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THE CHIRAL ANOMALY IN NON-LEPTONIC WEAK INTERACTIONS *

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ABSTRACT

The interplay between the chiral anomaly and the non-leptonic weak Hamiltonian is studied. The structure of the corresponding effective Lagrangian of odd intrinsic parity is established. It is shown that the factorizable contributions (leading in $1/N_C$) to that Lagrangian can be calculated without free parameters. As a first application, the decay $K^+ \to \pi^+\pi^0\gamma$ is investigated.

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1. The chiral anomaly [1] is a fundamental property of chiral quantum field theories such as the standard model. Although its origin as an intrinsically quantum mechanical violation of a classical symmetry is well understood, many aspects of the anomaly remain to be tested experimentally. For the strong, electromagnetic and semileptonic weak interactions, the manifestations of the chiral anomaly at low energies are completely determined by the Wess–Zumino–Witten (WZW) functional [2] in terms of pseudoscalar meson and external gauge fields.

The non-leptonic weak interactions, which are the subject of this letter, require a separate treatment. This can already be seen in the normal parity sector. To lowest order in chiral perturbation theory (CHPT), the strong, electromagnetic and semileptonic weak interactions of pseudoscalar mesons are governed by the effective chiral Lagrangian of $O(p^2)$ (in the notation of Ref. [3])

$$\mathcal{L}_2 = \frac{F^2}{4}\langle D_\mu U D^\mu U^\dagger + \chi U^\dagger + \chi^\dagger U\rangle, \tag{1}$$

where

$$D_\mu U = \partial_\mu U - i r_\mu U + i U l_\mu, \qquad \chi = 2B_0(s + ip), \tag{2}$$

and $\langle A\rangle$ stands for the trace of the matrix A; U is a unitary 3×3 matrix

$$U^\dagger U = \mathbf{1}, \qquad \det U = 1,$$

which transforms as

$$U \to g_R U g_L^\dagger \tag{3}$$

under $SU(3)_L \times SU(3)_R$ and incorporates the eight pseudoscalar Goldstone boson fields. The external 3×3 hermitian matrix fields l_μ, r_μ, s, p contain in particular the relevant gauge fields of the standard model for electromagnetic and semileptonic weak interactions

$$\begin{aligned} r_\mu = v_\mu + a_\mu &= eQA_\mu \\ l_\mu = v_\mu - a_\mu &= eQA_\mu + \frac{e}{\sqrt{2}\sin\theta_W}(W_\mu^+ T_+ + h.c.) \end{aligned} \tag{4}$$

$$Q = \frac{1}{3}\,\mathrm{diag}(2,-1,-1)$$

$$T_+ = \begin{pmatrix} 0 & V_{ud} & V_{us} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

where the V_{ij} are Kobayashi–Maskawa matrix elements. The parameters F and B_0 in the non-linear sigma model Lagrangian (1) are related to the pion decay constant ($F \simeq F_\pi = 93.2 MeV$) and to the quark condensate, respectively [3].

Although the Lagrangian (1) allows one in particular to calculate any mesonic amplitude with an external W to $O(p^2)$, one does not obtain the full non-leptonic mesonic amplitudes at low energies by simply contracting the W field. Instead, one first has to integrate out the W together with the heavy quarks in the fundamental theory to arrive at an effective $\Delta S = 1$ Hamiltonian [4]

$$\mathcal{H}_{eff}^{\Delta S=1} = \frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \sum_i C_i(\mu^2) Q_i + \text{h.c.} \tag{5}$$

The Wilson coefficients $C_i(\mu^2)$ are functions of the heavy masses, Λ_{QCD} and the renormalization scale μ. The Q_i are the standard four-quark operators which can be written as products of colour singlet quark bilinears.

Restricting ourselves to the dominant octet part, the effective Hamiltonian (5) has a unique realization at the mesonic level to lowest order in CHPT first given by Cronin [5]

$$\mathcal{L}_2^{\Delta S=1} = G_8 F^4 \langle \lambda D_\mu U^\dagger D^\mu U \rangle + \text{h.c.} \tag{6}$$

$$\lambda = \frac{1}{2}(\lambda_6 - i\lambda_7).$$

The only coupling constant at $O(p^2)$ can be determined from $K \to 2\pi$ decays to be

$$|G_8| \simeq 9 \times 10^{-6}\ \text{GeV}^{-2} \simeq 5 \times \frac{G_F}{\sqrt{2}} |V_{ud} V_{us}|. \tag{7}$$

G_8 exhibits the non-leptonic enhancement factor ($\Delta I = 1/2$ rule), but is subject to large higher-order corrections [6].

