Four-wave mixing of an intense continuous-wave pump beam with an ultrashort soliton signal in an optical fiber is theoretically analyzed. A novel class of stable two-color coupled solitary waves is found. These vector parametric solitons represent the optimal frequency conversion of an ultrashort pulse. © 1998 Optical Society of America

As is well known, optical phase conjugation undoes optical signal distortions on propagation in dispersive and nonlinear media.1 Generation of the conjugate replica of a signal at the middle of a fiber-optic transmission link can compensate for dispersive and nonlinear broadening.2 When it is applied to soliton systems, mid-span spectral inversion also removes other important transmission capacity-limiting effects such as pulse interactions.3 Four-wave mixing in fibers can also be exploited for wavelength shifting and time demultiplexing of high-bit-rate signals. For efficient and broadband frequency conversion, in which soliton pulses recirculate in a fiber loop of length L and a short section of cw-pumped fiber (of length Lp) provides parametric gain,8 Moreover, for λp close to the zero-dispersion wavelength, Vg ∼ Vc ∼ Vp, whereas β2s ∼ −β2c = β; with no restriction on the generality of our treatment, we can take β2s < 0. Under the above simplifying conditions, we average Eqs. (1) over a circulation inside the fiber loop and obtain dimensionless coupled nonlinear Schrödinger equations for the signal and conjugate pulses:

\[
i \frac{\partial E_s}{\partial z} + i \frac{\partial E_s}{\partial t} - \frac{\beta_{2s}}{2} \frac{\partial^2 E_s}{\partial t^2} + i \Gamma_p E_s + R \left[ |E_s|^2 + 2(|E_s|^2 + |E_p|^2) \right] E_s + R E_p^2 E_s^* \exp(-i\delta kz) = 0,\]

\[
i \frac{\partial E_c}{\partial z} + i \frac{\partial E_c}{\partial t} - \frac{\beta_{2c}}{2} \frac{\partial^2 E_c}{\partial t^2} + i \Gamma_c E_c + R \left[ |E_c|^2 + 2(|E_s|^2 + |E_p|^2) \right] E_c + R E_p^2 E_c^* \exp(-i\delta kz) = 0.\]
Moreover, $\Delta k = \delta k - 2RP \eta$ is the total wave-vector mismatch, $P = |E_0|^2$ is the cw pump power, and $\eta = l_p/l \leq 1$. In Eqs. (2) the loss coefficient is $\alpha = 1 - \nu_0$, the parametric coupling coefficient reads as $\gamma = \eta RP \nu_0$, and $\rho = - (\delta k/2 + \eta RP) \nu_0$.

We intend to study here the existence and stability of coupled solitary-pulse solutions of Eqs. (2). Next, we investigate the possibility of generating stable two-component waves by injecting a signal pulse into the fiber. The vector solitons thus represent an optimal solution for the phase conjugation of a stream of ultrashort pulses. Before we consider the pulselike solutions of Eqs. (2), a simple analysis permits us to identify the conditions for stability of the low-power background. We can find the conditions for damping the continuum in Eqs. (2) (neglecting at first the presence of solitons as source of radiation) by considering a cw solution in the form $(u, v) = (A, B) \exp(i \lambda z)$. Neglecting nonlinear terms and time derivatives, we find that the conditions $\alpha > 0$ and $\alpha^2 > \gamma^2 - \rho^2$ are necessary for the suppression of the cw growth.

Let us consider now the coupled bright solitary waves of Eqs. (2). Figure 1 shows the evolution with distance $Z$ of the amplitudes of a two-component stationary solution of the type

$$u(Z, T) = U(T) \exp(-iqZ), \quad v(Z, T) = V(T) \exp(-iqZ),$$

where $q$ is a real constant. We found the two complex amplitudes $U(T)$ and $V(T)$ and wave vector $q$ of the coupled pulselike stationary solutions numerically by inserting ansatz (3) into Eqs. (2) and by applying a shooting procedure to the resulting set of second-order ordinary differential equations. In Fig. 1 the stationary solution (with $q = 0.26$) was used as the initial condition for Eqs. (2), which was then integrated along distance $Z$ by a standard split-step beam propagation code. In Fig. 1 the linear loss $\alpha = 0.229$ exactly balances the parametric gain $\gamma = 0.4$, whereas the wave-vector mismatch $\rho = 1$ compensates for the self- and cross-phase modulation phase shifts between the signal and conjugate pulses. As shown in Fig. 1, the numerical integration of Eqs. (2) clearly shows that the stationary solution is spatially unstable: Both pulse components decay at $-Z = 30$. We investigated the linear stability of the two-component bright pulse solutions by considering the perturbed solutions

