Damping Injection by Reset Control

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This paper presents a method for using reset control as an alternative way of obtaining dissipation for a class of port-Hamiltonian systems. One advantage of this approach is the simplicity of its implementation, which requires only a velocity observer. Another advantage is its robustness to modeling uncertainties, since it can be calculated independently of the plant structure. A gantry crane is selected as case study, yielding simulation and experimental results. Finally, conclusions and guidelines for future work are given in Sec. 4.

2 Methods

2.1 Port-Controlled Hamiltonian Systems. The standard Euler–Lagrange equations are given as

\[
\frac{d}{dt} \left( \frac{\partial L}{\partial q} \right) - \frac{\partial L}{\partial \dot{q}} = \tau
\]

where \( q = (q_1, \ldots, q_n) \) are generalized configuration coordinates for the \( n \) degrees of freedom. The Lagrangian is \( L = T - V \), where \( T \) is the kinetic energy and \( V \) the potential energy; and \( \tau = (\tau_1, \ldots, \tau_n) \) is the vector of generalized forces. In standard mechanical systems, the kinetic energy is given by \( T = \frac{1}{2} M(q) \dot{q} \). The inertia matrix is symmetric and definite positive for all \( q \). The vector of generalized momenta \( p = (p_1, \ldots, p_n) \) is defined for as \( p = \frac{\partial L}{\partial \dot{q}} = M(q) \dot{q} \). The system vector as \( x = (q, p) \), the second order Lagrangian equations (1) transform into the 2n first order equations

\[
\dot{q} = \frac{\partial H}{\partial p}
\]

\[
\dot{p} = -\frac{\partial H}{\partial q} + \tau
\]

where the energy is given by the Hamiltonian function,

\[
H = \frac{1}{2} p^T M^{-1}(q) p + V(q)
\]

The system (2) is an example of a Hamiltonian system with collocated inputs and outputs, which is given more generally in the following form:

\[
\dot{q} = \frac{\partial H}{\partial p}
\]

\[
\dot{p} = -\frac{\partial H}{\partial q} + B(q)u
\]

\[
y = B^T(q) \frac{\partial H}{\partial p}
\]

Here, \( B(q) \) is the input force matrix, with \( B(q)u \) representing the generalized forces resulting from the control inputs.
where $J(x) = -J^T(x)$ and $R(x) = R^T(x) \succeq 0$ are, respectively, the interconnection and damping matrices.

### 2.1.1 Case Study: Port-Hamiltonian Model of a Gantry Crane

Let us model a 2 degrees of freedom gantry crane, such as the one depicted in Fig. 1, in the way described by Eq. (3). The Hamiltonian state coordinates are $q = (r, p, \theta, \rho, \rho_0, \rho_d)$. The cart with mass $m_c$ moves on the girder in the $r$-direction under the actuating force $F_r$, so its position coordinate is given by $r$. The payload is represented by a point mass $m_p$ hanging from a rope with variable length $\rho$. The payload and the cart are assumed to be connected by a massless, rigid rope, and the mass of the payload is assumed to be concentrated at a point. Adopting polar coordinates, with $\theta$ as the swing angle, the payload position is given by $r = \rho \sin \theta$. Thus, the payload moves in the $\rho$ direction under the actuating force $F_r$. The kinetic energy is given by

$$K = \frac{1}{2} (m_c \rho^2) + \frac{1}{2} (m_p \rho_d^2)$$

and the potential energy is $V = -m_p g \cos \theta$, where $g$ is the gravity acceleration. The Hamiltonian is $H(q, p) = \frac{1}{2} p^T M^{-1} q + V(q)$, where $M(q) = \frac{\partial^2 H}{\partial q \partial q}$ is the mass matrix, and $p$ is the momenta vector. As can be verified, $H(q, p)$ is a not a strictly positive energy function for any desired equilibrium point $q_d = (r_d, \rho_d, 0)$. The Hessian matrix of the uncontrolled plant at the equilibrium point is thus

$$\frac{\partial^2 H(q, p)}{\partial q \partial q} = \begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 \\\ng m_p \rho_d & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 2 \frac{m_c \rho_d}{m_c} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 2 \frac{m_c \rho_d}{m_c} \\
0 & 0 & 0 & 0 & 0 & 0 \\
\end{pmatrix}$$

(4)

The movement equations can be derived easily, obtaining Eq. (3) with

$$J = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad R = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \quad G = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

