Dyakonov surface wave resonant transmission

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Abstract: The role of Dyakonov surface waves in the transmission through structures composed of birefringent media is theoretically explored. In the case of structures using prisms, unexpected high transmission above the critical angle due to resonant excitation of Dyakonov surface waves is predicted. This transmission is produced only when TE polarized incident wave reaches the interface supporting the surface waves within a narrow interval of angles, for both the angle of incidence and the angle with respect to the optic axis of the birefringent medium. As a result, over 90% transmission for a single and isolated peak confined in the two transversal directions, with hybrid TE and TM polarization, can be obtained.

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References and links

1. Introduction

Light transmission through a multilayer structure is dictated by the interferences resulting from the multiple reflections at interfaces. Transmission is governed by Fresnel relations in the simplest case of one interface, and by resonant Fabry-Perot (FP) modes in the case of two interfaces [1]. Guided modes existing in the structure are not usually excited and do not play an important role in the transmission. An exception occurs when the wave vector component parallel to the interfaces of the incident field equals that of the waveguide-mode propagation constant in the structure, resulting in mode excitation. Various methods exist to excite desired modes, such as prism coupling (Otto or Kretschmann configurations) or grating coupling, among others. The effect of the mode excitation is better observed in reflection where for the proper angle of incidence or light wavelength, a dip in the reflected pattern is observed. In the case of transmission using grating coupling, an interesting effect referred to as extraordinary transmission, which involve the excitation of Surface Plasmons Polaritons (SPP) in metallic films, has been reported [2,3]. Here, light transmission through the structure normalized to the photon flux incident on the open subwavelength aperture can be several times larger than unity. In addition, by properly engineering the grating around a single aperture, the induced coupling between SPPs and radiation modes can be used to either enhance the total transmission or shape the transmitted beam [3,4].

Among the different types of surface waves [5], a special case of surface waves (SWs), referred to as Dyakonov SWs, is supported at the interfaces between an anisotropic cladding and an isotropic medium with refractive index \( n_t \) [6–10]. Such SWs form when the anisotropic cladding is (i) a positive uniaxial birefringent medium with the ordinary \( n_o \) and extraordinary \( n_e \) refractive indices satisfying the relation \( n_o < n_e < n_t \) [6]; (ii) a biaxial anisotropic medium with refractive indices \( n_o, n_{||}, \) and \( n_{\perp} \), fulfilling the condition \( n_1 > n_o > n_2 > n_3 \), where \( n_1 = \max(n_o, n_{||}, n_{\perp}) \) and \( n_3 = \min(n_o, n_{||}, n_{\perp}) \) [7]; (iii) at the interface between two birefringent media, either uniaxial [8] or biaxial [9]. They are lossless, hybrid (TE-dominant) polarized waves, very directional (since they only propagate in a narrow angular range with respect to the optical axis) and weakly localized compared with SPPs (see Ref. [10] for a review on these SWs). The difficulty to find natural materials fulfilling the relation among refractive indices described above and the narrow resonant dip obtained in Otto-Kretschmann excitation configuration (three orders of magnitude narrower than in standard SPPs excitation) made the experimental demonstration challenging. Recently, by using the polarization conversion effect [11], Dyakonov SWs were observed in an Otto–Kretschmann configuration [12].

Similar to SPPs, Dyakonov SWs can play an important role in the transmission through resonances in structures formed by birefringent media. Transmission through multilayer birefringent structures has been studied in the past (See, for example Refs. [1,13–15]). However, to the best of our knowledge, geometries supporting Dyakonov SWs have not been...
addressed. The objective of the present work is to numerically explore and illustrate the transmission characteristics caused by the excitation of Dyakonov SWs.

