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STRONG CP-VIOLATION IN AN EFFECTIVE CHIRAL LAGRANGIAN APPROACH

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ABSTRACT

Possible long-distance effects like $\eta \to \pi\pi$ and the electric dipole moment of the neutron (nEDM), which may originate in the strong CP-non-conserving sector of QCD, are re-examined in the light of an effective chiral Lagrangian approach. Contrary to recent claims, we show that it is possible to obtain reliable estimates for these processes from chiral symmetry and low-energy phenomenology (modulo the size of the θ-vacuum angle, of course). We also discuss how an improvement of the presented estimate of the nEDM could be made, if necessary.

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1. INTRODUCTION

One of the outstanding successes of QCD is the solution of the so-called $U(1)_A$ problem: why is the SU(3)-singlet $\eta'(958)$ so much heavier than the low-lying SU(3)-octet of pseudoscalars ? The two basic ingredients which are used in the explanation are the axial anomaly of the $U(1)_A$ current [1], and the non-perturbative structure of the QCD vacuum [2–4]. The solution within the QCD framework [5–9] implies, however, the existence of an effective additional term in the QCD Lagrangian ($\widetilde{G}^{(a)}_{\mu\nu} \equiv \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}G^{(a)\rho\sigma}$):

$$\mathcal{L}_\theta = \theta_0 \frac{g^2}{32\pi^2} \sum_a G^{(a)}_{\mu\nu}(x)\widetilde{G}^{(a)\mu\nu}(x), \tag{1.1}$$

with θ_0 , the so-called vacuum angle, a hitherto unknown parameter (for a good review article on the subject see ref. [10]).

The new term (1.1) violates P, T and CP and may lead to observable effects in flavour conserving transitions. It may generate, in particular, a sizeable neutron electric dipole moment (nEDM), which very refined experiments have constrained down to a very high precision [11,12]. With the standard definition ($F_{\mu\nu}(x) \equiv \partial_\mu \mathcal{A}_\nu(x) - \partial_\nu \mathcal{A}_\mu(x)$)

$$\mathcal{L}_{nEDM} = \frac{d_n^\gamma}{2} \overline{\Psi}_n(x) i\gamma_5 \sigma^{\mu\nu} \Psi_n(x)\, F_{\mu\nu}(x) \tag{1.2}$$

for an electric dipole moment coupling, the most accurate measurements give [12]

$$d_n^\gamma = (-3 \pm 5) \times 10^{-26} e\, cm\,; \tag{1.3}$$

and a further improvement in sensitivity by a factor of 10 is still expected from the same experiment within the next two years [13].

Early theoretical estimates of the size of d_n^γ induced by the θ_0-term in (1.1) range from $0.4 \times 10^{-16}\theta_0\, e\, cm$ to $20 \times 10^{-16}\theta_0\, e\, cm$ [14–17] , suggesting that the phase θ_0 is limited by experiments to be below 10^{-9}. In fact, the constrained parameter is not quite θ_0 but rather the combination

$$\theta \equiv \theta_0 + \arg(\det \mathcal{M}), \tag{1.4}$$

where $\mathcal{M}$ denotes the full mass matrix emerging from the Yukawa couplings of the light quarks in the electroweak sector. In full generality, $\mathcal{M}$ is non-diagonal and non-Hermitian:

$$\mathcal{L}_{Mass} = -\bar{q}_{Ri}\mathcal{M}_{ij}q_{Lj} - \bar{q}_{Li}\mathcal{M}^\dagger_{ij}q_{Rj}. \tag{1.5}$$

However, with the help of an appropriate $SU(3)_L \otimes SU(3)_R$ transformation one can always restrict $\mathcal{M}$ to the form

$$\mathcal{M} = \exp\{\frac{i}{3}\arg(\det \mathcal{M})\}\ \mathrm{diag}(m_u, m_d, m_s), \tag{1.6}$$

with m_u, m_d and m_s positive. In the absence of the θ_0-vacuum term in the QCD-Lagrangian, the phase $\arg(\det \mathcal{M})$ could be reabsorbed by a simple $U(1)_A$ rotation of the quark fields. However, because of the $U(1)_A$-anomaly, the θ_0-vacuum is not invariant under $U(1)_A$ transformations. The $U(1)_A$ rotation which eliminates $\arg(\det \mathcal{M})$ from the mass term generates a new θ_0-vacuum value, which is the combination appearing in eq. (1.4) above.

Of particular interest among the theoretical estimates of d_n^γ quoted above is the one by Crewther, Di Vecchia, Veneziano and Witten [15], hereafter referred to as CDVW. These authors pointed out that the coefficient of the leading chiral logarithm which contributes to d_n^γ is calculable in terms of known parameters (modulo the overall θ-phase factor of course), thus providing a reliable order-of-magnitude estimate of the d_n^γ expected from strong CP-violation. The possibility to check this result, by doing numerical simulations in lattice QCD [18,19] , has now triggered a renewal of interest in calculations of strong CP-violating processes [20–23].

We propose to take a fresh new look at this subject from the point of view of the chiral effective Lagrangian formulation of the strong CP-violation sector of QCD. The question of whether or not one can calculate long-distance effects like $\eta \to \pi\pi$ and d_n^γ, to some approximation at least, translates then into the question of to what level in a systematic chiral expansion there appear new constants in the effective Lagrangian not fixed by symmetry arguments alone, and unknown from phenomenology. In fact, the idea of using an effective Lagrangian framework to study strong CP-violating transitions is not new. Practically all that is needed here can be dug out from earlier papers by Di Vecchia, Veneziano and Witten [8,9,16] (see also ref. [24]) . Only the part of our formulation which includes baryons is perhaps new (to our knowledge) and provides an alternative to the early one offered in Di Vecchia's lectures [16].

We have organized the paper as follows. Section 2 is a short review of the QCD effective Lagrangian for Goldstone bosons, in the presence of the explicit breaking of chiral symmetry generated by the quark mass term in the QCD Lagrangian, and the breaking by the $U(1)_A$ anomaly. We also discuss the vacuum alignment in the presence of the θ-term.

The derivation of an effective QCD Lagrangian with inclusion of baryon fields is discussed in section 3. The possible couplings emerging from the underlying strong CP-violation in QCD, which can contribute leading chiral logarithms to the nEDM, are identified. The chiral loop calculation of d_n^γ (and d_Λ^γ) is reported in section 4. Section 5 contains a listing of the possible local counterterms, which contribute to the nEDM at $O(\mathcal{M})$ as well. We also discuss how these couplings could be fixed from phenomenology. Our conclusions and numerical results are given in section 6.

2. EFFECTIVE LAGRANGIAN FOR GOLDSTONE BOSONS; QUARK MASSES; LARGE N_c; AND THE $U(1)_A$ ANOMALY

In the limit where the number of colours N_c in QCD is taken to be large, the $U(1)_A$ anomaly is absent . If furthermore we also consider the chiral limit where all the light quark masses vanish, i.e. $m_u = m_d = m_s = 0$, the QCD Lagrangian

$$\mathcal{L}_{QCD}^{(0)} = -\frac{1}{4}\sum_a G_{\mu\nu}^{(a)} G^{(a)\mu\nu} + i\bar{q}\gamma^\mu D_\mu q \tag{2.1}$$

has then a $U(3)_L \otimes U(3)_R$ symmetry in flavour space, which is expected to be spontaneously broken down to $U(3)_V$. According to Goldstone's theorem , there appears then a nonet of massless pseudoscalar particles (π, K, η, η'). The fields of these particles can be conveniently collected in a 3×3 unitary matrix $\tilde{U}(\phi)$, which parametrizes the Goldstone excitations over the vacuum. Under the chiral group, $\tilde{U}(\phi)$ transforms as $\tilde{U} \to g_R \tilde{U} g_L^\dagger$ ($g_{R,L} \in U(3)_{R,L}$). It is convenient to factor out from the $\tilde{U}(\phi)$ matrix its vacuum expectation value, i.e.

$$\tilde{U}(\phi) = <\tilde{U}> U(\phi), \tag{2.2}$$

with $< U >= 1$. A useful parametrization for $U(\phi)$, which we shall adopt, is

$$U(\phi) \equiv \exp(-i\sqrt{2}\Phi(x)/f_\pi), \tag{2.3}$$

where ($\vec{\lambda}$ are Gell-Mann's matrices with $\mathrm{tr}\lambda_a\lambda_b = 2\delta_{ab}$)

$$\Phi(x) \equiv \frac{\phi^{(0)}}{\sqrt{3}} + \frac{\vec{\lambda}}{\sqrt{2}}\vec{\phi} = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} + \frac{\eta_1}{\sqrt{3}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} + \frac{\eta_1}{\sqrt{3}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2\eta_8}{\sqrt{6}} + \frac{\eta_1}{\sqrt{3}} \end{pmatrix}. \tag{2.4}$$

At low energies, it is possible to work out the consequences of the chiral symmetry properties of the underlying QCD-theory, by writing the most general effective Lagrangian involving the matrix $\tilde{U}$, which is consistent with the $U(3)_L \otimes U(3)_R$ symmetry. The Lagrangian is organized in terms of increasing powers of momentum or, equivalently, in terms of increasing number of derivatives. In the low energy domain we are interested in, the terms with a minimum number of derivatives will dominate. To lowest order in the number of derivatives, the effective chiral Lagrangian is uniquely given by the term

$$\mathcal{L}_{eff} = \frac{f_\pi^2}{4} \mathrm{tr}(\partial_\mu \tilde{U} \partial^\mu \tilde{U}^\dagger) . \tag{2.5}$$

The constant f_π, which is not fixed by symmetry requirements, is phenomenologically known from the decay $\pi \to \mu\nu$ ($f_\pi \approx 93.3 MeV$).

We need a generalization of eq. (2.5) which includes the explicit breaking of chiral symmetry generated by the quark mass term (1.5) in the QCD Lagrangian, and the breaking by the anomaly from $U(3)_L \otimes U(3)_R$ to $SU(3)_L \otimes SU(3)_R \otimes U(1)_V$. Moreover, we want to incorporate external gauge fields as well because eventually we shall have to consider chiral couplings of photons to pseudoscalars (and to baryons later). To the standard QCD Lagrangian $\mathcal{L}_{QCD}^{(0)}$ in (2.1) we then add the terms

$$\mathcal{L} = \mathcal{L}_{QCD}^{(0)} + \bar{q}\gamma^\mu\{v_\mu(x) + \gamma_5 a_\mu(x)\}q - \bar{q}_R \mathcal{M} q_L - \bar{q}_L \mathcal{M}^\dagger q_R + \theta_0 \frac{g^2}{32\pi^2} \sum_a G_{\mu\nu}^{(a)} \tilde{G}^{(a)\mu\nu} , \tag{2.6}$$

where the external fields $v_\mu(x)$ and $a_\mu(x)$ are Hermitian 3×3 matrices in flavour space. In the presence of these terms the full Lagrangian (2.6) is formally invariant with respect to local ($g_L(x)$,$g_R(x)$) chiral $U(3)_L \otimes U(3)_R$ transformations:

$$\begin{aligned} q_L &\to g_L q_L , \\ q_R &\to g_R q_R , \\ l_\mu \equiv v_\mu - a_\mu &\to g_L l_\mu g_L^\dagger + i g_L \partial_\mu g_L^\dagger , \\ r_\mu \equiv v_\mu + a_\mu &\to g_R r_\mu g_R^\dagger + i g_R \partial_\mu g_R^\dagger , \\ \mathcal{M} &\to g_R \mathcal{M} g_L^\dagger . \end{aligned} \tag{2.7}$$

To lowest order in the number of derivatives, and in powers of $\mathcal{M}$ and external v_μ and a_μ fields, the most general effective Lagrangian invariant under local chiral transformations is given by [24]

$$\mathcal{L}_{eff} = \frac{f_\pi^2}{4} \mathrm{tr}(D_\mu \tilde{U} D^\mu \tilde{U}^\dagger + \tilde{\chi}\tilde{U}^\dagger + \tilde{U}\tilde{\chi}^\dagger) , \tag{2.8}$$

where

$$D_\mu \tilde{U} \equiv \partial_\mu \tilde{U} - i r_\mu \tilde{U} + i \tilde{U} l_\mu \tag{2.9}$$

and $\tilde{\chi}$ is a 3×3 matrix proportional to $\mathcal{M}$,

$$\tilde{\chi} \equiv 2B_0 \, \mathcal{M} \, , \tag{2.10}$$

with B_0 a constant which, like f_π , is not fixed by symmetry requirements alone.

Once special directions in flavour space are selected for the external fields or for the matrix $\mathcal{M}$, chiral symmetry is of course explicitly broken. For instance, to introduce electromagnetic interactions one should take $l_\mu = r_\mu = eQ\mathcal{A}_\mu$, with $Q \equiv \mathrm{diag}(\frac{2}{3}, \frac{-1}{3}, \frac{-1}{3})$. The important point is that (2.8) then breaks chiral symmetry in exactly the same way as the fundamental Standard Model Lagrangian does.

Although the Lagrangian (2.6) is formally invariant under local chiral transformations, this is no longer true for the associated generating functional $\exp i\Gamma \equiv \int [\mathcal{D}G \, \mathcal{D}q \, \mathcal{D}\bar{q}] \exp\{i \int d^4x \mathcal{L}\}$. The anomalies of the fermionic determinant break chiral symmetry at the quantum level. Here, we are interested in the axial anomaly induced by $\mathrm{U}(1)_A$ chiral rotations, i.e. $g_R = g_L^\dagger = e^{i\beta} I$, which generate a contribution to the θ_0-term ($n_f = 3$ is the number of light quark flavours):

$$\theta_0 \rightarrow \theta_0 - 2n_f \beta \, . \tag{2.11}$$

As already mentioned before, the combination $\theta_0 + \arg(\det \mathcal{M})$ remains invariant under these transformations. To lowest non-trivial order in $1/N_c$, the chiral symmetry breaking effect induced by the $\mathrm{U}(1)_A$ anomaly can be taken into account in the effective Lagrangian, through the term (see Di Vecchia and Veneziano [9] and Witten [8]; see also ref. [25])

$$\mathcal{L}_{\mathrm{U}(1)_A} = -\frac{f_\pi^2}{4} \frac{a}{N_c} \{ \frac{i}{2} [\log(\det \tilde{U}) - \log(\det \tilde{U}^\dagger)] \}^2 \, , \tag{2.12}$$

which breaks $\mathrm{U}(3)_L \otimes \mathrm{U}(3)_R$ but preserves $\mathrm{SU}(3)_L \otimes \mathrm{SU}(3)_R \otimes \mathrm{U}(1)_V$. The parameter a has dimensions of mass squared and, with the factor $1/N_c$ pulled out, is booked to be of $O(1)$ in the large N_c counting rules. Its value is not fixed by symmetry requirements alone; it depends crucially on the dynamics of instantons. In the presence of the term (2.12), the η_1-field becomes massive even in the chiral limit ($m_{\eta_1^2} = \frac{3}{N_c} a + O(\mathcal{M})$).

Performing an appropriate chiral transformation, the quark mass matrix can be restricted to the form

$$\mathcal{M} = e^{i\theta/3} \, \mathrm{diag}(m_u, m_d, m_s) \equiv e^{i\theta/3} M \, , \tag{2.13}$$

with θ the full θ-vacuum angle in (1.4). In the absence of the term (2.12), the phase in (2.13) could be reabsorbed by the $U(1)_A$ transformation

$$\tilde{U} \;\rightarrow\; e^{i\theta/6}\,\tilde{U}\,e^{i\theta/6}\,. \tag{2.14}$$

In the presence of the $U(1)_A$ anomaly, and hence the term (2.12), this transformation generates new physical interactions. With (2.13) inserted in (2.8), and (2.14) applied to the $\mathcal{L}_{U(1)_A}$ Lagrangian in (2.12), the new form of the effective bosonic Lagrangian in the presence of the QCD θ-vacuum term is then

$$\mathcal{L}_{eff} \;=\; \frac{f_\pi^2}{4}\Big\{\mathrm{tr}(D_\mu\tilde{U}D^\mu\tilde{U}^\dagger \,+\, \tilde{\chi}^\dagger\tilde{U} \,+\, \tilde{U}^\dagger\tilde{\chi}) \,-\, \frac{a}{N_c}\{\frac{i}{2}[\log(\det\tilde{U}) - \log(\det\tilde{U}^\dagger)] - \theta\}^2\Big\}\,, \tag{2.15}$$

where now the matrix $\tilde{\chi}$ is real, positive and diagonal

$$\tilde{\chi} \;=\; \tilde{\chi}^\dagger \;=\; \mathrm{diag}(\chi_u^2, \chi_d^2, \chi_s^2)\,. \tag{2.16}$$

If the term proportional to a/N_c were absent, we could take without loss of generality $<\tilde{U}>= 1$ and the diagonal entries χ_i^2 would correspond to the Goldstone boson masses:

$$\begin{aligned} \chi_u^2 &= m_{\pi^+}^2 + m_{K^+}^2 - m_{K^0}^2\,, \\ \chi_d^2 &= m_{\pi^+}^2 + m_{K^0}^2 - m_{K^+}^2\,, \\ \chi_s^2 &= m_{K^+}^2 + m_{K^0}^2 - m_{\pi^+}^2\,. \end{aligned} \tag{2.17}$$

In this case, the constant B_0 introduced in eq. (2.10) directly relates the pseudoscalar masses to the current quark masses of the QCD Lagrangian:

$$B_0 \;=\; \frac{m_{\pi^+}^2}{m_u + m_d} \;=\; \frac{m_{K^+}^2}{m_u + m_s} \;=\; \frac{m_{K^0}^2}{m_d + m_s} \;=\; \frac{3m_{\eta_8}^2}{m_u + m_d + 4m_s}\,. \tag{2.18}$$

In the presence of the third term in eq. (2.15), $<\tilde{U}>$ cannot be set equal to the unit matrix , and therefore it is convenient, before applying this Lagrangian to calculate physical processes, to minimize the potential energy associated to $\mathcal{L}_{eff}$ in (2.15):

$$V(\tilde{U}) \;=\; -\frac{f_\pi^2}{4}\Big\{\mathrm{tr}(\tilde{\chi}^\dagger\tilde{U} + \tilde{U}^\dagger\tilde{\chi}) \,-\, \frac{a}{N_c}\{\frac{i}{2}[\log\Big(\frac{\det\tilde{U}}{\det\tilde{U}^\dagger}\Big)] - \theta\}^2\Big\}\,, \tag{2.19}$$

so as to fix $<\tilde{U}>$. With $\tilde{\chi}$ diagonal, $<\tilde{U}>$ can be restricted to be diagonal as well and of the form

$$<\tilde{U}> = \mathrm{diag}(e^{-i\varphi_u}, e^{-i\varphi_d}, e^{-i\varphi_s})\,. \tag{2.20}$$

The minimization conditions $\partial V/\partial\varphi_i = 0$ restrict the φ_i's to satisfy the Dashen [26] Nuyts [27] equations:

$$\chi_i^2 \sin\varphi_i = \frac{a}{N_c}(\theta - \sum_j \varphi_j), \qquad (i = u, d, s). \tag{2.21}$$

The φ_i's appearing in $\mathcal{L}_{eff}$ in eq. (2.15) can be reabsorbed in Hermitian matrices χ and H defined by

$$\begin{aligned} <\tilde{U}^\dagger> \tilde{\chi} &\equiv \chi + iH\,, \\ \tilde{\chi}^\dagger <\tilde{U}> &\equiv \chi - iH\,. \end{aligned} \tag{2.22}$$

In fact, eqs. (2.21) fix H to be proportional to the unit matrix,

$$H = \frac{a}{N_c}(\theta - \sum_j \varphi_j)\, I \equiv \frac{a}{N_c}\bar{\theta}\, I\,. \tag{2.23}$$

The effective bosonic Lagrangian as a functional of $U(\phi)$ (eq. (2.3)), with $< U >= 1$, is then ($\bar{\theta} \equiv \theta - \sum_j \varphi_j$)

$$\begin{aligned} \mathcal{L}_{eff} = \frac{f_\pi^2}{4}\Big\{ &\mathrm{tr}(D_\mu U D^\mu U^\dagger + \chi(U + U^\dagger)) - \frac{a}{N_c}\{\bar{\theta}^2 - \frac{1}{4}[\log(\frac{\det U}{\det U^\dagger})]^2\} \\ &- i\frac{a}{N_c}\bar{\theta}\{\mathrm{tr}(U - U^\dagger) - \log(\frac{\det U}{\det U^\dagger})\}\Big\}\,. \end{aligned} \tag{2.24}$$

We are now in the position to discuss two salient physical features of this Lagrangian.

i) In the chiral limit $\chi \to 0$, the singlet η_1-particle acquires a mass from the third term induced by the $\mathrm{U}(1)_A$-anomaly,

$$-\frac{1}{2}\frac{3a}{N_c}\eta_1(x)\eta_1(x)\,. \tag{2.25}$$

Furthermore, as stressed in ref. [28], the η_1-kinetic term in $\mathrm{tr}(D_\mu U D^\mu U^\dagger)$ decouples from the $\vec{\phi}$'s and the η_1 particle becomes stable in the chiral limit (but for strong CP-violating effects, which allow $\eta_1 \to \pi\pi$ and which shall be discussed next).

ii) The last term in $\mathcal{L}_{eff}$ above generates strong CP-violating transitions between pseudoscalar particles. In particular it induces the phase-space allowed decays $\eta_8 \to \pi^+\pi^-, \pi^0\pi^0$ and $\eta_1 \to \pi^+\pi^-, \pi^0\pi^0$. The transition amplitudes for $\eta \to \pi\pi$ can be readily obtained from this effective Lagrangian. (Notice that in the effective Lagrangian formulation of eq. (2.24) tadpole-like diagrams have been eliminated via the correct vacuum alignment.) The result is

$$T(\eta \to \pi^+\pi^-) = (\cos\theta_P - \sqrt{2}\sin\theta_P)\,\frac{a}{N_c}\,\frac{\bar{\theta}}{\sqrt{3}f_\pi}\,, \tag{2.26}$$

where we have taken into account the $\eta_1 - \eta_8$ mixing

$$\begin{aligned} \eta &= \eta_8 \cos\theta_P - \eta_1 \sin\theta_P \,, \\ \eta' &= \eta_8 \sin\theta_P + \eta_1 \cos\theta_P \,. \end{aligned} \tag{2.27}$$

The result in eq. (2.26) is in agreement with the earlier calculations in CDVW [15] and in ref. [29].

3. BARYONS AND CHIRAL DYNAMICS

The formal procedure to introduce baryons in effective Lagrangians was first discussed many years ago [30] (for an updated pedagogical introduction see the book of Howard Georgi [31]). In practice, this approach has been implemented in several papers where various pieces of low-energy phenomenology have been discussed [32]. Here we shall adopt the standard non-linear representation of baryons and recall only the basics.

The wanted ingredient for a non-linear representation of the chiral group is the compensating $\mathrm{U}(3)_V$ transformation $h(\phi, g)$ which appears under the action of the chiral $\mathrm{U}(3)_L \otimes \mathrm{U}(3)_R$ group on the left $\tilde{\xi}_L(\phi)$ and right $\tilde{\xi}_R(\phi)$ coset representatives ($g \equiv g_L \otimes g_R$):

$$\begin{aligned} \tilde{\xi}_L(\phi) &\to g_L \tilde{\xi}_L(\phi) h^\dagger(\phi, g) \,, \\ \tilde{\xi}_R(\phi) &\to g_R \tilde{\xi}_R(\phi) h^\dagger(\phi, g) \,. \end{aligned} \tag{3.1}$$

In terms of $\tilde{\xi}_L(\phi)$ and $\tilde{\xi}_R(\phi)$, the unitary matrix $\tilde{U}(\phi)$ introduced in the previous section (see eqs. (2.2) and (2.3)) is defined by the product

$$\tilde{U}(\phi) = \tilde{\xi}_R(\phi) \tilde{\xi}_L^\dagger(\phi) \,. \tag{3.2}$$

The octet of baryon fields is then collected in a 3×3 matrix

$$B(x) = \begin{pmatrix} \frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda^0}{\sqrt{6}} & \Sigma^+ & p \\ \Sigma^- & -\frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda^0}{\sqrt{6}} & n \\ \Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}}\Lambda^0 \end{pmatrix} , \tag{3.3}$$

which under $\mathrm{U}(3)_L \otimes \mathrm{U}(3)_R$ transforms non-linearly

$$B \to h(\phi, g)\, B\, h^\dagger(\phi, g) \,. \tag{3.4}$$

We look for the most general $\mathrm{U}(3)_L \otimes \mathrm{U}(3)_R$ invariant effective Lagrangian one can write in terms of the matrices $B(x)$, $\bar{B}(x) \equiv B(x)^\dagger \gamma_0$, $\tilde{\xi}_L(\phi)$ and $\tilde{\xi}_R(\phi)$; but (fortunately)

we wish to keep only the dominant terms which contribute to physical processes as leading powers in momenta and quark masses. The baryon-meson effective Lagrangian has a kinetic term

$$\mathcal{L}_{kin}^{(B)} = \mathrm{tr}(\bar{B} i\gamma^\mu D_\mu B) - M_B \mathrm{tr}(\bar{B}B), \tag{3.5}$$

where M_B is a common mass term to all baryons, and D_μ denotes here the covariant derivative

$$D_\mu B = \partial_\mu B + [\Gamma_\mu, B], \tag{3.6}$$

with

$$\Gamma_\mu \equiv \frac{1}{2}\{\tilde{\xi}_R^\dagger(\partial_\mu - ir_\mu)\tilde{\xi}_R + \tilde{\xi}_L^\dagger(\partial_\mu - il_\mu)\tilde{\xi}_L\}, \tag{3.7}$$

and r_μ , l_μ the external fields introduced in eq. (2.7). Notice that from the point of view of chiral power counting $\bar{B}$ and B are booked as $O(1)$; $D_\mu B$ and $M_B B$ as $O(1)$, but $i\gamma^\mu D_\mu B - M_B B$ is $O(p)$. Therefore, to the kinetic term in (3.5) we have to add possible interaction terms of $O(p)$ as well. These are

$$\mathcal{L}_{int}^{(B)} = -\frac{D}{2}\mathrm{tr}(\bar{B}\gamma^\mu\gamma_5\{\tilde{\xi}_\mu(\phi), B\}) - \frac{F}{2}\mathrm{tr}(\bar{B}\gamma^\mu\gamma_5[\tilde{\xi}_\mu(\phi), B]) + S\,\mathrm{tr}(\tilde{\xi}_\mu(\phi))\,\mathrm{tr}(\bar{B}\gamma^\mu\gamma_5 B), \tag{3.8}$$

with

$$\tilde{\xi}_\mu(\phi) \equiv i\,\{\tilde{\xi}_R^\dagger(\partial_\mu - ir_\mu)\tilde{\xi}_R - \tilde{\xi}_L^\dagger(\partial_\mu - il_\mu)\tilde{\xi}_L\}. \tag{3.9}$$

Under $\mathrm{U}(3)_L \otimes \mathrm{U}(3)_R$ gauge transformations,

$$\tilde{\xi}_\mu(\phi) \rightarrow h(\phi, g)\,\tilde{\xi}_\mu(\phi)\,h^\dagger(\phi, g). \tag{3.10}$$

The first two terms in (3.8) are the usual F and D couplings, which govern semileptonic hyperon decays. The third S coupling is specific to the axial flavour-singlet baryonic current.

Of special interest for our purposes are the possible terms generated by the explicit chiral symmetry breaking induced by quark mass terms in the underlying Lagrangian in eq. (2.6). The possible lowest $O(\mathcal{M})$ interactions induced in the effective meson-baryon Lagrangian are

$$\mathcal{L}_{\mathcal{M}}^{(B)} = -b_0\,\mathrm{tr}(\tilde{\chi}_+)\,\mathrm{tr}(\bar{B}B) - b_1\,\mathrm{tr}(\bar{B}\tilde{\chi}_+ B) - b_2\,\mathrm{tr}(\bar{B}B\tilde{\chi}_+), \tag{3.11}$$

where b_0, b_1 and b_2 are coupling constants with dimensions of an inverse mass, and $\tilde{\chi}_\pm$ is a shorthand notation for

$$\tilde{\chi}_\pm \equiv \tilde{\xi}_R^\dagger\tilde{\chi}\tilde{\xi}_L \pm \tilde{\xi}_L^\dagger\tilde{\chi}^\dagger\tilde{\xi}_R. \tag{3.12}$$

Under $U(3)_L \otimes U(3)_R$ gauge transformations,

$$\tilde{\chi}_\pm \;\rightarrow\; h(\phi,g)\,\tilde{\chi}_\pm\, h^\dagger(\phi,g)\,. \tag{3.13}$$

Terms with γ_5 and $\tilde{\chi}_-$ like e.g. $\mathrm{tr}(\bar{B}\tilde{\chi}_-\gamma_5 B)$ are $O(\mathcal{M}p)$, i.e. higher order in the chiral expansion, and will therefore not be kept at this stage. We shall however come back to possible physical effects from these terms at the end of this section.

With $\mathcal{M}$ restricted to the form in eq. (2.13), and in the absence of the $U(1)_A$ anomaly, the phase in (2.13) could be reabsorbed by the same $U(1)_A$ transformation as in (2.14) i.e.,

$$\tilde{\xi}_L(\phi) \;\rightarrow\; e^{-i\theta/6}\tilde{\xi}_L(\phi)h^\dagger(\phi,\theta) \qquad\qquad \tilde{\xi}_R(\phi) \;\rightarrow\; e^{i\theta/6}\tilde{\xi}_R(\phi)h^\dagger(\phi,\theta)\,. \tag{3.14}$$

In the presence of the $U(1)_A$ anomaly, and hence the term (2.12) in the mesonic Lagrangian, this transformation generates new physical interactions between mesons, as we have seen in the previous section, and new interactions as well between mesons and baryons, as we are going to see next. Again, before we proceed to analyze physical implications, it is convenient to rewrite the effective Lagrangian in a form compatible with the correct vacuum alignment.

