A new type of vortex merger is experimentally reported and numerically investigated. The merging process of two anticyclones under the influence of a cyclone (a three-vortex interaction) was observed in sea surface height (SSH) altimetry maps south of the Canary Islands. This three-vortex interaction is investigated using a process-oriented three-dimensional (3D), Boussinesq, and f-plane numerical model that explicitly conserves potential vorticity (PV) on isopycnals. The initial conditions consist of three static and inertially stable baroclinic vortices: two anticyclones and one cyclone. The vortex cores form a triangle in a configuration similar to that found south of the Canary Islands. The numerical results show, in agreement with SSH observations, that two corotating vortices, sufficiently close to each other and in presence of a third counterrotating vortex, merge, leading to a new elongated vortex, which couples with the counterrotating vortex, forming a dipole. Thus, the merging process occurred south of the Canary Islands is consistent with simplified vortex dynamics (basically PV conservation). The merging process depends on the initial PV density extrema, vertical extent, and the angle spanned by the corotating vortices. It is found that the presence of the third counterrotating vortex importantly affects the critical angle of merger and the processes of axisymmetrization and filamentation associated with the two corotating merging vortices. The torque exerted by the counterrotating vortex on the two corotating vortices delays, but does not prevent, their merger. [PUBLICATION ABSTRACT]
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1. Introduction
Oceanic mesoscale vortices are coherent vortical structures that frequently interact with other vortices as they move through background ocean currents. Vortex merger or coalescence is the vortex-vortex interaction occurring when two corotating vortices, close enough to each other, mix an important proportion of their vorticity core and form a single bigger vortex. This interaction, in spite of the scarce observational evidences in the ocean (Cresswell 1982; Yasuda et al. 1992; Schultz-Tokos et al. 1994; Carton et al. 2010), caused a great interest in the scientific community because of its important role in the transfer of energy and entropy across scales in turbulent flows (Kraichnan and Montgomery 1980; McWilliams 1984; Jiménez et al. 1996).

The merging process has motivated two scientific paradoxes. One of them, which prompted a large debate, was the energy paradox. A. E. Gill and R. W. Griffiths (1994, personal communication) showed that the final state of the merger of two vortices, if potential vorticity (PV) is conserved, contains more energy than the initial state, which means that spontaneous merger is prevented. On the other hand, laboratory and numerical experiments have reproduced satisfactorily the merger of two corotating vortices, which is at odds with the energy restriction. Griffiths and Hopfinger (1987) suggested that the additional energy is supplied by other corotating vortices generated at the upper levels because of the convergence of fluid in a finitely deep fluid. However, Nof and Simon (1987) showed that the additional energy proposal by Griffiths and Hopfinger (1987) is not sufficient to overcome the increase of energy forced by PV conservation and suggested that the PV of the vortices is altered during merger. Later, Nof (1988) speculated that the alteration of PV is produced by shock waves. Finally, Cushman-Roisin (1989) solves the paradox proposing a partial merger in the sense that the final state consists not only of a single vortex but also includes a pair of surrounding PV filaments.

The role of stratification was the second paradox concerning the merging process. Griffiths and Hopfinger (1987) found that the merging conditions strongly depend on the stratification profile, which is at odds with the results reported by Polvani et al. (1989). This apparent
contradiction was clarified by Verr?n et al. (1990), who showed that the results of Griffiths and Hopfinger (1987) and Polvani et al. (1989) were obtained with different initial conditions. Later, Valcke and Verr?n (1997) showed that the efficiency of merger of shielded vortices depends on their horizontal distribution of PV. They also showed that the increase of baroclinicity favors the merging efficiency. Consistently with these results, von Hardenberg et al. (2000) and Dritschel (2002) indicated that baroclinicity strongly affects the critical distance ratio \( dc \) of vortex merger.