2. The chiral anomaly enters at $O(p^4)$. Its contribution to strong, electromagnetic and semileptonic weak amplitudes is contained in the Wess–Zumino–Witten functional [2], which has the following explicit form in a scheme where the vector currents are conserved:

$$\begin{aligned} S[U,l,r]_{WZW} &= -\frac{iN_C}{240\pi^2} \int d\sigma^{ijklm} \langle \Sigma_i^L \Sigma_j^L \Sigma_k^L \Sigma_l^L \Sigma_m^L \rangle \\ &\quad -\frac{iN_C}{48\pi^2} \int d^4x \varepsilon_{\mu\nu\alpha\beta} \left(W(U,l,r)^{\mu\nu\alpha\beta} - W(\mathbf{1},l,r)^{\mu\nu\alpha\beta} \right) \end{aligned} \tag{8}$$

$$\begin{aligned} W(U,l,r)_{\mu\nu\alpha\beta} &= \langle U l_\mu l_\nu l_\alpha U^\dagger r_\beta + \frac{1}{4} U l_\mu U^\dagger r_\nu U l_\alpha U^\dagger r_\beta + iU \partial_\mu l_\nu l_\alpha U^\dagger r_\beta \\ &\quad + i\partial_\mu r_\nu U l_\alpha U^\dagger r_\beta - i\Sigma_\mu^L l_\nu U^\dagger r_\alpha U l_\beta + \Sigma_\mu^L U^\dagger \partial_\nu r_\alpha U l_\beta \\ &\quad - \Sigma_\mu^L \Sigma_\nu^L U^\dagger r_\alpha U l_\beta + \Sigma_\mu^L l_\nu \partial_\alpha l_\beta + \Sigma_\mu^L \partial_\nu l_\alpha l_\beta \\ &\quad - i\Sigma_\mu^L l_\nu l_\alpha l_\beta + \frac{1}{2} \Sigma_\mu^L l_\nu \Sigma_\alpha^L l_\beta - i\Sigma_\mu^L \Sigma_\nu^L \Sigma_\alpha^L l_\beta \rangle \\ &\quad - (L \leftrightarrow R) \end{aligned} \tag{9}$$

$$\Sigma_\mu^L = U^\dagger \partial_\mu U \qquad \Sigma_\mu^R = U \partial_\mu U^\dagger$$

$$N_C = 3 \qquad \varepsilon_{0123} = 1$$

where $(L \leftrightarrow R)$ stands for the interchange

$$U \leftrightarrow U^\dagger, \qquad l_\mu \leftrightarrow r_\mu, \qquad \Sigma_\mu^L \leftrightarrow \Sigma_\mu^R.$$

The anomaly also contributes to non-leptonic weak amplitudes starting at $O(p^4)$. The most obvious contribution is due to tree diagrams involving one WZW vertex and one vertex from the non-leptonic weak Lagrangian (6). Since $\mathcal{L}_2^{\Delta S=1}$ contains bilinear terms in the meson fields, there is a local part in the corresponding functional which can be given in explicit form by diagonalizing the kinetic part of the Lagrangians (1) and (6) simultaneously [7]. This local Lagrangian embodies the so-called pole contributions to anomalous non-leptonic weak amplitudes and is given by [8]

$$\mathcal{L}_{an}^{\Delta S=1} = -\frac{ieG_8}{8\pi^2 F}\widetilde{F}^{\mu\nu}\partial_\mu\pi^0 K^+ \overleftrightarrow{D}_\nu \pi^- + \frac{\alpha G_8}{6\pi f}\widetilde{F}^{\mu\nu}F_{\mu\nu}\left(K^+\pi^-\pi^0 - \frac{1}{\sqrt{2}}K^0\pi^+\pi^-\right) + \text{h.c.} \tag{10}$$

