

# A real Lorentz-FitzGerald contraction

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Many condensed matter systems are such that their collective excitations at low energies can be described by fields satisfying equations of motion formally indistinguishable from those of relativistic field theory. The finite speed of propagation of the disturbances in the effective fields (in the simplest models, the speed of sound) plays here the role of the speed of light in fundamental physics. However, these apparently relativistic fields are immersed in an external Newtonian world (the condensed matter system itself and the laboratory can be considered Newtonian, since all the velocities involved are much smaller than the velocity of light) which provides a privileged coordinate system and therefore seems to destroy the possibility of having a perfectly defined relativistic emergent world. In this essay we ask ourselves the following question: In a homogeneous condensed matter medium, is there a way for internal observers, dealing exclusively with the low-energy collective phenomena, to detect their state of uniform motion with respect to the medium? By proposing a thought experiment based on the construction of a Michelson-Morley interferometer made of quasi-particles, we show that a real Lorentz-FitzGerald contraction takes place, so that internal observers are unable to find out anything about their ‘absolute’ state of motion. Therefore, we also show that an effective but perfectly defined relativistic world can emerge in a fishbowl world situated inside a Newtonian (laboratory) system. This leads us to reflect on the various levels of description in physics, in particular regarding the quest towards a theory of quantum gravity.

**Keywords:** Einstein gravity; emergent phenomena; effective metric; Lorentz-FitzGerald contraction; Michelson-Morley experiment

PACS numbers: 04.20.Jb; 04.20.-q; 04.40.-b

## I. INTRODUCTION: EMERGENT METRICS IN CONDENSED MATTER SYSTEMS

It is by now a well known fact that the physics associated with fields, classical or quantum, in curved backgrounds can be reproduced in a large variety of condensed matter systems, the so-called analogue models of general relativity [1, 2]. The key point is the realization that some collective properties of these condensed matter systems satisfy equations of motion formally equivalent to that of a relativistic field in a curved spacetime. The simplest example is the equation describing a massless scalar field  $\phi$ ,

$$\frac{1}{\sqrt{-g}}\partial_\mu\sqrt{-g}g^{\mu\nu}\partial_\nu\phi = 0, \tag{1.1}$$

satisfied, for example, by acoustic perturbations in a moving perfect fluid. These acoustic perturbations therefore travel along the null geodesics of the acoustic metric  $g_{\mu\nu}$ . Generically, the most interesting relativistic behaviours show up in the analysis of collective excitations from the vacuum state of specific condensed matter systems [3]. In those cases, one can say that relativistic effects have emerged in the low-energy corner of the corresponding theory, since the relativistic symmetry was not present at higher energies nor at the microscopic level.

When looking at the analogue metrics one problem immediately comes to mind. The laboratory in which the condensed matter system is set up provides a privileged coordinate system. Thus, one is not really reproducing a geometrical configuration but only a specific metrical representation of it. This naturally raises the question whether diffeomorphism invariance is not lost in the analogue construction. Indeed, if all the degrees of freedom contained in the metric had a physical role, as opposed to what happens in a general relativistic context in which only the geometrical degrees of freedom (metric modulo diffeomorphism gauge) are physical, then diffeomorphism invariance would be violated.

The standard answer to this question is that diffeomorphism invariance is maintained but only for internal observers, i.e. those observers who can only perform experiments involving the propagation of the relativistic collective fields. In this essay we want to elaborate on this problem. Which are the necessary conditions in order for the world description developed by such internal observers to be completely relativistic?

Let us take the simplest geometrical configuration, Minkowski spacetime. Imagine for example that we have a homogeneous fluid (or more generally, a homogeneous medium) at rest in the laboratory. The analogue metric associated with this configuration would be the Minkowski metric. Is an internal observer capable of discerning whether he is at rest in the medium or moving through it at a certain uniform velocity?

