Hawking-like radiation does not require a trapped region

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We discuss the issue of quasi-particle production by “analogue black holes” with particular attention to the possibility of reproducing Hawking radiation in a laboratory. By constructing simple geometric acoustic models, we obtain a somewhat unexpected result: We show that in order to obtain a stationary and Planckian emission of quasi-particles, it is not necessary to create a trapped region in the acoustic spacetime (corresponding to a supersonic regime in the fluid flow). It is sufficient to set up a dynamically changing flow asymptotically approaching a sonic regime with sufficient rapidity in laboratory time.

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Introduction: It is by now well established that the physics associated with classical and quantum fields in curved spacetimes can be reproduced, within certain approximations, in a variety of different physical systems by constructing so-called “analogue spacetimes” 1,2. Among such systems the simplest example is provided by a barotropic, irrotational and viscosity-free fluid. Small classical (or quantum) disturbances propagating in this type of fluid behave equivalently to a linear classical (or quantum) field over an effective acoustic metric 3. In the case of a fluid flow along an infinitely long thin pipe, with density and velocity fields constant on any cross section orthogonal to the pipe, one can write the (1 + 1)-dimensional acoustic metric as

\[ g = \Omega^2 \left[ -(c^2 - v^2) dt^2 - 2v dt dx + dx^2 \right], \tag{1} \]

where \( t \in \mathbb{R} \) denotes laboratory time and \( x \in \mathbb{R} \) the physical distance along the pipe. In this expression \( c \) is the speed of sound, \( v \) is the fluid velocity, and \( \Omega \) is an unspecified non-vanishing function 3. In general, all these quantities depend on the laboratory coordinates \( x \) and \( t \). Here, we shall assume that \( c \) is a constant. Hence, it is the velocity \( v(x,t) \) that contains all the relevant information about the causal structure of the acoustic spacetime. (See Ref. 4 for a detailed analysis of the causal structure associated with a broad class of (1 + 1)-dimensional acoustic geometries, both static and dynamic.)

In this letter we analyze, in simple terms, the issue of quantum quasi-particle creation (in this specific case, phonon creation) by externally specified (1 + 1)-dimensional analogue geometries simulating the formation of black hole-like configurations. (Several additional cases and a more extensive discussion of the results can be found in Ref. 5.) In this analysis we have in mind, on the one hand, the possibility of setting up laboratory experiments exhibiting Hawking-like radiation 6,7 and, on the other hand, the acquisition of new insights into the physics of black hole evaporation in semiclassical gravity. In particular, we have found, quite surprisingly, that in order to produce a Hawking-like effect it is neither necessary to generate a supersonic regime (fluid velocity \( v \) strictly larger than sound velocity \( c \)), nor even a sonic point at finite laboratory time. All one needs is that a sonic point develops in the asymptotic future (that is, for \( t \to +\infty \)) with sufficient rapidity (we shall in due course explain exactly what we mean by this).

General framework: A sonic point in the flow, where \( v(t, x) = \pm c \), corresponds to a so-called acoustic trapping (or apparent) horizon for the Lorentzian geometry defined by the metric 1. See e.g. Ref. 2, Sec. 2.5, pp. 15–16. Consider a monotonically non-decreasing function \( \bar{v}(x) \) such that \( \bar{v}(0) = -c \) and \( \bar{v}(x) \to 0 \) for \( x \to +\infty \). If one chooses \( v(x, t) = \bar{v}(x) \) in 1, the corresponding acoustic spacetime represents, for observers with \( x > 0 \), a static black hole with the horizon located at \( x = 0 \) (in this case trapping and event horizon coincide), a black hole region for \( x < 0 \), and a (right-sided) surface gravity 8

\[ \kappa := \lim_{x \to 0^+} \frac{d\bar{v}(x)}{dx}. \tag{2} \]

Now, taking the above \( \bar{v}(x) \), let us consider \( t \)-dependent velocity functions

\[ v(x, t) = \begin{cases} \bar{v}(\xi(t)) & \text{if } x \leq \xi(t), \\ \bar{v}(x) & \text{if } x \geq \xi(t), \end{cases} \tag{3} \]

with \( \xi \) a monotonically decreasing function of \( t \), such that \( \lim_{t \to -\infty} \xi(t) = +\infty \) and \( \lim_{t \to +\infty} \xi(t) = 0 \). The first condition serves to guarantee that spacetime is flat at early times, whereas we impose the second one only for
In the case of the formation of what we called a critical acoustic black hole. There are basically two possibilities for $\xi$, according to whether the value $\xi = 0$ is attained for a finite laboratory time $t_H$ or asymptotically for an infinite future value of laboratory time.

In the first case $\xi(t_H) = 0$ and the corresponding metric represents the formation of a non-extremal black hole (in this letter we will consider only the case $\kappa \neq 0$, extremal black holes — zero surface gravity — are treated in [5]). For small values of $|t - t_H|$ we have

$$\xi(t) = -\lambda (t - t_H) + O(|t - t_H|^2), \quad (4)$$

where $\lambda$ is a positive parameter. Apart from this feature, the detailed behaviour of $\xi$ is largely irrelevant for our purposes.

If instead $\xi \to 0$ is attained only at infinite future time, that is $\lim_{t \to +\infty} \xi(t) = 0$, one is describing the asymptotic formation of what we shall call a critical black hole. In the following we shall restrict the discussion to the case in which for $t \to +\infty$, the behaviour of $\xi$ is exponential, that is $\xi(t) \sim A e^{-\kappa_D t}$, with $\kappa_D$ a positive constant, in general different from $\kappa$, and $A > 0$. Of course, other possibilities for the asymptotics can be envisaged; for example, in Ref. [5] the case of a power law $\xi(t) \sim B t^{-\nu}$, with $\nu > 0$ and $B > 0$, is also studied in detail.

Starting with a quantum scalar field in its natural Minkowskian vacuum at $t \to -\infty$, we want to know the total quantity of quasi-particle production to be detected at the right asymptotic region at late times, $t \to +\infty$, caused by the dynamical evolution of the velocity profile $v(x, t)$. This can be done by following the standard procedure in which one computes the Bogoliubov $\beta$ coefficients between modes that have positive frequency with respect to null coordinates $U$ and $u$, with $U$ regular on the acoustic event horizon $H$, whereas $u$ tends to $+\infty$ as $H$ is approached. (See, e.g., Ref. [5].) We must thus find the relation between $U$ and $u$ for a sound ray that is close to the horizon, i.e., in the asymptotic regime $u \to +\infty$. If this relation turns out to be exponential, it is then a well established result that a Planckian spectrum will be observed at late times, so Hawking-like radiation is recovered.

In the geometric acoustic approximation, a right-going sound ray is an integral curve of the differential equation

$$\frac{dx}{dt} = c + v(x, t), \quad (5)$$

which can be integrated exactly in order to find the values of $U$ and $u$ for a generic right-moving ray that crosses, at laboratory time $t_0$, the kink $x = \xi(t)$ (details can be found in Ref. [5]):

$$U = t_0 - \frac{\xi(t_0)}{c} + \frac{1}{c} \int_{-\infty}^{t_0} dt \bar{v}(\xi(t)) ; \quad (6)$$

$$u = t_0 - \frac{\xi(t_0)}{c} - \frac{1}{c} \int_{\xi(t_0)}^{+\infty} dx \frac{\bar{v}(x)}{c + \bar{v}(x)} . \quad (7)$$

Eliminating $t_0$ between Eqs. (6) and (7) provides the sought-for relation between $U$ and $u$.

**Formation of a non-extremal black hole:** In the case of the formation of a non-extremal black hole, the function $\bar{v}(x)$ must be such that for small values of $|x|$, one can write

$$\bar{v}(x) = -c + \kappa x + O(x^2). \quad (8)$$

In addition, the trapping horizon forms at a finite laboratory time, say at $t = t_H$, just when $\xi(t_H) = 0$. In this situation an event horizon always exists, generated by
the right-moving ray that eventually remains frozen on the trapping horizon, at $x = 0$. For such a ray $t_0 \to t_H$, and since $\xi(t_H) = 0$, the $U$ parameter has the finite value

$$U_H = t_H + \frac{1}{c} \int_{-\infty}^{t_H} dt \tilde{v}(\xi(t)).$$ \hspace{1cm} (9)

For a ray with $U < U_H$ we then obtain, combining Eqs. 3 and 5:

$$U = U_H + t_0 - t_H - \frac{\xi(t_0)}{c} - \frac{1}{c} \int_{t_0}^{t_H} dt \tilde{v}(\xi(t)).$$ \hspace{1cm} (10)

This exact equation is now in a form suitable for conveniently extracting approximate results in the region $t_0 \sim t_H$, corresponding to sound rays that “skim” the horizon. Using Eqs. 4 and 5 we find $U = U_H + \frac{1}{c} (t_0 - t_H) + O[(t_0 - t_H)^2]$. This provides us with the link between $U$ and $t_0$.

In order to link $t_0$ with $u$, consider the integral on the right hand side of Eq. 4. For $x \to +\infty$, the integrand function vanishes, while near $\xi(t_0)$ it can be approximated by $-c/(k\xi)$. Then the integral is just given by the difference of the corresponding integrals evaluated at $x = +\infty$ and $x = \xi(t_0)$, respectively, up to a possible finite positive constant. This gives $-\lambda(t_0 - t_H) \sim \text{const} e^{-\kappa u}$. Together with the previous link between $U$ and $t_0$ this leads to

$$U \sim U_H - \text{const} e^{-\kappa u}.$$ \hspace{1cm} (11)

This relation between $U$ and $u$ is exactly the one found by Hawking in his famous analysis of particle creation by a collapsing star \cite{Hawking1974}. It is by now a standard result that this relation implies the stationary creation of particles with a Planckian spectrum at temperature $T_H = \kappa/(2\pi) \cite{Hawking1974, Hawking1975, Hawking1975b}$.\footnote{Formation of a critical black hole}. Consider now the same type of function $\tilde{v}(x)$ but this time the sonic point is approached asymptotically in an infinite amount of laboratory time. Now the trapping horizon is just an asymptotic point located at $x = 0$, $t \to +\infty$, and in order to establish whether an event horizon does, or does not, exist one must perform an actual calculation of $U_H$ for the “last” ray that crosses the kink. The expression for $U_H$ is again obtained from Eq. 6, noticing that now $t_0 = +\infty$ along the generator of the would-be horizon, so

$$U_H = \lim_{t_0 \to +\infty} \left( t_0 + \frac{1}{c} \int_{-\infty}^{t_0} dt \tilde{v}(\xi(t)) \right).$$ \hspace{1cm} (12)

The necessary and sufficient condition for the event horizon to exist is that the limit on right hand side of Eq. 12 be finite. The integrand on right hand side of 12 can be approximated, for $t \to t_0 \to +\infty$, as $-c + \kappa \xi(t)$, while for $t \to -\infty$ it just approaches zero. Hence $U_H$ is, up to a finite constant, equal to $\kappa/c$ times the integral of $\xi$, evaluated at $t \to +\infty$. For an exponential behaviour, $U_H$ turns out to be finite.

For another right-moving sound ray that corresponds to a value $U \lesssim U_H$, combining Eqs. 3 and 12, realizing that $v \sim -c + \kappa \xi(t)$ for $\xi(t)$ close to zero, and using expansion 5, we find

$$U \sim U_H - \frac{\xi(t_0)}{c} - \frac{\kappa}{c} \int_{t_0}^{+\infty} dt \xi(t).$$ \hspace{1cm} (13)

For the asymptotically exponential $\xi(t)$ considered in this letter this implies

$$U \sim U_H - \frac{A}{c} \left( 1 + \frac{\kappa}{\kappa_D} \right) e^{-\kappa_D t_0}. \hspace{1cm} (14)$$

For the link between $t_0$ and $u$ we obtain

$$u \sim t_0 - \frac{1}{\kappa} \ln \xi(t_0),$$ \hspace{1cm} (15)

as one can easily check by inserting the appropriate asymptotic expansions into Eq. 7. Using Eqs. 13 and 14 we finally find

$$U \sim U_H - \text{const} \exp \left( \frac{-\kappa \kappa_D}{\kappa + \kappa_D} u \right). \hspace{1cm} (16)$$

Therefore, we have shown that by setting up a fluid flow that reaches a sonic regime asymptotically with exponential rapidity in laboratory time one can produce a stationary and Planckian spectrum of quasi-particles at asymptotic infinity with temperature $T_{\text{eff}} = \kappa_{\text{eff}}/(2\pi)$, where

$$\kappa_{\text{eff}} := \frac{\kappa \kappa_D}{\kappa + \kappa_D}. \hspace{1cm} (17)$$

When $\kappa_D \gg \kappa$, $\kappa_{\text{eff}} \simeq \kappa$ and thus, our result is operationally indistinguishable from that of Hawking. In this sense neither the existence of an ergoregion nor that of an apparent horizon are needed for the simulation of Hawking radiation. Remarkably this is true also for a “double-sided” velocity profile in which the asymptotic sonic regime is limited to a single spatial point \cite{Kubiznak2013}.

