SHEAR RESONANCE MODE DECOUPLING TO DETERMINE THE CHARACTERISTIC MATRIX OF PIEZOCERAMICS FOR 3-D MODELLING

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IEEE Transactions on Ultrasonics, Ferroelectrics and Frequency Control
(ISNN: 0885-3010, ESSN: 1525-8955)
Vol. 58 issue 3, pp. 646-657 (2011)
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Abstract

The determination of the characteristic frequencies of an electromechanical resonance is not enough data to obtain material properties of piezoceramics, including all losses, from complex impedance measurements. Besides, values of impedance around resonance and antiresonance frequencies are also required to determine the material losses. Uncoupled resonances are needed for this purpose. The shear plates used for the material characterization present unavoidable mode coupling of the shear mode and other modes of the plate. A study of the evolution of the complex material coefficients as the coupling of modes evolves with the change in the aspect ratio (lateral dimension/thickness) of the plate is presented here. These are obtained by Alemany et al. software. A soft commercial PZT ceramic was used in this study and a number of shear plates amenable to material characterization were obtained in the range of aspect ratios below 15. The validity of the material properties for 3-D modelling of piezoceramics is assessed by means of Finite Element Analysis (FEA), which shows that uncoupled resonances are virtually pure thickness driven shear modes.

IEEE Keywords: Ceramics, Finite Element Methods, Piezoelectric materials, Resonance

¹ This work was funded by Spanish CSIC project #201060E069 and European Network of Excellence (NoE-MIND CE FP6 515757-2), that promotes the European Institute of Piezoelectric Materials and Devices (Piezoinstitute).
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1. Introduction

Piezoelectric characterization in the linear range of poled ferroelectric ceramics, piezoceramics, which are anisotropic materials of 6mm symmetry, is commonly achieved by using standard methods [1] from measurements of complex impedance at resonance and the analysis of these data. Piezoceramics are also lossy materials. In the 1960s it was suggested to express their properties as complex values [2] to account with all material losses: dielectric, elastic and piezoelectric.

Standard methods are limited when dealing with material losses [3]. Iterative and fitting methods of analysis of the impedance curve at resonance are at present used by a number of authors to calculate elastic, dielectric and piezoelectric properties, including all losses. To determine the set of ten independent material coefficients in complex form, that fully characterizes piezoceramics and some other materials of the same symmetry, as 0-3 and 1-3 composites, is possible nowadays by the resonance method. For this, the use of the four resonators recommended by the standards [4] or just three of them [5] (Fig.1(a) to (c)) is needed. Methods for determination of complex parameters from resonance have been developed for these resonator shapes and resonance modes [3,6-13]. Nevertheless, the number of works published on the determination of the full set of coefficients, including all losses, of piezoelectric ceramics and composites is still very scarce [4,5,15-18], despite of their key value in their three dimensional (3-D) modelling.

The characteristic matrix for PZ27, a soft commercial Pb(Zr,Ti)O$_3$ (PZT) ceramic (Ferroperm piezoceramics A/S, Denmark), has been determined using Alemany et al. software on the analysis of three resonators (Fig. 1(a) to (c)) and four resonance modes [5]. In order to simplify the process by reducing the number of samples to study, this method avoids the use of standard, thickness-poled, rectangular bars by extracting all possible parameters from the planar and thickness resonances of thin disks [19].

Due to the intrinsic dynamical clamping of the in-plane poled shear plate (Fig. 1(b)) [20-22] and aiming to a precise determination of shear coefficients, the authors have tested previously [23, 24] the alternative use of a thickness-poled shear plate excited across one of the longest dimensions (Fig. 1(d)). Software for impedance data analysis using Alemany et al. iterative method for this resonator has been developed [23]. A refined matrix of PZ27 material parameters, modified with shear data from the thickness-poled plate, was obtained and FEA calculations were carried out based on this [23]. A good agreement was found
between FEA computed impedance spectra and measured ones in a wide frequency range (6 kHz - 3 MHz).

To take into account precise material coefficients is a key issue for their use in carrying out accurate 3-D numerical computations. When a piezoceramic resonator is used to determine material properties, the determination of the characteristic frequencies is needed. Besides, values of impedance around resonance and antiresonance frequencies are required to determine the material losses. Uncoupled resonances are mandatory in order to obtain material properties by the resonance method. It must also be noted that the method is strictly valid only for uncoupled resonances.

The impedance spectra of the shear plates always show additional peaks, satellite resonances, around that of the main resonance. This makes difficult to determine the characteristic frequencies of the resonance needed to determine the electromechanical coupling factor ($k_{13}$). For this reason, the standards [1] recommend to use the dielectric method and determine this from measurements of the corresponding free and clamped dielectric permittivity values ($\varepsilon_{11}^T$, $\varepsilon_{11}^S$). Such method has the additional problem of the indetermination in the frequency at which the clamped value has to be measured due to the number of resonances and their overtones found at frequencies above that of the fundamental shear resonance.

