CP Violation in the B Meson System and Prospects at an Asymmetric B Meson Factory

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ABSTRACT

We present an overview of the expected phenomenology of CP violation in the B system. The prospects for observing CP-violating signals at an asymmetric B-Factory are analyzed. We discuss how these phenomena can be used to test the unitarity of the Cabibbo-Kobayashi-Maskawa matrix, and to either verify the Standard Model mechanism of CP violation or provide clear evidence for new physics.

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1 Introduction

Symmetries are one of the most useful tools to elaborate theories in particle physics, although most of these symmetries are not preserved in Nature. Understanding why and how these violations occur allows one to powerfully discriminate between various hypothesis and in, general, a seed for new ideas. Charge conjugation (C) and Parity (P) are known to be drastically violated \(^1\) by the weak interactions since 1957, and observations of these violations have been made in many different systems. In contrast their product (CP), thought to be conserved until 1964, is only violated very slightly \((\approx 2 \times 10^{-5})\) in the decay of neutral kaons \([9]\). Since then, no observation of this phenomenon has been seen in a different system. Therefore, we do not know yet whether it is a fundamental property of Nature or simply an accident proper to the neutral kaon. Is CP violation only due to the fact that the \(K^+\) oscillates from the particle to its anti-particle, \(K^0\), as proposed in the superweak model \([7]\)? Or is it the weak interaction, responsible of the kaon decay and for which we have so far a consistent theory - the so-called Standard Model (SM) \([4]\), which is the underlying origin of this phenomenon? Getting an answer to these questions would be by itself sufficient to justify a large effort in this field during this decade.

The issue gets even more spectacular if one considers that CP violation, as well as baryon number violation and C violation in an expanding universe (with deviations from thermal equilibrium), is an essential ingredient to generate the Baryon Asymmetry of the Universe (BAU) \([5]\). For a while, it was thought that the BAU could only be generated at the scale of the Grand Unification \((\approx 10^{15-16} \text{ GeV})\). In 1985 it was realized \([7]\) that the electroweak phase transition could have dramatic consequences on any baryon asymmetry generated at higher temperature, and could even be at the origin of the observed number of baryon to photon ratio \(n_b/n_\gamma \approx 10^{-10}\) through anomalous electroweak baryon-number violation \([6]\). CLEARLY SUCH A POSSIBILITY MAKES THE STUDY OF THE "Standard" scenario producing CP non-conservation at the electroweak scale \([9]\), and for which we have definite predictions, extremely exciting. However, as today, it seems very difficult to produce large enough BAU in the minimal SM \(\nu\text{(1)}\) Standard Model with one scalar Higgs doublet \([10]\). Verifying the description of CP violation by the SM is therefore of prime importance.

Non conservation of the CP symmetry has been introduced in the SM in 1973 \([9]\) by requiring three massless families of quark doublet before the spontaneous \(SU(2) \times U(1)\) symmetry breaking by the Higgs mechanism \([11]\). After this phase transition, the remaining neutral Higgs particle allows one to generate a mass for each fermion through its Yukawa type coupling. The interaction Lagrangian is

\[
L_{Yukawa} = \left( V_{ts} m_{H^0} + V_{tc} m_{H^0} + \frac{s}{c} \right) \phi \psi + \left( 1 + \frac{c}{s} \right) \phi \psi,
\]

where \(\phi\) stands for the scalar Higgs field and \(\psi\) is its value in the new vacuum. The quark fields \(u_{L,R}\) and \(d_{L,R}\) are the 3-component vectors in flavour space for the up- and down-type quarks respectively,

\[
u_{L,R} = \left( \begin{array}{c} 1 \pi \varepsilon & \frac{n}{n} \\ \frac{n}{n} & \frac{n}{n} \end{array} \right), \quad d_{L,R} = \left( \begin{array}{c} 1 \pi \varepsilon & \frac{n}{n} \\ \frac{n}{n} & \frac{n}{n} \end{array} \right)
\]

