Hadronic $B_c$ decays as a test of $B_c$ cross section

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This paper focuses on disagreement between theoretical predictions and experimental results of the production properties of $B_c$ meson. Hadronic decays of $B_c$ are used to separate predictions of production cross section and predictions of branching ratio. The branching ratios of $B_c$ decays to $J/\psi + \pi$ and to $J/\psi + 3\pi$ are also presented.

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I. INTRODUCTION

Study of $B_c$ meson is important because it stands out of the crowd of other heavy-quark mesons. This is the only meson consisting of two different heavy quarks. Also, the lighter $c$ quark has a decay rate ($\sim 65\%$) [1] larger than heavier $b$ quark, which is uncommon for heavy-quark mesons. The mass and lifetime of $B_c$ meson have been measured by CDF [2,3] and D0 [4,5] in decays $B_c \rightarrow J/\psi \pi$ and $B_c \rightarrow J/\psi \ell$. They are in pretty good agreement with theory [1,6] (see Table I).

Also, the production properties of $B_c$ meson have been measured and compared to that of $B$ meson [7]:

$$R_e = \frac{\sigma(B_c) \cdot Br(B_c \rightarrow J/\psi e^\pm \nu)}{\sigma(B) \cdot Br(B \rightarrow J/\psi K^\pm)} = 0.282 \pm 0.038 \pm 0.074$$

and

$$R_\mu = \frac{\sigma(B_c) \cdot Br(B_c \rightarrow J/\psi \mu^\pm \nu)}{\sigma(B) \cdot Br(B \rightarrow J/\psi K^\pm)} = 0.249 \pm 0.045^{+0.107}_{-0.076}$$

in the kinematic region $p_T(B_{(c)}) > 4.0$ GeV and $|y(B_{(c)})| < 1.0$. Using the theoretical predictions for the branching fraction $Br(B_c \rightarrow J/\psi e^\pm \nu) = 2 \cdot 10^{-2}$ [1,8] and taking into account well-measured branching $Br(B^+ \rightarrow J/\psi K^+) = (1.007 \pm 0.0035) \cdot 10^{-3}$ [9], one can obtain the ratio of the production cross sections:

$$\frac{\sigma(B_c)}{\sigma(B)} = R_e \cdot \frac{Br(B \rightarrow J/\psi K^\pm)}{Br(B_c \rightarrow J/\psi e^\pm \nu)} \approx 1.4 \cdot 10^{-2}.$$  

Comparing this result with theoretical predictions of $B_c$ cross section [10–13] and of the ratio of production cross section $\sim 10^{-3}$ we see that $B_c$ semileptonic branching fraction has to be an order of magnitude larger than theoretical prediction, about 20%. This is a significant discrepancy between theory and experiment. Another discrepancy comes from the measurement of the production properties of $B_c$ in CDF data collected in Run I [14]. CDF presented a 95% C.L. on $\sigma(B_c) \cdot Br(B_c \rightarrow J/\psi e^\pm \nu)/\sigma(B_{\psi}) \cdot Br(B^+ \rightarrow J/\psi K^+)$ as a function of $B_c$ lifetime (see Fig. 1).

Using known $B_c$ lifetime $(0.46 \pm 0.07)$ ps we clearly see an order of magnitude disagreement between the theoretical prediction of $B_c$ properties with experimental results from Tevatron.

<table>
<thead>
<tr>
<th>Source</th>
<th>$B_c$ mass (MeV/c$^2$)</th>
<th>$B_c$ lifetime (ps)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CDF [2,3]</td>
<td>$6285 \pm 5.3$ (stat) $\pm 1.2$ (sys)</td>
<td>$0.463^{+0.077}_{-0.036}$ (stat) $\pm 0.036$ (sys)</td>
</tr>
<tr>
<td>D0 [4,5]</td>
<td>$6300 \pm 14$ (stat) $\pm 5$ (sys)</td>
<td>$0.448^{+0.037}_{-0.032}$ (stat) $\pm 0.032$ (sys)</td>
</tr>
<tr>
<td>Theory [1,6]</td>
<td>$6278$</td>
<td>$0.48 \pm 0.05$</td>
</tr>
</tbody>
</table>