These satellite resonances can also be found in the spectrum obtained by FEA modelling of the ideally homogeneous shear plate that is the object of the computation. This is observed both for the standard [20,21] and the alternative thickness-poled [23] samples. That indicates that such a satellite resonances cannot be ascribed to material inhomogeneities, but correspond to natural modes of vibration of plates, such as contour modes, associated to the shear geometry and a given aspect ratio, that are excited simultaneously to the shear mode. Contradictory results were found by previous authors [25-27] concerning the issue of the proper selection of the aspect ratio of the thickness-poled plate to have a virtually monomodal sample with a resonance of well defined characteristic frequencies.

Such a disagreement has prevented the wide use and exploitation of the advantages [17, 28] of the thickness-poled shear plate with respect to the in-plane poled one. First, thickness-poling is a simpler process than in-plane poling. This is true, in particular, for composites, porous ceramics or those having high coercive fields, as the family of Bismuth Layer Structure Ferroelectrics (BLSF). Second, using these shear plates, the consistency of the coefficients obtained from this plate and the data from the identically poled thin disks is higher. Besides, both resonators, disk and shear plate, can be obtained from just one thin disk
[28], which is of practical interest in the first stages of development of new piezoceramic materials.

In this study we illustrate an additional advantage of the thickness-poled shear plate, namely, the easy reduction of the coupling of modes, by fine tuning of the sample thickness. The shear mode related coefficients of PZ27 ceramic are obtained, by Alemany et al. software, as a function of the ratio between lateral dimension (L,w) and thickness (t) of square plates (Fig. 1(d)). The observed evolution of the spectra in this process gives explanation for the contradictory previous results about the optimum sample aspect ratio. A precise matrix of coefficients is obtained for PZ27 using those from the shear plates that show the minimum mode coupling. 3-D modelling is carried out by FEA using such a matrix. The results are compared with the experimental data to assess the validity of the matrix for accurate 3-D numerical computations.

2. Experimental Procedure

2.1. Automatic iterative method applied to the resonance of a thickness-poled shear plate

Ceramic square plates of soft commercial PZ27 of 15x15mm and 2.0, 1.5 and 1.0mm thickness, thus with L,w/t aspect ratios of 7.5, 10 and 15, were poled to saturation after applying electrodes at the major faces. These electrodes were removed and new ones were attached at two of the perpendicular surfaces for the electrical measurements. In order to study the evolution of the material coefficients, one of the plates was reduced in thickness from 2.00 to 1.00 mm in steps of 0.02 mm. The impedance spectrum was recorded and calculations of the coefficients were carried out at each step.

Measurements at resonance were carried out using an HP4192A impedance analyzer controlled by a computer via a GPIB-PCIIA (National Instruments) interface board. Alemany et al. software was used here for the calculation of the complex piezoelectric, elastic and dielectric coefficients, as well as for the determination of the corresponding electromechanical coupling factors. Details of the calculations are explained elsewhere [8,23]. After measurement of the admittance (Y), plots of the peaks of the a.c. resistance (R) and conductance (G), real parts of the impedance (Z=R+iX) and admittance (Y=G+iB), respectively, are traced as a function of the frequency. The maxima of these two peaks determine two of the characteristic frequencies of the resonance, $f_p$ and $f_m$, respectively, that are needed for the calculation. The software automatically determines the rest of the frequencies. In this software, the material coefficients in complex form ($P^*=P'+iP''$) are determined by solving a set of non-linear equations that results when experimental impedance
data at a number of frequencies are introduced into the analytical solution of the wave equation for this resonance mode when the aspect ratio given by \( L/w >> t \) is fulfilled [24]:

\[
Y = G + iB = i \frac{2\pi f L \varepsilon_{11}^s}{w} + i \frac{2t \varepsilon_{15}^E}{w} \sqrt{\frac{s_{55}^E}{\rho}} \tan \left( 2\pi f t \sqrt{\frac{\rho s_{55}^E}{w}} \right)
\]  

(1)

where \( \varepsilon_{11}^S \), \( \varepsilon_{15} \) and \( s_{55}^E \) are the relevant dielectric, piezoelectric and elastic complex material coefficients that can be directly obtained for this mode; \( \rho \) is the ceramic density; \( t \) is the thickness for poling and \( w \) the distance between the electrodes for the electrical measurements (Fig. 1(d)).

After the calculation of the coefficients, the resonance spectrum is reconstructed using Eq.(1) as an accuracy test of the final set of these. This accuracy is also quantitatively characterized by the regression factor \( R^2 \) of such reconstruction to the experimental spectrum. A coupled resonance spectrum cannot be reconstructed with a single resonance model, like Eq.(1). Low \( R^2 \) values and lack of accuracy in the calculation result from the use of coupled resonances to obtain material coefficients. It has to be mentioned that there is a good agreement between the real part of the coefficients calculated by Alemany et al. software and those obtained by standard methods [8]. This method also provides the imaginary part of the complex parameters thus giving the additional information of all the material losses.

2.2. 3-D modeling of shear plates by Finite Element Analysis

The FEA modelling was done using ATILA software [29]. Originally developed for modelling sonar transducers, this program has the ability to include piezoelectric materials defined by a full data set of complex variables. The 3-D harmonic analysis used yields the impedance values, modulus and phase, in a given interval of frequencies, which can easily be plotted as R and G peaks to compare with the experimental spectra.