In terms of inverse temperatures, defined as $\beta_i = 1/T_i = 2\pi/\kappa_i$, we have the very suggestive result

$$\beta_{\text{eff}} = \beta_{\text{Hawking}} + \beta_D. \hspace{1cm} (18)$$

\textbf{Experimental realizability}. The critical black hole model seems worth taking into consideration in connection with the realizability of a Hawking-like flux in the laboratory. The creation of supersonic configurations in a laboratory is usually associated with the development of instabilities. There are many examples of the latter in the literature; e.g. in Ref. 8 it was shown that in an analogue spacetime based on ripplons on the interface between two different sliding superfluids (for instance, $^3$He-phase A and $^3$He-phase B), the formation of an ergoregion would make the ripplons acquire an amplification factor that eventually would destroy the configuration. Therefore,
this analogue system, although very interesting in its own right, will prove to be useless in terms of detecting a Hawking-like flux. However, by creating, instead of an ergoregion, a critical configuration one should be able to at least have a better control of the incipient instability, while at the same time producing a dynamically controllable Hawking-like flux.

Nevertheless, the actual realization of a critical configuration might also appear problematic for entirely different reasons. The corresponding velocity profiles are characterized by discontinuities in the derivatives, so one might wonder whether they would be amenable to experimental construction, given that the continuum model is only an approximation. In particular, could it be that in a real experimental setting one does not observe the results predicted by our analysis, but those corresponding to models where the discontinuities have somehow been smoothed out?

Fortunately, this will not be the case. In realistic situations, what is relevant for Hawking radiation is a coarse-grained profile — obtained by averaging over a scale $\Delta$ larger than the one which characterizes the breakdown of the continuum model — which does not contain the unphysical small scale details of $v(x, t)$. This implies that the reliable results are those involving features recognizable over length scales of order $\Delta$, or larger $\Delta$. In particular, the relevant surface gravity will be defined by averaging the slope of the velocity profile over scales which are of order of $\Delta$. This averaged surface gravity will be non-zero for the critical profile, as well as for a smoothed one. Indeed, it will be approximately equal to the surface gravity at the horizon of the critical black hole, while it will obviously not coincide with the one of the smoothed profile (which is zero) in Ref. [12].

Hints for semiclassical gravity: The critical collapse result also suggests an alternative scenario for the semiclassical collapse and evaporation of black hole-like objects. Although somewhat speculative at this stage we believe it is worthwhile to describe it here.

Imagine a dynamically collapsing star. The collapse process starts to create particles dynamically before the surface of the star crosses its Schwarzschild radius. (This particle creation is normally associated with a transient regime and have nothing to do with Hawking’s Planckian radiation.) The energy extracted from the star in this way will (due to energy conservation) reduce its total mass, and so also its Schwarzschild radius, so $\kappa$ increases. By this argument alone, we can see that a process is established in which the surface of the star starts to "chase" its Schwarzschild radius while both collapse towards zero (this situation was already described by Boulware in Ref. [10]).

Now, contrary to the standard view, it could happen that the back-reaction on the geometry is such that it prevents the surface of the star from actually crossing its Schwarzschild radius (see [13] for a discussion of this possibility). The function $\xi(t)$ in our calculation would represent the radial distance between the surface of the star and its Schwarzschild radius. Then it is sensible to think that during the evaporation process any dynamical $\kappa_D$ would also depend on $t$. As the evaporation temperature increases ($\kappa$ increases) the back-reaction would become more efficient and therefore we might expect that $\kappa_D$ decreases. Then the evolution of the evaporation temperature would interpolate between an early-time temperature completely controlled by $\kappa(t)$, and a late-time temperature completely controlled by $\kappa_D(t)$. Therefore, this black hole-like object will have a temperature-decreasing phase, showing a possible semiclassical mechanism for regularizing the end point of the evaporation process.

In this scenario the complete semiclassical geometry will have neither a trapped region (and so by construction no trapping horizon) nor an event horizon. In this circumstance there would be no trans-Planckian problem, nor information loss associated with the collapse and evaporation of this black hole-like object. Whether this scenario is viable or not will be the subject of future work.

[12] Of course, in addition, in any realistic system one will have to take into consideration the existence of modified dispersion relations. This does not seem to alter the above scenario [13]. (For a discussion of these issues we refer the reader to [3].)