In this case, the DS retains the share \((1-\alpha)\) of the surplus \((b_h - c)\). The first-order condition for this problem is

\[
(4) \quad \left[1 - G_h(b_h^{-1}(b))\right] - \alpha \cdot (b - c) \cdot g_h(b_h^{-1}(b)) \cdot \frac{db_h^{-1}(b)}{db} = 0.
\]

In order to ensure that the first order conditions (2) and (4) are sufficient to define the equilibrium bidding functions of the HS, \(b_h(c)\), and the DS, \(b_d(c)\), we need some conditions on the distributions \(G_h\) and \(G_d\).

**Lemma:** Assume \(G_i\), for \(i=h,d\), is twice differentiable and with a decreasing inverse hazard rate \(\frac{1-G_i(x)}{g_i(x)}\). Also, assume that \(J_d(x) = x - \frac{1-G_d(x)}{g_d(x)}\) is convex.

Then an equilibrium of the FPA with bribery is \((b_h(c), b_d(c))\), where \(b_h(c)\) is implicitly defined by (2), \(b_d(c) = \max \{b_h(0), \bar{b}_d(c)\}\), and \(\bar{b}_d(c)\) is the solution to (4).

The function \(J_d(x)\) is equivalent to what is known as the "virtual valuation" in the literature on auction theory. The proof of the lemma is contained in the Appendix 1.\(^{11}\) The effects of the bribe share \(\alpha\) on the bidding behavior of both suppliers are easy to characterize.

**Proposition 1:** The bidding function of the honest supplier, \(b_h(c)\) is independent of the bribe share \(\alpha\). However, the bidding function of the dishonest supplier, \(b_d(c)\), shifts downward as the bribe share increases.

**Proof:** The first result is obvious from inspection of the first-order condition (2) defining \(b_h(c)\). The second result follows from differentiating the first-order condition (4) with respect to the bid \(b\) of the dishonest supplier and the bribe share \(\alpha\):

\(^{11}\) This lemma and the proof are due to Richard P. McLean at Rutgers University.
where $\partial^2 \pi_d / \partial b^2$ is obtained by differentiating the left-hand side of (4) with respect to $b$. This derivative is negative for interior solutions of (3). QED

The DS would never bid below the minimum bid of the HS. Otherwise, the DS could raise his bid without reducing the probability of winning the auction outright. Thus, if the solution to (4) is less than $b_h(0)$, the DS simply bids $b_h(0)$.

At the highest cost realization, both suppliers bid their cost: $b_h(1) = b_d(1) = 1$. When $\alpha = 0$, the DS bids unity for all cost realizations: $b_d(c) = 1$. Thus, with favoritism, the DS does not effectively bid for the contract, but is awarded the contract whenever his cost is below the bid of the HS. On the other hand, when $\alpha > 0$, the DS will bid to win the auction outright in order to avoid paying the bribe: $b_d(c) < 1$ for all $c < 1$. It is easy to show that the DS prefers a smaller bribe share.  

We can now examine the effect of bribery on the bidding strategies of the suppliers. Proposition 1 does not fully answer this question because no value of the bribe share corresponds to a fair auction without bribery or favoritism. Even in the case for which the DS pays the full surplus as a bribe to the auctioneer ($\alpha = 1$) and thus bids aggressively to win the contract outright with the low bid, the HS still bids against the cost of the DS, and not against the bid of the DS.

Consider the first-order conditions (2) and (4) for the symmetric case $G_d = G_h$. The first term of each condition is the probability of winning the contract outright for any given bid. This is the incentive to raise the bid because the marginal increase in expected profit from a higher bid is the probability of winning the contract outright. In other words, higher bids increase the profit on contracts that the supplier wins outright in the auction. For a given cost realization common to both suppliers, the first term in (2) for the HS is always less than the corresponding term in (4) for the DS. This follows immediately from symmetry ($G_d = G_h$) and the fact that $b_h^{-1}(b) < b$. The second term in the two first-order conditions is the disincentive to raise the bid. Higher bids increase the probability of losing the auction and losing the corresponding profit on those contracts. The second term for the DS is multiplied by the bribe share $\alpha$ because the DS loses

\[
\frac{db_d(c)}{d\alpha} = -(b-c) \cdot g_h(h^{-1}(b)) \cdot \frac{db^{-1}(b)}{\partial^2 \pi_d / \partial b^2} < 0,
\]

12 For any given cost realization $c$ and bid $b$, the profit function (3) of the DS is decreasing in $\alpha$. As a result, the envelope theorem implies that a lower bribe share will increase the expected profits of the DS.
only the fraction $\alpha$ of the difference between his bid and his cost on the contracts for which his bid exceeds the bid of the HS. Thus, when $\alpha < 1$, the DS has a smaller disincentive to raise his bid, inducing him to bid less aggressively.

These two forces suggest that the DS bids less aggressively than the HS. However, there is an additional factor in the second term of the first-order condition of the DS: $db_h^{-1}(b)/db$. This factor is the slope of the inverse bidding function of the HS. When the DS increases his bid marginally above $b$, this factor is defined as the additional cost realizations of the HS for which the HS submitted a bid lower than $b$. For these contracts which are no longer won outright, the DS loses the fraction $\alpha$ of the surplus when he obtains these contracts by a bribe. This factor may well be greater than unity, in which case the DS experiences a greater probability of losing the auction outright when he increases his bid marginally. Thus, this factor increases the disincentive of the DS to raise his bid. This third force works against the two previous forces, and is the reason why there is no general result that the DS bids less aggressively than the HS.

We can also compare the bidding function of the HS to the bidding function of a supplier in a symmetric FPA without bribery. The first-order condition for the bidding function of $i$th supplier without bribery is

$$
1 - G_j(b_j^{-1}(b)) - (b - c) \cdot g_j(b_j^{-1}(b)) \cdot \frac{db_j^{-1}(b)}{db} = 0,
$$

where $j$ denotes the other supplier. Comparing this condition to the first-order condition (2) for the HS, we see that the same tradeoffs apply. The first term representing the incentive to raise the bid would be larger without bribery. However, the second term representing the disincentive to raise the bid can also be larger because the factor $db_h^{-1}(b)/db$ may be greater than 1. Thus, it is not clear in general whether favoritism toward the DS will cause the HS to bid more aggressively than he would in a FPA without bribery.

Even if the DS and HS both had the same cost distribution, there is no one-sided result on whether the HS would bid more or less aggressively than in a FPA without bribery. For this symmetric case, Arozamena and Weinschelbaum (2004) and Porter and Shoham (2004) have shown that convexity (or concavity) of the inverse hazard rate of the cost distribution is the condition that would determine whether one or more honest suppliers would bid more (or less) aggressively against a remaining supplier who had a right of first refusal at the cost distribution. However, the second term representing the disincentive to raise the bid can also be larger because the factor $db_h^{-1}(b)/db$ may be greater than 1. Thus, it is not clear in general whether favoritism toward the DS will cause the HS to bid more aggressively than he would in a FPA without bribery.

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13 This factor will be greater than unity whenever the margin between bid and cost of the HS, $bh(c) - c$, is decreasing in $c$. For example, this will be true when $g_h(c)$ is non-decreasing in $c$. It will also be true for the family of cost distributions defined in Section 6.
lowest bid of the honest suppliers. In our asymmetric case in which the HS and DS have different cost distributions, this condition is not sufficient to determine the bidding behavior of the HS.

In Burguet and Perry (1999), we examined the model with both type I and type II discretion by the auctioneer. With the addition of type I discretion, the price paid by the buyer is always equal to the bid of the HS. Given the form of the bribe, the resulting bidding behavior of the DS is solely designed to reduce the bribery payments to the auctioneer. If the DS must pay the same bribe share when type I discretion is exercised, \( \alpha (b_h - b_d) \), the bidding function of the DS would then be the same as the solution to (4) for \( \alpha = 1 \). There are also two other simple cases. If the DS must pay a bribe share for type I discretion, but not for type II discretion, the DS would bid his highest possible cost realization, \( b_d(c) = 1 \), the same as the solution to (4) for \( \alpha = 0 \). As such, the DS would only obtain the contract with the exercise of type II discretion by the auctioneer. Conversely, if the DS must pay a bribe share for type II discretion, but not for type I discretion, the DS would bid his cost (or lower) and only obtain the contract with the exercise of type I discretion by the auctioneer.

