Abstract: The ability to control light over very small distances is a problem of fundamental importance for a vast range of applications in communications, nanophotonics, and quantum information technologies. For this purpose, several methods have been proposed and demonstrated to confine and guide light, for example in dielectric and surface plasmon polariton (SPP) waveguides. Here, we study the interaction between different kinds of planar waveguides, which produces dramatic changes in the dispersion relation of the waveguide pair and even leads to mode suppression at small separations. This interaction also produces a transfer of power between the waveguides, which depends on the gap and propagation distances, thus providing a mechanism for optical signal transfer. We analytically study the properties of this interaction and the power transfer in different structures of interest including plasmonic and particle-array waveguides, for which we propose an experimental realization of these ideas.
Optical excitations propagating in planar waveguides find application to optical-signal processing [1], and they pose interesting questions from a fundamental viewpoint regarding the behavior of light over small distances. Different realizations of planar waveguiding include light confinement in dielectric films [2], surface plasmon polaritons (SPPs) [3–5], and planar particle arrays [6]. Despite ohmic losses in metals, SPPs have the advantage of concentrating light energy over small regions compared to the wavelength [7]. In contrast, dielectric films can propagate light over much longer distances and are versatile elements used in integrated optics [8]. Particle arrays have been demonstrated in 1D chains [9, 10], and they are intimately related to 2D photonic crystal waveguides [11]. Guided modes in the latter depend strongly on the environment and their existence can undergo a sudden transition as the degree of dielectric asymmetry is varied [6].

Here, we study the interaction between planar waveguides placed in close proximity. The guided modes of these systems are a combination of the modes in the individual waveguides, which are hybridized due to their mutual interaction. We focus on the transfer of optical energy between two parallel waveguides as a major in-out problem of signal processing and communications, and we consider the particular case of two gently curved waveguides, so that a parallel waveguide configuration can be assumed at each position along the guiding direction as an adiabatic approximation [12]. We show the energy transfer to strongly depend on the bending radius of the waveguides and their minimum gap separation. For small gaps, only one mode is supported, thus producing a sharp transition in the output power.

Planar waveguides are structures that confine light in one spatial direction and allow free propagation along the remaining two dimensions. The normal wave vector $\kappa_z$ is imaginary outside the waveguide, thus producing evanescent fields that cannot propagate away from it. These evanescent tails spread out of the guiding structure with a characteristic decay penetration distance $L_z = 1/\kappa_z$ [13].

In Fig. 1(a-c) we show characteristic dispersion relations for several kinds of waveguides as a function of wave vector $k_\parallel$ parallel to the plane of the structure. Particle array waveguides (a) have a confining resonance within the First Brillouin Zone (FBZ). This resonance defines a dispersion relation with vanishing group velocity when the parallel wave vector approaches $\pi/a$ and degenerate TE and TM modes when the particles are spherical [6]. Dielectric slabs (b) show a variety of TM and TE modes, with the fundamental TM mode having no cutoff wavelength. These modes lie in between the light line in the surrounding medium and the light line in the dielectric slab material. Finally, metal-dielectric interfaces (c) and metal films can sustain propagating surface-plasmon polaritons (SPPs) [3].

The confining planar waveguides in a parallel-waveguide configuration interact with each other when placed in close proximity via the evanescent tails of the modes. The resonances of the resulting structure are then determined by the Fabry-Perot condition [15]