The main condition for vortex merger is that the aspect ratio \( \frac{a}{b} \) between radius \( a \) and the separation distance between vortices \( b \) exceeds \( dc \). This critical distance is related to the existence or nonexistence of steady equilibrium states (e.g., Saffman and Szeto 1980; Overman and Zabusky 1982; Dritschel 1985, 1995; Melander et al. 1988; Reinaud and Dritschel 2005). For ratios shorter than \( dc \), the vortices develop a steady equilibrium state and merger is prevented. For ratios larger than \( dc \), the vortices become unstable and merger takes place. The determination of \( dc \) has been investigated through numerous numerical experiments (e.g., Overman and Zabusky 1982; Dritschel 1985, 1986; Yasuda and Flierl 1997; Reinaud and Dritschel 2002, 2005), laboratory experiments (e.g., Griffiths and Hopfinger 1987), and theoretical analysis (e.g., Melander et al. 1988; Saffman and Szeto 1980). These studies show that the critical distance ratio \( dc \) varies between 0.20 and 0.33.

The small discrepancies are due to the different vorticity profiles. The transient features of the merging process have been extensively described in the scientific literature. For example, Nof (1988) proposed that, when two vortices, in an infinitely barotropic in viscous hydrostatic fluid, get in contact and then a common boundary is established, vorticity tentacles (or arms) are generated. These tentacles become larger and ultimately wrap around each other, leading to two intermingled spirals that conform the merged vortex. Pavia and Cushman-Roisin (1990), using a reduced gravity model and a particle-in-cell method, established three stages in the merging process. In the first stage, intrusions from each vortex wrap around the other, substituting the particles of one vortex location almost completely with those of the companion vortex. The second stage is the merger per se, which ends up leaving a new elongated vortex. In the third stage, the vortex stabilization proceeds with the axisymmetrization and ejection of filaments. Numerical simulations by Masina and Pinardi (1993) using a regional quasigeostrophic (QG) dynamical model showed two phases in the merging process. The first one corresponds to the clove shaping of the vorticity front of the vortices with a diffusive mixing of vorticity, whereas the second one consists of the sliding of the remaining vorticity core with a second diffusive mixing of the interior vorticity field.

In this paper, we report observations and analyze using idealized numerical simulations a new type of vortex merger that occurred south of the Canary Islands. This event was experimentally observed through in situ hydrography and remote sea surface height (SSH) satellite data. The experimental data show the merger of two anticyclones under the influence of a third vortex (a cyclone). Thus, in this sense, this merging process involves a three-vortex interaction, which, though being a new vortex interaction, has some similarities
with the merger of two isolated vortices described above. In particular, these characteristics include, as introduced earlier, PV filamentation, baroclinicity, the critical distance ratio \( d_c \), and vortex axisymmetrization. The numerical analysis is carried out through a series of numerical experiments using a 3D process-oriented circulation model. Threecorotating vortices interaction has been reported previously by Carton et al. (2002). They observed experimentally the interaction between two meddies and a deep cyclone in the southern Gulf of Cadiz. In their case, the vortices distribution was different from that found south of the Canary Islands, and vortex merger, as investigated using numerical simulations, was not prognosticated.

In section 2, we provide essential technical information on the satellite data analysis, present the hydrographie data, and describe the experimental observations. In section 3, we briefly introduce the numerical method, explain the initial conditions, and describe the results. Our objective is to verify that the merging process is consistent with simplified PV conservation dynamics. We do not therefore attempt to exhaust the huge parameter space of three-vortex interactions. The initial conditions consist of three PV ellipsoids (corresponding to two anticyclones and one cyclone) whose PV centroids form a triangle in a configuration similar to the observations found south of the Canary Islands. Next, the merger between identical and unequal vortices is investigated through a series of numerical cases in which the importance of PV extrema, initial distance between corotating vortices, and the vertical aspect ratio are considered. We found that the third vortex (cyclone) importantly affects the merging process of the two corotating vortices (anticyclones) but does not prevent it. Finally, concluding remarks are given in section 4.