$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the electromagnetic field strength tensor, $\widetilde{F}_{\mu\nu} = \varepsilon_{\mu\nu\rho\sigma}F^{\rho\sigma}$ its dual, and $D_\nu\varphi^\pm$ denotes the covariant derivative $(\partial_\nu \mp ieA_\nu)\varphi^\pm$. In the limit of CP conservation, the anomalous Lagrangian (10) contributes only to the decays (with real or virtual photons)

$$K^+ \to \pi^+\pi^0\gamma,\ \pi^+\pi^0\gamma\gamma \quad \text{and} \quad K_L \to \pi^+\pi^-\gamma\gamma. \tag{11}$$

There is, however, an additional source of non-leptonic anomalous amplitudes of $O(p^4)$, which was not taken into account in Ref. [8]. Diagrammatically, those contributions can be pictured as arising from contraction of the W field between Green functions due to the anomaly on the one side and the Lagrangian $\mathcal{L}_2$ of Eq. (1) on the other side. However, as in the normal parity sector discussed before, such a procedure would not give the correct amplitudes at the hadronic scale. Rather, we must use again the operator product expansion first and realize the corresponding operators at the bosonic level in the presence of the anomaly.

3. A possible framework to implement the bosonization of four-quark operators was formulated in Ref. [9]. Let us first recall the standard bosonization of (left-handed) quark currents in CHPT. The Green functions of quark currents can be expressed as functional integrals in the fundamental theory

$$\begin{aligned}\langle 0|T\{\overline{q_{jL}}\gamma^\mu q_{iL}\ldots\}|0\rangle &= N^{-1}\int [DqD\overline{q}DG]\overline{q_{jL}}\gamma^\mu q_{iL}\ldots e^{i\int d^4x\mathcal{L}} \\ &= (-i\frac{\delta}{\delta l_{\mu,ji}})\ldots \mathrm{e}^{\mathrm{i}Z[\mathrm{l},\mathrm{r},\mathrm{s},\mathrm{p}]}\end{aligned} \tag{12}$$

where [3]

$$\mathcal{L} = \mathcal{L}^0_{QCD} + \overline{q}\gamma^\mu\{v_\mu + \gamma_5 a_\mu\}q - \overline{q}\{s - i\gamma_5 p\}q \tag{13}$$

is the QCD Lagrangian with massless light quarks in the presence of external fields l, r, s, p. The basic tenet of CHPT is that the generating functional $Z[l, r, s, p]$ [3] can be calculated in the effective theory in terms of the chiral effective action $S[U, l, r, s, p]$ as

$$\mathrm{e}^{\mathrm{iZ[l,\,r,\,s,\,p]}} = \mathrm{N}^{-1}\int [\mathrm{DU}]\mathrm{e}^{\mathrm{iS[U,\,l,\,r,\,s,\,p]}} \, . \tag{14}$$

The bosonized form of the quark currents in the low-energy theory is then given by

$$\overline{q_{jL}}\gamma^\mu q_{iL} \leftrightarrow \frac{\delta S[U, l, r, s, p]}{\delta l_{\mu,ji}}. \tag{15}$$

The insertion of a four-quark operator Q_i can be treated in a similar fashion [9]. In the effective chiral theory, a four-quark operator [1] corresponds to the insertion [9]

$$\langle 0|T\{\overline{q_{lL}}\gamma^\mu q_{kL}\overline{q_{jL}}\gamma_\mu q_{iL}\dots\}|0\rangle = N^{-1}\int [DUDG]\left\{\frac{\delta\Gamma}{\delta l_{\mu,lk}}\frac{\delta\Gamma}{\delta l^\mu_{ji}} - i\frac{\delta^2\Gamma}{\delta l_{\mu,lk}\delta l^\mu_{ji}}\right\}\dots \mathrm{e}^{\mathrm{i}\Gamma} \tag{16}$$

where $\Gamma[U, l, r, s, p; G]$ is the effective action before the gluons are integrated out. There is a difference between the bosonization of a current (15) and a four-quark operator (16) due to the anomalous dimension of the four-quark operator. The gluonic integral includes soft gluons dressing the four-quark operator [recall that only hard gluons were integrated out to arrive at the Hamiltonian (5)] to produce the necessary μ-dependence of the operator in its bosonized form. Of course, neither $S[U, l, r, s, p]$ nor $\Gamma[U, l, r, s, p; G]$ can be calculated directly from QCD at this time. However, very encouraging progress in this direction has recently been made in the context of a model incorporating a specific mechanism for spontaneous chiral symmetry breaking [10, 9, 11].