From eqs. (2.22) and (3.2) it follows that

$$<\tilde{\xi}_R^\dagger>\tilde{\chi}<\tilde{\xi}_L>=<\tilde{\xi}_L><\tilde{\xi}_R^\dagger>\tilde{\chi} \;=\; \chi + iH\,, \tag{3.15}$$

where we have used the fact that $\tilde{\chi}$ is diagonal and , without loss of generality, $<\tilde{\xi}_L>$ and $<\tilde{\xi}_R>$ can be restricted to be diagonal as well. It is then convenient to introduce field matrices ξ_L and ξ_R, so that

$$\tilde{\xi}_L \;=<\tilde{\xi}_L>\xi_L \qquad \text{and} \qquad \tilde{\xi}_R \;=<\tilde{\xi}_R>\xi_R\,, \tag{3.16}$$

with $<\xi_L>=<\xi_R>=1$ and $U=\xi_R\xi_L^\dagger$. Furthermore, it is always possible to choose a coset representative such that

$$\xi_R \;=\; \xi_L^\dagger \;=\; \xi \qquad ; \qquad U \;=\; \xi^2\,. \tag{3.17}$$

We then have

$$\tilde{\chi}_\pm \;=\; \chi_\pm \;+\; i\frac{a}{N_c}\bar{\theta}(U^\dagger \mp U)\,, \tag{3.18}$$

where

$$\chi_\pm \;\equiv\; \xi^\dagger\chi\xi^\dagger \;\pm\; \xi\chi\xi\,. \tag{3.19}$$

Inserting (3.18) in $\mathcal{L}_{\mathcal{M}}^{(B)}$ as given in (3.11), leads to CP non-conserving meson-baryon interaction terms modulated by the coupling $\frac{a}{N_c}\bar{\theta}$:

$$\begin{aligned}\mathcal{L}_{\mathcal{M}}^{(B)} &= -b_0\,\mathrm{tr}(\chi_+)\,\mathrm{tr}(\bar{B}B) - b_1\,\mathrm{tr}(\bar{B}\chi_+B) - b_2\,\mathrm{tr}(\bar{B}B\chi_+) \\ &\quad - i\frac{a}{N_c}\bar{\theta}\left\{b_0\,\mathrm{tr}(U^\dagger - U)\,\mathrm{tr}(\bar{B}B) + b_1\,\mathrm{tr}(\bar{B}(U^\dagger - U)B) + b_2\,\mathrm{tr}(\bar{B}B(U^\dagger - U))\right\}.\end{aligned} \tag{3.20}$$

The first three terms in $\mathcal{L}_{\mathcal{M}}^{(B)}$ give contributions to baryon masses. From the experimentally known baryon mass splittings it is then possible to obtain the couplings b_1 and b_2, with the result (for $m_u = m_d = m$)

$$b_1 = \frac{M_\Xi - M_\Sigma}{4(m_K^2 - m_\pi^2)} \approx 0.14\,\mathrm{GeV}^{-1} \quad , \quad b_2 = \frac{M_N - M_\Sigma}{4(m_K^2 - m_\pi^2)} \approx -0.28\,\mathrm{GeV}^{-1}. \tag{3.21}$$

The term with b_0 gives an overall contribution to the baryon mass M_B, and therefore cannot be extracted from baryon mass splittings.

The interesting terms for our purposes are the ones proportional to $\frac{a}{N_c}\bar{\theta}$. The interactions with the lowest possible number of fields, induced by these terms, are of the type

$$\begin{aligned}\mathcal{L}_{\bar{\theta}}^{(B)} = -i\frac{a}{N_c}\bar{\theta}\,\frac{2i\sqrt{2}}{f_\pi}\Big\{&\frac{n_f}{\sqrt{3}}b_0\,\eta_1\,\mathrm{tr}(\bar{B}B) + b_1\,\mathrm{tr}(\bar{B}\Phi B) + b_2\,\mathrm{tr}(\bar{B}B\Phi) \\ &+ O(\bar{B}B\phi\phi\phi/f_\pi^2)\Big\}.\end{aligned} \tag{3.22}$$

If we further restrict $\bar{B}$ to be the $\bar{n}$-field, and ϕ to be a charged field, we find as possible interaction terms

$$\mathcal{L}_{\bar{\theta}}^{(n)} \doteq \frac{2\sqrt{2}}{f_\pi}\,\frac{a}{N_c}\bar{\theta}\left\{b_1\,\bar{n}p\pi^- + b_2\,\bar{n}\Sigma^-K^+\right\} + h.c.. \tag{3.23}$$

Since we are also interested in a calculation of the electric dipole moment of the Λ^0 expected from strong CP-violation, we shall also give the couplings when $\bar{B}$ is restricted to be the $\bar{\Lambda}^0$-field, and ϕ is restricted to charged fields:

$$\begin{aligned}\mathcal{L}_{\bar{\theta}}^{(\Lambda)} \doteq \frac{2}{f_\pi\sqrt{3}}\,\frac{a}{N_c}\bar{\theta}\,\Big\{&b_1\,\bar{\Lambda}^0[-2pK^- + \Xi^-K^+ + \Sigma^+\pi^- + \Sigma^-\pi^+] \\ &+ b_2\,\bar{\Lambda}^0[pK^- - 2\Xi^-K^+ + \Sigma^+\pi^- + \Sigma^-\pi^+]\Big\} + h.c..\end{aligned} \tag{3.24}$$

We finally wish to discuss possible strong CP-violation effects which may originate from baryon couplings with a γ_5 and linear in $\tilde{\chi}_-$; i.e., terms of $O(\mathcal{M}p)$ which we have neglected so far. The possible terms of this type are

$$\mathcal{L}_{5\mathcal{M}}^{(B)} = -c_0\,\mathrm{tr}(\tilde{\chi}_-)\,\mathrm{tr}(\bar{B}\gamma_5B) - c_1\,\mathrm{tr}(\bar{B}\gamma_5\tilde{\chi}_-B) - c_2\,\mathrm{tr}(\bar{B}\gamma_5B\tilde{\chi}_-), \tag{3.25}$$

Upon redefinition of $\tilde{\chi}_-$ in (3.18) they generate new strong CP-violating interactions

$$\begin{aligned}\mathcal{L}_{5\mathcal{M}}^{(B)} \rightarrow & -i\frac{a}{N_c}\bar{\theta}\left\{c_0\,\mathrm{tr}(U^\dagger+U)\,\mathrm{tr}(\bar{B}\gamma_5 B) + c_1\,\mathrm{tr}(\bar{B}\gamma_5(U^\dagger+U)B)\right.\\ & \left. + c_2\,\mathrm{tr}(\bar{B}\gamma_5 B(U^\dagger+U))\right\} \qquad (3.26)\\ = & -2i\frac{a}{N_c}\bar{\theta}\,(3c_0+c_1+c_2)\,\mathrm{tr}(\bar{B}\gamma_5 B) + i\frac{a}{N_c}\bar{\theta}\,O(\bar{B}\gamma_5\phi\phi B)\,.\end{aligned}$$

Terms $O(\bar{B}\gamma_5\phi\phi B)$ can only contribute to d_n^γ via two chiral loops at least and therefore will be neglected. The term proportional to $i\mathrm{tr}(\bar{B}\gamma_5 B)$, when lumped together with the baryon mass term $M_B tr(\bar{B}B)$ in the kinetic term (3.5), becomes (terms $O(\bar{\theta}^2)$ and higher are always neglected)

$$-M_B\mathrm{tr}(\bar{B}B) - 2i\frac{a}{N_C}\bar{\theta}\,(3c_0+c_1+c_2)\,\mathrm{tr}(\bar{B}\gamma_5 B) \simeq -\mathrm{tr}(\bar{B}'M_B B')\,, \qquad (3.27)$$

where

$$B' \equiv \exp\left\{i\frac{a}{N_c}\frac{\bar{\theta}}{M_B}(3c_0+c_1+c_2)\gamma_5\right\}B\,. \qquad (3.28)$$

This baryon field redefinition leaves the kinetic term $\mathrm{tr}(\bar{B}i\gamma^\mu D_\mu B)$, as well as the D, F and S couplings in (3.8) invariant. It generates new interactions when applied to the $\mathcal{L}_{\mathcal{M}}^{(B)}$ term in (3.20). The new strong CP-violating interactions are however higher order in the chiral expansion, they are $O(\mathcal{M}\bar{\theta})$ at least, and therefore will be neglected as well.

We conclude that the leading effective CP-violating couplings which can contribute to d_n^γ and d_Λ^γ via one chiral loop are only those in $\mathcal{L}_{\bar{\theta}}^{(n)}$ and $\mathcal{L}_{\bar{\theta}}^{(\Lambda)}$ in (3.23) and (3.24).

4. THE CHIRAL LOOP CALCULATION OF d_n^γ

We have now all the ingredients to do this calculation. The possible Feynman diagrams which can generate a nEDM at the one-loop level are shown in fig. 1 The continuous line represents a baryon, the dashed line a meson, the wavy line the photon. The vertex with a dot is the CP-violating interaction induced by one of the couplings in eq. (3.23).

The normal CP-conserving $\bar{B}B\phi$ vertex interactions are those generated by the interaction Lagrangian $\mathcal{L}_{int}^{(B)}$ in (3.8), with ξ_μ in eq. (3.9) restricted to the term

$$\xi_\mu \doteq \frac{\sqrt{2}}{f_\pi}(\partial_\mu\Phi - ie\mathcal{A}_\mu[Q,\Phi])\,, \qquad (4.1)$$

where Φ is the 3×3 matrix in (2.4), $\mathcal{A}_\mu$ the electromagnetic field and $Q = \mathrm{diag}(\frac{2}{3}, \frac{-1}{3}, \frac{-1}{3})$ the electric charge matrix. The fact that the propagating pseudoscalar has to be charged

restricts the CP-conserving $\bar{B}B\phi$ vertex interactions to those of F and D type only. There is no contribution from the S-coupling at the one-loop level.

We also require the $\phi\phi\gamma$ and $\bar{B}B\gamma$ vertices which follow from the first term in (2.24),

$$\mathcal{L}_{em} \doteq -ie\mathcal{A}_\mu(\phi^+\partial^\mu\phi^- - \phi^-\partial^\mu\phi^+) + e^2\mathcal{A}_\mu\mathcal{A}^\mu\phi^+\phi^-, \tag{4.2}$$

with $\phi = \pi, K$; and from the kinetic term $\mathcal{L}^{(B)}_{kin}$ in (3.5), i.e.

$$\mathcal{L}^{(B)}_{em} = e\mathcal{A}_\mu \,\mathrm{tr}\Big(\bar{B}\gamma^\mu[\frac{1}{2}(\xi^\dagger Q\xi + \xi Q\xi^\dagger), B]\Big). \tag{4.3}$$

The calculation is then rather straightforward. Only diagrams (a) and (b) give chiral logarithms, plus constant terms which we keep and higher order terms in the chiral expansion which are neglected. Diagrams (c) and (d) are suppressed by an additional baryon propagator; they don't give chiral logarithms, but produce constant terms which happen to cancel the constant terms from (a) and (b). Diagrams (e) and (f) cancel each other. The final result from the one-loop calculation is then

$$d_n^\gamma = \frac{a}{N_c}\bar{\theta}\,\frac{e}{16\pi^2 f_\pi^2}\left\{\frac{M_\Xi - M_\Sigma}{m_K^2 - m_\pi^2}(D+F)\log(\frac{M_N^2}{m_\pi^2}) + \frac{M_\Sigma - M_N}{m_K^2 - m_\pi^2}(D-F)\log(\frac{M_\Sigma^2}{m_K^2})\right\}. \tag{4.4}$$

The baryon mass in the chiral logarithm acts as an ultra-violet cut-off and should be considered as such. It is simply telling us that a complete calculation of d_n^γ to $O(p^2)$ must necessarily bring new local counterterms of the same chiral order, with coupling constants not fixed by symmetry arguments alone and probably rather hard to determine from phenomenology. We examine this question in the next section.

5. LOCAL COUNTERTERMS

We are interested in all the possible local couplings which could generate a tree-level contribution to d_n^γ (and d_Λ^γ) to order $O(\mathcal{M})$. Some must certainly be there to renormalize the UV-behaviour of the contribution from the chiral loop reported in the previous section. There may be others as well with scale-independent coupling constants.

A possible set of terms are those with factors $\bar{B}\sigma^{\mu\nu}\gamma_5 B$, $\tilde{\chi}_-$ and $F_{\mu\nu}$ as necessary ingredients. In order to build chiral invariants we have to use, rather than $F_{\mu\nu}$, the quantities

$$f_\pm^{\mu\nu} \equiv \tilde{\xi}_L^\dagger F_L^{\mu\nu}\tilde{\xi}_L \pm \tilde{\xi}_R^\dagger F_R^{\mu\nu}\tilde{\xi}_R, \tag{5.1}$$

where $F^{\mu\nu}_{L,R}$ are the left (right) field strength tensors associated with the l_μ and r_μ external gauge field 3×3 matrices. Under $\mathrm{U}(3)_L \otimes \mathrm{U}(3)_R$ gauge transformations,

$$f^{\mu\nu}_{\pm} \;\rightarrow\; h(\phi,g)\, f^{\mu\nu}_{\pm}\, h^{\dagger}(\phi,g)\,. \tag{5.2}$$

When restricted to the electromagnetic field tensor, $f^{\mu\nu}_{\pm}$ reduces to

$$\begin{aligned} f^{\mu\nu}_{\pm} &= e\,(\tilde{\xi}^{\dagger}_L Q \tilde{\xi}_L \,\pm\, \tilde{\xi}^{\dagger}_R Q \tilde{\xi}_R)\, F^{\mu\nu} \\ &= e\,(\xi Q \xi^{\dagger} \,\pm\, \xi^{\dagger} Q \xi)\, F^{\mu\nu}\,, \end{aligned} \tag{5.3}$$

where we have used the fact that Q is diagonal .

There are 10 possible invariants of this type:

$$\begin{aligned} \mathcal{L}^{(B)}_C \;=\; \frac{1}{16\pi^2 f_\pi^2}\Big\{ & \delta_1\, \mathrm{tr}(\bar{B}\sigma_{\mu\nu}\gamma_5\{\tilde{\chi}_-, f^{\mu\nu}_+\}B) \,+\, \delta_2\, \mathrm{tr}(\bar{B}\sigma_{\mu\nu}\gamma_5 f^{\mu\nu}_+ B\tilde{\chi}_-) \\ & + \delta_3\, \mathrm{tr}(\bar{B}\sigma_{\mu\nu}\gamma_5 \tilde{\chi}_- B f^{\mu\nu}_+) \,+\, \delta_4\, \mathrm{tr}(\bar{B}\sigma_{\mu\nu}\gamma_5 B\{\tilde{\chi}_-, f^{\mu\nu}_+\}) \\ & + \delta_5\, \mathrm{tr}(\tilde{\chi}_-)\, \mathrm{tr}(\bar{B}\sigma_{\mu\nu}\gamma_5 B f^{\mu\nu}_+) \,+\, \delta_6\, \mathrm{tr}(\tilde{\chi}_-)\, \mathrm{tr}(\bar{B}\sigma_{\mu\nu}\gamma_5 f^{\mu\nu}_+ B) \\ & + \delta_7\, \mathrm{tr}(f^{\mu\nu}_+)\, \mathrm{tr}(\bar{B}\sigma_{\mu\nu}\gamma_5 B\tilde{\chi}_-) \,+\, \delta_8\, \mathrm{tr}(f^{\mu\nu}_+)\, \mathrm{tr}(\bar{B}\sigma_{\mu\nu}\gamma_5 \tilde{\chi}_- B) \\ & + \delta_9\, \mathrm{tr}(f^{\mu\nu}_+ \tilde{\chi}_-)\, \mathrm{tr}(\bar{B}\sigma_{\mu\nu}\gamma_5 B) \,+\, \delta_{10}\, \mathrm{tr}(f^{\mu\nu}_+)\, \mathrm{tr}(\tilde{\chi}_-)\, \mathrm{tr}(\bar{B}\sigma_{\mu\nu}\gamma_5 B)\Big\}\,. \end{aligned} \tag{5.4}$$

Since Q is traceless, so is the restricted form of $f^{\mu\nu}_+$ in (5.3), and the couplings δ_7, δ_8 and δ_{10} are inoperative. We are interested only in the CP-violating terms generated by the replacement of $\tilde{\chi}_-$ by the r.h.s. in eq. (3.18), when ξ is furthermore restricted to the constant term I in the Φ expansion. In that case δ_9 is also inoperative, and the rest of the couplings can be collected in two sets :

$$\begin{aligned} \mathcal{L}^{(B)}_C \;\simeq\; i\frac{a}{N_c}\bar{\theta}F^{\mu\nu}\frac{4e}{16\pi^2 f_\pi^2}\Big\{ & (2\delta_1 + \delta_2 + 3\delta_6)\, \mathrm{tr}(\bar{B}\sigma_{\mu\nu}\gamma_5 QB) \\ & + (\delta_3 + 2\delta_4 + 3\delta_5)\, \mathrm{tr}(\bar{B}\sigma_{\mu\nu}\gamma_5 BQ)\Big\}\,. \end{aligned} \tag{5.5}$$

The tree-level contribution to the neutron (lambda) electric dipole moment can now be read out from this effective Lagrangian, with the result

$$d^{\gamma}_n \;=\; 2d^{\gamma}_{\Lambda} \;=\; -\frac{a}{N_c}\bar{\theta}\frac{8}{3}\frac{e}{16\pi^2 f_\pi^2}(2\delta_1 + \delta_2 + \delta_3 + 2\delta_4 + 3\delta_5 + 3\delta_6)\,. \tag{5.6}$$

Possible terms with $\bar{B}\gamma_5(\gamma_\mu D_\nu - \gamma_\nu D_\mu)B$ (instead of $\bar{B}\sigma_{\mu\nu}\gamma_5 B$), $\tilde{\chi}_-$ and $f^{\mu\nu}_{\pm}$ are not independent. They are related to the ones in (5.4) because $(\gamma_\mu D_\nu - \gamma_\nu D_\mu)B = \frac{-1}{2}M_B\sigma_{\mu\nu}B +$ terms $O(p)$, which follows from the baryon-field equations of motion.

The other possible set of counterterms are those generated by the B-field redefinition in (3.28) when applied to magnetic-moment-like couplings of the type

$$\mathcal{L}^{(B)}_{\mu_B} = \frac{M_B}{16\pi^2 f_\pi^2}\Big\{\kappa_1 \,\mathrm{tr}(\bar{B}\sigma_{\mu\nu} f_+^{\mu\nu} B) + \kappa_2 \,\mathrm{tr}(\bar{B}\sigma_{\mu\nu} B f_+^{\mu\nu})\Big\}, \tag{5.7}$$

with κ_1 and κ_2 dimensionless couplings, finite in the chiral limit, which to lowest order in Chiral Perturbation Theory (ChPT) are fixed by the baryon magnetic moments. For the neutron and the lambda we have in particular

$$\mu_n = 2\mu_\Lambda = -\frac{8}{3}(\kappa_1+\kappa_2)\frac{M_B^2}{16\pi^2 f_\pi^2}\frac{e}{2M_B}. \tag{5.8}$$

The baryon field redefinition (3.28), when applied to (5.7) above, generates direct tree-level electric dipole moment couplings

$$\mathcal{L}^{(B)}_{\mu_B} \to -i\frac{a}{N_C}\bar{\theta}\, 4(3c_0+c_1+c_2)\frac{e}{16\pi^2 f_\pi^2}F^{\mu\nu}\Big\{\kappa_1 \,\mathrm{tr}(\bar{B}\sigma_{\mu\nu}\gamma_5 QB) + \kappa_2 \,\mathrm{tr}(\bar{B}\sigma_{\mu\nu}\gamma_5 BQ)\Big\}, \tag{5.9}$$

from which we can read out the contributions

$$d_n^\gamma = 2d_\Lambda^\gamma = \frac{a}{N_c}\bar{\theta}\frac{8}{3}\frac{e}{16\pi^2 f_\pi^2}(\kappa_1+\kappa_2)(3c_0+c_1+c_2). \tag{5.10}$$

The total tree-level contribution to d_n^γ to lowest order in the ChPT expansion is then

$$d_n^\gamma\big|_{tree} = 2d_\Lambda^\gamma\big|_{tree} = -\frac{a}{N_c}\bar{\theta}\frac{8}{3}\frac{e}{16\pi^2 f_\pi^2}\delta(\nu^2), \tag{5.11}$$

with $\delta(\nu^2)$ the combination of couplings

$$\delta \equiv 2\delta_1+\delta_2+\delta_3+2\delta_4+3\delta_5+3\delta_6-(\kappa_1+\kappa_2)(3c_0+c_1+c_2), \tag{5.12}$$

at a renormalization scale ν^2. This contribution, when added to the result in eq.(4.4) from the chiral loop, gives our final scale-independent result

$$d_n^\gamma = 2d_\Lambda^\gamma = \frac{a}{N_c}\bar{\theta}\frac{e}{16\pi^2 f_\pi^2}4\Big\{b_1(D+F)\log(\frac{\nu^2}{m_\pi^2}) - b_2(D-F)\log(\frac{\nu^2}{m_K^2}) - \frac{2}{3}\delta(\nu^2)\Big\}. \tag{5.13}$$

Of the coupling constants appearing in $\delta(\nu^2)$ in (5.12), only $\kappa_1+\kappa_2$ is known. How could the δ_i's and the c_i's be determined phenomenologically? This is the final question which we wish to discuss here.

The δ_i's in the interaction Lagrangian $\mathcal{L}_C^{(B)}$ in (5.4) also contribute to CP-conserving transitions via the term χ_- in the r.h.s. of $\tilde{\chi}_-$ in eq. (3.18). By construction χ_- requires at least one Goldstone field to be present. Therefore the δ_i's contribute to physical processes like pion-photoproduction ($\gamma N \to \pi N$) at tree level. In fact they renormalize the UV-behaviour of chiral loop corrections to the lowest order contribution; loops like the one in the Feynman graph of fig. 2 . The loop structure in this diagram is in fact the same as the one in the diagrams of fig. 1. (Notice that the $N\pi N\pi$ vertex in fig. 2 comes from the CP-conserving part of the same interaction Lagrangian $\mathcal{L}_{\mathcal{M}}^{(B)}$ which contributes to the vertex with a dot in the chiral loop evaluation of d_n^γ .)

The c_i's in the interaction Lagrangian $\mathcal{L}_{5\mathcal{M}}^{(B)}$ in (3.25) also contribute to CP-conserving transitions via the term χ_- in the r.h.s. of $\tilde{\chi}_-$ in eq. (3.18). Again χ_- requires an odd number of Goldstones to be present. Therefore the c_i's contribute to effective $\bar{B}B\pi$ vertices and could in principle be extracted from low energy πN phenomenology by a comparison of data, say $\pi N \to \pi N$, with ChPT predictions at the one-loop level and with the inclusion of tree-level terms like those in (3.25).

6. CONCLUSIONS AND NUMERICAL RESULTS

The main conclusion from the analysis reported here is that the coefficient of the leading chiral logarithm ($\log m_\pi$) which contributes to the nEDM, as a result of strong CP-violation in the Standard Model, is unambiguously fixed by low energy phenomenology. The expression we find in eq. (4.4) has all the expected factors present: i) it vanishes in the large N_c limit; ii) it vanishes in the chiral limit where $\bar{\theta} \to 0$ [see eqs. (2.21) and (2.23)] ; iii) it is proportional to the couplings b_1 and b_2 [see eq. (3.21)] responsible for the baryon mass splitting; iv) it is proportional to the baryon D and F couplings; and v) it has the suppression factor $\frac{1}{16\pi^2 f_\pi^2}$ characteristic of a chiral loop.