## II. THE MICHELSON-MORLEY INTERFEROMETER

We were told at school that a way to distinguish whether one is moving or not with respect to a medium is to set up a Michelson-Morley type of experiment [4]. In our analogue version of this experiment an interferometer will be used with two arms of perfectly equal length which are placed perpendicularly and along which “acoustic signals” are sent (here we will designate as acoustic signal any signal in the form of a relativistic massless field perturbation; these signals travel at a constant speed, the speed of sound  $c_s$ ). Mirrors placed at the ends of these arms reflect the acoustic signals back to a common point  $p$ . If a displacement of the interference fringes at  $p$  is observed when the interferometer is rotated, then one can conclude that the velocity along let’s say the  $x$  axis is different from that along the  $y$  axis.

However, there is a hidden assumption in this experiment. Although the internal observers seem to be using only acoustic signals, the interferometer itself is an apparatus completely alien to the medium and its excitations. By assuming the availability of this interferometer for the usage of internal observers, one is also assuming that they are in contact with the outside or external world (anything not describable in terms of collective excitations within the system). A genuinely internal observer should be confined to the manipulation of objects strictly within his own realm. Consistently, internal observers can only use an interferometer if it could have been created by themselves. So, imagine that instead of using an interferometer in which the two rigid arm structures are built by putting together particle after particle, they were using a quasi-interferometer created by using *quasi-particles* as building blocks.

A quasi-particle is a collective excitation of the system with properties similar to those of particles. Imagine that among the emergent features of the condensed matter-like system there exist some quasi-particles with mass so that they can be at rest within the system. These massive quasi-particles could be genuine, as in a two-component Bose–Einstein condensate [5], or complex systems formed themselves by elementary quasi-particles. Imagine also that these quasi-particles interact with each other through the distortion of the acoustic fields, as is the case e.g. for quasi-neutrinos in  ${}^3\text{He-A}$  [3]<sup>1</sup>.

We are going to show that these two hypotheses generically imply that the physical length of a quasi-interferometer arm, as measured in the lab, would shrink by an acoustic Lorentz factor  $\gamma = (1 - v^2/c_s^2)^{-1/2}$  when moving at a velocity  $v$  with respect to the medium ( $c_s$  is the velocity of sound in the medium).

## III. A REAL LORENTZ-FITZGERALD CONTRACTION

Let us consider the following specific situation: We have *i*) a medium at rest, *ii*) some relativistic collective excitations described by a field  $A_\mu$  and *iii*) a massive quasi-particle acting as a source with charge  $q$  of the field  $A_\mu$ , so that the equations satisfied by the relativistic field are

$$\square A_\mu - \partial_\mu(\partial^\nu A_\nu) = j_\mu; \quad j_\mu = \{-q\delta^3[\vec{x} - \vec{x}(t)], q(\vec{v}/c_s)\delta^3[\vec{x} - \vec{x}(t)]\}, \quad (3.1)$$

with  $\vec{x}(t)$  the trajectory of the massive quasi-particle and  $\vec{v} = d\vec{x}(t)/dt$  its velocity. The d’Alembertian corresponds to that of a Minkowskian acoustic metric so that by using the three Cartesian coordinates of the lab and the absolute laboratory time it can be written in the standard Cartesian form. The field equations are expressed in this way only when using these lab coordinates and when the medium is at rest. Note that we are considering a vector field because of its parallelism with the (electromagnetic) field responsible for the existence of solid bodies (and so of rigid rods) in nature. However, many of the following arguments could have been presented using a single massless scalar field. Let us also point out that there exist specific condensed matter systems (for example  ${}^3\text{He-A}$  [3]) in which fields and field equations like these emerge in the low-energy description of the system.

Let us first solve the system when the source is at rest ( $\vec{v} = 0$ ). A solution for  $A_\mu$  is then

$$A_0 = -\frac{q}{[(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2]^{1/2}}; \quad A_i = 0. \quad (3.2)$$

There are other solutions to the previous equations, but these are all related to one another by a gauge transformation  $A'_\mu = A_\mu + \partial_\mu\chi$ . For simplicity, let us choose the Lorenz gauge  $\partial^\mu A_\mu = 0$  and continue with the discussion.

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<sup>1</sup> In this essay we will not discuss the feasibility of the combination of these two hypotheses in current realistic condensed matter systems; we just take the view that within a sufficiently complicated condensed matter system it should be possible to fulfil them both.

We now seek for a solution of the same system of equations but with a source moving uniformly in the  $x$  direction:

$$j_\mu = \{-q\delta[x - (x_0 + vt)]\delta(y - y_0)\delta(z - z_0), \\ q(v/c_s)\delta[x - (x_0 + vt)]\delta(y - y_0)\delta(z - z_0), 0, 0\}. \quad (3.3)$$

The solution can be found in many text books (see for example [6]) and reads

$$A_0(x) = -\frac{q\gamma}{[(\gamma(x - vt) - \gamma x_0)^2 + (y - y_0)^2 + (z - z_0)^2]^{1/2}}; \\ A_x(x) = \frac{q\gamma(v/c_s)}{[(\gamma(x - vt) - \gamma x_0)^2 + (y - y_0)^2 + (z - z_0)^2]^{1/2}}; \\ A_y(x) = A_z(x) = 0. \quad (3.4)$$

The only difference with the standard solution for a proper electromagnetic field is that in this solution the gamma factor is defined with the velocity of sound  $c_s$  rather than with the velocity of light. The important point we want to highlight here is that the fields decay faster in the  $x$  direction than in the orthogonal  $y, z$  directions, the ratio of the two decays being given by a sonic  $\gamma$  factor.

The next step in our construction of a quasi-interferometer is to use these emergent vector fields and sources to produce a rigid bar. The simplest model one can think of for that task is provided by dipolar interactions between globally neutral objects. Imagine that it is possible to create, starting from elementary charged sources (i.e. charged quasi-particles), a neutral super-structure. This super-structure would correspond to something that we could call a ‘quasi-atom’. The existence of rigid structures in nature is based on the possibility for atoms to be arranged in a regular and stable way. To make this possible, two atoms must find their most stable configuration when they are separated from each other a distance  $a_0$ . Here we are going to reproduce with quasi-atoms in our condensed-matter model what is found in nature for atoms in solids.

Take two of these neutral objects (quasi-atoms) and place them at rest with respect to the medium, both along the  $x$  axis, and at a distance  $a$  with respect to each other. If one associates an energy density  $e$  with the vector field created by the total system, then this energy density will be a function of the location in space and of the distance  $a$ :  $e = e(\vec{x}, a)$ . Integrating over the entire space one finds an interaction energy

$$E_I(a) = \int dx^3 e(\vec{x}, a). \quad (3.5)$$

In suitable situations one will obtain that this interaction energy has a sharp minimum at  $a = a_0$ , exactly what is needed to guarantee the rigidity of the two-component system.

Now, assume that the two neutral objects are moving uniformly in the  $x$  direction while maintaining their relative distance. The energy density will now depend on  $a$ ,  $\vec{x}$  and  $t$ . Again, after integration one will find

$$E'_I(a) = \int dx^3 e'(\vec{x}, t, a). \quad (3.6)$$

According to the result obtained for the one-particle case, namely that the vector field associated with a moving charge acquires a gamma factor in its decay in the  $x$  direction compared to the case for the charge at rest, it is natural to expect that this interaction energy will be precisely  $E'_I(a) = E_I(\gamma a)$ . The minimum of this potential energy satisfies the condition

$$0 = \frac{dE'_I(a)}{da} = \gamma \frac{dE_I(a')}{da'}; \quad a' = \gamma a \quad (3.7)$$

Thus, the minimum now occurs when  $a' = a_0$ , that is, when  $a = \gamma^{-1}a_0$ . In this way we have shown that the real distance (i.e. the distance measured in the lab) between the two quasi-atoms has decreased by an acoustic Lorentz factor  $\gamma$  due to their velocity with respect to the medium. And obviously, when adding more quasi-particles to construct the quasi-interferometer arm, the Lorentz factor in the  $x$  direction will persist.

This acoustic gamma factor is precisely the length contraction – the contraction of an interferometer arm oriented in the direction of motion – which is needed in order for the interference pattern to remain unaffected by such a uniform motion. So a Michelson-Morley type of experiment using a quasi-interferometer does not make it possible for internal observers to distinguish whether they are at rest or uniformly moving with respect to the medium. Therefore, a natural description of the internal world (based on operational definitions) as experienced by its internal inhabitants is provided by relativity theory.

In this simple model, this relativistic world in a fishbowl is itself immersed in a Newtonian external world (the laboratory). Remarkably, all of relativity (at least, all of special relativity) could be taught as an effective theory by using only Newtonian language.<sup>2</sup>

So we have explicitly demonstrated here for the case of a flat spacetime what was suggested earlier in, for example, [8] and [3], namely that Lorentz invariance is not broken, i.e., that an internal observer cannot detect his absolute state of motion. The argument is the same for curved spacetimes: The internal observer would have no way to detect the “absolute” or fixed background. So the apparent background dependence provided by the (non-relativistic) condensed matter system will not violate diffeomorphism invariance, at least not for these internal inhabitants. These internal observers will then have no way to collect any metric information beyond what is coded into the intrinsic geometry (i.e., they only get metric information up to a gauge or diffeomorphism equivalence factor). Internal observers would be able to write down diffeomorphism invariant Lagrangians for relativistic matter fields in a curved geometry. The dynamics of this geometry, however, is a different issue. It is a well-known problem that the expected relativistic dynamics, i.e. the Einstein equations, have not been reproduced in any known condensed matter system so far.

#### IV. SEARCHING IN THE PAST

In a way, the model we are discussing here could be seen as a variant of the old ether model. At the end of the 19th century, the ether assumption was so entrenched in the physical community that, even in the light of the null result of the Michelson-Morley experiment, nobody thought immediately about discarding it. Until the acceptance of special relativity, the best candidate to explain this null result was the Lorentz-FitzGerald contraction hypothesis. What Lorentz and FitzGerald proposed was essentially what we have described in the previous section, namely<sup>3</sup>:

“one would have to imagine that the motion of a solid body (...) through the resting ether exerts upon the dimensions of that body an influence which varies according to the orientation of the body with respect to the direction of motion.”[9]

There are important differences between our thought model and the idea of a luminiferous ether as it was held at the end of the 19th century. For example, in our model everything — spacetime, electromagnetism and (quasi-)matter — arises effectively from the same condensed matter ‘ether’, whereas the luminiferous ether supposedly only affected electromagnetic phenomena. In addition, we consider our model of a relativistic world in a fishbowl, itself immersed in a Newtonian external world, as a source of reflection, as a *Gedankenmodel*. By no means are we suggesting that there is a world beyond our relativistic world describable in all its facets in Newtonian terms.

Coming back to the contraction hypothesis of Lorentz and FitzGerald, it is generally considered to be *ad hoc*. However, this might have more to do with the caution of the authors, who themselves presented it as a hypothesis<sup>4</sup>, than with the naturalness or not of the assumption. The contraction hypothesis is in fact quite natural if one assumes that not only electromagnetism but also matter (and in our condensed matter model, even spacetime itself) are effective phenomena emerging from the ether. As observed by Lorentz<sup>5</sup> :

“Surprising though this hypothesis may appear at first sight, yet we shall have to admit that it is by no means far-fetched, as soon as we assume that molecular forces are also transmitted through the ether, like the electrical and magnetic forces. [Then] the translation will very probably affect the action between two molecules or atoms in a manner resembling the attraction or repulsion between charged particles.”[9]

The reason that special relativity was considered a better explanation than the Lorentz-FitzGerald hypothesis can best be illustrated by Einstein’s own words:

“The introduction of a ‘luminiferous ether’ will prove to be superfluous inasmuch as the view here to be developed will not require an ‘absolutely stationary space’ provided with special properties.”[14]

The ether theory had not been *disproved*, it merely became *superfluous*. Einstein realised that the knowledge of

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<sup>2</sup> A similar point of view was defended by Bell [7].

<sup>3</sup> FitzGerald published a similar suggestion without formally working it out [10]. Curiously, even FitzGerald himself was not certain whether it had effectively been published or not and this was only ‘discovered’ in 1967 [11].

<sup>4</sup> Lorentz uses the term *Hülfs-hypothese*. Holton [12] has observed that Lorentz introduces at least eleven hypotheses in [13]. Although some of these seem much more natural than others, Lorentz introduces all of them in the same cautious way: as hypotheses that are needed to advance with the theory.

<sup>5</sup> Or in FitzGerald’s words: “We know that electric forces are affected by the motion of the electrified bodies relative to the ether, and it seems a not improbable supposition that the molecular forces are affected by the motion, and that the size of a body alters consequently” [10].

the elementary interactions of matter was not advanced enough to make any claim about the relation between the constitution of matter (the ‘molecular forces’), and a deeper layer of description (the ‘ether’) with certainty. Thus his formulation of special relativity was an advance within the given context, precisely because it avoided making any claim about the fundamental structure of matter, and limited itself to an *effective* macroscopic description. That this was very clearly realised by Einstein himself can be seen from the following quote.

“The next position which it was possible to take up in face of this state of things [the acceptance of the special theory of relativity] appeared to be the following. The ether does not exist at all. (...) More careful reflection teaches us, however, that the special theory of relativity does not compel us to deny ether. We may assume the existence of an ether; only we must give up ascribing a definite state of motion to it.”

And he continues with the following model:

“Think of waves on the surface of water. Here, we can describe two entirely different things. Either we may observe how the undulatory surface forming the boundary between water and air alters in the course of time; or else – with the help of small floats, for instance – we can observe how the position of the separate particles of water alters in the course of time. If the existence of such floats for tracking the motion of the particles of a fluid were a fundamental impossibility in physics – if, in fact, nothing else whatever were observable than the shape of the space occupied by the water as it varies in time, we should have no ground for the assumption that water consists of movable particles. But all the same we could characterise it as a medium.” [15]

## V. SOME FINAL REMARKS

In Einstein’s pragmatic approach for explaining the null result of the Michelson-Morley experiment<sup>6</sup>, and in particular in his analogy of the waves on the surface of water described in the previous paragraph, lie the essential lessons that we can learn from the fishbowl model presented in this essay. As long as we have no possibility to directly track the elementary level of description that we are looking for in a quantum theory of gravity (the particles that constitute the water, in Einstein’s analogy), maybe we should limit ourselves to a description of how spacetime (the undulatory surface at the boundary between water and air) and matter effectively emerge from this elementary level (the ether or medium), without making any more assumptions about the water particles themselves than strictly necessary.

The thought model that we have presented shows that, as long as there are no direct experimental constraints on this elementary description, one should take care when postulating which aspects of the currently known ‘effective’ physics (i.e. of the internal world) should also be taken as fundamental in the elementary description (the external world). As a matter of fact, in our *Gedankenmodel*, we showed that this fundamental external world can even be Newtonian (the laboratory), while still reproducing a relativistic behaviour with respect to an internal observer. Only if this internal observer were able to probe the external world, e.g. through observation of phenomena linked directly to a quantum regime of gravity, would he be able to really know anything about the fundamental physics. Of course, it is far from certain that nature will actually allow us to probe this deeper layer of description, i.e. maybe

“tracking the motion of the particles on a fluid [is] a fundamental impossibility in physics.” [15]

But even if this were the case, this would not necessarily mean the end of the story. If it turns out that nature’s defences are strong enough to prevent us from probing the quantum scale of gravity, then maybe this intrinsic protection should precisely be used as a new theoretical guideline.

So when Einstein concluded in [15] that

“according to the general theory of relativity space without ether is unthinkable. (...) But this ether should not be thought of as endowed with the quality characteristic of ponderable media, as consisting of parts which may be tracked through time. *The idea of motion may not be applied to it,*”

we would add that the idea of motion may not be applied to it in the same sense as it is for an internal observer. As long as the fundamental physics remains hidden from direct observation by internal observers, there could be *plenty of states of motion that may be applied to it.*

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<sup>6</sup> Although historically, when formulating special relativity, Einstein was barely aware of, let alone concerned with, the Michelson-Morley experiment in particular [12].

### Acknowledgements

The authors wish to thank Andrés Cano and Luis J. Garay for very useful comments and suggestions. This work has been funded by the Spanish MEC under project FIS2005-05736-C03-01. G.J. was also supported by CSIC grant I3P-BPD2005 of the I3P programme, cofinanced by the European Social Fund.

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