The three samples simulated were the plates of 15x15 mm and 2.0, 1.5 and 1.0 mm thickness. The mesh used for the shear elements was a 30x30x3, consisting of 2700 hexagonal 20 node elements. This fine mesh was used to include the satellite resonances found in the experimental spectra, which has a wavelength much smaller than that of the main resonance. The number of frequencies analyzed was chosen to get a compromise between the time of the simulation and the required resolution of the calculated spectrum, depending of its complexity. The shear elements were simulated as full 3-D object, resulting in some 15
minutes calculation time for each discrete frequency point with a Pentium IV, 3GHz, processor.

3. Experimental Results

Fig. 2 shows the experimental impedance spectra (R and G peaks) and the reconstructed one, after material coefficients calculation, for the main shear resonance of square plates of aspect ratio values of 7.5, 10 and 15. To increase L, w/t of the plates does not cause the expected [30] progressive elimination of the mode coupling. On the contrary, this is more pronounced for the intermediate ratio of 10, as the corresponding value of the regression factor of the reconstructed to the experimental spectra ($R^2=0.8279$), the lowest of the three samples, shows.

Fig. 3 shows the experimental spectra for samples with some intermediate L, w/t values of those of Fig. 2, namely, 8.33 (t=1.80 mm), 9.26 (t=1.60 mm) and 12.12 (t=1.24 mm), in the process of reduction of the thickness of the sample that originally had t=2.00 mm. At this coarse scale of changes in the dimensions of the plate there is also a change of the mode coupling in the process, with regression factors changing yet without an apparent trend, instead of the continuous elimination of the coupling. For values of aspect ratios of 10 and 15 the spectra repeat again those features for the previously measured samples shown in Fig.2 (b) and (c), respectively, they are identical to all practical purposes. The periodicity of the mode coupling is obvious in the plot of the regression factor ($R^2$) as a function of L, w/t of the shear plate during the whole process of thickness reduction, also shown in Fig. 3(d). The periodicity can just be observed using the very fine scale of thickness variation of 0.02 mm step.

Fig. 4 shows the evolution of the real and imaginary parts of the directly obtained coefficients ($\varepsilon_{11}^S$, $h_{15}$ and $c_{55}^D$ and those additional coefficients showed in Table 1 are calculated using

Table 1 shows the shear resonance related coefficients that are directly calculated for the PZ27 plates in Fig. 2, compared with data from reference [23] and from the in-plane poled sample [5]. The regression factors of the reconstructed spectrum to the experimental one, the characteristic frequencies, coupling factors and frequency numbers of the shear resonance are also shown in Table 1. For the in-plane poled shear plate the directly obtained coefficients are $\varepsilon_{11}^S$, $h_{15}$ and $c_{55}^D$ and those additional coefficients showed in Table 1 are calculated using
known equations of relations among material coefficients [1, 5]. For the thickness-poled shear plate, similarly, the equations used here are:

\[ d_{15} = e_{15} s_{55}^{E} \]  \hspace{1cm} (2)

\[ c_{55}^{E} = \frac{1}{s_{55}^{E}} \]  \hspace{1cm} (3)

\[ h_{15} = \frac{e_{15}^{E}}{e_{11}^{S}} \]  \hspace{1cm} (4)

\[ c_{55}^{D} = c_{55}^{E} + h_{15} e_{15} \]  \hspace{1cm} (5)

\[ s_{55}^{D} = \frac{1}{c_{55}^{D}} \]  \hspace{1cm} (6)

\[ e_{11}^{T} = \frac{e_{11}^{S}}{1 - h_{15}^{2} s_{55}^{D} e_{11}^{S}} \]  \hspace{1cm} (7)

\[ g_{15} = \frac{d_{15}}{e_{11}^{T}} \]  \hspace{1cm} (8)

Table 2 shows all the shear resonance related coefficients for the four samples whose spectra correspond to the maximum regression factors, and thus minimum mode coupling, of the four periods observed in the Fig. 3(d). Table 2 includes the standard deviation of these coefficients as a fraction of the mean value also.

4. Discussion

4.1. Periodicity of the mode coupling as a function of the thickness of the plate

The study of the evolution of the coupling of the modes of PZ27 thickness-poled shear plate has been previously carried out by the authors by laser interferometry and impedance spectroscopy as a function of the ratio between lateral dimensions (L,w) and thickness (t) of square plates [24]. A periodical coupling of the thickness shear mode with other plate modes was found as L,w/t increases, instead of a continuous change towards a pure shear mode. Fig. 4(d) shows that the main resonance moves to higher frequency as the thickness decreases. Therefore, the main resonance is a thickness driven one. Around the peaks of the main resonance there are satellite peaks that changes just slightly their position as the thickness decreases. This indicates that satellite resonances are linked to the unchanged lateral dimensions of the sample. The electrically driven main thickness-shear mode excites, mechanically, the different overtones of plate waves (n, n+1, n+2, etc.) when corresponding frequencies get close to each other as thickness changes. Due to this mechanism, thickness
shear and plate waves couple necessarily in a periodic manner, regardless of the increase of the aspect ratio.