The control by interconnection was first proposed and developed in Ref. [15]. A method to shape the Hamiltonian was given by Eq. (3), regarded as a plant system to be controlled. Recall the well-known result that the standard feedback interconnection of two passive systems is again a passive system [15]. A method to shape the energy function via interconnection was first proposed and developed in Ref. [13]. The main idea of this method is to interconnect the plant system (3) with a source system given by

$$\begin{align*}
\dot{\xi} &= J_1(\xi) \frac{\partial H_1}{\partial \xi} + G_1(\xi) u_c \\
\dot{u}_c &= G_1^T(\xi) \frac{\partial H_1}{\partial \xi} 
\end{align*}$$

regarded as the controller system, via the standard feedback interconnection

### 2.2 Interconnection and Damping Assignment-Passivity-Based Control

Let $x_d$ be a desired configuration in the state space for a plant described in the port-Hamiltonian framework as in Eq. (3). The control goal is to find a state feedback law $u = \tilde{\beta}(x)$ such that the dynamics of the resulting closed loop system is given by

$$\dot{x} = [J(x) - R(x)] \frac{\partial H_d}{\partial x}$$

where $J_d(x)$ and $R_d(x) \succeq 0$ are desired interconnection and damping matrices, respectively. This desired energy function $H_d(q, p)$ can be represented as

$$H_d(q, p) = \frac{1}{2} p^T M^{-1} q + V_d(q)$$

The plant can be regulated to $x_d$ in a passive way if the desired energy function $H_d(q, p)$ has a minimum in the state space. This procedure is called interconnection and damping assignment (IDA), and it can be applied jointly with passivity-based control (PBC) [13]. In PBC, the control input is naturally decomposed into two terms, $u = u_d(q, p) + u_a(q, p)$, where

$$u_d(q, p) = -K_c G^2 \frac{\partial H_d}{\partial p}$$

with $K_c > 0$ responsible for damping injection. Energy shaping is obtained with $u_a = (G^2)^T \frac{\partial H_d}{\partial q} \frac{\partial H_d}{\partial q}^T$, as in Ref. [12].

### 2.3 Control by Interconnection

Consider a port-controlled Hamiltonian system given by Eq. (3), regarded as a plant system to be controlled. Recall the well-known result that the standard feedback interconnection of two passive systems is again a passive system [15]. A method to shape the energy function via interconnection was first proposed and developed in Ref. [13]. The main idea of this method is to interconnect the plant system (3) with a source system given by

$$\dot{\xi} = J_1(\xi) \frac{\partial H_1}{\partial \xi} + G_1(\xi) u_c \\
\dot{u}_c = G_1^T(\xi) \frac{\partial H_1}{\partial \xi}$$

Fig. 1 2D overhead crane arrangement

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Assuming that there are no external disturbances ($e = 0, \epsilon_v = 0$), the closed loop takes the form
\[
\begin{pmatrix}
\dot{x} \\
\dot{\xi}
\end{pmatrix} = \begin{pmatrix}
J(x) & -G(x)G^T(\xi) \\
G_c(\xi)G^T(x) & I_c(\xi)
\end{pmatrix} \begin{pmatrix}
\frac{\partial H}{\partial x} \\
\frac{\partial H_c}{\partial \xi}
\end{pmatrix}
\]
\[
\begin{pmatrix}
y \\
y_c
\end{pmatrix} = \begin{pmatrix}
G(x) & 0 \\
0 & G_c(\xi)
\end{pmatrix} \begin{pmatrix}
\frac{\partial H}{\partial x} \\
\frac{\partial H_c}{\partial \xi}
\end{pmatrix}
\]
and the closed loop energy function is
\[
H_c(x, \xi) = H(x) + H_c(\xi)
\]

2.4 Reset Control. A resetting differential equation consists of three elements:

(1) A continuous-time dynamical equation, which governs the motion of the system between resetting events.

(2) A difference equation, which governs the way the states are instantaneously changed when a resetting event occurs.

(3) A criterion for determining when the states of the system are to be reset.

Thus, a resetting differential equation has the form
\[
\dot{x}(t) = f(x(t)), (t, x(t)) \not\in S
\]
\[
\Delta x(t) = \rho(x(t)), (t, x(t)) \in S
\]
where $t \geq 0, x(t) \in \mathbb{R}^d, f: \mathbb{R}^d \rightarrow \mathbb{R}^d$ is Lipschitz continuous and satisfies $f(0) = 0$; $\rho: \mathbb{R}^d \rightarrow \mathbb{R}^d$ is such that $\rho(0) = 0$ and $S \subseteq [0, \infty) \times \mathbb{R}^d$ is the resetting set. We refer to the first equation in Eq. (8) as the continuous-time dynamics, and to the second equation in Eq. (8) as the resetting law. For our purposes, the following result for the stability of the zero solution is needed.

**Theorem 1.** Suppose there exists a continuously differentiable function $V: \mathbb{R}^d \rightarrow [0, \infty)$ satisfying $V(0) = 0, V(x) > 0, x \neq 0$, and
\[
\frac{\partial V}{\partial x} f(x) \leq 0, x \not\in S
\]
Then the zero solution of Eq. (8) is Lyapunov stable. Furthermore, if the inequality in Eq. (9) is strict for $x \neq 0$, then the zero solution is asymptotically stable [16].