$$1 - r_1^2 r_2^2 \exp(-2\kappa_z d) = 0,$$

(1)
**Fig. 1.** (a-c) Schematic view and dispersion relations for different planar waveguide structures. The guided mode dispersion relations are shown in red in these frequency-momentum plots. (a) Planar array of dielectric particles. The light cone and the diffracted light line are shown as black lines. (b) Dielectric slab. Straight lines show the light cones in vacuum (black) and the dielectric medium (green). (c) SPP supporting metal-dielectric interface. Straight lines indicate the light cone (solid line) and the electrostatic surface-plasmon frequency (dashed line). (d-f) Interaction between different combinations of confining structures placed at varying distance $d$. A similar behavior is observed in all cases, with a strong repulsion between modes at small distances. (d) Two planar arrays of silicon ($\varepsilon_{\text{Si}} = 12$) spheres of radii 200 nm and 175 nm, respectively, embedded in silica ($\varepsilon_{\text{SiO}_2} = 2$) and arranged in square lattices of period 1 $\mu$m. The light wavelength is $\lambda = 4$ $\mu$m. (e) Planar array of silicon spheres of radius 200 nm arrange in a square lattice of period 1 $\mu$m placed near a dielectric slab of thickness 70 nm and permittivity $\varepsilon = 6$. (f) Slot waveguide consisting of a silver-air-silver structure [14] illuminated with $\lambda = 1550$nm light.

where $r_{\sigma}^i$ is the reflectivity of guide $i$ for $\sigma$-polarized light and $d$ is the separation distance. This model can be easily generalized to an arbitrary number of waveguides. The results of this model are shown in Fig. 1(d-f) for different combinations of waveguides. The effect of interaction is qualitatively the same in all of these cavities: for large distances, the modes are at the position of the original non-interacting resonances; however, as the separation is decreased, mode repulsion takes place down to a critical distance below which one of the modes is pushed beyond the light line of the surrounding medium, thus being broadened and effectively non contributing to propagating optical signal due to coupling to radiation; in contrast, the remaining mode is still pushed towards larger parallel wave vectors that increase its degree of confinement. This
repulsive interaction is a common effect in many optical and plasmonic hybridized systems when similar modes are placed at interacting distances [16]. In the particular case of symmetric structures (e.g., Fig. 1(f)), the interaction breaks the mode degeneracy and produces characteristic symmetric and antisymmetric field distributions. In the metal-insulator-metal (MIM) layer structure, the metal prevents radiation losses, so that the lowest- \( k \parallel \) mode in Fig. 1(f) (the symmetric mode) is well defined down to \( k \parallel = 0 \). Nevertheless, there is a critical distance \( d \) below which the mode is suppressed, which occurs at \( k \parallel = 0 \). This point describes non-propagating modes of vanishing group velocity.

As a result of the mismatch in parallel wave vector for a given separation \( d \), the two modes accumulate different phase as they propagate along the structure. Therefore, the power density moving in each part (i.e., each individual waveguide) of the interacting structure (\( P_i \)) varies along the propagation direction when both modes are excited, thus producing power transfer back and forth between both waveguides. For simplicity, we focus on symmetric geometries (e.g., the MIM structure or a double layer of equal-size particles), in which the field amplitudes \( c_i(x) \) in waveguide \( i = 1, 2 \) at the position \( x \) along the propagation direction satisfies the equations

\[
\begin{align*}
c_1(x) &= \frac{c_s(x) + c_a(x)}{\sqrt{2}}, \\
c_2(x) &= \frac{c_s(x) - c_a(x)}{\sqrt{2}},
\end{align*}
\]

where \( c_s \) and \( c_a \) are the complex amplitudes of the symmetric and antisymmetric modes.

We consider two neighboring curved waveguides, in which the distance between them \( d \) varies very smoothly along the propagation direction. In such a system, the interaction (and therefore, also the wave vector of the modes) varies adiabatically along the propagation direction \( x \). Within the adiabatic regime, we can neglect losses and propagation constant shifts if the penetration distance of the modes outside the waveguide is much smaller than the bending radius. Treating as a perturbation all terms in the Maxwell equations that differ from the penetration distance of the modes outside the waveguide is much smaller than the bending radius. Starting with a mode prepared in guide 1 at large distances between the waveguides at position \( x_0 \), the net power in each guide at a subsequent position \( x \) reduces to [18, 19]