This periodical coupling can be seen in Figs. 2(a) and (b) and in Figs. 3(a) and (c), all which show multiple resonances. The periodically obtained spectra as a function of L/w/t that correspond to the higher values of the regression factors, see Fig. 2(c) and Fig. 3(b), also correspond to similar high values of $R_{\text{max}} (>500k\Omega)$ and $G_{\text{max}} (>1.1\ mS)$. These values correspond to the occurrence for these aspect ratios of a virtually uncoupled shear mode, for which the energy provided by the electrical excitation is transferred only to this resonance mode via electromechanical coupling. The maxima of the $R$ and $G$ peaks of the main resonance take place at increasing frequencies as the thickness decreases (see Figs. 3 (a) to (c)). The plots of $f_p$ and $f_s$ of the main shear resonance as a function of L/w/t (Fig. 4(d)) show periodical spurious values in coincidence with the minimum regression factors that correspond to severe mode coupling. These frequency jumps towards higher frequencies can be explained as the consequence of the displacement of the resonance [31] due to stiffening of the plate under mode coupling that is observed in Fig. 4(c).

4.2. Accuracy of the coefficients from the shear resonance of the plate

Results shown in Table 1 indicate that the coupling factors and the piezoelectric coefficients $e_{15}$, $h_{15}$ and $d_{15}$ obtained from the thickness-poled sample have higher real part than those obtained from the in-plane poled plate for PZ27. Whereas the elastic constants obtained from both types of plates are almost identical, there is an increase in the dielectric permittivity for thickness-poled plates. All this reveals the dynamical clamping and, most probably, also the lower level of polarization in the in-plane poled plate that causes the comparatively small increase in $g_{15}$ (see Eq. (8)). A noticeable result shown also in Table 1 is the dispersion of the calculated coefficients, strong in the values of the imaginary part of these. This dispersion can be explained when the periodic coupling of the resonances and its influence in the material parameters is taken into account. As said before, since this periodicity can be observed just when using the very fine scale of the thickness variation at 0.02 mm steps (steps of $\Delta(L,w/t)=\Delta a= 0.08$), the study of a discrete number of samples with ratios $a$, $a+1$, $a+2$ ... leads to the apparent dispersion of values observed in Table 1 and to the contradictory results found by previous authors [25-27].

The evolution of the real and imaginary parts of the PZ27 coefficients in Fig. 4 for aspect ratios between 7.5 and 9 occurs in a rather scattered way. The dispersion of the values is very high for the imaginary part of the permittivity, $\varepsilon_{11}^{S_{\cdot\cdot}}$, and the piezoelectric coefficient,
e_{15}^\prime\prime\prime$, whereas is lower, but still high, for the elastic compliance. The dispersion of the real values of the coefficients is not remarkably higher in this interval than in the complete interval of aspect ratios analyzed, whereas the dispersion in the imaginary values decreases noticeably for L,w/t > 9. For aspect ratios between 7.5 and 9, all results have $R^2 < 0.95$, and decoupling of shear with other plate modes is not obtained. The laser interferometry study for the sample of ratio 7.5 showed an out-of-plane displacement pattern of interference between two standing waves with perpendicular directions, which shows that the main resonance is not a shear mode [24]. The mode of resonance corresponding to 7.5 ratio strongly affects the impedance measurement and subsequent material coefficients calculation.

For L,w/t > 9 (Fig. 4), we found a variation of the parameters in a more or less systematic way as the mode coupling, characterized by the regression factor, changes periodically. The high dispersion found in the material coefficients, particularly in their imaginary part (Fig. 4), should not be merely understood as the error associated with their calculation procedure, as can be wrongly concluded if only few values of aspect ratios are analyzed (Table 1). Contrarily, this dispersion is the result of the systematic evolution with the mode coupling that can be observed when a fine scale of changes of L,w/t is studied (Fig. 3(d)). This is clearly shown in the plots for $s_{55}^{E'}$ and $s_{55}^{E''}$ (Fig. 4(c)) that follows the same periodicity as $R^2$. The measurements around the L,w/t values for which there is a clear mode coupling, which occurs periodically, leads to the higher uncertainty in the material coefficients. Table 2 shows that only those calculations carried out for situations of negligible coupling, characterized by a high $R^2$ and also high $G_{\text{max}}$ and $R_{\text{max}}$ values, give place to material coefficients determination, both in real and imaginary parts, with relatively low dispersion of values arising from the accuracy of the calculation method. For those parameters calculated from the directly obtained ones, there is a certain propagation of the errors of the coefficients involved, that shows a maximum for $g_{15}$, which calculation involves all the previous ones, as Eqs. (2) to (8) shows.

4.3. Validity of the material coefficients from resonance for 3-D modelling

The material coefficients used for the 3-D FEA computations are those reported in [20] except for the shear related ones, which were substituted by the values for thickness-poled shear plates.