and \(m, m, \) are \(3 \times 3\) mass matrices of arbitrary complex numbers, the elements of which are \(m_{ij} = -\frac{\sqrt{j}}{\sqrt{j}} Y_{H_u}^T\) and \(m_{ij} = -\frac{\sqrt{j}}{\sqrt{j}} Y_{H_u}^T\) where \(Y_{H_u}\) and \(Y_{H_u}\) are the Yukawa coupling constants with \(i = u, c\) or \(t\) and \(j = d, s\) or \(b\). Since \(m\) and \(m\) are not diagonal, one needs to define the physical fermion fields \(u_{L,R}\) and \(d_{L,R}\) as \(V_{ij} u_{L,R}\) and \(d_{L,R}\) as \(V_{ij} u_{L,R}\) where \(V_{ij}, V_{ij}\) are unitary matrices. Writing \((1)\) in terms of these new fields diagonalizes the mass matrices with \(m_{ij} = V_{ij} M_{ij} V_{ij}\) and \(m_{ij} = V_{ij} M_{ij} V_{ij}\). The coupling of the physical quarks to the neutral \(Z\) preserves the observed absence of flavour-changing neutral currents (FCNC), while their coupling to the charged \(W^\pm\) introduces the mixing between families. The charged-current couplings are

\[
L_{W} = \frac{\sqrt{3}}{2} \left( \bar{u} \gamma^\mu W^+_{\mu} \psi d + \bar{d} \gamma^\mu W^-_{\mu} \psi u \right),
\]

where \(V \equiv V_{ij} V_{ij}\) is an unitary \(3 \times 3\) matrix called the quark-mixing or Cabibbo-Kobayashi-Maskawa (CKM) matrix \([9,12]\):

\[
V = \left( \begin{array}{ccc} v_2 & v_3 & v_4 \\
 v_3 & v_4 & v_5 \\
 v_4 & v_5 & v_6 \end{array} \right). \quad (4)
\]

Obviously this picture can be extended to more than three families and could be valid (with massive neutrino) for the leptonic sector.

We have briefly sketched how the CKM matrix is generated to outline its interconnection with the generation of the mass of the fermions. Unfortunately at present, there exists no obvious way to deduce the values of the elements of \(V\) from this mechanism and we are left (in the three generation case) with ten arbitrary parameters accounting for the mass and the mixing of the quarks. The most general expression of \(V\) can be derived from \(n(n-1)/2\) mixing angles and \((n-1)(n-2)/2\) phases where \(n\) is the number of fermion families; several equivalent representations can be found in the literature. With more than two families, the elements of \(V\) can be complex numbers which allow the possibility for generating CP violation through the interference of two diagrams involving different matrix elements.

So far, with three families, the three mixing angles and the single phase are fundamental parameters of the theory. Attempts to connect these parameters to the quark masses have been proposed in the framework of particular models \([13]\). It is therefore very important to measure the matrix elements \(V_{ij}\) in order to probe the predicted relations and obtain some hints on the nature of the quark-mass matrices \(m\) and \(m\). In the SM, however, the only informations that are available on \(V_{ij}\) are derived from the unitarity condition \(V_V V = 1:\)

\[
V_{ij} V_{ij} + V_{ij} V_{ij} + V_{ij} V_{ij} = 0,
\]

\[
V_{ij} V_{ij} + V_{ij} V_{ij} + V_{ij} V_{ij} = 0,
\]

\[
V_{ij} V_{ij} + V_{ij} V_{ij} + V_{ij} V_{ij} = 0.
\]

These relations can be visualized by triangles in a complex plane \([14]\). In order to see how these triangles
look like, it is practical to use the so-called Wolfenstein approximate representation of $V$ [15],

$$
V = \begin{bmatrix}
1 - \frac{\lambda^2}{2} & \lambda & \lambda \lambda^2 (\rho - i\eta) \\
-\lambda & 1 - \frac{\lambda^2}{2} & \lambda \lambda^2 \\
\lambda \lambda^2 (1 - \rho - i\eta) & -\lambda \lambda^2 & 1
\end{bmatrix} + \mathcal{O}(\lambda^4),
$$

(6)

where $\lambda = \sin \theta_{C} \cong 0.23$, $\lambda = \sin \theta_{23} / \lambda$, $\rho = \sin \theta_{23} \cos (\Delta \lambda^2 / 2)$ and $\eta = \sin \theta_{13} \sin \delta / (\Delta \lambda^2)$ ($\delta_0$ is the mixing angle between quarks $i$ and $j$, and $\delta$ the CP-violating phase, in the standard [16] parametrization of the CKM matrix). The values of $\lambda$, $\rho$ and $\eta$ are poorly measured but are in the range 0.7–1.1 for $\lambda$ and smaller than 0.5 for $\rho$ and $\eta$. These values translate into very flat triangles for the two first equations in (5) while the third expression leads to the interesting possibility of having angles significantly different from 0 or $\pi$. Experimental verification of the consistency between the sides and the angles of this unitary triangle can therefore be envisaged and would be a very powerful test of the Standard Model. A closer look at the third relation in (6) shows that the sides of the triangle can be measured by studying $B$ meson physics. Indeed $C_{u}$ and $C_{d}$ can be obtained by measuring $B$ decays into charmed and non-charmed mesons respectively while $V_{ud}$ is extracted from $B^{*}-\bar{B}^{0}$ mixing [17]. The angles $\alpha$, $\beta$ and $\gamma$ of the triangle are also directly connected to $B$ physics and can be in principle extracted from the measurements of CP violation in $B$ decays as it is emphasized in the present paper. To summarize this paragraph, we show in Fig. 1 the unitarity triangle together with the various topics in $B$ physics that allow one to measure its sides and angles. Needless to say that measuring CP violation in the $B$ sector is an inevitable challenge that experimentalists will have to face and, since it probes a totally unexplored field, could lead to very large effects.

![Unitarity Triangle](image1)

Figure 1: The unitarity triangle

2 CP-violation mechanisms

Any observable CP-violating effect is generated by the interference between different amplitudes contributing to the same physical transition. This interference can occur either through $B^{*}-\bar{B}^{0}$ mixing or via final-state interactions, or by a combination of both effects [18].

2.1 Indirect CP violation

The general formalism to describe mixing among the neutral $B^{*}$ and $\bar{B}^{0}$ mesons is completely analogous to the one used in the kaon sector. Assuming CP symmetry to hold, the $2 \times 2$ $B^{*}-\bar{B}^{0}$ mixing matrix can be written as

$$
M = \begin{pmatrix}
M_{11} & M_{12} \\
M_{21} & M_{22}
\end{pmatrix}
$$

(7)

where the diagonal elements $M$ and $\Gamma$ are real parameters. If CP were conserved, $M_{12} = M_{21}$ would also be real. The physical eigenstates of $M$ are

$$
|B_{k}\rangle = \frac{1}{\sqrt{|\rho| + |\gamma|}} \begin{pmatrix}
|\rho| B^{*} \pm \gamma \bar{B}\n\end{pmatrix},
$$

(8)

where

$$
\rho \equiv \frac{M_{12} - i \Gamma_{12}}{1 + \Gamma_{12}}
$$

(9)

Clearly if $M_{12}$ and $\Gamma_{12}$ were real, then $\rho = 1$ and $|B_{k}\rangle$ would be CP eigenstates.

The departure of $\rho$ from unity could be measured, in principle, by looking to a CP-violating asymmetry in flavour-specific decays, i.e. decays to final states which can only be reached from an initial $B^{*}$ (or $\bar{B}^{0}$) but not from both. The most convenient example is given by the semileptonic decays

$$
B^{*} \rightarrow l^{+} \nu_{l} X, \quad \bar{B}^{0} \rightarrow l^{-} \bar{\nu}_{l} X.
$$

(10)

The asymmetry between the number of $l^{+}l^{+}$ and $l^{-}l^{-}$ pairs produced in the processes $e^{+}e^{-} \rightarrow B^{*}\bar{B}^{0} \rightarrow l^{+}l^{-}X$ is easily found to be

$$
\mathcal{A}_{L} \equiv \frac{N(l^{+}l^{+}) - N(l^{-}l^{-})}{N(l^{+}l^{+}) + N(l^{-}l^{-})} = \frac{\rho |\rho| - |\gamma| |\gamma|}{|\rho| + |\gamma|} \approx 4 \Re(\rho/\gamma).
$$

(11)

![Mixing Diagrams](image2)