Our result in (4.4) confirms the basic claims of the early work by CDVW [15] and Di Vecchia [16]. The result (2.26) for the $\eta \to \pi\pi$ decay amplitude, which follows from the effective bosonic QCD Lagrangian obtained in eq. (2.24), is also in agreement with the earlier calculations in CDVW [15] and in ref. [29].

We have identified the set of local couplings which can also give contributions to the nEDM at the same $O(\mathcal{M})$ in ChPT. They are listed in eqs. (5.4) and (5.7). Their overall contribution to the nEDM is proportional to the combination $\delta(\nu^2)$ given in eq. (5.12). Only $\kappa_1 + \kappa_2$ is explicitly known at present, from the neutron (and lambda)

magnetic moment measurements. The couplings δ_i and c_i could however be determined from a phenomenological comparison of data on $\gamma N \to \pi N$ and $\pi N \to \pi N$ reactions near threshold, with ChPT predictions at the one-loop level, where these constants contribute as local counterterms.

We have also calculated the electric dipole moment of the Λ, to lowest $O(\mathcal{M})$, with the result

$$d_\Lambda^\gamma = \frac{1}{2} d_n^\gamma . \tag{6.1}$$

In order to make a numerical estimate of d_n^γ, it is convenient to use the empirical fact that

$$\chi_u^2, \chi_d^2 \ll \chi_s^2 , \frac{a}{N_c} \tag{6.2}$$

and use the approximate relation ($m_u \approx m_d = m \ll m_s$)

$$\frac{a}{N_c}\bar{\theta} \approx \frac{\theta}{\sum_i \frac{1}{\chi_i^2} + \frac{N_c}{a}} \approx \frac{1}{2} m_\pi^2 \theta . \tag{6.3}$$

We also need values for the D and F couplings. Jenkins and Manohar [33], in a recent phenomenological analysis of hyperon semileptonic decays within the framework of ChPT, have pointed out that the values obtained at the tree level have rather large logarithmic corrections. They propose a new determination, which incorporates the effect of Δ's as well as N's, with the result [34]

$$D = 0.61 \pm 0.04 \qquad \text{and} \qquad F = 0.40 \pm 0.03 . \tag{6.4}$$

Using these values and the physical nucleon mass in the chiral logarithms, we get from the chiral loop expression in (4.4) the result

$$d_n^\gamma \big|_{Loop} = 3.3 \times 10^{-16}\, \theta \; e\, cm , \tag{6.5}$$

which is remarkably close to Di Vecchia's estimate in ref. [16].

As an estimate of the error coming from the unknown contribution of the constant $\delta(\nu^2)$ in (5.11), we propose to vary the scale in the chiral logarithm between the value of the constituent quark mass, $M_Q \sim 320\, MeV$, and the average mass scale $M_\Delta \sim 1500\, MeV$ of the baryon decuplet. This gives as our final estimate

$$d_n^\gamma = (3.3 \pm 1.8) \times 10^{-16}\, \theta \; e\, cm . \tag{6.6}$$

From a comparison between this result and the experimental upper limit in eq. (1.3), we conclude that

$$|\theta| < 5 \times 10^{-10} . \tag{6.7}$$

This is a much more stringent limit, of course, than the one obtained from $\eta \to \pi^+\pi^-$. The predicted branching ratio here (using $\theta_P = -20°$ for the $\eta_1 - \eta_8$ mixing angle) is

$$\mathrm{Br}(\eta \to \pi^+\pi^-) = 1.8 \times 10^2\, \theta^2 ; \tag{6.8}$$

while the present experimental upper limit is

$$\mathrm{Br}(\eta \to \pi^+\pi^-) < 1.5 \times 10^{-3} , \tag{6.9}$$

from which θ is limited to be $|\theta| < 3 \times 10^{-3}$.

The comparison between our result for the ΛEDM (6.1) and the present experimental upper bound [35], $d_\Lambda^\gamma < 1.5 \times 10^{-16}\, e\, cm$ (95 % C.L.) , only limits θ to $|\theta| < 2$.

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Figure Captions

Fig. 1. Feynman diagrams contributing to the nEDM at the one-loop level. The continuous line represents a baryon, the dashed line a meson, the wavy line the photon. The vertex with a dot is the CP-violating interaction induced by one of the couplings in eq. (3.23).

Fig. 2. Feynman graph contributing to ($\gamma N \to \pi N$) at the one-loop level .

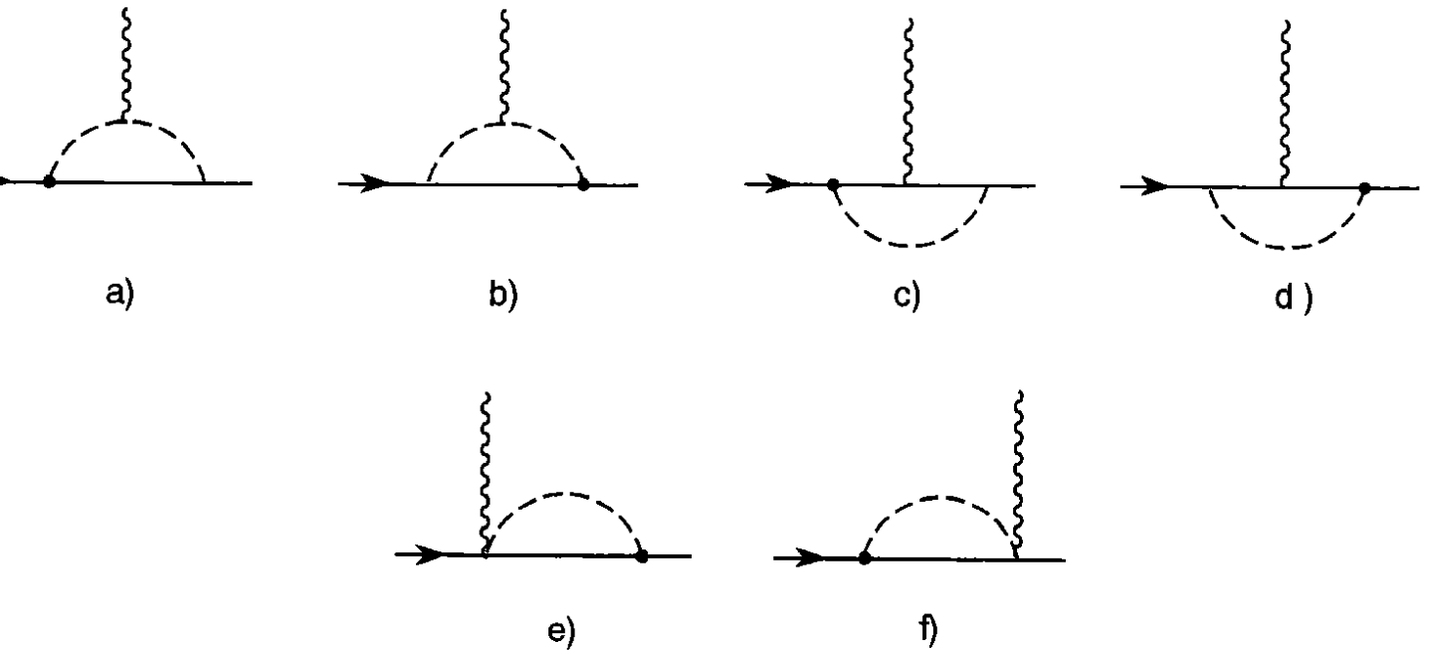

Fig. 1

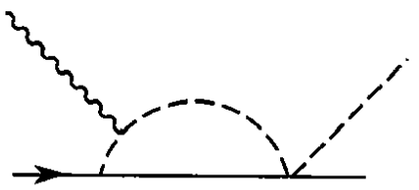

Fig. 2