Fig. 5 shows the FEA calculated impedance spectra for the same samples as in Fig. 2 (L,w/t values of 7.5, 10 and 15), which show different degrees of mode coupling using two sets of material coefficients. Results plotted in Figs. 5(a), (b) and (c) were obtained using a
matrix including coefficients from the plates with \((L,w/t)=10\) [23], which is in accordance with the standards, but shows signs of mode coupling (Fig. 2(b)). Results plotted in Figs. 5(d), (e) and (f) were obtained using coefficients determined in this work (Table 2) from plates having the optimized value of \((L,w/t)=13.4\) \((R^2=0.9832)\) to reduce mode coupling. The main characteristics of the spectra, experimentally measured and FEA generated ones, are given for comparison in Figs. 2 and 5. Both simulations provide reasonable agreement between experimental and computed values of the characteristic frequencies of the shear resonance and the main satellite one. However, the reproduction of the maximum \(G\) and \(R\) values, as well as the reproduction of the secondary peaks, is better when the computation is based on the matrix using data from the optimum ratio plate. In particular, the FEA spectra of the sample with \((L,w/t)=15\) is simulated with higher accuracy when using data obtained from an uncoupled shear mode (Fig. 5(f) and Fig. 2(c)).

Fig. 6 shows the FEA calculated displacement patterns in the two directions perpendicular to the poling direction, \(X\) and \(Y\) (perpendicular and parallel to the measuring electrodes, respectively), and in the poling direction, \(Z\), for the same samples as in Fig. 2. 3-D modelling was carried out using the matrix determined in this work with a double purpose. First, it is a useful tool in the analysis of the resonance modes at each stage of coupling corresponding to a given dimensional ratio of the resonator. The calculations shown here correspond to the frequency of maximum conductance \((f_s)\) of the FEA generated spectra obtained using the matrix of material coefficients from an optimum ratio plate (Figs. 5(d), (e) and (f)). Second, the comparison between the experimental results and these FEA results is a validity criterion of the matrix of coefficients used. It must be noted here that the precise material characterization is the ultimate purpose of the study carried out here.

Laser interferometry measurements reported elsewhere [24] for the samples in Fig. 6 at \(f_s\) showed displacement patterns in the normal direction to their major surfaces that are identical to those shown here for the calculated displacement in the \(Z\) direction.

It can be observed that for the sample with \((L,w/t)=7.5\) the \(Z\) displacement (Fig. 6(c)) shows the previously mentioned interference of two standing plate waves with perpendicular directions. This, as explained before, strongly affects the impedance measurement and its analysis up to ratios of \((L,w/t) \sim 9\) (Fig. 4(a) to (c)). Fig. 6(f) shows 10 wide bands parallel with the electrodes, in and out of plane maxima of displacement, of an uncoupled shear wave of 5 wavelengths. Fig. 6(i) shows for the \(Z\) displacement of the sample 16 bands parallel with the electrodes, corresponding to 8 wavelengths. These features of the two displacement patterns follow reasonably well the ratio between the thickness of the two samples, indicating
also that for both samples this is a thickness driven mode of motion. For all samples the shear displacement in the X direction (Figs. 6 (a), (d) and (g)), the direction of the a.c. driving field, is more important that the one in the Y direction (Figs. 6 (b), (e) and (h)). The displacement in the X direction is, due to the lower mode coupling, more homogeneous in the sample with (L,w/t)=15 (Fig. 6(g)), and this sample can be considered virtually monomodal.

To obtain precise values of material coefficients from shear plates, both in-plane and thickness-poled, it is a key issue to have resonances with minimum coupling. Our experimental data and 3-D modelling results show that for the thickness-poled shear resonators the reduction of the mode coupling is feasible by fine tuning of the thickness, from any starting ratio within 9<(L,w/t)<15, due to the periodicity of the coupling. It has to be pointed out that such a process for the in-plane poled sample will require the replacement of the eliminated electrode used for the thickness electrical excitation and measurement at each step of the reduction of the thickness. Therefore, new sources of error could be introduced in such procedure, otherwise unpractical and time consuming for in-plane poled samples.

Conclusions

Coupling of the shear resonance mode of piezoceramic plates, excited perpendicularly to the poling direction, with other types of plate modes is an unavoidable periodical phenomenon as a function of the aspect ratio of the plate. It was shown here that this periodicity can be used to minimize the mode coupling in thickness-poled plates by fine-tuning of the thickness, thus the L,w/t, of the plate. This process leads to virtually uncoupled thickness-shear resonances for which the single resonator model used in the formulation of the analytical solution needed in the parameter calculation can be considered valid. Four optimum dimensional ratios were found in the four periods of the mode coupling analyzed here. One of these is below the standard threshold value of 10 and all of them below the value of 20 suggested in previous literature.

Around these optimum dimensional ratios (for \( R^2 > 0.97 \)) there is a systematic change in the measured impedance spectra and the coefficients obtained from these, which is the result of the systematic evolution of the mode coupling. This fact, together with the strict validity of the method only for monomodal samples, results in the apparently high dispersion of the calculated parameters when compared just using the criteria of a threshold value of dimensional ratio for validity of the results. This effect is particularly high for the losses, because they require accurate impedance values for their accurate determination.
For the optimum ratios (for $R^2 > 0.97$), dispersion in the calculated parameters arising from the accuracy of the calculation method are of 0.4% for $k_{15}$ and for the directly calculated parameters, $s_{55}^E$, $e_{15}$ and $\varepsilon_{11}^S$, of less than 2%, in their real part, and 10%, 12% and 13% in their imaginary part, respectively.