Figure 2: $B^{*}-\bar{B}^{0}$ mixing diagrams

Unfortunately, this $\Delta \rho = 2$ asymmetry is expected to be quite tiny in the Standard Model, because $|\Delta \rho| \ll |M_{12}|$. This can be easily understood, by looking to the relevant box diagrams contributing to the $B^{*}-\bar{B}^{0}$ transition (Fig. 3); the mass mixing is
dominated by the top-quark graph, while the decay amplitudes get obviously its main contribution from the $b \rightarrow c$ transition. Thus,
\[ \frac{\Gamma_{12}}{\Gamma} = \frac{3 \pi m_t^2}{2 \pi m_b^2} \frac{1}{B'(x)} < < 1, \]  
(12)
where \[B'(x)\] is a slowly decreasing function of \[x = m_t^2/M_W^2\], \[B'(0) = 1, B'(1) = 3/4, B'(\infty) = 1/4\]. From Eq. (9), one has then
\[ \left| \frac{\Gamma_{12}}{\Gamma} \right| \approx \frac{1}{2} \frac{\Gamma_{12}}{\Gamma} \sin \phi_{b\rightarrow c}, \]  
(13)
where
\[ \phi_{b\rightarrow c} \equiv \arg \frac{\Gamma_{12}}{\Gamma}. \]  
(14)
The factor \[\sin \phi_{b\rightarrow c}\] involves an additional GIM suppression,
\[ \sin \phi_{b\rightarrow c} \approx \frac{8 m_t^2 - m_b^2}{3 m_t^2} \left( \frac{V_{ts} V_{tb}^*}{V_{td} V_{tb}^*} \right), \]  
(15)
implying a value of \[\phi_{b\rightarrow c}\] very close to 1. Here, \[g = 2, 3\] denote the corresponding CKM matrix elements for \[b \rightarrow s\] mesons. Therefore, one expects
\[ \left| \frac{\Gamma_{12}}{\Gamma} \right| \leq \begin{cases} 10^{-3} & (B'), \\ 10^{-4} & (B') \end{cases}. \]  
(16)

The observation of an asymmetry \[\alpha_{b\rightarrow s}\] at the percent level, would then be a clear indication of new physics beyond the Standard Model.

2.2 Direct CP violation

When a final state \(j\) tags the beauty content (or \(\bar{j}\) of a \(B\) meson, any observed difference in the rates \(B \rightarrow j\) and its conjugate mode \(\bar{B} \rightarrow \bar{j}\) indicates that CP violation directly originates from the weak interactions. Since at least two interfering amplitudes are needed to generate a CP-violating effect, we get the amplitudes for the transition \(B \rightarrow j\) and \(\bar{B} \rightarrow \bar{j}\) as
\[ A(B \rightarrow j) = g_1 M_1 \text{e}^{i\alpha_1} + g_2 M_2 \text{e}^{i\alpha_2}, \]  
\[ A(\bar{B} \rightarrow \bar{j}) = g_1 M_1 \text{e}^{-i\alpha_1} + g_2 M_2 \text{e}^{-i\alpha_2}, \]  
(17)
where \(g_1, g_2\) denote the weak couplings, \(\alpha_1, \alpha_2\) strong final-state phases (and/or strong phases between \(S\) and \(P\)-wave contributions in the case of \(b\)-baryon decays), and \(M_1, M_2\) the moduli of the matrix elements. The rate difference is then proportional to
\[ \Gamma(B \rightarrow j) - \Gamma(\bar{B} \rightarrow \bar{j}) \propto \Im(g_1 g_2^*) \sin(\alpha_1 - \alpha_2) M_1 M_2, \]  
(18)
i.e., the two interfering amplitudes need to have different weak phases as well as different strong phases. Moreover, in order to get a sizeable asymmetry (rate difference / sum), the two amplitudes \(M_1, M_2\) should be of comparable size. These types of decays are often referred to as self-tagging modes.

One example is the decay \(B^- \rightarrow K^-\pi^0\) which proceeds via a tree- and a penguin-diagram (Fig. 3), the weak couplings of which are given by \(V_{td} V_{ts}^* \approx A X (s_{13})\) and \(V_{td} V_{ts}^* \approx \bar{A} X, \text{respectively}^4\). Although the penguin contribution is of higher-order in the strong coupling, and suppressed by the loop factor \(1/(16\pi^2)\), one could expect both amplitudes to be of comparable size, owing to the additional \(1/4\) suppression factor of the tree diagram. The needed strong-phase difference can be generated through the absorptive part of the penguin diagram, corresponding to on-shell intermediate particle rescattering. Therefore, one could expect a sizeable asymmetry, provided the strong-phase difference is not too small. However, a very large number of \(B^+\) is required, because the branching ratio is quite suppressed \((\sim 10^{-4})\). Other decay modes such as \(B^- \rightarrow K^-\pi^0, K^-\pi^+\) [21] involve the interference between penguin diagrams only and might show sizeable CP-violating asymmetries as well, but the corresponding branching fractions are expected to be even smaller than the previously discussed one.