3 Damping by Reset Interconnection

In this section, it is shown how, by interconnecting the plant to a reset controller, it is possible to achieve the desired damping injection effect. Instead of using Eq. (6), the dissipation is injected through an energy absorber device, characterized by a resetting oscillator. The controller system interconnected with the plant is given by Eq. (7), with the energy function being
\[
H_c = \frac{1}{2} (q^T K_c q_c + p^T M_c^{-1} p_c)
\]
This controller corresponds physically to a mass–spring system, with $K_c$ and $M_c$ being the (constant, definite positive) virtual rigidity and mass controller matrices. Since it is a reset controller, its dynamic equations corresponding to Eq. (8) are
\[
\dot{q}_c = M_c^{-1} p_c
\]
\[
\dot{p}_c = -K_c q_c + y_c(q_c, p_c) \not\in S
\]
\[
u_c = K_c q_c
\]
\[
\Delta q_c = -q_c
\]
\[
\Delta p_c = -p_c(q_c, p_c) \in S
\]
\[
u_c = K_c q_c
\]
Notice that, without taking reset into consideration, the controller does not include any energy-dissipating elements. The set $S$ is characterized as those $(q_c, p_c)$ for which $\frac{\text{d}H_c(q_c, p_c)}{\text{d}t} < 0$. As stated in Theorem 1, this resetting controller asymptotically stabilizes the
plant for \( V(x, \xi) = H_d(x) + H_c(\xi) \). Deriving along an orbit \( \dot{V}(x, \xi) = H_d(x) + H_c(\xi) \) and calculating the theorem conditions, we obtain

\[
(x, \xi) \notin S \Rightarrow \dot{V}(x, \xi) = 0, \text{ (lossless)}
\]
\[
(x, \xi) \in S \Rightarrow \Delta V(x, \xi) = -H_c(\xi) < 0, \forall \xi \neq 0
\]

Notice that, since the flow is lossless, the first inequality in Eq. (9) is not strict and we cannot prove asymptotic stability. However, in practice, we have found that the dissipation is complete and asymptotic stability is achieved, as intuitively expected. A rigorous prove of this property deserves further research.

3.1 Case Study: Damping by Resetting for a Gantry Crane. We show now how the procedure can be applied to our case study. For the gantry crane, \( q_c = (r, \rho) \) are the controller configuration variables and \( p_c = (p_r, p_\rho) \) its momenta. The resetting law is calculated with

\[
\frac{dH_c}{dt} = \frac{2((m_22\dot{r} - m_12\dot{\rho})p_{r1} + (m_11\dot{\rho} - m_12\dot{r})p_{\rho1})}{m_1m_22 - m_11^2}
\]

We perform simulations using the values \( m_c = 1.155 \), \( m_b = 0.5 \), \( g = 9.8 \), \( K_r = 3; K_q = 2; K_s_r = 1; K_s_q = 1 \), with initial conditions \( r_0 = 0, \rho_0 = 1, \dot{r}_0 = \pi/4, \dot{\rho}_0 = 0, p_r = 0, p_\rho = 0, \). The desired equilibrium point \( (r_d = 2, p_d = 2, \theta_0 = 0) \), and the controller parameters are

\[
K_c = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}, M_c = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}
\]

The simulation results in Fig. 2 show the good performance of the adopted solution. In Fig. 3, the evolution of the plant’s energy is plotted.

It should be noticed that the use of dissipation injection as given in Eq. (6), that is,
requires to know precise values for the momenta, entailing the
adoption of a full state observer. As can be concluded from the use
of the resetting controller, only the plant velocities ($\dot{r}, \dot{h}$) are needed
for its implementation; thus, only a velocity observer is required.

The dissipation injection as given by Eq. (11) has two tuning
parameters ($K_r, K_q$) while the resetting controller has a richer pa-
parameter space ($K, \lambda$) to enhance transient characteristics.

The controller has also been applied to a real gantry crane, Inte-
co’s 3DCrane model (depicted in Fig. 4, see also http://www.inte-
co.com.pl/ for details). Experimental results can be found in
Fig. 5, where the evolution of the ($r, q$) coordinates (cart position
and cable length) are plotted both for the case that no reset is
applied (dashed lines) and for the resetted controller (solid lines).

The evolution of the controller’s energy is pictured in Fig. 6,
where the abrupt changes due to reset of the controller states can
be noticed.

4 Conclusions

In this paper, a new strategy for injecting dissipation into port-
controlled Hamiltonian systems has been designed. The controller
synthesis procedure is as follows: first, a physical controller is
developed, which is characterized as a port-Hamiltonian system
itself. This controller has in principle no damping terms, and it is
connected to the plant to be controlled in an energy-conserving
way. Dissipation is then achieved by resetting the controller states
every time that the controller’s energy is going to decrease. It has
been shown that the effect of this reset (and hence, nonlinear) con-
troller is equivalent to injecting damping to the plant at some
required moments, thus leading to performance improvements.

An advantage of this alternative way of performing damping
injection is the simplicity of its implementation.

Acknowledgment

This work was supported by the Spanish Ministry of Science
and Education, Grant No. DPI-2007-66455-C02-02.

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