Quasi-particle creation by analogue black holes

Carlos Barceló *, Stefano Liberati †, Sebastiano Sonego ‡, and Matt Visser §

*Instituto de Astrofísica de Andalucía, CSIC, Camino Bajo de Huétor 50, 18008 Granada, Spain

†International School for Advanced Studies, Via Beirut 2-4, 34014 Trieste, Italy
and
INFN, Trieste, Italy

‡Università di Udine, Via delle Scienze 208, 33100 Udine, Italy

§ School of Mathematics, Statistics, and Computer Science, Victoria University of Wellington, Wellington, New Zealand

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Abstract

We discuss the issue of quasi-particle production by "analogue black holes" with particular attention to the possibility of reproducing Hawking radiation in a laboratory. By constructing simple geometric acoustic models, we obtain a somewhat unexpected result: We show that in order to obtain a stationary and Planckian emission of quasi-particles, it is *not* necessary to create an ergoregion in the acoustic spacetime (corresponding to a supersonic regime in the flow). It is sufficient to set up a dynamically changing flow *either* eventually generating an arbitrarily small sonic region $v = c$, but without any ergoregion, *or* even just asymptotically, in laboratory time, approaching a sonic regime with sufficient rapidity.

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*carlos@iaa.es
†liberati@sissa.it; http://www.sissa.it/~liberati
‡sebastiano.sonego@uniud.it
§Matt.Visser@mcs.vuw.ac.nz; http://www.mcs.vuw.ac.nz/~visser

1 Introduction

It is by now well established that the physics associated with classical and quantum fields in curved spacetimes can be reproduced, within certain approximations, in a variety of different physical systems — the so-called "analogue models of General Relativity (GR)" [1, 2]. The simplest example of such a system is provided by acoustic disturbances propagating in a barotropic, irrotational and viscosity-free fluid.

In the context of analogue models it is natural to separate the kinematical aspects of GR from the dynamical ones. In general, within a sufficiently complex analogue model one can reproduce any pre-specified spacetime — and the kinematics of fields evolving on it — independently of whether or not it satisfies the classical (or semiclassical) Einstein equations [3]. Indeed, to date there are no analogue models whose effective geometry is determined by Einstein equations. In this sense we currently have both analogue spacetimes and analogues of quantum field theory in curved spacetimes, but (strictly speaking) no analogue model for GR itself [4].

In order to reproduce a specific spacetime geometry within an analogue model, one would have to take advantage of the specific equations describing the latter (for example, for fluid models, the Euler and continuity equations, together with an equation of state), plus the possibility of manipulating the system by applying appropriate external forces. In the analysis of this paper we will think of the spacetime configuration as "externally given", assuming that it has been set up as desired by external means — any back-reaction on the geometry is neglected as in principle we can counter-balance its effects using the external forces. In the context of analogue models this is not merely a hypothesis introduced solely for theoretical simplicity, but rather a realistic situation that is in principle quite achievable.

Specifically, in this paper we analyze in simple terms the issue of quantum quasi-particle creation by several externally specified $(1+1)$-dimensional analogue geometries simulating the formation of black hole-like configurations. (In a previous companion paper [5] we investigated the causal structure of these, and other, spacetimes.) In this analysis we have in mind, on the one hand, the possibility of setting up laboratory experiments exhibiting Hawking-like radiation [6, 7] and, on the other hand, the acquisition of new insights into the physics of black hole evaporation in semiclassical gravity. All the discussion holds for a scalar field obeying the D'Alembert wave equation in a curved spacetime. This means that we are not (for current purposes) considering the deviations from the phononic dispersion relations that show up at high energies owing to the atomic structure underlying any condensed matter system. We shall briefly comment on these modifications at the end of the paper. For simplicity, throughout the paper we adopt a terminology based on acoustics in moving fluids (we will use terms such as acoustic spacetimes, sonic points, fluid velocity, etc.), but our results are far more general and apply to many other analogue gravity models not based on acoustics. We summarise the main conclusions below.

First of all, we recover the standard Hawking result when considering fluid flows that generate a supersonic regime at finite time. (That is, we recover a stationary creation of quasi-particles with a Planckian spectrum.) We then analyze the quasi-particle creation associated with other types of configurations. In particular, we shall discuss in detail a "critical black hole" — a flow configuration that presents an acoustic horizon without an associated supersonic region. From this analysis we want to highlight two key results:

- The existence of a supersonic regime (sound velocity c strictly smaller than fluid velocity v) is not needed in order to reproduce Hawking's stationary particle creation. We demonstrate this fact by calculating the quantity of quasi-particle production in an evolving geometry which generates only an isolated sonic point ($v = c$), but without a supersonic region, in a finite amount of laboratory time.

- Moreover, in order to produce a Hawking-like effect it is not even necessary to generate a sonic point at finite time. All one needs is that a sonic point develops in the asymptotic future (that is, for $t \to +\infty$) *with sufficient rapidity* (we shall explain in due course what we exactly mean by this).

From the point of view of the reproducibility of a Hawking-like effect in a laboratory, the latter result is particularly interesting. In general, the formation of a supersonic regime in a fluid flow — normally considered to be the crucial requirement to produce Hawking emission — is associated with various different types of instability (Landau instability in superfluids, quantized vortex formation in Bose–Einstein condensates, etc.) that could mask the Hawking effect. To reproduce a Hawking-like effect without invoking a supersonic regime could alleviate this situation.

From the point of view of GR, we believe that our result could also have some relevance, as it suggests a possible alternative scenario for the formation and semiclassical evaporation of black hole-like objects.