3-D FEA simulation of the impedance spectra, in a range between 325 kHz and 1.2 MHz, and of the displacement patterns for PZ27 shear plates at different stages of mode coupling ($L,w/t$ values of 7.5, 10 and 15), was carried out and compared with the corresponding experimental values. The matrix of material coefficients here obtained, using shear coefficients from a plate with the optimum ($L,w/t$)=13.4, was used in this computation. Specifically, the FEA simulated impedance spectrum of the plate of ($L,w/t$)=15, which corresponds to the highest frequency range considered, shows accurately reproduced maxima of the $G$ and $R$ peaks and also ratios between such a maxima for the main shear resonance and the satellite ones. Finally, the FEA calculated strain, in agreement with previously reported laser interferometry measurements shows that uncoupled resonances of thickness-poled shear plates are virtually pure thickness driven shear modes.
References


Figure captions

Figure 1. Ceramic resonators used for piezoceramic characterization [5]: (a) thickness-poled and excited thin disk (radial and thickness extensional modes), (b) in-plane poled and thickness excited plate (shear mode) and (c) length poled and excited long rod or bar (length extensional mode) (c). The ceramic resonator under study in this work: (d) thickness-poled and length excited plate (shear mode).

Figure 2. Resistance (R) and conductance (G) for the fundamental resonance of thickness-poled plates with L,w/t values of: (a) 7.5 (b) 10 and (c) 15. The symbols represent the experimental data and the lines are the reconstructed spectra after determination of the complex parameters by Alemany et al. method. The regression factor ($R^2$) of the reconstructed to the experimental spectra is shown for each spectrum.

Figure 3. Measured (symbols) and reconstructed (lines) spectra for the shear plate that initially had a 7.5 aspect ratio (t=2.00mm) in the process of reduction of its thickness, at the steps with aspect ratio of: (a) 8.33 (t=1.80mm), (b) 9.26 (t=1.60mm) and (c) and 12.12 (t=1.24mm); (d) the regression factor of the experimental to the reconstructed spectra as a function of L,w/t.

Figure 4. Evolution of the real and imaginary parts of the (a) dielectric, $\varepsilon_{11}^T$, (b) piezoelectric, $\varepsilon_{15}$, and (c) elastic, $s_{55}^T$, coefficients directly calculated by the software and (d) of the electromechanical coupling factor and the frequencies of maximum R, $f_p$, and maximum G, $f_s$. Evolution is shown as a function of the aspect ratio in the process of the thickness reduction of the plate. The corresponding regression factor of the experimental to the reconstructed spectrum is also shown in each plot.

Figure 5. FEA generated resonance spectra. Left column: simulations with the matrix of material coefficients including shear data for (L,w/t)=10 plate. Plates with aspect ratio of: (a) 7.5, (b) 10 and (c) 15. Right column: simulations with the matrix including shear data for (L,w/t)=13.4 plate. Plates with aspect ratio of: (d) 7.5, (e) 10 and (f) 15.

Figure 6. FEA generated displacement in “X”, “Y” and “Z” directions at the frequency of the maximum G, $f_s$, for the main resonance of thickness-poled shear plates with L,w/t values of 7.5, 10 and 15. The colour code indicates the value of the displacement ($U_{x,y,z}$) in arbitrary units per input volt. Displacement is highly exaggerated. Sample is poled in the “Z” direction and the sample is electrically excited using electrodes in the two faces perpendicular to the “X” direction.

Tables captions

Table 1. Thickness shear resonance related parameters of PZ27 from the spectra in Fig.2 compared with some values previously reported. The directly calculated parameters are shown in bold.

Table 2. Thickness shear resonance related parameters, and their standard deviation as a percentage of the average value, of PZ27 thickness-poled plates of negligible coupling at the four periods of $R^2$ found as L,w/t increases. The directly calculated parameters are shown in bold.
### Table 1

