Electronic collective excitations in compressed lithium from \textit{ab initio} calculations: Importance and anisotropy of local-field effects at large momenta

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According to our \textit{ab initio} calculations, the recently reported undamped plasmon in compressed fcc lithium emerges not only due to the fact that the dielectric matrix determinant vanishes at long wavelengths at the plasmon energy, i.e., satisfying the ideal condition for an undamped plasmon in a crystal but also extends at large momenta specially along the IL direction. Different from the case of simple metals, the local-field effects dominate, leading to the striking periodicity exhibited by the low-energy plasmon for momentum transfers beyond the first Brillouin zone. Remarkably, pressure-induced electronic anisotropy and localization increase the impact of local-field effects on the electronic response even in the compact fcc structure. Interestingly, the importance of local-field effects is similar in lithium under pressure and in materials with confined geometries such as MgB\textsubscript{2}.

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I. INTRODUCTION

The intense research on simple materials under pressure of the last decade has taught physicists not to consider any longer alkaline metals as model free-electron-like systems. In fact, as a consequence of the induced $s$-$p$ orbital mixing,\textsuperscript{1} pressure makes them depart from their simplicity at ambient conditions to become metals with rather complex behavior.\textsuperscript{2} Not only superconductivity arises under pressure in lithium with a large $T_c$\textsuperscript{3,4} but, even more strikingly, both lithium and sodium become semiconductors when further compressed\textsuperscript{5,6} confirming the predictions made by Neaton and Ashcroft.\textsuperscript{7,8} Moreover, contradicting common sense, the melting curve of the alkalies shows a negative slope of the melting temperature as a function of pressure in a wide pressure range\textsuperscript{9–13} confirming the predictions made by Neaton and Ashcroft.\textsuperscript{7,8} Therefore, alkaline metals exemplify how pressure can become a catalyst of complexity in different systems.

The departure of lithium from nearly free-electron behavior is reflected as well in its energy-loss spectra. Lithium transforms from its equilibrium body-centered cubic (bcc) phase into the face-centered cubic (fcc) one at 8 GPa and into a series of low-coordinated complex structures above 40 GPa.\textsuperscript{15} Coinciding with the bcc-fcc phase transition, beyond the conventional free-electron-like plasmon, the energy-loss spectra shows that a low-energy undamped plasmon emerges.\textsuperscript{16,17} The origin of this remarkable extra collective excitation is associated to interband electronic transitions around the L point that are enhanced by the pressure-induced anisotropic flattening of the bands. Interestingly, this plasmon is similar to the low-energy plasmon found experimentally in the good superconductor MgB\textsubscript{2} (Ref. 18) that was predicted theoretically in advance.\textsuperscript{19,20} Remarkably, as it was found by Cai et al.\textsuperscript{21} in a joint experimental and theoretical work, such a plasmon in MgB\textsubscript{2} extends beyond the first Brillouin zone (1BZ) presenting a striking periodic dispersion as a function of momentum $\mathbf{q}$. Here, based on \textit{ab initio} calculations, we predict that the lower energy plasmon found in compressed fcc lithium\textsuperscript{16} appears beyond the 1BZ as well. Hence, present results demonstrate that such plasmon dispersions are not restricted to anisotropic layered geometries and that long-lived plasmons in subsequent BZs can also be present in a compact cubic structure like the fcc lattice.

The paper is organized as follows. In Sec. II we briefly describe details of a self-consistent calculation of dynamical response properties within the \textit{ab initio} approach. The calculation results for Li in the fcc lattice compressed up to 40 GPa are discussed in Sec. III. Summary and conclusions are given in Sec. IV. Unless stated otherwise, we use atomic units (a.u.) throughout, i.e., $\hbar = e^2 = m_e = 1$.

II. CALCULATION DETAILS

A plasmon is defined as a self-sustained charge oscillation, that is, as an oscillating electronic potential that is setup without the application of an external potential. This implies that the plasmon dispersion is ideally defined by the set of $(\mathbf{k}, \omega)$ points at which\textsuperscript{22}

$$\text{det} \mathbf{\varepsilon}_{GG}^{(1)}(\mathbf{k}, \omega) = 0.$$ \textsuperscript{1}

In Eq. (1) $\mathbf{\varepsilon}_{GG}^{(1)}(\mathbf{k}, \omega)$ is the dielectric matrix and the reciprocal lattice vectors $\mathbf{G}$ serve as its indices. According to this
In Eq. (1), the exchange-correlation effects have been approximated within the random-phase approximation (RPA). The lattice parameter used here is \( a = 6.2739 \) a.u., which approximately corresponds to an applied pressure of 40 GPa, close to the value at which the fcc Li phase transforms into lower coordinated structures.