2. Experimental data and observations

a. Altimetry and hydrographie data

We use weekly maps of blended Jason-1, Environmental Satellite (Envisat), and Geosat Follow-On (GFO) sea level data. The time period spans from 15 October to 30 November 2008, and the usual geophysical corrections were applied (LeTraon and Ogor 1998). Data were carefully intercalibrated and homogenized at Collecte Localisation Satellites (CLS) by performing a global crossover adjustment of the Envisat and GFO orbits, using Jason-1 data as a reference. Sea level anomaly (SLA) was computed by removing a 7-yr mean. To reduce the measurement noise, SLA was filtered with a 35-km median filter and a Lanczos filter with a cutoff wavelength of 42 km. Finally, the data were subsampled at one point in two. We use the Archiving, Validation, and Interpretation of Satellite Oce?anographie data (AVISO) global sea level anomalies product (1A0 X 1A0 resolution on a Mercator grid) that is obtained using a suboptimal space/time objective analysis (Bretherton et al. 1976). This analysis is performed applying space and time correlation functions with zero-crossing points at 100 km and 10 days, respectively. Quasi-real-time altimetry data were received on board the R/V Sarmiento de Gamboa during the oce?anographie cruise Corriente de Afloramiento del Noreste Africano (CANOA-08) south of the Canary Islands and were used in the hydrographie sampling strategy.

On 11 November 2008, four CTD casts were performed crossing an anticyclonic eddy. On 28 November 2008, the ship revisited the area to collect four additional CTD casts.
hydrographie data were processed following standard procedures (removing spikes, thermal lag correction, etc.). The CTD casts horizontal resolution is about 24 km, and, after averaging, the final profiles have a vertical resolution of 1 m. Further details on the hydrographie sampling design and data processing can be found in Emelianov et al. (2009).

b. Experimental observations

SSH altimetry maps for October and November 2008 report the stages of the merging process of two anticyclones eddies under the influence of a cyclone (Fig. 1). On 15 October, altimetry maps show an anticyclonic eddy of about 125-km diameter located around 26°N, 16°W (hereafter AE1). Between AE1 and the western Canary Islands, a second anticyclonic eddy of around 100 km diameter is centered roughly at 27.5°N, 17.2°W (hereafter AE2). Maximum geostrophic speeds about 35 cm s\(^{-1}\) are located at the edges of both structures. In the eastern part of the domain, between the Canary Islands and the African coast, a cyclone eddy (hereafter CE1) of approximately the same size than AE1 is also identified. Its maximum geostrophic velocity (20 cm s\(^{-1}\)) is slightly weaker than the velocities associated to the anticyclones.

From 22 October to 5 November, AE2 remains approximately at the same location. However, during the first week of November, AE1 moves northwestward with a translation speed of about 2 km day\(^{-1}\). At this stage, the interaction between the edges of both anticyclones is quite apparent. A week later, the merger of the two anticyclones is already advanced and the signature of the two eddies interacting is weaker but still noticeable. Cyclone CE1 moves slightly southward, occupying part of the area AE1 occupied 2 weeks before.

By 19 November, the fusion of AE1 and AE2 has occurred and the resulting eddy has an elongated shape with a north-south diameter of ~160 km. The magnitude of the surface geostrophic velocities associated to the new structure is similar to the observed before the merger.

At the upper layer, temperature in the AE1 core is 22.2°C (CTD-4; Fig. 2). This temperature is slightly lower at the edge of the eddy where values of about 21.7°C are found (CTD-I). Note also that the stratification is deeper near the center (125 m) than in the edge (75 m). After the fusion of the two eddies, this stratification depth remains almost at the same depth (CTD-8); however, the temperature in the upper core of the resulting eddy is lower (21.8°C) than the original temperature of AE1 (Fig. 2).

3. Numerical experiments

Our main objective in this section is to verify that the merging process occurred south of the Canary Islands is consistent with a simplified vortex dynamics involving PV conservation. With this purpose, we analyze the evolution of two corotating vortices under the influence of a third counterrotating vortex through a series of numerical experiments.

a. Numerical model and parameters

We use a 3D numerical model to simulate rotating, volume-preserving, nonhydrostatic, stably stratified flows under the /-plane and Boussinesq approximations (Dritschel and Viudez 2003). The use of an /-plane numerical model is justified here because the horizontal scale of the vortices is L = 100 km, so that the ratio LIRq <· 1, where Rq is the earth's radius. On
two-dimensional vortices, the effect may noticeably alter the vortex merger (e.g., Bertrand and Carton 1993; Velasco-Fuentes and Velazquez 2003). The main characteristics of this model are the explicit conservation of PV via its advection on isopycnal surfaces using the contour advection algorithm of Dritschel and Ambaum (1997) and the use of a vector potential f defining both the 3D velocity and density anomaly. This approach allows long time integrations of the vortical flow with minimum numerical diffusive effects. This makes this numerical algorithm very convenient to simulate inviscid vortex interactions involving vortex merger and dipole formation.