<table>
<thead>
<tr>
<th>PZ27 plates</th>
<th>Thickness-poled plate 2.0x15x15mm (L,w/t)= 7.5</th>
<th>Thickness-poled plate 1.5x15x15mm (L,w/t)= 10.0</th>
<th>Thickness-poled plate 1.0x15x15mm (L,w/t)=15.0</th>
<th>Thickness-poled plate 0.9x8.97x8.1 mm (L/t)=10.0 (ref. 23)</th>
<th>In-plane poled plate 0.59x5.9x5.9mm (L,w/t)= 10 (ref. 20)</th>
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</thead>
<tbody>
<tr>
<td>$f_B$ (kHz)</td>
<td>430</td>
<td>561</td>
<td>868</td>
<td>921</td>
<td>1567</td>
</tr>
<tr>
<td>$f_p$ (kHz)</td>
<td>538</td>
<td>705</td>
<td>1092</td>
<td>1151</td>
<td>1921</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.9200</td>
<td>0.8280</td>
<td>0.9591</td>
<td>0.8800</td>
<td>0.9582</td>
</tr>
<tr>
<td>$k_{15}$ (%)</td>
<td>67.29</td>
<td>67.80</td>
<td>67.55</td>
<td>67.02</td>
<td>61.86</td>
</tr>
<tr>
<td>$N_{15}$ (kHz.mm)</td>
<td>860</td>
<td>842</td>
<td>868</td>
<td>829</td>
<td>925</td>
</tr>
<tr>
<td>$c_{55}^B$ ($10^{10}$ N.m$^{-2}$)</td>
<td>4.14 + 0.039i</td>
<td>4.04 + 0.089i</td>
<td>4.27 +0.044i</td>
<td>3.84 + 0.063i</td>
<td>3.99 + 0.09i</td>
</tr>
<tr>
<td>$c_{55}^E$ ($10^{10}$ N.m$^{-2}$)</td>
<td>2.27 + 0.055i</td>
<td>2.18 + 0.055i</td>
<td>2.32 + 0.046i</td>
<td>2.11 + 0.058i</td>
<td>2.44 + 0.11i</td>
</tr>
<tr>
<td>$s_{55}^D$ ($10^{12}$ m$^2$.N$^{-1}$)</td>
<td>24.12 - 0.23i</td>
<td>24.73 – 0.54i</td>
<td>23.40 – 0.24i</td>
<td>26.06 – 0.43i</td>
<td>25.25 - 0.54i</td>
</tr>
<tr>
<td>$s_{55}^E$ ($10^{12}$ m$^2$.N$^{-1}$)</td>
<td>44.06 –1.07i</td>
<td>45.77 –1.14i</td>
<td>43.03 – 0.26i</td>
<td>47.30 – 1.29i</td>
<td>40.85 - 1.82i</td>
</tr>
<tr>
<td>$h_{15}$ ($10^5$ V.cm$^{-1}$)</td>
<td>16.33 + 0.13i</td>
<td>16.51 + 0.32i</td>
<td>17.18 + 0.39i</td>
<td>15.56 + 0.24i</td>
<td>15.54 + 0.16i</td>
</tr>
<tr>
<td>$e_{15}$ (C.m$^{-2}$)</td>
<td>11.49 – 0.19i</td>
<td>11.25 – 0.01i</td>
<td>11.34 – 0.27i</td>
<td>11.07- 0.14i</td>
<td>9.74 - 0.26i</td>
</tr>
<tr>
<td>$d_{15}$ ($10^{12}$C.N$^{-1}$)</td>
<td>506.11 - 20.74i</td>
<td>514.91 – 13.17i</td>
<td>487.87 – 21.16i</td>
<td>523.5 – 20.73i</td>
<td>397.5 – 28.22i</td>
</tr>
<tr>
<td>$g_{15}$ ($10^3$ mV.N$^{-1}$)</td>
<td>39.39 – 0.04i</td>
<td>40.86 – 0.11i</td>
<td>40.21 + 0.49i</td>
<td>40.57 + 0.05i</td>
<td>39.25 + 0.44i</td>
</tr>
<tr>
<td>$e_{11} (e_0)$</td>
<td>1451.2 – 57.8i</td>
<td>1423.4 – 32.6i</td>
<td>1369.4 – 76.3i</td>
<td>1457.6 – 55.8i</td>
<td>1144.4 – 68.5i</td>
</tr>
<tr>
<td>$e_{11} (e_0)$</td>
<td>794.7 – 19.9i</td>
<td>769.2 – 15.5i</td>
<td>744.9 – 34.4i</td>
<td>803.3 – 21.9i</td>
<td>707.8 – 25.9i</td>
</tr>
<tr>
<td>PZ27 Thickness-poled plates (original w=2.0mm)</td>
<td>1.60x15x15mm (L,w/t)= 9.26</td>
<td>1.34x15x15mm (L,w/t)= 11.94</td>
<td>1.12x15x15mm (L,w/t)= 13.4</td>
<td>1.00x15x15mm (L,w/t )= 15.00</td>
<td>Standard deviation of the parameter</td>
</tr>
<tr>
<td>---------------------------------------------</td>
<td>-----------------------------</td>
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<tr>
<td>---------------------------------------------</td>
<td>-----------------------------</td>
<td>-----------------------------</td>
<td>-----------------------------</td>
<td>-----------------------------</td>
<td>--------------------------------</td>
</tr>
<tr>
<td>Physical properties</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Real</td>