III. RESULTS AND DISCUSSION

The dynamical structure factor of lithium at 40 GPa, including results at large momenta with \( q \) beyond the 1BZ, is presented in Fig. 1. As it has been already noted,\(^{17}\) the undamped plasmon has similar dispersion along \( \Gamma X [100] \), \( \Gamma K [110] \), and \( [111] \) directions while \( q \) lays in the 1BZ. As can be seen in Figs. 1(a)–1(c), the dynamical structure factor shows that along these three directions the plasmon starts at

\[
\chi^0(k,\omega) = \sum_{\mathbf{G}} \frac{1}{\Omega} \sum_{\mathbf{G}} \frac{f_{\mathbf{k}n} - f_{\mathbf{k}n^{-1} + \mathbf{G}}}{\varepsilon_{\mathbf{k}n} - \varepsilon_{\mathbf{k}n^{-1} + \mathbf{G}} + i\eta} \langle \psi_{\mathbf{k}n^{-1} + \mathbf{G}} | e^{i(k+\mathbf{G})r} | \psi_{\mathbf{k}n^{-1} + \mathbf{G}} \rangle,
\]

where \( |\psi_{\mathbf{k}n}\rangle \) and \( \varepsilon_{\mathbf{k}n} \) are, respectively, the single-electron Bloch wave functions and energies, \( f_{\mathbf{k}n} \) the Fermi-Dirac distribution function, and \( \eta \) a positive infinitesimal. In this work, instead of calculating directly Eq. (5) we first obtain the imaginary part of \( \chi^0 \) as

\[
-\frac{1}{\pi} \text{Im} \chi^0_{\mathbf{k}}(k,\omega) = \text{Im} \sum_{\mathbf{G}} \frac{1}{\Omega} \sum_{\mathbf{G}} \frac{f_{\mathbf{k}n} - f_{\mathbf{k}n^{-1} + \mathbf{G}}}{\varepsilon_{\mathbf{k}n} - \varepsilon_{\mathbf{k}n^{-1} + \mathbf{G}} + i\eta} \langle \psi_{\mathbf{k}n^{-1} + \mathbf{G}} | e^{i(k+\mathbf{G})r} | \psi_{\mathbf{k}n^{-1} + \mathbf{G}} \rangle,
\]

and then calculate its real part using the Kramers-Kronig relation. Once \( \chi^0 \) is calculated, the response function can be obtained from Eq. (4) and, thereafter, the loss function from Eq. (3).
\( \approx 6 \) eV and disperses smoothly until it decays via Landau damping into the electron-hole single-particle continuum at approximately 0.4 a.u.\(^{-1}\) along \( \Gamma K \) and \( \Gamma X \) and around 0.5 a.u.\(^{-1}\) along \( \Gamma L \). This slight difference is attributed to the anisotropy of the electron-hole excitation continuum, given by \(-\text{Im} \chi^0\) and depicted in Figs. 1(d)–1(f), that shows a small valley at energies around \(6\) eV along \( \Gamma L \) from which the plasmon can spread longer. Interestingly, within the 1BZ, a very similar curve is obtained if one inverts directly the dielectric function-like peak with a Lorentzian of a width of 25 meV and a height that allows the \(f\) sum rule\(^{23}\) in the form

\[
\int_{-\infty}^{\infty} d\omega \, \omega [ - \text{Im} \epsilon_{G_G}^0(\mathbf{k}, \omega)] = 2 \pi^2 n_0
\]

with \( n_0 \) the average electron density and \( \mathbf{k} \) in the 1BZ, to be satisfied. The fact that both definitions agree at almost the same energy explains why LFEs are not important for mo-