FIG. 1 (color online). The circular points show the different 95% C.L. on the ratio of cross section times branching fraction for $B_c^+ \rightarrow J/\psi \pi^+$ relative to $B_{\psi}^+ \rightarrow J/\psi K^+$ as a function of the $B_c^+$ lifetime. The dotted curve represents calculation of this ratio based on the assumption that the $B_c^+$ is produced $1.5 \times 10^{-3}$ times as often as all other $B$ mesons and that $\Gamma(B_c^+ \rightarrow J/\psi \pi^+) = 4.2 \times 10^9$ s$^{-1}$. |
I. THEORETICAL BASEMENT

In this paper we will use the fact that a hadronic matrix element of heavy-quark current might be written in a simple form if expressed in terms of the velocities of heavy particles [8,15,16]. Also, we will base on definition of nonrecoil form factor. The validity of using the nonrecoil approximation is strongly supported by the fact that the kinematic variable \(\omega = v_1 \cdot v_2\) is restricted to values close to unity (indexes 1 and 2 do mean initial and final hadrons, respectively). Let heavy-quark \(Q_i\) undergo a weak decay to \(Q_f\) with a spectator quark \(Q_s\). At \(s = (p_1 - p_2)^2 = 0\), we have

\[
(v_1 \cdot v_2)_{\text{max}} = 1 + \frac{(m_1 - m_2)^2}{2m_1 m_2} - 1 + \frac{(m_Q - m_Q')^2}{2(m_Q + m_Q')(m_Q + m_Q')},
\]

where the vector and axial-vector currents are \(V^\mu = \bar{Q}_f \gamma^\mu Q_i\) and \(A^\mu = \bar{Q}_f \gamma^\mu \gamma_5 Q_i\) and \(\eta_{12}\) is a form factor playing the role of Isgur-Wise functions for transition between initial and final states of hadrons. Here, \(\eta_{12}\) can be parametrized as

\[
\eta_{12} = \left(\frac{2 \beta_1 \beta_2}{\beta_1^2 + \beta_2^2}\right)^{1/2}.
\]

For the case of \(B_c\) decays to \(J/\psi\), the parameters \(\beta_1\) and \(\beta_2\) are equal to 0.82 and 0.66, respectively.

II. DECAY \(B_c^+\) TO \(J/\psi + \pi^+\)

The amplitude of this decay includes two factors, one of them is a pionic decay amplitude, and the other is the formfactor appearing in semileptonic decay. This gives us a direct relation between pionic and semileptonic decays. In the case \(s = m_{B_c}^2 \approx 0\), the width of pionic decay may be given as [17]

\[
\frac{\Gamma(B_c \rightarrow J/\psi + \pi)}{d\Gamma/ds_{\rightarrow 0}(B_c \rightarrow J/\psi + \ell \nu)} \approx 6\pi^2 f_{\pi}^2 |V_{ud}|^2 \approx 1 \text{ GeV}^2.
\]

Upon contracting Eq. (2) with leptonic current \(\bar{\ell} \gamma^\mu (1 - \gamma_5) \nu\), the width of \(B_c \rightarrow J/\psi + \ell \nu\) is [8]

\[
\frac{d\Gamma}{ds} \approx 3 \cdot \frac{G_F^2 (\lambda^{3/2} + 12s^2 m_{B_c}^2)^{1/2}}{576\pi^3 m_{B_c}^5} \cdot \frac{m_{J/\psi}}{m_{B_c}} \eta_{B_c/J/\psi} |V_{uJ/\psi}|^2,
\]

where \(\lambda = m_{J/\psi}^2/m_{B_c}^2\) is “triangle” Källen function denoted as

\[
\lambda(x, y, z) = (x^2 + y^2 + z^2 - 2xy - 2yz - 2zx)^{1/2}.
\]

Combining Eqs. (3) and (4) and using \(B_c\) lifetime \(\tau = 0.46 \text{ ps}\), we may expect the pionic decay branching ratio to be

\[
Br(B_c^+ \rightarrow J/\psi + \pi^+) \approx 0.2\%.
\]

This result is in good agreement with other results (see [1,8] and references therein).