2. Structure geometry and analysis

The excitation of Dyakonov surface waves can either be performed using grating or prisms. In the first case the simplest structure is two layers with a grating written at the interface (see Fig. 1(a)). However, in this case transmission through the structure is the superposition of both Dyakonov SW and FP modes, hiding the characteristics of the former [16]. Excitation using prisms have the advantage that the two kinds of modes are spectrally separated: Dyakonov SWs are excited above and FP modes below the incident critical angle [12]. Therefore, the properties of the Dyakonov SW resonant transmissions are better described in this second excitation configuration. Here, to decouple the SW allowing for the transmission, an additional output prism is required, resulting in a two layer structure (with the interface supporting the Dyakonov SWs) sandwiched between two prisms. Figures 1(b) and 1(c) show the different and simplest possible configurations (more layers can be added). In this paper, for simplicity and without loss of generality, we focus on the configuration described in Fig. 1(c). This figure corresponds to two identical uniaxial birefringent media with the optic axis parallel to the interface. The two optic axes form an angle $\Theta$ between them. The coordinate system is taken so that the SW propagates along the $x$ axis, which is contained in the interface and forms an angle $\theta$ with respect to the optic axis (OA$_1$) of the top layer. The $z$ axis is orthogonal to the interface, directed to the bottom output prism. The $y$ axis is also contained in the interface as shown in Fig. 1(d). The angle of incidence $\phi_i$ is measured from the normal to the interface in the coupling prism (negative $z$ axis). In this geometry, Dyakonov SWs propagates only within a range of existence angles $\Delta \theta$ centered at the bisector angle $\theta = \Theta/2$ (measured from the optic axis at the upper layer), as Fig. 1(d) shows.

The electric field amplitude in the prism coupler, assuming harmonic dependence $E_p(x,z) = \exp[i(Nk_0x-\omega t)]$, with $k_0$ the free space wave number, $n_p$ the refractive index in the prism, and $\gamma_p = \sqrt{n_p^2 - N^2}$, with $N = n_p \sin \phi$ being the effective refractive index, can be written in compact form in terms of the incident and reflected $TE$ and $TM$ wave as,

$$E_p(x,z) = \begin{bmatrix} 0 & \gamma_p & A_{TE}^* \exp[ik_0\gamma_p z] \\ 1 & 0 & 0 \\ 0 & N & A_{TM}^* \exp[ik_0\gamma_p z] \end{bmatrix} + \begin{bmatrix} 0 & -\gamma_p & A_{TE} \exp[-ik_0\gamma_p z] \\ 1 & 0 & 0 \\ 0 & N & A_{TM} \exp[-ik_0\gamma_p z] \end{bmatrix}$$

(1)
This equation is also valid for the bottom output coupler substituting $A_{tE,TE}^r \rightarrow A_{tE,TE}^r$, $A_{tE,TM}^r \rightarrow 0$, and $z \rightarrow z - d_1 - d_2$. For the geometry considered here, the relative permittivity tensor for the birefringent layer is symmetric. This tensor for the top layer has the form:

$$
\varepsilon_1 = \begin{pmatrix}
\varepsilon_e \sin^2 \theta + \varepsilon_o \cos^2 \theta & (\varepsilon_e - \varepsilon_o) \sin \theta \cos \theta & 0 \\
(\varepsilon_e - \varepsilon_o) \sin \theta \cos \theta & \varepsilon_o \cos^2 \theta + \varepsilon_e \sin^2 \theta & 0 \\
0 & 0 & \varepsilon_e
\end{pmatrix},
$$

(2)

where $\varepsilon_e = n_e^2$ and $\varepsilon_o = n_o^2$ are the relative permittivities of the ordinary and extraordinary waves, respectively. Since the considered structure supports SWs, we chose to express the field in both birefringent media as a superposition of evanescent ordinary and extraordinary waves, which are truncated by the presence of the prism. The field amplitude in the top layer can be written as

$$
E_{\theta}(x, z) = \begin{pmatrix}
\gamma_o \tan \theta & -\gamma_o^2 \\
\gamma_o n_o^2 \tan \theta & A_{o1} \exp[-k_0 o n_o^2z] \\
iN \tan \theta & iN \gamma_e
\end{pmatrix} + \begin{pmatrix}
\gamma_o \tan \theta & -\gamma_o^2 \\
\gamma_o n_o^2 \tan \theta & A_{o1} \exp[-k_0 o n_o^2z] \\
iN \tan \theta & -iN \gamma_e
\end{pmatrix} \begin{pmatrix}
\gamma_o \tan \theta & -\gamma_o^2 \\
\gamma_o n_o^2 \tan \theta & A_{o1} \exp[-k_0 o n_o^2z] \\
iN \tan \theta & -iN \gamma_e
\end{pmatrix},
$$

(3)

where $\gamma_o$ and $\gamma_e$ are the ordinary and extraordinary decaying constants. From the wave equation one gets

$$
\gamma_o = \sqrt{N^2 - n_o^2} \quad \text{and} \quad \gamma_e = \frac{n_e}{n_e(\theta)} \sqrt{N^2 - n_e^2(\theta)},
$$

(4)

with

$$
\frac{1}{n_e(\theta)} = \frac{\cos^2 \theta}{n_o^2} + \frac{\sin^2 \theta}{n_e^2},
$$

(5)

$n_o(\theta)$ being the refractive index of the extraordinary wave propagating in an unbounded uniaxial crystal. Equations (2) to (5) are also valid for the bottom birefringent layer by substituting $\theta \rightarrow (90^\circ - \theta)$ and $z \rightarrow z - d_1$.

By imposing boundary conditions at the different layers, the eigenvalue relations can be found. To solve the transmission and reflection problem through the structure, a practical procedure is to use a matrix formalism such as the one developed by Hodgkinson et. al. [15]. This formalism can efficiently deal with multilayer structures formed by birefringent media, providing the reflectance and transmittance (irradiance) coefficients. By varying the angle of incidence $\phi_i$ and propagation angle $\theta$, reflectance and transmittance spectra can be obtained. As a reference in spectra in Figs. 2 to 4, the critical angles for the extraordinary refractive index $n_e(\theta)$ are denoted as a green line in the top layer given by $\phi_e = \sin^{-1}(n_e/n_o(\theta))$ and as a blue line for the bottom layer, obtained substituting $\theta \rightarrow (90^\circ - \theta)$.

3. Results

As a model structure we consider a SF11 prisms with refractive index $n_p = 1.77862$. The two uniaxial layers are considered identical with refractive index $n_o = 1.520$ and $n_e = 1.725$, corresponding to a E7 liquid crystal. The angle between optical axes of the two birefringent layers is chosen to be $\Theta = 90^\circ$. The thickness of the two layers are considered equal, $d_1 = d_2 =$
$4\lambda$, where $\lambda$ is the wavelength of the incident field. The resulting reflection and transmission spectra are shown in Fig. 2.

![Fig. 2. Reflectance and transmittance spectra for a Prism-Uniaxial-Uniaxial-Prism configuration. The two prisms and uniaxial layers are identical with $n_p = 1.77862$, $n_o = 1.520$, $n_e = 1.725$, $d_b = 4\lambda$. $\theta$, is the angle between the top uniaxial optical axis OA$^1$ and the in-plane component of the incident wavevector. The angle between optical axes is $\Theta = 90^\circ$. Reflection for (a) TE$^-\text{TE}^-$, (b) TM$^-\text{TM}^-$, and (c) TE$^-\text{TM}^-$, input-output polarizations. Transmission for (d) TE$^-\text{TM}^-$, (e) TE$^-\text{TE}^-$, and (f) TM$^-\text{TM}^-$, input-output polarizations. Magnification of red rectangle in (e) is shown in Fig. 3(a). The green (blue) line shows the critical angles for the extraordinary refractive index $n_e(\theta)$ for the top (bottom) birefringent layer.](image)

The reflection spectra maintaining the polarization are composed of dark regions or dips of low reflection (Figs. 2(a) and 2(b)). These dips in reflectance are caused by the coupling to FP modes (below the green and blue lines showing the critical angles) and the excitation of other modes supported by the structure, namely leaky guided modes supported by the top birefringent layer (dips above the blue critical angle line) and Dyakonov SWs (dips above both, the green and blue critical angle lines). Note that since this structure supports TE dominant hybrid modes, TE input results in a more efficient excitation of all the existing modes (lower reflection for the corresponding dip) than TM input. Interestingly, excitation of birefringent layers results in a polarization conversion effect (Fig. 2(c)) [17,18]. This
conversion appears as peaks in the intensity of reflected light and is identical for both input polarization but stronger for propagating modes (leaky or Dyakonov) than for FP modes. This effect was used to demonstrate the existence of Dyakonov SWs [12] and has extensively been discussed in Ref. [11].