FIG. 1. (Color online) Normalized dynamical structure factor, \( S(q, \omega)/\Omega \), for fcc lithium at 40 GPa within the RPA along (a) \( \Gamma K [110] \), (b) \( \Gamma X [100] \), and (c) \( \Gamma L [111] \) symmetry directions. The LFEs are included. The undamped plasmon peak is denoted with \( A \) in the 1BZ and with \( A' \) beyond the 1BZ. \(-\text{Im} \chi_{00}^0(\mathbf{q} - \mathbf{G}_q, \omega)\), which informs about the electron-hole excitations in the system, is presented in (d), (e), and (f) for \( \Gamma K \), \( \Gamma X \), and \( \Gamma L \) directions, respectively. Note the presence of interband transitions at small momenta in (d), (e), and (f) with energies around \(-8\) eV. The dotted green lines in all the figures show the boundaries for the intraband electron-hole excitations in the jellium model when the effective mass values of 2.10 (along \( \Gamma K \)), 2.04 (along \( \Gamma X \)), and 2.16 (along \( \Gamma L \)) are employed.
momenta in the 1BZ. At higher momenta, \(k \approx 0.2 \text{ a.u.}^{-1}\), as soon as the plasmon curve enters into the electron-hole damping region and \(\text{Im } \varepsilon\) is no longer zero, the condition of Eq. (1) is not satisfied and the loss function shows a damped peak as depicted in Fig. 2(b). Remarkably, the low-energy plasmon of MgB\(_2\) also shows a weak influence of LFE for momenta in the 1BZ.\(^{21,35,36}\)

Although in the 1BZ LFEs do not modify dramatically the plasmon dispersion, they are crucial to make the undamped plasmon reappear beyond the 1BZ as it happens in MgB\(_2\).\(^{21}\)

In fact, no resonant peak is observed in the loss function when the inversion is performed as in Eq. (7) for large \(q\)’s. This can be understood easily if one reverses the inverse dielectric matrix by the exact expression\(^{21}\)

\[
epsilon_{G_q}^{-1}(q-G_q,\omega) = \frac{1}{\varepsilon_{G_q}G_q(q-G_q,\omega)} + F(q,\omega)\varepsilon_{G_q}^{-1}(q-G_q,\omega),
\]

In Eq. (10) the first term of the right-hand side corresponds to the inversion without LFE appropriate for homogeneous media and the second one contains all the contribution of the LFE. The material- and direction-dependent coupling function \(F(q,\omega)\), given by

\[
F(q,\omega) = \frac{M_{G_G}G_G(q-G_q,\omega)}{M_{G_G}(q-G_q,\omega)} - \frac{\det \varepsilon_{G_G}(q-G_q,\omega)}{M_{G_G}(q-G_q,\omega)\varepsilon_{G_q}G_q(q-G_q,\omega)},
\]

drives how the peak in the 1BZ at \(q-G_q\) which is ascribed to the imaginary part of \(\varepsilon_{G_q}^{-1}(q-G_q,\omega)\), reappears at \(q\) in higher Brillouin zones. In Eq. (11) \(M_{G_G}(q-G_q,\omega)\) is the minor of the \(GG’\) element of the dielectric matrix \(\varepsilon_{GG’}(q-G_q,\omega)\). \(F(q,\omega)\) gathers all the contribution of the LFE and vanishes in a homogeneous media. A peak in the dynamical structure factor \(S(q,\omega)\) of Eq. (2), with \(q\) beyond the 1BZ, will reappear only if \(\text{Re } F(q,\omega)\) is large enough so that it does not suppress the peak in \(\text{Im } \varepsilon_{G_q}^{-1}(q-G_q,\omega)\).

Comparing Figs. 1(a)–1(c), it can be observed that, when \(q\) is beyond the 1BZ, the strength of the A’ plasmon peak is very direction dependent. Figure 3 shows the contribution of the plasmon peak, both A and A’, to the f sum rule, \(f(q) = \int_{\omega_{\text{min}}}^{\omega_{\text{max}}} \text{d}\omega \varepsilon_{G_q}G_q(q-G_q,\omega)\) with \(\omega_{\text{min}}\) and \(\omega_{\text{max}}\) the boundaries of the peak, and demonstrates that the A’ peak along \(\Gamma L\) is approximately two and four orders of magnitude larger than along \(\Gamma X\) and \(\Gamma K\), respectively. The plasmon along \(\Gamma L\) starts to reappear when the electron-hole damping is suppressed as can be seen in Fig. 1(f). As a consequence of the pressure-induced band flattening, the electron-hole excitation spectra is not well described within the jellium model at high momenta and a region without possible excitations is present inside the boundaries of intraband transitions in jellium. It is precisely in this region where the plasmon reappears along \(\Gamma L\) and shows a large amplitude. Note as well the presence of interband transitions around 8 eV in the electron-hole excitation spectra of Figs. 1(d)–1(f) that, as it has been stated already, are responsible for the presence of the undamped plasmon.