The plan of the paper is the following: In the next section we introduce the various acoustic spacetimes on which we focus our attention, spacetimes that describe the formation of acoustic black holes of different types. In section 4 we present separately the specific calculations of redshift for sound rays that pass asymptotically close to the event horizon of these black holes. By invoking standard techniques of quantum field theory in curved spacetime, one can then immediately say when particle production with a Planckian spectrum takes place. Finally, in the last section of the paper we summarise and discuss the results obtained.

2 Acoustic black holes

Associated with the flow of a barotropic, viscosity-free fluid along an infinitely long thin pipe, with density and velocity fields constant on any cross section orthogonal to the pipe, there is a (1+1)-dimensional *acoustic spacetime* $(\mathcal{M}, g)$, where the manifold $\mathcal{M}$ is diffeomorphic to $\mathbb{R}^2$. Using the laboratory time $t \in \mathbb{R}$ and physical distance $x \in \mathbb{R}$ along the pipe as coordinates on $\mathcal{M}$, the *acoustic metric* on $\mathcal{M}$ can be written as

$$g = \Omega^2 \left[-\left(c^2 - v^2\right) \mathrm{d}t^2 + 2\, v \,\mathrm{d}t \,\mathrm{d}x + \mathrm{d}x^2 \right] = \Omega^2 \left[-c^2\, dt^2 + (\mathrm{d}x + v\,\mathrm{d}t)^2 \right] , \tag{2.1}$$

where c is the speed of sound, v is the fluid velocity, and Ω is an unspecified non-vanishing function [8]. In general, all these quantities depend on the laboratory coordinates x and t. Here, we shall assume that c is a constant. Hence, it is the velocity $v(x,t)$ that contains all the relevant information about the causal structure of the acoustic spacetime $(\mathcal{M}, g)$. We direct the reader to the companion paper [5] for a detailed analysis of the causal structure associated with a broad class of $(1 + 1)$-dimensional acoustic geometries, both static and dynamic.

2.1 Apparent horizon

The sonic points, where $v(t, x) = \pm c$, correspond to the so-called acoustic apparent horizons — apparent horizons for the Lorentzian geometry defined on $\mathcal{M}$ by the metric (2.1). The fact of having an underlying Minkowski structure associated with the laboratory observer makes the definition of apparent horizons in acoustic models less troublesome than in GR (see *e.g.* reference [2], pp. 15–16).

Consider a monotonically non-decreasing function $\bar{v}(x)$ such that $\bar{v}(0) = -c$ and $\bar{v}(x) \to 0$ for $x \to +\infty$. If one chooses $v(x, t) = \bar{v}(x)$ in (2.1), the corresponding acoustic spacetime represents, for observers with $x > 0$, a static black hole with the horizon located at $x = 0$ (in this case apparent and event horizon coincide), a black hole region for $x < 0$, and a (right-sided) surface gravity

$$\kappa := \lim_{x \to 0^+} \frac{\mathrm{d}\bar{v}(x)}{\mathrm{d}x} \,. \tag{2.2}$$

We can, moreover, distinguish three cases:

- $\kappa \neq 0$ and $\bar{v}(x) < -c$ for $x < 0$: a non-extremal black hole;
- $\kappa \neq 0$ and $\bar{v}(x) = -c$ for $x < 0$: a “critical” black hole;
- $\kappa = 0$ and $\bar{v}(x) = -c$ for $x < 0$: an extremal black hole.

Now, taking the above $\bar{v}(x)$, let us consider t-dependent velocity functions

$$v(x, t) = \begin{cases} \bar{v}(\xi(t)) & \text{if} \quad x \leq \xi(t) \,, \\ \bar{v}(x) & \text{if} \quad x \geq \xi(t) \,, \end{cases} \tag{2.3}$$

with ξ a monotonically decreasing function of t, such that $\lim_{t \to -\infty} \xi(t) = +\infty$ and $\lim_{t \to -\infty} \dot{\xi}(t) = 0$. (The first condition serves to guarantee that spacetime is flat at early times, whereas we impose the second one only for simplicity. All the analysis in the paper could be performed without adopting this assumption, leaving the physical results unchanged. However, that would require more case-by-case splitting, only to cover new situations without physical interest.) There are basically two possibilities for ξ, according to whether the value $\xi = 0$ is attained for a finite laboratory time t_{H} or asymptotically for an infinite future value of laboratory time.

In the first case $\xi(t_{\mathrm{H}}) = 0$ and the corresponding metric (2.1) represents the formation of a non-extremal, critical, or extremal black hole, respectively. For small values of $|t - t_{\mathrm{H}}|$ we have

$$\xi(t) = -\lambda\,(t - t_{\mathrm{H}}) + \mathcal{O}([t - t_{\mathrm{H}}]^2) \,, \tag{2.4}$$

where λ is a positive parameter. Hence the function ξ behaves, qualitatively, as shown in figure 1. Apart from this feature, the detailed behaviour of ξ is largely irrelevant for our purposes.

If instead $\xi \to 0$ is attained only at infinite future time, that is $\lim_{t \to +\infty} \xi(t) = 0$, one is describing the asymptotic formation of either a critical black hole (if $\kappa \neq 0$; obviously, in this case choosing the non-extremal or the critical $\bar{v}(x)$ profile is irrelevant) or an extremal black hole (if $\kappa = 0$). Now the function ξ behaves, qualitatively, as shown in figure 2. The relevant feature of $\xi(t)$ is its asymptotic behaviour as $t \to +\infty$. In the following we shall consider two possibilities for this asymptotics, although others can, of course, be envisaged: