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QCD DUALITY ANALYSIS OF B^0-$\bar{B}^0$ MIXING

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ABSTRACT

Using finite energy sum rules, the hadronic matrix element of the $\Delta B = 2$ operator governing the mixing between the B^0 and $\bar{B}^0$ mesons is evaluated. The result is very sensitive to the input value of the bottom quark mass. Allowing it to vary in the range m_b=(4.70±0.14)GeV, one finds $\xi_B \equiv |f_B| \sqrt{|B_B|}$ = (0.13±0.05)GeV.

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Important experimental progress in our knowledge of flavour changing phenomena has been achieved during the last year. Together with previous electroweak measurements, the new experimental results provide useful constraints on the parameters of the standard model. However, in order to completely pin down the remaining uncertainties in these parameters (or perhaps even to disprove the model), it is first necessary to reduce the size of the theoretical errors entering the analysis of weak transitions. With the expected increasing accuracy of forthcoming experiments, the improvement of our theoretical ability to make definite predictions for these processes is even more strongly called for.

The $B^0-\bar{B}^0$ system is a good example of this situation. While it is clear that the recent observation of a relatively large amount of $B^0_d-\bar{B}^0_d$ mixing [1] seems to require [2] a rather heavy top quark, $m_t \gtrsim 50-100$ GeV, the actual value of the extracted m_t lower bound is quite uncertain. The biggest source of error originates from the matrix element of the $\Delta B = 2$ operator

$$O_{\Delta B=2} \equiv (\bar{b}_L \gamma_\mu d_L)\ (\bar{b}_L \gamma^\mu d_L) \tag{1}$$

between the B^0 and $\bar{B}^0$ states, which is usually parametrized as

$$\langle \bar{B}^0 | O_{\Delta B=2} | B^0 \rangle = \left(\frac{4}{3} f_B^2 M_B^2 \right) B_B \tag{2}$$

Here, f_B is the decay constant of the B^0-meson (the normalization corresponds to $f_\pi \approx 93.3$ MeV) and the factor B_B takes into account the discrepancy between the true matrix element and the result $(4/3)f_B^2M_B^2$ obtained by inserting the vacuum in all possible ways.

At present, the best evaluation of the B^0-meson decay constant f_B comes from QCD sum rules. Although calculations by different groups gave initially quite different results [3], the situation seems to be clarified by recent analyses [4,5] showing that various versions of QCD-duality sum rules lead to a unique prediction [5]

$$f_B \simeq (129 \pm 13)\ \mathrm{MeV} \tag{3}$$

The error bar appears to be dominated by the input value of the bottom quark mass (f_B decreases for increasing values of m_b) which has been taken to be*) $m_b = (4.71 \pm 0.13)$ GeV.

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*) m_b refers to the "on-shell" or "pole" mass $m_b \equiv m_b\ (s = m_b^2)$.

Concerning the B_B- factor, one would naively expect that the vacuum insertion approximation becomes more accurate with the increase of the meson mass, so $B_B = 1$ is usually assumed. However, a quantitative analysis of this statement is still lacking.

The purpose of this letter is to present a direct calculation of the B^0-$\bar{B}^0$ matrix element, without relying on the vacuum insertion approximation [6]. This can be done by studying the two-point function

$$\psi_{\Delta B=2}(q^2) \equiv i \int d^4x \; e^{iqx} \; \langle 0 | T(O_{\Delta B=2}(x) \; O_{\Delta B=2}(0)^{+}) | 0 \rangle \tag{4}$$

constructed with the $\Delta B = 2$ operator of Eq. (1), and writing a system of finite energy sum rules relating integrals of the corresponding hadronic spectral function to their QCD counterparts.

A similar correlator was used in Ref. [7] to compute the B_K parameter governing the K^0-$\bar{K}^0$ mixing. However, due to the higher mass scale involved, the analysis of $\psi_{\Delta B=2}(q^2)$ requires a somewhat different approach. Unlike the kaon case, the onset of the asymptotic continuum can be taken here very near the physical threshold $s_{th} = 4M_B^2$. One can then approximate the hadronic spectral function $(1/\pi)\,\mathrm{Im}\psi_{\Delta B=2}(s)$ by the lowest intermediate state contribution, which is obviously related to the matrix element of Eq. (2). The effect of higher mass intermediate states is taken into account by the QCD continuum.

Using for convenience the effective realization

$$O^{eff}_{\Delta B=2} = \frac{2}{3}\, g_{\Delta B=2} \; \partial_\mu B^0 \; \partial^\mu B^0 \tag{5}$$

where

$$g_{\Delta B=2} = f_B^2 \; B_B \tag{6}$$

one immediately gets

$$\frac{1}{\pi}\,\mathrm{Im}\,\psi^{eff}_{\Delta B=2}(s) = \theta(s-4M_B^2)\,\frac{2\,|g_{\Delta B=2}|^2}{9\,(16\pi^2)}\; s^2\,(1-2M_B^2/s)^2\,\sqrt{1-4M_B^2/s} \tag{7}$$

The short distance calculation of $\psi_{\Delta B=2}(q^2)$ is rather involved. As shown in Fig. 1, the lowest order contribution contains four internal quark lines. Working in the $m_d = 0$ limit, we have still to take care of the exact m_b-dependence coming from the two heavy quark propagators. Nevertheless, it is possible to do analytically all the momentum integrations in order to get a compact expression in parametric form:

$$\frac{1}{\pi}\,\mathrm{Im}\,\Psi^{AF}_{\Delta B=2} = \theta(s-4m_b^2)\;\frac{s^4}{(16\pi^2)^3}\quad A(s) \tag{8}$$

where

$$A(s) = 2\left(1+\frac{1}{3}\right)\int_{\delta}^{(1-\sqrt{\delta})^2} dz \int_{\delta}^{(1-\sqrt{z})^2} du\quad \lambda^{1/2}(1,z,u)\quad (1-\delta/z)^2\,(1-\delta/u)^2$$

$$\cdot\left\{(1-z-u)^2\,(1+2\delta/z)\,(1+2\delta/u) + 8zu - 2\delta\,(z+u) - 4\,\delta^2\right\} \tag{9}$$

Here, $\delta \equiv m_b^2/s$ and $\lambda(x,y,z) \equiv x^2+y^2+z^2-2xy-2xz-2yz$.

The leading non-perturbative corrections come from dimension four operators appearing in the operator product expansion of the T-product of Eq. (4). The heavy quark condensate contribution, $m_b\langle 0|b\bar{b}|0\rangle$, is already taken into account by the Wilson coefficient of the $\langle 0|G^2|0\rangle$ term, and a possible $m_b\,\langle 0|\bar{d}d|0\rangle$ correction is absent because of the helicity projectors appearing in the $0_{\Delta B=2}$ operator. The $\Delta B = 2$ spectral function can then be written, to this order, as

$$\frac{1}{\pi}\,\mathrm{Im}\,\Psi^{QCD}_{\Delta B=2}(s) = \theta(s-4m_b^2)\frac{s^4}{(16\pi^2)^3}\left\{A(s) + \frac{16\pi^2}{s^2}\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle B(s) + O\!\left(\frac{1}{s^3}\right)\right\} \tag{10}$$

The gluon condensate coefficient B(s) is more easily computed by writing the quark propagators in an external background gluonic field and using the standard fixed-point gauge techniques [8]. The result one finds is

$$\begin{aligned} B(s) = &\int_{\underline{X}}^{1} dx \int_{Y_-}^{Y_+} dY \left\{ -\frac{\Delta}{2Y^2}\left[\Delta - Y(1-Y)\right]\left[2XY + (1-X)^2(1-Y)\right] \right.\\ &\left. \qquad + \frac{\delta X}{3Y^3}\,(1-X)^2(1-Y)^3\left[2\Delta - Y(1-Y)\right]\right\} \\ &- \int_{\delta}^{(1-\sqrt{\delta})^2} dz\quad z\quad (1-\delta/z)^2\quad \lambda^{1/2}(1,z,\delta) \end{aligned} \tag{11}$$

where

$$\Delta \equiv \delta \left(\frac{Y}{x} + 1 - Y \right) - Y(1-Y) \tag{12}$$

and the parametric integration limits are given by

$$\begin{aligned} \underline{X} &= \frac{\delta}{(1-\sqrt{\delta})^2} \\ Y_{\pm} &= \frac{1}{2} \left\{ 1 + \delta \left(1 - \frac{1}{x} \right) \pm \lambda^{1/2}(1, \delta, \delta/x) \right\} \end{aligned} \tag{13}$$

Let us consider now the integrals

$$G_n(s_0) \equiv \int_0^{s_0} ds \quad s^n \quad \frac{1}{\pi} \, \mathrm{Im}\, \Psi_{\Delta B=2}(s) \tag{14}$$

for different values of n, and write the duality constraints

$$G_n(s_0)^{eff} = G_n(s_0)^{QCD} \tag{15}$$

which relate the $G_n(s_0)$ moments computed with the hadronic parametrization of Eq. (7), $G_n(s_0)^{eff}$, to the corresponding short-distance calculation, $G_n(s_0)^{QCD}$, using Eq. (10).

The ratio

$$R_n(s_0) \equiv G_{n+1}(s_0) \,/\, G_n(s_0) \tag{16}$$

does not depend on $g_{\Delta B=2}$ and therefore can be used to fix the optimal duality region on the s_0 variable. For numerical convenience, we normalize it to the asymptotic freedom behaviour $R_n(s_0)^{AF}$, obtained with the spectral function of Eq. (8),

$$\mathcal{R}_n(s_0) \equiv R_n(s_0) \,/\, R_n(s_0)^{AF} \tag{17}$$

and use the eigenvalue condition

$$\mathcal{R}_n(s_0)^{eff} = \mathcal{R}_n(s_0)^{QCD} \tag{18}$$

Inserting the value of s0 thus obtained in Eq. (15) results then in a prediction for $|g_{\Delta B=2}|$ and hence for the matrix element (2) that we are looking for.

With the input value $M_{B^0} = 5.275$ GeV, the functions $\chi_n(s_0)$ are plotted in Fig. 2, for n = -8 and using three different values of the bottom quark mass which cover the range [9]

$$m_b = (4.70 \pm 0.14) \quad GeV \tag{19}$$

In view of the recent claims [10] that the gluon condensate has been usually underestimated in the literature, we have allowed it to vary within a factor of two, i.e., we have taken

$$\langle \frac{\alpha_s}{\pi} G^2 \rangle = (0.012 \quad GeV^4) \times w \quad , \quad (w = 1-2) \tag{20}$$

where w = 1 corresponds to the so-called standard value.

In the absence of QCD corrections $\chi_n(s_0)$ would be one (the dashed lines in Fig. 2). With inclusion of the calculated $\langle \alpha_s/\pi \; G^2 \rangle$ contributions, these lines become the continuous (w=1) and dot-dashed (w=2) curves which approach asymptotically the value one at large s_0. The curves corresponding to the hadronic parametrization of the spectral function, given in Eq. (7), are the lines of dots shown in the figure.

The duality test of Eq. (18) turns out to be quite sensitive to the actual value of the gluon condensate. With the standard value w=1, the eigenvalue solutions are found to be around $r \equiv s_0/4M_B^2 \sim 1.15$, 1.10 and 1.05, for m_b = 4.56, 4.70 and 4.84 GeV respectively. Taking w = 2 and the same values of m_b, the best duality regions move to $r \sim 1.13$, 1.05 and 1.10. Note that for the biggest value of m_b the duality behaviour is better with w = 1, while smaller quark masses seem to prefer higher values of the gluon condensate. The central mass value m_b = 4.70 GeV shows indeed quite a good matching between the hadronic and QCD curves for w = 2.

From Eq. (15) one can obtain

$$\xi_B \equiv \sqrt{|g_{\Delta B=2}|} = |f_B| \sqrt{|B_B|} \tag{21}$$

as function of r. With n = -8, this is shown in Fig. 3 for w = 1 (continuous curves) and w = 2 (dot-dashed ones). When r approaches the value one, the QCD calculation is unable to reproduce the physical threshold $s_{th} = 4M_{B^0}^2$ and $|\xi_B|$ grows to infinity. On the other hand, if one chooses too high values for the onset of the asymptotic continuum s_0, the resulting ξ_B starts growing because the effect of higher mass intermediate states, not contained in the parametrization of Eq. (7), becomes important.

To study the stability of ξ_B versus the election of the power n, we have solved the eigenvalue equation (18) and inserted the value of s_0 thus obtained in Eq. (15) for different values for n. The resulting predictions for ξ_B are plotted in Fig. 4, which shows a remarkable stability for the central value of the bottom quark mass. The cases (m_b = 4.56 GeV, w = 1) and (m_b = 4.84 GeV, w = 2) look however worse.

From Figs. 2 to 4 one then gets the results

$$\xi_B \simeq \begin{cases} 0.17 - 0.18 \quad \text{GeV} & , \quad (m_b = 4.56 \quad \text{GeV}) \\ 0.12 - 0.13 \quad \text{GeV} & , \quad (m_b = 4.70 \quad \text{GeV}) \\ 0.075 - 0.085 \quad \text{GeV} & , \quad (m_b = 4.84 \quad \text{GeV}) \end{cases} \tag{22}$$

The allowed changes on the value of $\langle \alpha_s/\pi \, G^2 \rangle$ have produced an effect of no more than 10% in these numbers.

Due to the anomalous dimension of the $\Delta B = 2$ operator, the parameter ξ_B is renormalization scale dependent. The numbers given in Eq. (22) correspond to the scale $\mu^2 = s_0 \simeq 4M_B^2$. Since the Wilson coefficient of $O_{\Delta B=2}$ is usually computed at $\mu^2 = m_b^2$, the results (22) should be rescaled by a factor $[\alpha_s(m_b^2)/\alpha_s(s_0)]^{3/23}$. However, this amounts to a 3% effect only which is negligible.

The largest source of uncertainty in our results is by far the input value of the bottom quark mass. Allowing it to vary in the range m_b = (4.70±0.14) GeV, we can give as a final result

$$\xi_B = (0.13 \pm 0.05) \quad \text{GeV} \tag{23}$$

Comparing our prediction for ξ_B with the f_B-values given in Eq. (3), one finds a good agreement for the central value of the quark mass m_b = 4.70 GeV, which could give some support to the validity of the vacuum saturation approximation in this case. However, the B_B parameter resulting from this comparison is strongly m_b-dependent and therefore no firm conclusion can be extracted.

Our analysis could obviously be improved by computing α_s corrections and higher order condensate contributions to the two-point function (4). However, the expected effect of these additional terms is smaller than the m_b uncertainty already reflected in the error bar of Eq. (23). In order to reduce this large error, a better determination of the bottom quark mass is then mandatory.

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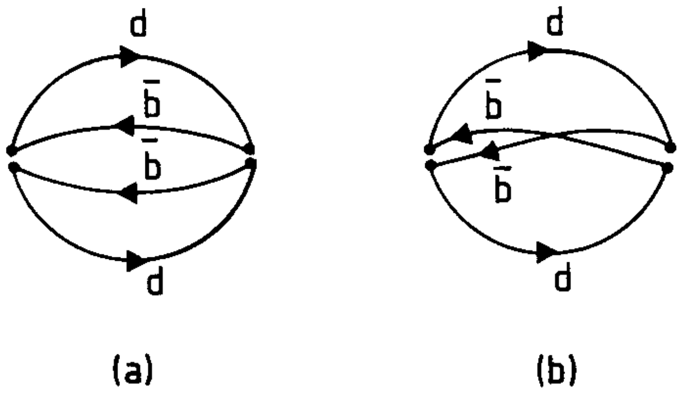
FIGURE CAPTIONS

Fig. 1 : Lowest order contributions to the $\Delta B = 2$ correlator of Eq. (4).

Fig. 2 : The ratio $\chi_n(s_0)$ defined in Eq. (17) is plotted versus $r \equiv s_0/4M_B^2$, for $n = -8$ and three different values of the bottom quark mass. The continuous (dot-dashed) curves represent the QCD predictions obtained with the parametrization of Eq. (10) and taking the value $w = 1$ ($w=2$) for the strength of the gluon condensate. They approach asymptotically the value one - the dashed lines in the figure - at large r. The lines of dots are the values obtained using the hadronic spectral function given in Eq. (7).

Fig. 3 : The quantity $\xi_B \equiv |f_B|\sqrt{|B_B|}$ obtained from Eq. (15) is plotted versus $r \equiv s_0/4M_B^2$ for $n = -8$ and three different values of the bottom quark mass. The continuous and dot-dashed curves correspond to the values $w = 1$ and $w = 2$ respectively, for the strength of the gluon condensate.

Fig. 4 : Behaviour of the ξ_B-value, obtained at the optimal duality s_0-point, versus n, for different bottom quark masses. The continuous and dot-dashed curves correspond to the values $w = 1$ and $w = 2$ respectively, for the strength of the gluon condensate.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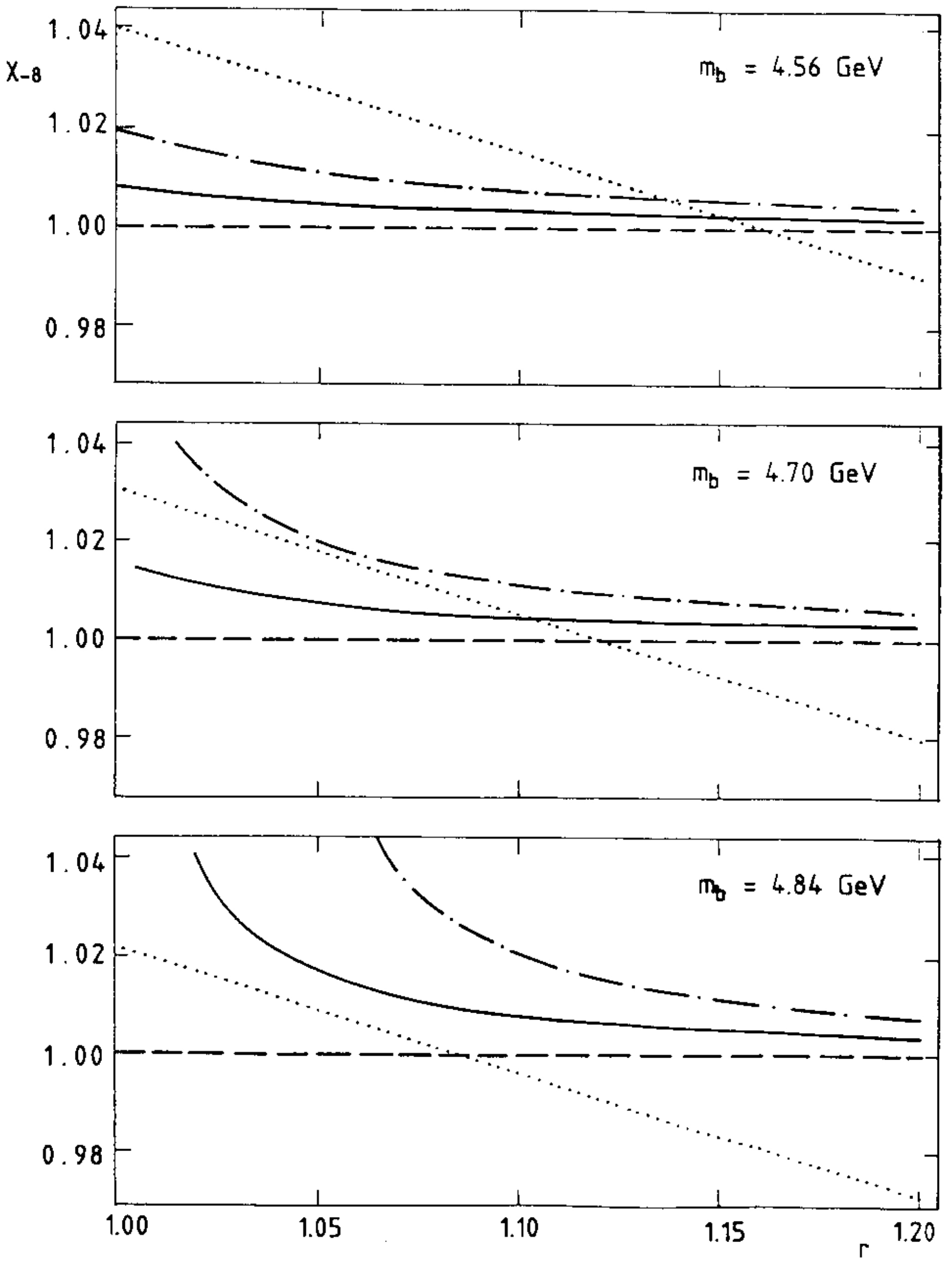
(b)