III. DECAY \(B_c^+\) TO \(J/\psi + \pi^+\pi^-\pi^+\)

A. Axial current

The amplitude of \(B_c^+ \rightarrow J/\psi + \pi^+\pi^-\pi^+\) is

\[
A \sim \langle J/\psi |A^\mu|B_c\rangle < \pi^+\pi^-\pi^+ |J^\mu_{\text{axial}}(0)|0>,
\]

where \(\langle J/\psi |A^\mu|B_c\rangle = 2 \eta_{B_c/J/\psi} |V_{uJ/\psi}|^2\). \(A^\mu = \bar{c} \gamma^\mu \gamma_5 b\) is the axial-vector current and \(\epsilon^\mu_{J/\psi}\) presents the polarization four-vector of \(J/\psi\). Let us remind the reader that the phase space can be represented as

\[
dPS(B_c \rightarrow J/\psi + 3\pi) = \frac{dS}{2\pi} dPS(W^+ \rightarrow 3\pi),
\]

where the three-pion phase space is

\[
\frac{1}{2\pi} \int dPS(3\pi) < 0 |J_{\mu}|3\pi > < 3\pi |J_{\nu}|0 > = q_{\mu} q_{\nu} p_{0}(s) + (q_{\mu} q_{\nu} - g_{\mu\nu} q^2) p_{1}(s).
\]
where \( q \) the four momentum vector of the three-pion state and \( s = q^2 \). It is easy to show that \( \rho_0 = 0 \).

We have the same situation in \( \tau^+ \rightarrow \nu_\tau + \pi^- \pi^+ \pi^- \) decay, therefore we will follow \( a_1 \) meson domination model of Ref. [18] (\( a_1 \) dominance is also discussed in [19,20], angular distributions of \( \tau \rightarrow \nu + 3\pi \) are discussed in [21]). The spectral function \( \rho_1(s) \) can be cast into the form

\[
\rho_1(s) = \frac{1}{6} \frac{1}{(4\pi)^4} \frac{8}{9g^2} |\text{BW}_{a_1}(s)|^2 \frac{g(s)}{s}.
\] (7)

The Breit-Wigner function \( \text{BW}_{a_1} \) is parametrized including energy dependent width \( \Gamma_{a_1}(s) \):

\[
\text{BW}_{a_1} = \frac{m_{a_1}^2}{m_{a_1}^2 - s - i\sqrt{s}\Gamma_{a_1}(s)}.
\]

\[
\Gamma_{a_1}(s) = \frac{m_{a_1}}{\sqrt{s}} \frac{g(s)}{g(m_{a_1})},
\] (8)

where \( m_{a_1} = 1251 \pm 13 \text{ GeV}, \Gamma_{a_1} = 599 \pm 44 \text{ MeV}, \) and the function \( g(s) \) has been calculated in Ref. [18] and is derived from the observation that the axial-vector resonance \( a_1 \) decays predominately into tree pions. In this way, the branching is

\[
Br(B_c^+ \rightarrow J/\psi + \pi^+ \pi^- \pi^+) \approx 0.3\%.
\]

B. Vector current

The other possibility to observe three charged pions in fully reconstructed mode is \( B_c \) decay to \( J/\psi + \omega \pi \), where \( \omega \) decays to \( \pi^+ \pi^- \). However, the simple analysis of similar decay \( \tau^+ \rightarrow \nu_\tau + \omega \pi^- \), \( \omega \rightarrow \pi^+ \pi^- \) decay shows that this mode gives a too small contribution.

V. SUMMARY

Current theoretical and experimental knowledge about \( B_c \) meson suggests that either we do not understand the production cross section or semileptonic branching fraction of \( B_c \) (see Sec. I). In our paper we propose to measure the branching fractions for \( B_c \) decays into final states \( J/\psi + \pi \) and \( J/\psi + 3\pi \) to resolve this issue. Since the decays to \( J/\psi + \pi \) and \( J/\psi + \ell \nu \) are correlated (as discussed in Sec. III) the decay into \( J/\psi + 3\pi \) has a special meaning, allowing for independent test of \( B_c \) production cross section. The predictions of the branching fractions of \( B_c \) decays into these final states are also obtained.

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