The two contributions in Eq. (16) are denoted [9] as factorizable (leading in $1/N_C$) and non-factorizable (non-leading in $1/N_C$), respectively. Let us now consider the odd-intrinsic parity parts (containing the ε tensor)

$$\Gamma^-[U, l, r, s, p; G] \qquad \text{and} \qquad S^-[U, l, r, s, p]. \tag{17}$$

There is obviously no term of $O(p^2)$ in Γ^-. Nor does chiral symmetry allow a chiral invariant $O(p^4)$ term. Instead, Γ^- is given to $O(p^4)$ by the chiral anomaly in terms of the WZW functional $S[U, l, r]_{WZW}$ and by a term accounting for the gluonic component of the chiral $U(1)$ anomaly. Except for this latter part, which will not play any special rôle in the following, the remainder $\Gamma^-_{rem}[U, l, r, s, p; G]$ defined via

$$\Gamma^-[U, l, r, s, p; G] = S[U, l, r]_{WZW} + \Gamma^-_{rem}[U, l, r, s, p; G] \tag{18}$$

[1] We restrict the analysis to products of left-chiral currents.

is chiral-invariant. The anomaly can be viewed as arising from the chiral non-invariance of the fermionic measure in the path integral [12]. The non-renormalization theorem [13] of the chiral anomaly then tells us that there is a similar decomposition for the effective action S^- with odd intrinsic parity,

$$S^-[U,l,r,s,p] = S[U,l,r]_{WZW} + S^-_{inv}[U,l,r,s,p]. \tag{19}$$

The $O(p^4)$ part of S^- is unaffected by the gluonic path integral and is again given by $S[U,l,r]_{WZW}$. The remainder $S^-_{inv}[U,l,r,s,p]$ starts at $O(p^6)$ and is $SU(3)_L \times SU(3)_R$ invariant [14].

Looking at Eq. (16), we find that the WZW functional in Eq. (18) only contributes to the factorizable part because there is no possible contribution of $O(p^2)$ with an ε tensor. The non-factorizable contribution is determined entirely by Γ^-_{rem} and it can only be calculated with a special model for spontaneous chiral symmetry breaking (cf. Refs. [10, 9, 11]).

To lowest order, $O(p^4)$, the factorizable contribution can be given exactly because the WZW functional can be pulled out of the gluonic path integral. The bosonized form of the four-quark operator in the anomalous parity sector is [factorizable contribution of $O(p^4)$] :

$$\overline{q_{lL}}\gamma^\mu q_{kL}\overline{q_{jL}}\gamma_\mu q_{iL} \leftrightarrow \frac{\delta S_{WZW}}{\delta l^\mu_{lk}}\frac{\delta S_2}{\delta l_{\mu,ji}} + (lk \leftrightarrow ji) \tag{20}$$

where

$$\begin{aligned} \frac{\delta S_2}{\delta l_{\mu,ji}} &= -\frac{F^2}{2}\left(L^\mu\right)_{ij} \\ L^\mu &= iU^\dagger D^\mu U \end{aligned} \tag{21}$$

is the left-chiral current of lowest order p corresponding to the chiral Lagrangian (1). The anomalous current [of $O(p^3)$] has the following form

$$\begin{aligned} \frac{\delta S_{WZW}}{\delta l_{\mu,ji}} &= \frac{1}{16\pi^2}\varepsilon^{\mu\nu\alpha\beta}J^{an}_{\nu\alpha\beta,ij} \\ J^{an}_{\nu\alpha\beta} &= iL_\nu L_\alpha L_\beta + \left\{F^L_{\nu\alpha} + \frac{1}{2}U^\dagger F^R_{\nu\alpha}U, L_\beta\right\} \\ &\quad + \text{a chirally non-covariant polynomial} \\ &\quad\ \text{in the external fields } l, r, \end{aligned} \tag{22}$$

where F^L, F^R are the non-Abelian field strengths associated with the fields l, r [3]. The anomalous current (22) has a well-known structure [15, 16]: it consists of a chirally covariant piece written explicitly in (22) and a local polynomial in the external gauge fields l, r, which is not chirally covariant. Only the covariant anomalous current has direct physical significance. Changing the local polynomial in l, r amounts to different regularization schemes which cannot modify the physical content of the anomaly. Moreover, in the regularization scheme chosen in Eq. (8), the local polynomial disappears for external vector gauge fields which is exactly the case relevant for us: all external gauge fields are photons (radiative non-leptonic kaon decays).

4. We can now construct the bosonization of the dominant octet operator in (5)

$$\begin{aligned} Q_- &= Q_2 - Q_1 \\ Q_1 &= \overline{s}\gamma^\mu(1-\gamma_5)d\overline{u}\gamma_\mu(1-\gamma_5)u \\ Q_2 &= \overline{s}\gamma^\mu(1-\gamma_5)u\overline{u}\gamma_\mu(1-\gamma_5)d \end{aligned} \tag{23}$$

in the factorizable approximation for the odd-parity part of $O(p^4)$. The final result is

$$Q_-(\text{fact}) \leftrightarrow -\frac{F^2}{8\pi^2}\varepsilon^{\mu\nu\alpha\beta}\left[\langle\lambda\{L_\mu, J^{an}_{\nu\alpha\beta}\}\rangle - \langle\lambda L_\mu\rangle\langle J^{an}_{\nu\alpha\beta}\rangle\right] \tag{24}$$

in terms of the covariant anomalous current (22). This result may be written in a more explicit form in terms of four chiral operators of $O(p^4)$ of odd intrinsic parity as [2]

$$\begin{aligned} Q_-(\text{fact}) \quad \leftrightarrow \quad & \frac{F^2}{16\pi^2}\Big(2i\varepsilon^{\mu\nu\alpha\beta}\langle\lambda L_\mu\rangle\langle L_\nu L_\alpha L_\beta\rangle \\ & + \langle\lambda[U^\dagger\tilde{F}^{\mu\nu}_R U, L_\mu L_\nu]\rangle \\ & + 3\langle\lambda L_\mu\rangle\langle(\tilde{F}^{\mu\nu}_L + U^\dagger\tilde{F}^{\mu\nu}_R U)L_\nu\rangle \\ & + \langle\lambda L_\mu\rangle\langle(\tilde{F}^{\mu\nu}_L - U^\dagger\tilde{F}^{\mu\nu}_R U)L_\nu\rangle\Big)\,. \end{aligned} \tag{25}$$

The last three terms in Eq. (25) were already obtained by Cheng [17], although not in the explicitly covariant form given here. They were also discussed in Ref. [8] since they contribute to the decays $K \to \pi\pi\gamma(\gamma)$ considered there. Contrary to the conjecture made in Ref. [8], the coefficients of these terms are by no means small, since they are actually generated by the chiral anomaly although the operators are completely chirally covariant. The first term in Eq. (25) only contributes to K decays with at least three pions in the final state. In fact, in the limit of CP conservation it only contributes to the decay $K_L \to \pi^+\pi^-\pi^0\gamma$ [18].

We have therefore established that the factorizable contribution to Q_- in the anomalous parity sector can be completely determined to $O(p^4)$ as given by Eq. (25). Moreover, of all possible octet operators of $O(p^4)$ proportional to the ε tensor [19], only the four operators appearing in (25) can contribute to processes where all external gauge fields are photons. Since the WZW functional cannot contribute to the non-factorizable part, the latter automatically has the right chiral transformation property of an octet operator. Therefore, even if we

[2] We neglect a term proportional to $\langle[\lambda, \tilde{F}^{\mu\nu}_L]L_\mu L_\nu\rangle$, which cannot contribute for the case of external photons.

cannot calculate the non-factorizable contributions in a model-independent way, we know they must be of the form (25), albeit with different coefficients. In fact, those coefficients will be μ-dependent to cancel the μ-dependence of the Wilson coefficients. This cancellation was discussed in Ref. [9] for the normal-parity octet Lagrangian of $O(p^2)$ in (6).