Fig. 1


1.04
X_{-8}
1.02
1.00
0.98
m_b = 4.56 GeV
1.04
1.02
1.00
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m_b = 4.70 GeV
1.04
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0.98
m_b = 4.84 GeV
1.00
1.05
1.10
1.15
1.20
r

Fig. 2

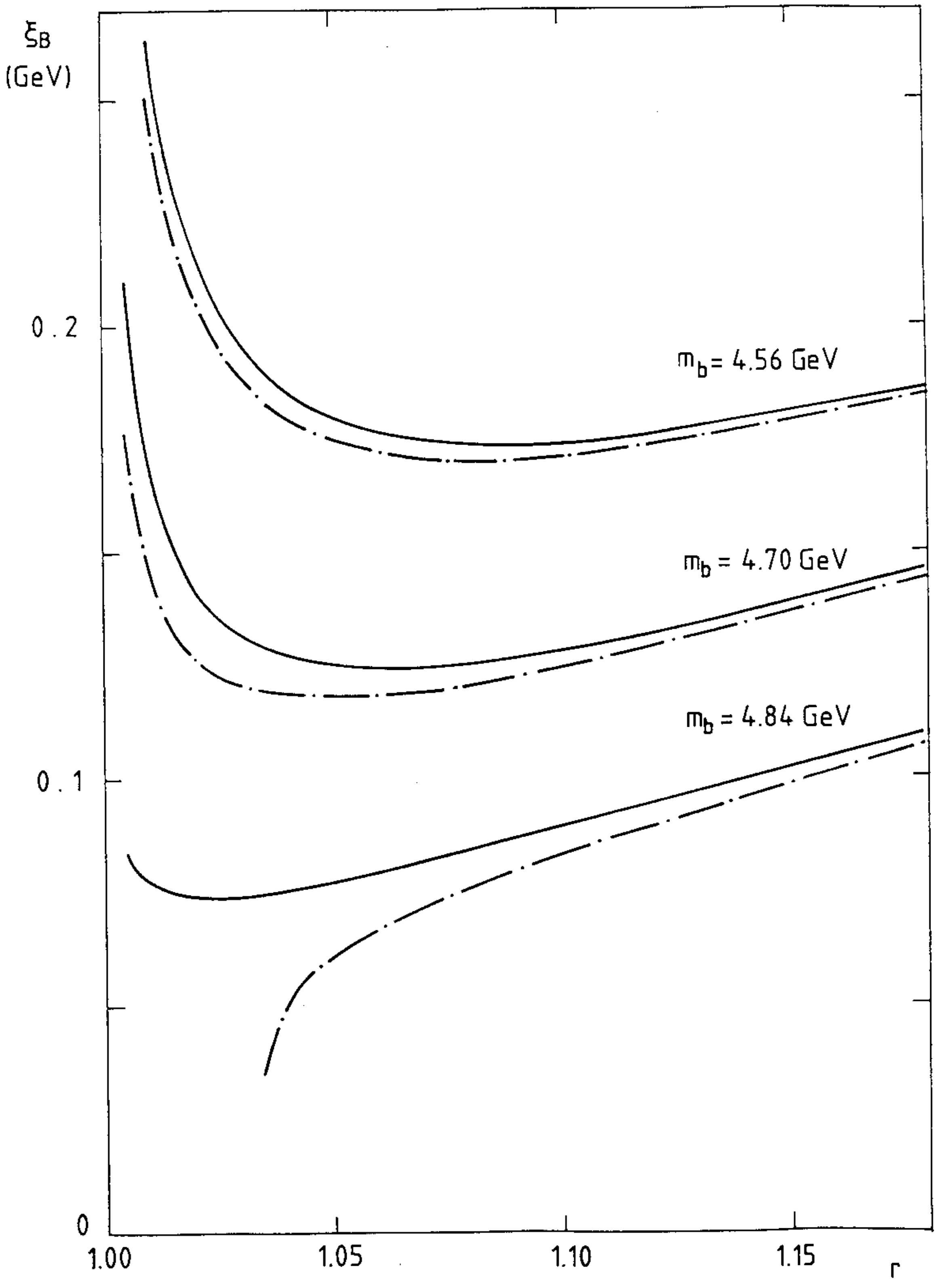

Fig. 3

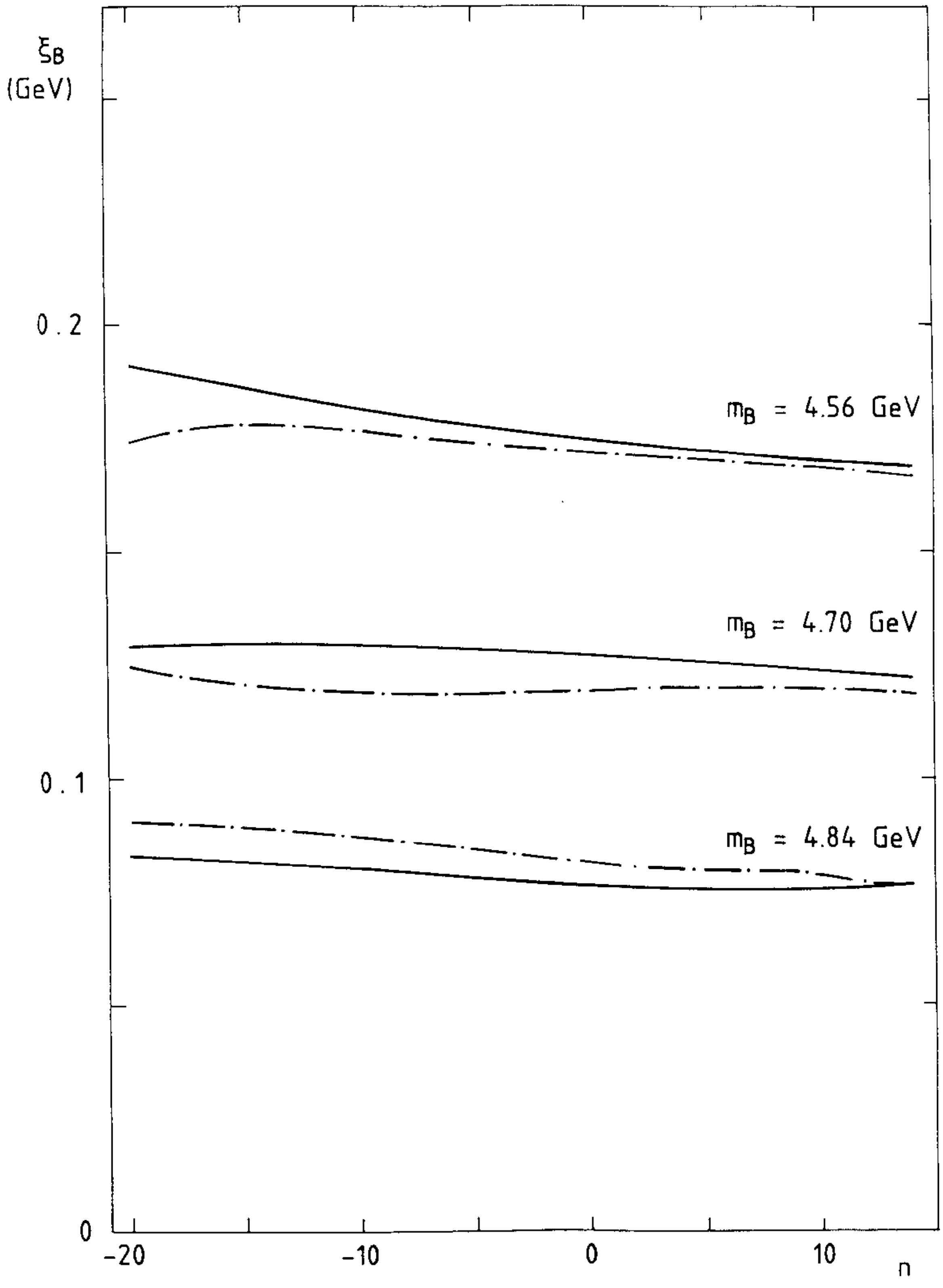

Fig. 4