The state variables are the components of the vector potential \( \mathbf{A} = (A, B, [\text{straight } \phi]) \), which provide the velocity \( \mathbf{u} = -\mathbf{f} \times \mathbf{v} \) and the vertical displacement of isopycnals \( \mathbf{V} = -\mathbf{\nabla}^2 \phi^* \), where the Prandtl ratio \( \mathbf{\nabla}^2 = c = N/f \) is the ratio between background Brunt-Väisälä and Coriolis frequencies. Note that the total Brunt-Väisälä frequency \( \mathbf{\omega} \) is not a constant because it depends on \( D(x, t) \),

\[ \mathbf{\omega} = \mathbf{\omega}^0 + \mathbf{\omega}^k \]

Static stability is always assumed so that \( \mathbf{\omega}^k < 1 \), which implies that \( N \) is a real number.

The prognostic scalar equations are the rate of change of the dimensionless ageostrophic horizontal vorticity \( \mathbf{A}^h = (\mathbf{A}, B) = (\mathbf{w}^h - \mathbf{w}^g \mathbf{h}^g) / f \), where \( \mathbf{w}^h \) is the relative horizontal vorticity, \( \mathbf{w}^g \mathbf{h}^g \) is the geostrophic horizontal vorticity, and the material conservation of PV anomaly \( m \) on isopycnals is \( dm/dt = 0 \). The horizontal potential \( \mathbf{\phi}^h = (\phi^h, \phi^v) \) is obtained every time step by inverting \( \mathbf{A}^h \), whereas the vertical potential \( [\text{straight } \phi] \) is recovered from the inversion of the dimensionless PV anomaly,

\[ \mathbf{\phi} = \mathbf{\phi}^h + \mathbf{\phi}^v \]

where \( \mathbf{\phi} \) is the dimensionless PV and \( \mathbf{w} = (\phi^h, \phi^v) \) is the relative vorticity vector. More details of the theoretical basis of the numerical model are given in Viudez and Dritschel (2003) and Dritschel and Viudez (2003).

We use a triply periodic numerical domain with \( (n^x, n^y, n^z) = (128, 128, 128) \) grid points; vertical extent \( L^z = 2 \) (which defines the unit of length); and horizontal extents \( L^x = L^y = cL^z \), with a Prandtl ratio \( c = Nf = 10 \). We take the buoyancy period as the unit of time by setting \( N = 2 \), so that one inertial period \( T^b \) equals 10 buoyancy periods \( T^b \). The number of isopycnal surfaces \( n^l \) is equal to the number of grid points in any direction, \( n^l = n^x = 128 \). The time step \( t = 0.1T^b \) and the initialization time \( t^i = 5T^b \). The numerical simulations are initialized using the PV initialization approach (Viudez and Dritschel 2003). The initialization time is the minimum time interval required for the fluid to reach its prescribed initial state with minimal generation of inertia-gravity waves. For simplicity, the values of all dimensional magnitudes are relative to the above space and time scales and units are not explicitly written. To recover the physical units of any quantity, given a dimensional domain depth \( H \) and mean latitude \( \phi_0 \), we need to multiply the numerical value by the spatial and time scale conversion factors \( S^z = H/ \) and \( S^t = 1 \) day/2c sin\( \phi_0 \), respectively, elevated to the appropriate powers to match the physical
b. Numerical results

1) INITIAL CONDITIONS

The initial conditions consist of three static and inertially stable baroclinic vortices: namely, two anticyclones and one cyclone. The vortices centers form the vertices of a triangle (Fig. 3) in a configuration similar to that found south of the Canary Islands. The anticyclones centers are initially separated from the cyclone center a distance $d = 1.02c$. The PV surfaces have initially an ellipsoidal geometry, where $m$ is constant on ellipsoidal surfaces and varies linearly with the ellipsoidal volume, with $m = 0$ on the outermost surface of semiaxes $(a_x, a_y, a_z)$ and $m = m_j$ in the vortex core. The isopycnal surface at middepth (layer index $i_0 = 65$) has $n_c = 45$ PV contours. The PV increment $\delta m^2$ across each PV contour (or PV jump) is fixed $\delta m^2 = m_j \ln n$ for every contour, except for the outermost contour, where $\delta m^2 = m_j \ln (2n)$. Note that our numerical model is triply periodic. However, the initial conditions (PV initial distribution) are symmetric with respect to the horizontal plane $z = Q$ ($U = 65$). This implies that all the fields remain symmetric (or antisymmetric: e.g., the vertical displacement $V$ or the vertical velocity $w$) during the complete numerical simulation. Thus, one may consider the horizontal plane $z = 0$, where $V = 0$ and $w = 0$, as the ocean surface under the rigid lid approximation. In this case, one disregards the upper half domain ($z > 0$) and analyzes the lower half domain ($z \leq 0$), where vortices speed increase with $z$. Alternatively, one may keep the whole domain to investigate interactions between subsurface eddies.