Peaks in the transmission spectra are obtained at some angles for TE input polarization. Similarly to the reflection response, a portion of the TE (TM) incident wave is converted to TM (TE) output polarization (Fig. 2(d)). Note that for TM excitation, transmission maintaining the polarization is three orders of magnitude lower than for TE excitation (Fig. 2(f)), and the only perceptible transmitted light is due to the polarization conversion effect (Fig. 2(d)). In general, transmission for any input polarization, mainly occurs when the angle of incidence $\varphi$ are either below (related with FP modes) or above (related with Dyakonov SWs) the two critical angles showed by the green and blue lines in Figs. 2(d)–2(f). Therefore, the existence of leaky modes does not result in transmission. To show the properties associated with Dyakonov SW-resonant transmission, the magnification of the transmission peak is presented in Fig. 3(a). In this figure, we show as a reference the theoretical Dyakonov-SW range of existence $\Delta \theta$ (denoted by the two red lines) considering two semi-infinite uniaxial materials. The resulting range is $\Delta \theta = 1.48^\circ$, which is expected to be slightly modified by the presence of the prism. Transmission is only obtained within this range, and mainly between the two lines showing the critical angle, showing that transmission is driven by the existence of Dyakonov SWs supported by the studied structure. This is in contrast to the reflectance spectrum, where there is not a clear distinction between the dip corresponding to Dyakonov SWs and the leaky modes of the structure. This can be observed in the dip (or peak in polarization conversion) related with Dyakonov SW above the two critical angle lines in Figs. 2(a)–2(c), which continuously evolves crossing the green line, transforming itself into a leaky mode. This result shows that transmission analysis can be an appropriate method for Dyakonov SWs spectroscopy.

The Dyakonov SW resonant transmission is a localized resonance in both, the angle $\theta$ with respect to the optical axis and the angle of incidence. In order to better understand this localization, we show slices of Fig. 3(a) maintaining one of the two angles fixed. In the first case, the angle of incidence is fixed at $\varphi = 65.147^\circ$. Here, only one transmission peak is obtained, with the maximum at $\theta = 45^\circ$ (corresponding to the center of the existence domain of Dyakonov SW), with a TE and converted TM transmission of ~80% and ~10%, respectively (~90% of total transmission), and a width of $\Delta \varphi_p = 0.4^\circ$. This situation corresponds to the broadest peak in $\theta$. For higher angles of incidence, the peak splits into two narrower peaks with lower transmission coefficient. In the second slice, the angle with respect to the optical axis is fixed at $\theta = 45^\circ$. In Fig. 3(c), the resonance peaks are shown in an extended range $\varphi$ considered in Fig. 2. This clearly shows the difference between the FP resonances (broad peaks with a background-to-peak ratio higher that 20% in all cases) and the Dyakonov SW resonant transmission (much narrower and with no surrounding background). Figure 3(d) shows a magnification of the Dyakonov peak, whose main difference with respect to Fig. 3(b) is that in this case the peak width is much narrower, of $\Delta \varphi_p = 0.006^\circ$. Such a narrow peak could seem extremely hard to experimentally detect. However, peaks of similar width were detected by the polarization conversion method used to observe Dyakonov SWs [12]. As in that experiment, the key point is that the null background surrounding the peak offers enough contrast for the peak detection. Therefore, the resulting narrow peak implies a high sensitivity to parameters variations in the structure.
Fig. 3. (a) Magnification of the transmission spectrum associated with Dyakonov SWs. The two red vertical lines represent the theoretical existence domain (lower and upper cutoff angles) for two semi-infinite uniaxial media. (b) TE and TM transmission in terms of $\theta$ for a fix angle of incidence $\phi = 65.147^\circ$. Transmission for a fix value of $\theta = 45^\circ$ for (c) the extended range of incident angles and (d) the magnification of the peak associated with the Dyakonov SW highlighted by black rectangle in (c).