5. The Effect of Bribery on the Allocation of Contracts

With bribery, the allocation of contracts is independent of the bidding behavior by the DS, and the bribe share. As such, the allocation is determined solely by the bidding function of the HS. The bidding function of the HS depends on the distribution of costs of the DS, \( G_d \) (see equation (2)). A change in \( G_d \) that causes the HS to bid more aggressively reduces the expected allocative distortion for any given value of \( c_d \). When the difference between the bid and the cost of the HS is smaller, it is less likely that the cost of the HS \( c_d \) lies between the two. In fact, if the change in the distribution of the dishonest supplier lowers the inverse hazard rate, the HS unambiguously bids more aggressively for any cost realization. Thus, by inspection of equation (2), we obtain the following proposition.

**Proposition 2:** Given the cost realization of the dishonest supplier, the allocative distortion from bribery is larger for cost distributions of the dishonest supplier with a lower inverse hazard rate \( \frac{1 - G_d(c)}{g_d(c)} \). The allocative distortion is independent of the bribe share \( \alpha \).

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14 In Section 6, we use the power family of cost distributions to illustrate the effects of bribery on the expected price paid by the buyer. This family of cost distributions has the special property that the inverse hazard rate is linear in the cost. Thus, if the DS and HS have the same cost distribution within this family, the HS would bid the same in a FPA with or without bribery.
Asymmetric FPAs generate allocative distortions because a supplier who bids more aggressively will be awarded the contract in some situations where his costs are higher than that of the rival who bids less aggressively. As a consequence, the allocative distortion in an asymmetric FPA without bribery occurs by awarding the contract to the weaker supplier in cases where he has a higher cost.\textsuperscript{15}

In contrast, an asymmetric FPA with bribery allocates the contract to the DS whenever the cost of the DS is below the bid of the HS. Since the HS bids above his cost, the FPA with bribery distorts the allocation by awarding the contract to the DS in cases where he has a higher cost. However, this distortion occurs irrespective of whether the DS is stronger or weaker than the HS.

When the HS is stronger, a FPA with or without bribery favors the DS by allocating the contract to him in some cases when he has a higher cost. However, unlike a FPA without bribery, the allocative distortion in a FPA with bribery does not vanish as the costs of the suppliers approach the lowest possible realization.

As in a FPA, an optimal mechanism favors allocating the contract to the weaker supplier.\textsuperscript{16} Then, once again, a FPA with bribery distorts the allocation in the opposite direction from the optimal mechanism when DS is the stronger supplier. Moreover, even when the DS is the weaker supplier, the distortion with bribery is very different from the optimal distortion. Indeed, in order to reduce informational rents with the smallest negative impact on efficiency, an optimal mechanism typically distorts the allocation in favor of the weaker DS more at the higher cost realizations for the HS, whereas the distortion disappears as the cost approaches the lowest possible realization. In a precise sense, bribery distorts the allocation in the opposite direction from the optimal allocation mechanism.

Indeed, if the cost distribution of the DS is characterized by a monotone inverse hazard rate, i.e., if \( \frac{1-G_d(c)}{g_d(c)} \) is decreasing, then the margin \( b_h(c) - c \) is decreasing in \( c \). This follows immediately from (2). That is, \( b_h(c) - c \) attains its highest value at \( c = 0 \), and then decreases to zero as the cost \( c \) increases to one. Therefore, when the cost of the HS is close to 1, the HS almost always wins the contract whenever it is efficient for him to win. However, when the cost of the HS is close to 0, the DS obtains the contract frequently when his cost is higher than the cost of the HS: \( b_h(c_h) > c_d > c_h \).

\textsuperscript{15} See Lebrun (1999) and Waehrer (1999) for a more general examination of this issue.

\textsuperscript{16} See Myerson (1981), and McAfee and McMillan (1989). Rothkopf, Harstad, and Fu (2003) discuss a model in which a particular form of type I corruption can be used by a buyer to “subsidize” a weaker supplier with higher costs. They find that the buyer can benefit by increasing the price paid to the weaker supplier above his winning bid.
This discussion demonstrates that bribery is not an alternative way of introducing distortions in the allocation of the contracts that will reduce the rents of suppliers and reduces the expected price paid by the buyer in line with optimal mechanisms.\footnote{When suppliers have uniform distributions with different lower bounds on their domains, Lee (2004) has shown that awarding a right of first refusal ($\alpha=0$) to the weaker supplier, introduces precisely the type of distortions that an optimal auction introduces, and may result in a lower price for the buyer as well.} This notwithstanding, the next section shows that bribery need not result in a higher expected price paid by the buyer.

6. The Effect of Bribery on the Expected Price Paid by the Buyer

In this section, we examine the effect of bribery on the expected price. One might expect that the expected price paid by the buyer would increase with bribery. First, bribery induces non-optimal allocative distortions. Second, the bribery payment to the auctioneer is a tax on the procurement transaction. One might also expect these problems to be more acute when the bribe share paid to the auctioneer is higher.

For several reasons, we will show that these results need not obtain. First, bribery may induce the HS to bid more aggressively. Second, from Proposition 1, the DS bids more aggressively when the bribe share is higher. In fact, we can show that

**Proposition 3:** The expected price paid by the buyer is lower when the bribe share $\alpha$ is larger.

**Proof:** Notice that the bid of the HS is independent of $\alpha$. On the other hand, from our previous lemma, we found that $\frac{db_d(c)}{da} < 0$ for $c < 1$. Since the price paid by the buyer is simply the $\min \{b_d, b_h\}$, the expected price is lower when $\alpha$ is larger for any realization of the cost $c_h$. QED.

Now, if we again consider equations (2) and (4), we notice that for $\alpha = 1$, (4) is the same first-order condition that DS solves when competing in a FPA without bribery. Equation (2), on the other hand, is the bidding function of a supplier competing against a “virtual” competitor that bids its cost. We know that the HS need not bid uniformly more aggressively than in a FPA without bribery. Even if the HS does bid more aggressively, the DS need not bid uniformly more aggressively. However, both the HS and DS could bid more aggressively over a
sufficient range of costs such that the expected price paid by the buyer would decline.

In order to examine this possibility, we define the one-parameter power family of distribution functions \( G(c;t) = 1 - [1 - c]^t \) over the support \([0,1]\) where \( c \) is the cost and \( t > 0 \) is a parameter which can vary between the suppliers.\(^{18}\) The corresponding density function is \( g(c;t) = t \cdot [1 - c]^{t-1} \). For higher \( t \), \( G(c;t) \) is higher and \( \frac{1 - G_d(c)}{g_d(c)} \) is lower. The supplier with a higher \( t \) is stronger in the sense of both first-order stochastic dominance and hazard rate dominance.

Let \( t_h \) define the cost distribution of the HS and \( t_d \) define the cost distribution of the DS, and denote \( G_d(c) = G(c;t_d) \) and \( G_h(c) = G(c;t_h) \). The equilibrium bidding function of the HS from (2) has the following linear form:

\[
(7) \quad b_h(c) = \frac{1}{1 + t_d} + \frac{t_d}{1 + t_d} \cdot c .
\]

The equilibrium bidding function of the DS from (4) is slightly more complicated. It takes the following form:

\[
(8) \quad b_d(c;\alpha) = \frac{1}{1 + \alpha \cdot t_h} + \frac{\alpha \cdot t_h}{1 + \alpha \cdot t_h} \cdot c \quad \text{for } c > c',
\]

\[
= \frac{1}{1 + t_d}, \quad \text{for } c \leq c',
\]

where \( c' = \max\{0, \frac{\alpha \cdot t_h - t_d}{\alpha \cdot t_h \cdot (1 + t_d)}\} \). Figure 1 depicts representative bidding functions of the suppliers for the two expressions of \( c' \) in (8). If the two suppliers are symmetric within this power family of cost distributions \( t = t_d = t_h \), then the HS bids just as he would in a FPA without bribery.\(^{19}\) The DS bids uniformly less aggressively than the HS for all bribe shares \( \alpha < 1 \). However, if the auctioneer extracts all the surplus from the DS \( (\alpha = 1) \), then both suppliers bid just as they

\(^{18}\) Waehrer and Perry (2003) show that this distribution function is not as restrictive as it might seem. Distributions of this form follow directly from natural properties, particularly a property corresponding to constant returns to scale.