Since all octet operators in $\mathcal{H}_{eff}^{\Delta S=1}$ produce the same structure (25), we can write down the final representation of the $\Delta S = 1$ effective Lagrangian in the anomalous parity sector to $O(p^4)$

$$\begin{aligned}\mathcal{L}_{eff}^{\Delta S=1} = & \frac{G_8 F^2}{16\pi^2}\Big(2a_1 i\varepsilon^{\mu\nu\alpha\beta}\langle\lambda L_\mu\rangle\langle L_\nu L_\alpha L_\beta\rangle \\ & + a_2\langle\lambda[U^\dagger \tilde{F}_R^{\mu\nu}U, L_\mu L_\nu]\rangle \\ & + 3a_3\langle\lambda L_\mu\rangle\langle(\tilde{F}_L^{\mu\nu} + U^\dagger\tilde{F}_R^{\mu\nu}U)L_\nu\rangle \\ & + a_4\langle\lambda L_\mu\rangle\langle(\tilde{F}_L^{\mu\nu} - U^\dagger\tilde{F}_R^{\mu\nu}U)L_\nu\rangle\Big) + h.c. \qquad (26)\end{aligned}$$

From the dominance of the octet operator Q_- we expect the dimensionless coefficients a_i to be positive and of order one. In fact, they can be expected to be somewhat smaller than one because there is no factorizable contribution to (26) from the dominant penguin operator Q_6 unlike the situation in the normal parity sector at $O(p^2)$ [9].

5. The phenomenological implications of the chiral anomaly for non-leptonic K decays will be treated in more detail elsewhere [18]. Here, we confine ourselves to a brief application of our findings to the decay $K^+ \to \pi^+\pi^0\gamma$. The available experimental evidence is consistent with a dominant magnetic part for the direct emission amplitude (non-bremsstrahlung)[20]. In Ref. [8], the magnetic amplitude due to the Lagrangian (10) was found to be

$$M = -\frac{eG_8 M_K^3}{2\pi^2 F}, \qquad (27)$$

whereas (26) contributes

$$M = \frac{3eG_8 M_K^3}{2\pi^2 F}(\frac{a_2}{2} - a_3). \qquad (28)$$

Moreover, it was shown in [8] that the V-exchange corrections of $O(p^6)$ to M are much smaller than the anomalous amplitude (27) of $O(p^4)$. For $a_i \simeq 1$, there is positive interference between the two amplitudes [17]. Assuming that the direct emission rate is purely magnetic, the experiments of Ref. [20] have found a corresponding branching ratio (the average is from Ref. [21])

$$BR(K^+ \to \pi^+\pi^0\gamma)_{DE} = (1.8 \pm 0.4) \times 10^{-5} \qquad (29)$$

for a certain cut in the kinetic energy of the charged pion. With the total magnetic amplitude given by the sum of (27) and (28), we find

$$BR(K^+ \to \pi^+\pi^0\gamma)_M = 2.2\times 10^{-5} \left(\frac{2+6a_3-3a_2}{5}\right)^2 (G_8/9\times 10^{-6}\ \mathrm{GeV}^{-2})^2 \quad (30)$$

for the same cuts. The good agreement between (29) and (30) gives support to the theoretical expectation

$$a_i \lesssim 1. \quad (31)$$

Further experimental work isolating the competing electric direct emission amplitude [8] is necessary to make this agreement more conclusive.

Conclusions

i. The non-Abelian chiral anomaly contributes to non-leptonic weak processes in two different ways at $O(p^4)$:

   - CHPT amplitudes involving one WZW vertex and one vertex from $\mathcal{L}_2^{\Delta S=1}$ (6). The so-called pole contributions are given in closed form by the Lagrangian (10).
   - Direct anomalous amplitudes due to chirally covariant octet operators of $O(p^4)$ in (25) and (26).

ii. In the non-leptonic sector, the anomaly contributes only to radiative kaon decays.

iii. The factorizable contributions of odd intrinsic parity, which are unambiguously calculable, already produce all possible terms of $O(p^4)$ relevant for radiative processes where all external gauge fields are photons. Due to the non-renormalization of the chiral anomaly, there are no QCD corrections to the factorizable part at $O(p^4)$.

iv. The non-factorizable terms (non-leading in $1/N_C$) compensate the scale dependence of the Wilson coefficients. They contribute to the coefficients of the factorizable terms but they cannot generate new ones.

v. Although the coefficients of the four possible chiral operators are not calculable in a model-independent way, the overall scale is expected to be of $O(G_8)$ or somewhat smaller. The octet enhancement of the Wilson coefficients appears in this scale, but the dominant penguin operator Q_6 does not contribute to the factorizable part, because the WZW functional is independent of external scalar/pseudoscalar fields.

vi. The phenomenological implications are encouraging for the decay $K^+ \to \pi^+\pi^0\gamma$ and will be considered in more detail elsewhere [18].

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