\[
\begin{align*}
P_1(x) &= \cos^2 \left( \int_{x_0}^{x} d' \Delta k_{||} (d'(x')) / 2 \right), \\
P_2(x) &= \sin^2 \left( \int_{x_0}^{x} d' \Delta k_{||} (d'(x')) / 2 \right),
\end{align*}
\]

where \( \Delta k_{||}(d) \) is the wave vector difference between the two modes, which is calculated from the Fabry-Perot condition (Eq. (1)), rendering \( k_{||} \) for each mode as a function of separation \( d \) (see Fig. 1). In the derivation of these equations, we consider the incident wave to be equally split between in-phase symmetric and antisymmetric modes at \( x_0 \) (this places the weight of the incident wave only in waveguide 1 according to Eqs. (2)), we follow the adiabatic evolution
Fig. 2. (a) SPP energy transfer between neighboring silver cylinders. A SPP is assumed to be excited at line A (e.g., by external illumination over a grating parallel to the left cylinder), so that it propagates along the surface polar direction, as shown by arrows. As the SPP reaches the gap between both wires, their interaction produces power transfer to the right cylinder. (b) Left: Power transfer under the configuration of (a) as a function of cylinder radius $R$ for a gap distance $d = 3 \mu m$. Right: power transfer as a function of gap distance $d$ for cylinder radius $R = 200 \mu m$. Black dots display the line A where the modes are excited.

of the modes (i.e., they pick up a phase as they propagate), and we recombine the resulting amplitudes using Eqs. (2) to obtain the power density $P_i \propto |c_i|^2$.

We have solved Eq. (3) for planar waveguides curved onto the surface of large-radius cylinders, with the guided modes propagating along the polar direction of the latter, for both particle arrays and metal-dielectric SPP-supporting interfaces (Fig. 2(a)). The results are shown in Fig. 2(b) for different values of the cylinder radius (left) and the gap distance (right) in silver-air SPP waveguides. For a fixed gap distance, the power is only partially transferred when the bending radius is small. When the radius increases so does the interaction region, thus enhancing the power transfer, until 100% transfer is achieved at $R \approx 200 \mu m$ for a $d = 3 \mu m$ gap. If we continue increasing the radius, total transfer occurs at intermediate propagation distances, from where the power would be transferred back to the first waveguide. The corresponding oscillations of the power back and forth between both curved waveguides along the propagation direction in the interaction arc is clearly observed for a large value of the radius in Fig. 3(c).

The same pattern occurs when the gap is reduced and the radius is kept constant. For large separations, the interaction is weak and only a fraction of the power is transferred between the waveguides. When the gap is small enough ($d \approx 3 \mu m$ for $R = 200 \mu m$), the power is completely transferred to the second waveguide. Smaller gap values produce several cycles of power transfer and, when the gap distance is smaller than a critical value, the suppression of one of the hybrid modes leads to a featureless distribution of power on both sides of the system. In Fig. 3(a), we show that the power changes rapidly between both waveguides when we increase the bending radius $R$ or decrease the gap distance $d$. When the constitutive guiding structure is
surrounded by a dielectric material, such as in Fig. 3(b), the modes become leaky below the light line and the gap distance has larger critical values, and therefore the changes in the output signal are slower and occur for larger values of the radius.

In summary, we have analyzed the dispersion relations of planar waveguides placed in close proximity. The interaction produces repulsion between the modes of the individual waveguides. The hybridized modes of the interacting structures propagate at different phase velocities, thus producing a power transfer between both waveguides that depends on the traveled distance. When the gap distance is varied adiabatically (e.g., by gently bending the waveguides so that they form cylinders of large radius compared to the minimum gap separation between them), a net power transfer is produced that exhibits a smooth dependence on bending radius and minimum gap distance. Eventually, when the gap is below a critical value, only a single mode is able to propagate inside the waveguide and the system sharply changes to an equally distributed power output. The present study is relevant for advancing in the in/out coupling problem, which is still a major issue in the design of integrated optics waveguides based upon plasmons and particle arrays.

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