2) INTERACTION BETWEEN IDENTICAL ANTICYCLONES

We describe first the merger of identical anticyclones (reference case $R$; Table 1). Initially, the two anticyclones approach each other and depart from the initial elliptical PV configuration evolving into an elongated spoon-like shape (Fig. 4a). This process is typical of baroclinic vortex merger (von Hardenberg et al. 2000). Next, the anticyclones slide one on the other, sharing a common boundary (Figs. 4a,b). In this phase, as we will see later, the flow undergoes a large acceleration. Finally, each anticyclone wraps around the other, mixing their respective volumes (exception is the thin PV filaments) and leading to a single and elongated anticyclone (Figs. 4c, 5a). Both anticyclones contribute equally to the new vortex because they have identical $\omega_0$ and $d$ (the initial PV configuration is symmetric relative to the axis $A$ in Fig. 3). At the same time, the cyclone is deformed, and, once the anticyclones merge, a vortex dipole is formed. Comparison with other merger events (e.g., Pavia and Cushman-Roisin 1990; Masina and Pinardi 1993) indicates a substantial agreement in the kinematics of this process.

Merger of two isolated vortices takes place when the distance between them is equal or smaller than a critical distance. This critical distance, as previously mentioned, is associated with the margin of stability for two vortices in mutual equilibrium (e.g., Overman and Zabusky 1982; Dritschel 1985). On the other hand, Huang (2005) suggests that the critical distance is related to the mutual attraction and the self-induced rotation of vortices. He shows that merger starts when the self-induced rotation is overcome by the attraction of vortices,
resulting in deformation of the vorticity field. This deformation induces an inward flow at the position of the other vortex, the vortices become unstable, and merger takes place. In our three-vortex configuration, we found that merger occurs when the angle a spanned by the anticyclones is equal or smaller than the critical angle \( \alpha_c = 65^\circ \). For a \( \alpha > \alpha_c \) (case A; Table 1), the self-induced rotation of vortices dominates over the torque that each vortex exerts on the other so that the anticyclones may initially approach and interchange some PV but merger is prevented (Fig. 5b). For a \( \alpha < \alpha_c \), vortex merger is favored and the merging time is reduced. This is because, in this case, the torque that each vortex exerts on the other is large enough to dominate over their self-induced rotation. The critical angle \( \alpha_c \) depends on the maximum PV \( w_0 \) and vertical extent \( a_z \) of the merging vortices. Increasing \( w_0 \) (case B; Table 1) increases \( \alpha_c \), the vortices undergo larger interaction, and the vortices therefore can merge when the distance between them is larger (Fig. 5c). Larger \( a_z \) implies a larger amount of PV and therefore a larger \( \alpha_c \). The torque exerted by each anticyclone on its companion increases, as a result of the increase in the amount of PV (increase in \( \text{ero} \) or \( a_z \)). This implies that the mutual attraction of the anticyclones dominates over the self-induced rotation in a wider range of distances. Note that the amount of PV, because the flows are closely in QG balance, allows us to investigate the torque exerted by the vortices.

On the other hand, \( \alpha_c \) decreases with the presence of the cyclone. This fact is clearly noticeable comparing the merger of two anticyclones in presence and absence of the cyclone with a \( \alpha > \alpha_c \) (cases A and E in Table 1 and Figs. 5b,d). Thus, large \( |w_0| \), small \( a \), and large \( a_z \) favor the vortex merger, whereas the amount of positive \( \text{ero} \) of a third counterrotating vortex works against anticyclone merger.