Figure 4 shows that at large momenta the A’ plasmon peak in the loss function, or in \(S(q,\omega)\), comes from the second term of the right-hand side of Eq. (10). Therefore, as it has been already pointed out, only through LFE can the plasmon reappear in higher BZs and present a periodic dispersion. In Fig. 4, \(f(q,\omega)\) and the first and second terms of the right-hand side of Eq. (10) are depicted at momenta beyond the 1BZ that show a slightly damped plasmon. These figures point out as well that the height of the A’ peak is very different.
along the other directions. Nevertheless, as can be observed in Fig. 5, this function is intrinsically anisotropic and completely responsible for the differences observed in the plasmon dispersions along different directions and also consistent with the behavior of $I(q)$ shown in Fig. 3.

Additional insight can be gained making use of a simplified $2 \times 2$ dielectric matrix model. In this case, the $F$ function adopts the simple form and is plotted in Fig. 4 for some $q$’s beyond the 1BZ. Strikingly, along $\Gamma L$ the $F$ function calculated from Eq. (12) perfectly agrees with the one obtained with the full dielectric matrix and, thus, it is confirmed that $\epsilon_{G,0}$ elements with $G = 2\pi/a(1,1,1)$ are crucial. Along $\Gamma X$ the curve obtained with the $F_{2 \times 2}$ approximation deviates substantially from the value of $F$ and along $\Gamma K$ the latter function cannot be approximated by $F_{2 \times 2}$ at all. This is consistent with the fact that the lower the value of $F$, the worse this approximation. The relative importance of the nondiagonal terms with $G = 2\pi/a(1,1,1)$ is related to strong Fourier components with this $G$ of the pseudopotential. As well as the electronic $s$-$p$ mixing, these Fourier components are strengthened with pressure strongly increasing the electronic inhomogeneity and, although the stable structure (fcc) under study is compact, electronic states localize in the [111] plane at the interstitial regions. As a consequence, bands flatten along $\Gamma L$ and show a wide band gap at the L point while remaining more free-electron like along $\Gamma X$ and $\Gamma K$ with a vanishing band gap at K (Refs. 16, 17, and 40) demonstrating the anisotropy of the pseudopotential under pressure. It is precisely along $\Gamma L$ where the plasmon at large momenta reappears periodically in successive BZs with a large amplitude, being suppressed in the other directions. Indeed, the polarization induced in real space by this plasmon is expected to be especially localized in the [111] plane so that several Fourier components along this direction are needed to describe properly the induced charge making crucial the LFE.

**IV. CONCLUSIONS**

In this paper we have presented a theoretical ab initio study of the electronic excitations in fcc compressed lithium. We have focused on the properties of the previously reported low-energy undamped plasmon. This plasmon not only satisfies the condition ascribed to the homogeneous electron gas, but, considering that both real and imaginary parts of the determinant of the dielectric matrix vanish at the $(k, \omega)$ points that form the dispersion of the plasmon when $k \leq 0.2 \text{ a.u.}^{-1}$, it is an example of a perfectly defined plasmon in a solid. Moreover, we have shown that this plasmon reappears periodically in successive BZs only if LFEs are taken into account. Furthermore, it has been demonstrated that the different behavior of the LFE along $\Gamma K$, $\Gamma X$, and $\Gamma L$ makes the plasmon dispersion very anisotropic beyond the 1BZ.
Interestingly, pressure-induced electronic anisotropy and localization, even in compact structures, increase the impact of LFE on the electronic response function. In fact, LFEs in MgB$_2$, where a similar collective excitation arises along the direction perpendicular to the basal boron layers (the $\Gamma A$ direction). This is confirmed considering that the $F$ function reaches values of the same order of magnitude along $\Gamma L$ in Li and along $\Gamma A$ in MgB$_2$.\textsuperscript{21} Therefore, similar plasmon dispersions to the one described here are expected in a wide range of materials under pressure. Considering that x-ray scattering has made possible the measurement of plasmon dispersions under pressure,\textsuperscript{41} it would be interesting to check the presence of the undamped mode in order to confirm the anisotropic behavior predicted here.

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