\(^{19}\) This finding follows from that fact that the inverse hazard rate is linear for the power family of cost distributions. See Arozamena and Weinschelbaum (2004) and Porter and Shoham (2004).
would in a symmetric FPA without bribery. In Appendix 2, we generalize these bidding functions to the case of multiple symmetric honest suppliers.

\[ b_d(c;0) = 1 \]

\[ b_d(c;1/9) \]

\[ b_h(c) \]

\[ b_d(c;1/3) \]

\[ b_d(c;1) \]

\[ c' \]

\[ 1 \]

\[ td = 1 \text{ and } th = 3 \]

\[ \alpha = \{0, 1/9, 1/3, 1\} \]

**Figure 1: Bidding Functions**

We can now examine the expected price with this family of distributions. We first compare the expected price in the FPA with bribery to the expected price that would arise with an efficient auction, like the second-price auction (SPA).\(^{20}\) Let \( E_p(\alpha; t_d, t_h) \) be the expected price in the FPA with bribery, and \( E_{pe}(t_d, t_h) \) be the expected price in an efficient auction. See Appendix 2 for the expressions of these expected prices. The following proposition provides comparisons of these two expected prices.

**Proposition 4:** (i) When \( t_d > t_h \), the expected price in a first-price auction with bribery can be below the expected price in a second-price auction. In particular,

\(^{20}\) Without bribery, the allocation of the contract virtually determines the total surplus that can be divided between the buyer and the suppliers, and how it is divided. This is an implication of the Revenue Equivalence Theorem. See Myerson (1981). Thus, since a SPA allocates the contract to the lowest cost supplier, the expected price in a SPA provides the natural reference point for the division of total surplus in any efficient auction.
there exists a set of bribe shares \([\alpha, 1]\) such that for any \(\alpha\) in this set, 
\[E_{p}(t_d, t_h) > E_{p}(\alpha; t_d, t_h).\]

(ii) When \(t_d \leq t_h\), the expected price in a first-price auction with bribery is above the expected price in a second-price auction. In particular,
\[E_{p}(t_d, t_h) \leq E_{p}(\alpha; t_d, t_h) \quad \text{for all } \alpha \leq 1, \text{ and}
E_{p}(t_d, t_h) < E_{p}(\alpha; t_d, t_h) \quad \text{for } t_d \neq t_h \text{ or } \alpha < 1.

The proof of Proposition 4 is contained in Appendix 2. Appendix 2 also provides a generalization of Proposition 4 to the case of multiple symmetric honest suppliers.

Proposition 4 states that, for this power family of cost distributions, the distortions that bribery introduces (discussed in Section 4) help reduce the price paid by the buyer when the DS is the stronger supplier. As we discussed before, the optimal rent-reducing distortion would favor allocation of the contract to the weaker supplier in order to reduce the information rents of the stronger supplier. However, allowing the opportunity for the stronger supplier to bribe the auctioneer can also achieve the ultimate goal of a lower expected price. This may seem paradoxical. The explanation is that, when the auctioneer appropriates part of the rents in the form of bribe, the buyer should also be concerned that these rents will cause an increase in the expected price. As a result, the auction-theoretic device of decomposing the expected price into the sum of the expected cost and the information rents is less helpful in analyzing the effect of distortions on expected prices in auctions with bribery. Instead we should consider the expected price directly as the winning bid. By doing so, we realize that the only advantage of bribery for the buyer is that the HS faces fiercer competition in that he must beat the cost of the DS instead of the "bid" of this DS.21 Consider the case \(\alpha = 1\). If the DS is the weaker supplier, bribery can only have a small impact on the bid of the stronger HS. The reason is that the "bid" of the weaker DS is very close to his cost. Thus, competing against the cost of the DS instead of his "bid" provides very little incentive for the stronger HS to bid more aggressively. The situation is reversed if the DS is the stronger supplier. In an (efficient) auction there is a large gap between the cost and the "bid" of the stronger DS. Thus, the competition faced by the weaker HS becomes much tougher when he has to beat the cost of the DS instead of the "bid" of the DS. This provides a stronger incentive for the weaker HS to bid more aggressively.