During the merging process, PV filaments are generated (Fig. 6). Melander et al. (1987) showed that the generation of these vorticity structures is the main mechanism in the evolution of the new elongated asymmetric vortex toward axisymmetry. Later, Dritschel (1998) and LeDizes (2000) showed the existence of persistent nonaxisymmetric vortices in two-dimensional flows. In our case R, the new anticyclone evolves toward an asymmetric PV configuration because of the presence of the cyclone. When two isolated anticyclones merge (case F in Table 1 and Fig. 6a), PV filaments around the new anticyclone are generated. These filaments stabilize and favor the axisymmetrization of the new anticyclone. However, in the three-vortex case (Fig. 6b), the filaments and the new anticyclone are affected by the torque exerted by the cyclone. The filaments become unsteady and are expelled out of the place where they were generated.

The vertical velocity \( w \) has a complicated structure displaying alternating upwelling and downwelling cells (Fig. 7). However, as time goes on and the anticyclones merge, the \( w \) pattern (Fig. 7; \( t = 21T_{\text{sub ip}} \)) becomes similar to the quadrupolar pattern typical of mesoscale QG balanced dipoles (Pallès-Sanz and Viúdez 2007). During the vortex merger, \( w \) maxima are located along the common boundary between anticyclones reaching \( w_{\text{max}} = \max(|w|) = 1.4 \times 10^3 \) when the anticyclones wrap around the other. The \( w \) in the merging process is similar to the QG vertical velocity \( w_{\text{sup q}} \) obtained here solving the QG omega equation. The maximum differences \( \max(|w - w_{\text{sup q}}|) = 5 \times 10^3 \) indicate
that the flow is largely in QG balance. Note that this predominance of QG balance could justify the use of a simpler continuously stratified QG model to investigate this kind of three-vortex interaction. The horizontal speed $u = u^h$ in the anticyclones is larger than in the cyclone, with $u^h = \max(|w|) \approx 0.69$ located at the edges of both anticyclones (Fig. 8). This is consistent with the experimental and hydrographic observations south of the Canary Islands during the cruise CANOA08 described in the previous section. Both $w^\text{max}$ and $u^\text{max}$ increase during the first 12T sub ip, so that the fluid accelerates during the merging process (Fig. 9). The presence of the cyclone intensifies the flow (case R) as compared with the merger of the anticyclones in absence of the cyclone (case F). In this isolated vortex merger case, $w^\text{max} = 1.3 \times 10^3$ and $u^\text{max} = 0.49$.

3) INTERACTION BETWEEN UNEQUAL ANTICYCLONES

We describe next the merger of unequal anticyclones. The satellite images (Fig. 1) suggest that the anticyclone AE2 is created by the Canary Current (CC) interacting with the Canary archipelago, whereas vortices AE1 and CE1 result from the meandering of the Canary Upwelling Current (CUC) off the African Coast. The CUC is more intense than the CC, suggesting that AE1 and CE1 are more energetic than AE2. For this reason, we now consider three vortices with identical extent $(a_x, a_y, a_z) = (0.5c, 0.5c, 0.5)$ but different PV extrema, $w^0_1 = -0.5$ (weak anticyclone), $w^0_2 = -0.8$ (strong anticyclone), and $w_0 = 0.8$ (case C; Table 1).

Similarly to what happens to the identical anticyclones merger (case R), the anticyclones in this case approach each other, establish a mutual boundary, and finally merge leading to a new and larger anticyclone (Fig. 10a). However, contrary to what happens in case R, the strong anticyclone dominates over the weak anticyclone and attracts it quickly at the same time that strongly couples the cyclone forming a dipole that moves southward. The weak anticyclone therefore undergoes a large deformation and eventually splits, being partially absorbed by the strong anticyclone (partial vortex merger) and losing some PV through filamentation (Figs. 10a, 11).