</tr>
<tr>
<td>f&lt;sub&gt;p&lt;/sub&gt;(kHz)</td>
<td>530.5</td>
<td>636.5</td>
<td>761.2</td>
<td>850.9</td>
<td></td>
</tr>
<tr>
<td>f&lt;sub&gt;0&lt;/sub&gt;(kHz)</td>
<td>663</td>
<td>792.5</td>
<td>946</td>
<td>1056.7</td>
<td></td>
</tr>
<tr>
<td>R&lt;sup&gt;2&lt;/sup&gt;</td>
<td>0.9698</td>
<td>0.9832</td>
<td>0.9822</td>
<td>0.9805</td>
<td></td>
</tr>
<tr>
<td>k&lt;sub&gt;15&lt;/sub&gt; (%)</td>
<td>66.84</td>
<td>66.42</td>
<td>66.36</td>
<td>66.10</td>
<td>0.4%</td>
</tr>
<tr>
<td>N&lt;sub&gt;15&lt;/sub&gt; (kHz.mm)</td>
<td>848.80</td>
<td>852.9</td>
<td>852.5</td>
<td>850.9</td>
<td>0.2%</td>
</tr>
<tr>
<td>c&lt;sub&gt;55&lt;/sub&gt;&lt;sup&gt;D&lt;/sup&gt; (10&lt;sup&gt;10&lt;/sup&gt; N.m&lt;sup&gt;-2&lt;/sup&gt;)</td>
<td>4.01 +0.043i</td>
<td>4.01 + 0.040i</td>
<td>3.99 + 0.044 i</td>
<td>3.96 + 0.038 i</td>
<td></td>
</tr>
<tr>
<td>c&lt;sub&gt;55&lt;/sub&gt;&lt;sup&gt;E&lt;/sup&gt; (10&lt;sup&gt;10&lt;/sup&gt; N.m&lt;sup&gt;-2&lt;/sup&gt;)</td>
<td>2.22 +0.039i</td>
<td>2.24 +0.043i</td>
<td>2.23 + 0.051 i</td>
<td>2.23 + 0.041 i</td>
<td></td>
</tr>
<tr>
<td>s&lt;sub&gt;55&lt;/sub&gt;&lt;sup&gt;D&lt;/sup&gt; (10&lt;sup&gt;-12&lt;/sup&gt; m&lt;sup&gt;2&lt;/sup&gt;.N&lt;sup&gt;-1&lt;/sup&gt;)</td>
<td>24.88-0.27i</td>
<td>24.92-0.25i</td>
<td>25.04 - 0.27 i</td>
<td>25.23 - 0.24 i</td>
<td></td>
</tr>
<tr>
<td>s&lt;sub&gt;55&lt;/sub&gt;&lt;sup&gt;E&lt;/sup&gt; (10&lt;sup&gt;-12&lt;/sup&gt; m&lt;sup&gt;2&lt;/sup&gt;.N&lt;sup&gt;-1&lt;/sup&gt;)</td>
<td>45.97-0.79 i</td>
<td>44.59 –0.85i</td>
<td>44.74 – 1.01i</td>
<td>44.77 – 0.82i</td>
<td>1.2%</td>
</tr>
<tr>
<td>h&lt;sub&gt;15&lt;/sub&gt; (10&lt;sup&gt;8&lt;/sup&gt; V.cm&lt;sup&gt;-1&lt;/sup&gt;)</td>
<td>16.36+0.31i</td>
<td>16.39 +0.23i</td>
<td>16.18 + 0.22 i</td>
<td>16.39 + 0.33i</td>
<td></td>
</tr>
<tr>
<td>e&lt;sub&gt;15&lt;/sub&gt; (C.m&lt;sup&gt;-2&lt;/sup&gt;)</td>
<td>10.97-0.18i</td>
<td>10.80-0.17i</td>
<td>10.87 - 0.19i</td>
<td>10.56 -0.23i</td>
<td></td>
</tr>
<tr>
<td>d&lt;sub&gt;15&lt;/sub&gt; (10&lt;sup&gt;12&lt;/sup&gt; C.N&lt;sup&gt;-1&lt;/sup&gt;)</td>
<td>493.0 -16.7i</td>
<td>481.3-16.7i</td>
<td>486.00-19.4i</td>
<td>472.5 - 19.0i</td>
<td></td>
</tr>
<tr>
<td>g&lt;sub&gt;15&lt;/sub&gt; (10&lt;sup&gt;-3&lt;/sup&gt; mVN&lt;sup&gt;-1&lt;/sup&gt;)</td>
<td>40.72 +0.32i</td>
<td>40.86 +0.17i</td>
<td>40.52 + 0.10 i</td>
<td>41.34 + 0.44 i</td>
<td></td>
</tr>
<tr>
<td>ε&lt;sub&gt;11&lt;/sub&gt;&lt;sup&gt;T&lt;/sup&gt; (ε&lt;sub&gt;0&lt;/sub&gt;)</td>
<td>1366.93 –56.99i</td>
<td>1329.97–51.73i</td>
<td>1354.39 - 57.28 i</td>
<td>1289.96 - 65.49 i</td>
<td>2.2% ±9%</td>
</tr>
<tr>
<td>ε&lt;sub&gt;11&lt;/sub&gt;&lt;sup&gt;S&lt;/sup&gt; (ε&lt;sub&gt;0&lt;/sub&gt;)</td>
<td>756.54-26.4i</td>
<td>743.50-22.1i</td>
<td>758.34 - 23.2i</td>
<td>727.1-30.5i</td>
<td>1.7% ±13%</td>
</tr>
</tbody>
</table>

Table 2
Figure 1.

“Shear resonance mode decoupling to determine…..
by L. Pardo et al.
“Shear resonance mode decoupling…”
by L. Pardo et al.
Figure 3.
"Shear resonance mode decoupling to determine….." by L. Pardo et al.
Figure 4. “Shear resonance mode decoupling to determine……” by L. Pardo et al.
Figure 5.

“Shear resonance mode decoupling to determine…..”
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Figure 6.

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