$$u(Z, T) = [U(T) + f(Z, T)] \exp(-iqZ), \quad v(Z, T) = [V(T) + g(Z, T)] \exp(-iqZ),$$

where

$$f(Z, T) = f_0(T) \exp(\delta Z), \quad g(Z, T) = g_0(T) \exp(\delta Z).$$

By inserting Eq. (4) into Eqs. (2) and after linearizing the equations for perturbations $f$ and $g$ about a vector stationary wave solution, we obtained the profile of the most unstable perturbation eigenmode and its corresponding growth rate $\delta$ by means of a Crank–Nicholson procedure as described in detail else-

where. For example, for the case in Fig. 1 one finds the exponential growth rate coefficient $\delta = 0.473$, in full agreement with numerical estimations obtained by integration of Eqs. (2) with different step sizes.

By slightly increasing the linear loss to $\alpha = 0.277$ while keeping the other parameters fixed, we found that the numerical integration of Eqs. (2) with an initial condition given by the unstable stationary wave profile (in this case, $q = 0.21$) with an additional small field proportional to the corresponding eigener-perturbation evolved into a new stable coupled pulselike solution, as shown in Fig. 2. Note that the $V$ component of the vector soliton has a dip in its center and is temporally broader than its $U$ counterpart. This stable vector soliton represents a pair of signal and conjugate pulses of comparable amplitude and time width. Therefore, under such conditions, the signal pulse propagates along the fiber unchanged, along with a similar phase-conjugate replica. Clearly, each individual pulse of this coupled solitary pair is not in itself a soliton solution of the scalar nonlinear Schrödinger equation.

We numerically investigated (Fig. 3) the basin of attraction from the unstable into the stable coupled solitary waves as a function of both parametric gain $\gamma$ and loss $\alpha$ coefficients. As can be seen from Fig. 3, there is a relatively narrow stripe in the $(\gamma, \alpha)$ plane (the shaded area), which leads to decay from an unstable into a stable pulse pair. Above the boundary of existence of these asymptotically stable solutions, the unstable waves evolve into $Z$-periodic localized solutions, or breathers.
of a single pulse injected at the signal wavelength (along with the cw pump). We thus performed extensive numerical solutions of Eqs. (2) with an initial signal pulse of the form $u(Z = 0, T) = A \, \text{sech}(\omega T)$, and vanishing idler $v(Z = 0, T) = 0$. The filled circles in Fig. 5(a) illustrate the basin of attraction from the above initial conditions into stable coupled solutions as a function of initial amplitude $A$ and inverse width $\omega$. The open triangles represent the initial conditions that led to decay of both pulses. The dotted curve in Fig. 5(a) illustrates the relation $2A^2/\omega = E_{ss}$, where $E_{ss}$ is the energy corresponding to the stable stationary solution. Quite remarkably, Fig. 5(b) reveals that the basin of attraction into stable conjugate solitary waves is even broader when the input pulse is injected into the $v$ component, that is, for a signal in the normal GVD regime (and a conjugate pulse in the anomalous GVD). Indeed, the solitonic pulse compression mechanism inherent in the anomalous GVD regime enhances the reorganization of energy at the idler frequency into a narrow pulse. In real units, with $t_0 = 3.1$ ps, $\beta = -1$ ps$^2$/km, fiber loss 0.24 dB/km, and $R = 3.3$ (W/km)$^{-1}$, we obtain $z_0 = 10$ km and $\alpha = 0.277$. Moreover, with $n = 0.1$, the conditions $\gamma = 0.4$ and $\rho = 1$ are satisfied with pump power $P = 120$ mW, $\beta_{sp} = -0.2$ ps$^2$/km, and a pump–signal wavelength spacing of 1.8 nm.

We have shown the existence of stable coupled conjugate pulses in fiber four-wave mixing. These waves can be generated by injection of a single pulse at the signal wavelength and permit time localization to be maintained in the ultrashort conjugate pulse.

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