Fig. 4. (a) Transmission spectra (TE$_{in}$-TE$_{out}$) for a Prism-Uniaxial-Uniaxial-Prism configuration when the thicknesses of the two uniaxial layers are different. $d_b1 = 6\lambda$ and $d_b2 = 3\lambda$. The rest of the parameters are same as Fig. 2. (b) Magnification of the transmission spectrum associated with Dyakonov SWs. The red vertical lines represent the angular existence domain for semi-infinite layers of two uniaxial media. (c) Transmission in terms of $\theta$ when the angle of incidence is fixed at $\phi = 65.185^\circ$ (red line in (b)). (d) Transmission in terms of $\phi$ when the angle of incidence is fixed at $\theta = 45.5^\circ$ (blue line in (b)).
For practical purposes, transmission control by changing the angle with respect to the optic axis $\theta$ (which can be performed by rotating the structure while maintaining the output beam direction) can be more convenient than changing the angle of incidence $\phi_i$ (which results in the change of the transmitted direction). However, as discussed above, the structure analyzed in Fig. 3 does not offer a high sensitivity ($\Delta \theta_p = 0.4^\circ$) and in addition it can result in two transmission peaks. This can be solved by using an asymmetrical structure, for example, by using different thicknesses for the two birefringent layers. The results considering $d_1 = 6\lambda$ and $d_2 = 3\lambda$, and maintaining the same value for all the other parameters, are shown in Fig. 4. In this case, the different layer thickness breaks the symmetry of the transmission spectrum with respect to the angle $\theta$, as can be seen by comparing Fig. 2(e) and Fig. 4(a). The increase in the top layer thickness moves the Dyakonov SWs cutoff (by radiation to the coupling prism) to higher angles $\theta$. In a similar way, the cutoff by radiation to the output prism moves to higher angles $\theta$ by the decrease in the bottom layer thickness (compare Fig. 3(a) and 4(b)). As a result, the transmission can be engineered to obtain a narrow transmission peak in the angle $\theta$.

For the particular case analyzed here, the transmission is maintained up to 80% for TE$_{\text{in}}$–TE$_{\text{out}}$ and 10% for TE$_{\text{in}}$ – TM$_{\text{out}}$ polarization conversion. The maximum transmission is translated to higher values of $\theta$, in this case around $\theta = 45.5^\circ$ with a width one order of magnitude narrower, $\Delta \theta_p = 0.04^\circ$ (compare Fig. 3(b) and 4(c)), while the width for the angle $\phi_i$ remain in the same order, $\Delta \phi_p = 0.005^\circ$ (compare Fig. 3(d) and 4(d)).

The results reported here correspond to a specific structure, however there is room for optimization and engineering for a given particular purpose. In general the total transmission depends on the range of existence of the Dyakonov SWs and the polarization conversion on its degree of hybridity. For example, by increasing the angle between the two optic axis to $\Theta = 120^\circ$, the range of existence of Dyakonov SWs increases, resulting in a higher TE output transmission (90%) and a lower TM output transmission (6%).

4. Conclusion and discussion

We have theoretically studied the effect of Dyakonov SWs in the transmission through dielectric structures. We have shown that it is possible to obtain high transmission for TE excitation, resulting in ~80% for TE output and a small polarization conversion, i.e., ~10% for TM output. However, transmission maintaining the polarization by TM excitation is marginal, only resulting in polarization conversion. Such transmission is produced only in a certain range of excitation angles in the two transversal directions, namely angle of incidence $\phi_i$ and angle with respect the optic axis $\theta$. The range of transmission angles are $\Delta \phi_p = 0.006^\circ$ and $\Delta \theta_p = 0.4^\circ$ for a typical symmetric structure, while in an asymmetrical structure the range in $\theta$ can be reduced by one order of magnitude to $\Delta \theta_p = 0.04^\circ$.

Potential applications of the effect reported here include directional spatial filters and sensing applications. In the first case the structure considered here is the most suitable, since it shows very well defined transmission characteristics. In the second case, a structure formed by a birefringent and an isotropic layer (instead of two birefringent layers) is more adequate since the isotropic layer could be easily substituted by the medium to be sensed. In this last situation, when compared with sensors based on surface plasmons polaritons (SPP), the low localization of the Dyakonov SW at the surface decreases the its sensitivity, however, this can be compensated to some extent by the narrower peak showed by the Dyakonov SW resonant transmission, which is two orders of magnitude narrower than the typical resonance dip in SPP.

The transmission in general can be engineered by changing the different parameters, such as the birefringent layer thicknesses, the angle $\Theta$ between optical axes or the prism refractive index. In addition, new opportunities appear in combination with other material and geometrical properties, such as nanostructured materials exhibiting tunable form-birefringence [19], magnetic materials [20], thin films [21], nonlinear media [22], or electro-optic effect [23], which may afford further control of the transmission characteristics. A special important
case is the combination with SPP. Recent works have proposed the use of dielectric layers on top of a thin metal to enhance the TE transmission in structures where the existence of SPP results in TM extraordinary transmission [24]. The combination of Dyakonov SWs and SPP opens new opportunities to achieve this objective.

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