\[21\] Since we are discussing the comparison to an efficient auction here, "bid" should be understood as the expected price conditional on winning.
As in Section 4, Proposition 4 illustrates the direct effect of bribery on how aggressive the DS bids. But there is also a "slope effect". The change in the slope of the bidding function of the HS has less intuitive effects on the bidding function of the DS. The importance of these effects is clearer when we consider the effects of bribery on a FPA. Proposition 4 only compares the asymmetric FPA with bribery to an efficient auction with no allocative distortions. However, it is also interesting to compare the expected prices in an FPA with and without bribery. Let $Ep_1(t_d,t_h)$ be the expected price paid by the buyer in a FPA without bribery. This comparison is straight-forward for the symmetric case. When $\alpha < 1$, $Ep(\alpha;t,t) > Ep(1;t,t) = Ep_1(t_d,t_h) = Ep_2(t,t)$. For the asymmetric case, the comparison is more difficult because the asymmetric FPA without bribery does not have analytic solutions for the bidding functions even with this power family of cost distributions. Nevertheless, using numerical computations, we can show that bribery may increase or decrease the expected price paid by the buyer in a FPA when $\alpha$ is close to 1. When $t_d = 4$ and $t_h = 1$, the expected price paid by the buyer is .4943 in a FPA without bribery. (See Marshall, Meurer, Richard, and Stromquist, 1994.) Alternatively, with bribery and $\alpha = 1$, the expected price is .4958. Thus, bribery increases the expected price paid by the buyer, even though $\alpha = 1$. However, when $t_d = 3$ and $t_h = 2$, the expected prices in a FPA without and with bribery are .4125 and .4122, respectively. Thus, bribery reduces the expected price for this case. Notice that both cases are in the region $t_d > t_h$. Thus, even though the stronger supplier is the DS in both cases, the effect of bribery on the expected price can have either sign.

We can further illustrate the comparison using the Bidcomp2 program of Li and Riley (1999) to compute the asymmetric FPA without bribery. Figure 2 illustrates the expected price with and without bribery for values of the capacity parameters $(t_d,t_h)$ with $\alpha = 1$. 

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Figure 2: Expected Prices
First-Price Auction with ($\alpha = 1$) and without Bribery

The expected prices are equal along the diagonal (a property special to the power family of cost distributions), and the expected price with bribery is always greater when $t_d < t_h$. However, when $t_d > t_h$, the expected price with bribery is lower when $t_d$ is only somewhat larger than $t_h$. The expected price with bribery is again higher in the lower right region where $t_d$ is substantially greater than $t_h$. The previous examples from Marshall, Meurer, Richard, and Stromquist(1994) are marked by asterisks in Figure 2.

7. Conclusion

We have examined the effects of bribery on the behavior of suppliers and the outcome of a FPA. In the particular form of bribery that we have considered, one supplier bribes the auctioneer in order to revise his bid downward when this is necessary to win the contract and profitable. We have shown that this form of bribery need not make the dishonest supplier bid less aggressively or the honest supplier bid more aggressively. Bribery distorts the allocation of the contract. In cases where the dishonest supplier is the ex ante stronger supplier, bribery reverses the distortion that would exists in a FPA without bribery. Even when the dishonest supplier is ex ante weaker, bribery alters the allocation of the contract in fundamental ways. In particular, an inefficient allocation of the contract to the
weaker dishonest supplier occurs with high probability even when the cost of the stronger honest supplier is very low. For similar reasons, these properties of the allocation in the FPA with bribery contrast sharply with the allocation induced by optimal mechanisms. The effects of bribery on the expected price paid by the buyer are subtle. When the auctioneer appropriates a large fraction of the surplus, the price for the buyer may in fact be lower than in the absence of bribery. This in fact occurs when the stronger supplier is the dishonest supplier with the opportunity to bribe the auctioneer.