In this case, $a^c = 80^\circ$, which, as expected, is larger than in the case R ($a^c = 65^\circ$) because now one of the anticyclones is stronger. In general, $ac$ in unequal anticyclones $(|c7o2| > |o7o1|)$ is larger than in the case of identical anticyclones (where $|w^0_2|$ decreases to match $|c7o1|$). Also, the merging process is faster than in the case R as can be noticed when comparing Figs. 5a and 10a. The filaments are more unsteady and, once expelled out of the place where they were generated, may develop small satellite vortices (Fig. 11; $t = 15\text{ Tip}^\circ$). On the other hand, $u^h$ and $w$ have the same pattern as in case R (Fig. 9). As it happens in identical anticyclones merger, the speed increases in the first phases of merger and reaches its maximum value when each anticyclone wraps around the other. However, in this case, the flow is more intense with $w^\text{max} = 2.1 \times 10^3$ and $u^\text{max} = 0.95$.

As previously commented, the origin of AE1 and CE1 seems to be associated with the upwelling jet of the CUC. This upwelling jet is very shallow, suggesting that AE2, besides being less energetic, is deeper than AE1. For this reason we now consider anticyclones with different PV maxima and different vertical extent (case D; Table 1). The anticyclones,
although they have different ... and $a^z$, exert globally a similar influence on each other because they have a similar amount of PV. Indeed, the torque that each anticyclone exerts on the other is similar to the torque in the case R, so that $a_c$ is very similar in both cases. The merger (Fig. 10b) is therefore very similar to the identical anticyclone merger (Fig. 5a). The main difference is the generation of two large filaments (Fig. 12). These filaments originate in the deep anticyclone and are caused by the torque exerted by the shallow anticyclone. On the other hand, as it happens in identical anticyclone merging case R, the flow speed increases in the first phases of merger (Fig. 9) and reaches its maximum value when each anticyclone wraps around the other. However, in this case, the flow speed $(w^\text{sub max} = 1.7 \times 10^{-3}$ and $u^\text{sub max} = 0.7)$ is a bit larger than that in the case R where $w^\text{sub max} = 1.4 \times 10^{-3}$ and $u^\text{sub max} = 0.69$.

4. Concluding remarks

Numerical results show, in agreement with experimental observations south of the Canary Islands, that two corotating vortices, sufficiently close to each other and in presence of a third counterrotating vortex, approach each other and ultimately merge, leading to a new elongated vortex, which couples with the counterrotating vortex forming a dipole. The fluid contribution of each corotating vortex to the new vortex depends on the torque that each vortex exerts on the other. An equal PV contribution from each corotating vortex happens when the vortices exert the same torque on each other, whereas an unequal contribution happens otherwise. Loss of PV through filamentation always occurs.

Obviously, the vortex merging process depends on the initial conditions. Vortex merger is favored when the corotating vortices have initially a large amount of PV (because they have either a large PV density or a large vertical extent) and span an angle $a$ smaller than a critical angle $a_c$.

The typical speed $\|u\|$ increases during the merging process and decreases afterward, finally reaching a value similar to the initial one. The vertical velocity $w$ associated to this three-vortex interaction process has a complicated structure displaying alternating upwelling and downwelling cells and developing extrema value along the corotating vortex common boundary before these vortices wrap around each other.

The presence of a third counterrotating vortex affects the merging process of two corotating vortices in the following ways: 1) axisymmetrization, because asymmetry is favored because the counterrotating vortex prevents filaments to crowd around the new elongated vortex; 2) filamentation, because filaments become unsteady and, once expelled outside the place they were generated, may develop small satellite vortices; 3) the critical angle, because the torque exerted by the counterrotating vortex causes a decrease in the critical merging angle; and 4) the flow speed, because both $\|u\|$ and $w$ increase because of the presence of a third vortex. We note that the conclusions above have been obtained using a process-oriented numerical model of simplified dynamics with very idealized initial conditions. In particular, we have not taken into account several physical conditions present in the Canary basin such as background mean currents (e.g., the Canary Current), wind stress (e.g., due to the Trade Winds), or solid boundary conditions (e.g., the presence of the African coast). These facts
are probably important in the generation of the three-vortex PV configuration that we have taken as initial conditions but do not seem to be of primary importance in the process of vortex merger, which happens in a short time scale of about 15 inertial periods. The origin of these initial vortex conditions deserve more realistic numerical simulations including, of course, the presence of coastal boundaries, wind stress, and background currents. The influence of beta effect on the merger of large oceanic vortices should be also addressed in a three-dimensional Boussinesq model. New experimental data and more realistic numerical simulations will help us to understand the ways and frequency these corotating and counterrotating vortices interact south of the Canary Islands.

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