There are other interesting questions that could be addressed with variants of this model. We have used some to analyze the effect of bribery on the incentives to invest, and the resulting implications for trade (Burguet and Perry (2006)), and the incentives for the sale of preference by a buyer in the form of a right of first refusal (Burguet and Perry (2003, revised 2005)). Despite its simplicity, the model highlights subtle but intuitive consequences of auctioneer discretion in a wide range of contracting scenarios.

Appendix 1: Second-Order Conditions

Monotonicity of the inverse hazard rate implies (inverse) monotonicity of $J_d(b)$. The first order condition (2) can be written as:

$$g_d(b) \cdot [c - J_d(b)] = 0.$$  

The derivative of the left-hand side is

$$g'_d(b) \cdot [c - J_d(b)] - g_d(b) \cdot J'_d(b),$$

which, using the first-order condition, shows that the second-order conditions are satisfied under our assumptions. Thus, (2) defines an interior solution of (1). Let $b_h(c) = J_d^{-1}(b)$ as implicitly defined in (2). Note that since $J_d(b)$ is increasing and convex, $b_h(c)$ is increasing and concave. Now (4) can be written as

$$-g_h(J_d(b)) \cdot \left[ \frac{\alpha(b-c)}{b'_d(J_d(b))} - \frac{1 - G_h(J_d(b))}{g_h(J_d(b))} \right] = 0.$$

As before, the second-order conditions are satisfied if the term in brackets is increasing in $b$. Since $J_d(b)$ is increasing, it suffices to show that

$$\frac{\alpha(b_h(x)-c)}{b'_h(x)} - \frac{1 - G_h(x)}{g_h(x)}$$
is increasing in x. The second term is decreasing, and the first term is increasing since $b_d(c)$ is concave. Thus, second-order conditions are satisfied. The Lemma follows immediately.

**Appendix 2: Proof of Proposition 4**

Using the bidding functions from Section 6, we can calculate the expected price paid by the buyer in the FPA with bribery:

\[
\text{Ep}(\alpha; t_d, t_h) = \frac{1 + t_d + t_h}{(1 + t_d)(1 + t_h)} - \frac{t_d}{(1 + t_d)(1 + t_h)} \left( \frac{1 + t_d}{t_d} \right) \left( \frac{1 + \alpha \cdot t_h}{1 + \alpha \cdot t_h} \right)^{1 + t_h}
\]

for $\alpha \leq \frac{t_d}{t_h}$ (Case 1).

\[
\text{Ep}(\alpha, t_d, t_h) = \frac{1}{1 + t_d} + \frac{t_d}{(1 + t_d)(1 + t_h)} \left[ \frac{t_d}{1 + t_d} \right]^{1 + \alpha \cdot t_h} \left( \frac{1 + \alpha \cdot t_h}{\alpha \cdot t_h} \right)^{t_d}
\]

for $\alpha \geq \frac{t_d}{t_h}$ (Case 2).

Also, we can calculate the expected price in a SPA:

\[
\text{Ep}_2(t_d, t_h) = \frac{1 + t_d + t_h}{(1 + t_d)(1 + t_h)} - \frac{t_d t_h}{(1 + t_d)(1 + t_h)(1 + t_h + t_d + t_h)}
\]

**Proof of Proposition (4):**

(i) When $t_d > t_h$, expression (A1) for the expected price applies over the entire range of the bribe share of $\alpha \leq 1$. This expected price is continuous and declines as $\alpha$ increases over this range. At $\alpha = 0$, this expected price is clearly greater than the expected price under a SPA from (A3). However, at $\alpha = 1$, the condition that $t_d > t_h$ implies that this expected price is lower than the expected price under a SPA. Thus, there exists an $\bar{\alpha}$ such that for all $\alpha > \bar{\alpha}$, bribery would result in a lower expected price. The value of $\bar{\alpha}$ can be easily defined as

\[
\bar{\alpha} = \frac{1}{t_h} \cdot Q, \quad \text{where} \quad Q = \left[ \frac{t_h}{1 + t_d} \left( \frac{t_d}{1 + t_d} \right)^{t_h} \right]^{1/(1+t_h)}.
\]

Note that $\bar{\alpha}$ need not be close to unity. If $t_h = 1$, then $\bar{\alpha} = .89$ when $t_d = 2$, $\bar{\alpha} = .75$ when $t_d = 3$, $\bar{\alpha} = .67$ when $t_d = 4$. It is also easy to show that $\bar{\alpha}$ is decreasing in $t_d$.

(ii) When $t_d < t_h$, expression (A1) for the expected price applies only for $\alpha \leq t_d/t_h$. But under the condition that $t_d < t_h$, this expected price is greater that the expected
price under a SPA from (A3) at $\alpha = t_d/t_h$, and thus for all $\alpha \leq t_d/t_h$. When $t_d = t_h$, expression (A1) applies for all $\alpha \leq 1$, exceeds the expected price in a SPA for $\alpha < 1$, and equals the expected price in a SPA for $\alpha = 1$. Q.E.D.

With the power family of cost distributions, we can solve for the asymmetric case in which there is one dishonest supplier but two or more symmetric honest suppliers. If the DS has capacity $t_d$ and $n$ honest suppliers each have the same capacity $t_h$, the equilibrium bidding function of each HS will be $b_h(c;n) = (1+T_d c)/(1+T_d)$, where $T_d = t_d + (n-1)t_h$. Clearly, this bidding function becomes more aggressive as the number of honest suppliers is increased. Also note that in the fully symmetric case when $t_d = t_h$, the bidding function of the honest suppliers is the same as it would be in a FPA without bribery. This follows from the results in Arozamena and Weinschelbaum (2004) and Porter and Shoham (2004) because the inverse hazard rate of the power family of cost distributions is linearly decreasing in the cost. The equilibrium bidding function of the DS would then be $b_d(c; n) = (1+\alpha n t_h c)/(1+\alpha n t_h)$ for $c > c'$, and $b_d(c; n) = 1/(1+T_d)$, for $c \leq c'$, where $c' = \max\{0, \{\alpha n t_h - T_d\}/[\alpha n t_h (1+T_d)]\}$. Thus, the bidding functions are parallel to those from (7) and (8), with $T_d$ replacing $t_d$ and $nt_h$ replacing $t_h$. Proposition 4 still applies to this case with an arbitrary number $n$ of honest suppliers. Of course, the lower bound $\alpha$ on the set of bribe shares that generate a lower expected price would be modified. The expressions in (A1) and (A2) are modified as follows:

(A4) \[
Ep(\alpha; t_d, t_h, n) = \frac{1+T_d + nt_h}{(1+T_d)(1+nt_h)} - \frac{t_d}{(1+nt_h)(1+t_d + nt_h)} \left[ \frac{1+T_d}{T_d} \right]^{nt_h} \left[ \frac{\alpha \cdot nt_h}{1+\alpha \cdot nt_h} \right]^{1+nt_h}
\]
for $\alpha \leq T_d/(n \cdot t_h)$ (Case 1),

(A5) \[
Ep(\alpha, t_d, t_h, n) = \frac{1}{(1+T_d)} + \frac{T_d}{(1+T_d)(1+t_d + nt_h)} \left[ \frac{T_d}{1+T_d} \right]^{t_d} \left[ \frac{1+\alpha \cdot nt_h}{\alpha \cdot nt_h} \right]^{t_d}
\]
for $\alpha \geq T_d/(n \cdot t_h)$ (Case 2).

The expression for the expected price in a SPA auction is also easily modified as

(A6) \[
Ep_2(t_d, t_h, n) = \frac{1+T_d + nt_h}{(1+T_d)(1+nt_h)} - \frac{t_d \cdot nt_h}{(1+T_d)(1+nt_h)(1+t_d + nt_h)}
\]
The result that corresponds to Proposition 4(i) can be proved in the same manner as the case of \( n = 1 \). In particular, \( T_d \) replaces \( t_d \) and \( nt_h \) replaces \( t_h \) in the inequality comparing the expected prices when \( \alpha = 1 \). The result that there exists an \( \bar{\alpha}(n) < 1 \) for \( t_d > t_h \) follows from the fact that: \( t_d > t_h \) if and only if \( T_d > nt_h \).

The result that corresponds to Proposition 4(ii) can also be proved in the same manner as the case of \( n = 1 \).

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