The two interfering amplitudes can also be generated through other mechanisms. For instance, one can have an interplay between two different cascade processes [22, 23] like \(B^- \rightarrow D^-\pi^+ightarrow K_S\pi^+\) and \(B^- \rightarrow D^-\pi^+ \rightarrow K_S\pi^-\). Another possibility would be an interference between two tree-diagrams corresponding to two different decay mechanisms like direct decay (spectator) and weak annihilation [24]. Note that, for all these flavour-specific decays, the necessary presence of strong phases makes very difficult to extract useful information on the CKM mixing factors from their measured CP asymmetries. Nevertheless, the experimental observation of a non-zero CP-violating asymmetry in any of these decay modes would be a major milestone in our understanding of CP-violation phenomena, as it would clearly establish the existence of direct CP violation in the decay amplitudes.

The interpretation of a CP non-conserving effect in terms of the angles of the unitarity triangle may be better achieved by studying the charged \(B\) decays \(B^- \rightarrow D^-\pi^+\) [25], where \(\pi^+\) is any state with the flavor of a \(K^+\), and \(D^-\) denote the CP eigenstates \((D^0 \equiv \bar{D}^0)/\sqrt{2}\). Let's consider the decay \(B^- \rightarrow D^-K^+\), where the CP-even eigenstate \(D^-\) is identified by one of its CP-even decay products. There are two different amplitudes contributing to this process, which correspond to the transitions \(B^- \rightarrow D^+K^-\) and \(B^- \rightarrow \bar{D}^0K^+\) (Fig. 4). At the quark level, they are associated with the decays \(\bar{d} \rightarrow s\bar{c}d\) and \(\bar{d} \rightarrow s\bar{c}u\), which have different CKM factors \((V_{ts} V_{ts}^* \approx A X (s_{13})\) and \(V_{td} V_{td}^* \approx A X^3\) respectively). These two amplitudes can be separately identified by looking for events where the \(D^-\) decays semileptonically; the flavour states \(D^0\) and \(\bar{D}^0\) are distinguished by the charge of the final lepton.

\[ u \quad u \quad u \]  
\[ \bar{u} \quad \bar{u} \quad \bar{u} \]  
\[ \bar{u} \quad \bar{u} \quad \bar{u} \]

Figure 4: Feynman diagrams contributing to the \(B^- \rightarrow D^+K^-\) and \(B^- \rightarrow \bar{D}^0K^+\) decays

In the standard CKM parametrization the amplitudes of the three processes can be written in the form
\[ A_B \equiv A(B^- \rightarrow D^+K^-) = |A_B| \text{e}^{i\phi_B}, \]  
\[ A_{\bar{B}} \equiv A(B^- \rightarrow \bar{D}^0K^+) = |A_{\bar{B}}| \text{e}^{i\phi_{\bar{B}}}, \]  
(19)
\[ A_{B^*} \equiv A(B^+ \rightarrow D^-K^+) = |A_{B^*}| \text{e}^{i\phi_{B^*}}, \]  
(20)
\[ ^4\text{We neglect } D^0\bar{D}^0\text{ mixing and CP violation in } D\text{ decays.} \]
\[ A_0 = A(B^+ \rightarrow D^*_0 K^+) = \frac{1}{\sqrt{2}} (A_0 + A_\bar{0}), \]

(21)

where \(A_0\) and \(A_\bar{0}\) are the corresponding final-state interaction phases, and \(\gamma\) is one of the weak angles of the unitarity triangle. The amplitudes of the charge-conjugated processes \(B^+ \rightarrow D^0 K^+\) and \(B^- \rightarrow D^0 K^-\) and \(B^- \rightarrow D^*_0 K^-\) (\(A_0\), \(A_\bar{0}\), and \(A_{\bar{0}}\), respectively) are obtained from the A amplitudes by simply changing the sign of the CKM phase \(\gamma\). Note that \(|A_0| = |A_{\bar{0}}|, \frac{|A_{\bar{0}}|}{|A_0|} = |\lambda_0|\), but \(|A_0| \neq |A_{\bar{0}}|\) if \(\gamma \neq 0\).

From the two triangle relations relating the \(B^+ (B^-)\) decays into \(D^+, D^0\) and \(D^*_0\), one could obtain sin \(\gamma\) up to a four-fold ambiguity [29] (the ambiguity, which is due to the presence of final-state interactions, could be eliminated in principle, by studying different processes with the same weak phase). However, one expects \(A(B^+ \rightarrow D^*_0 K^+) \ll A(B^+ \rightarrow D^0 K^+)\), because the first amplitude is colour suppressed while the second one is not.

The self-tagging decays \(B^+_s \rightarrow D_{s0}^- K^{*-}\) could perhaps provide a better way to extract \(\gamma\) [29] by deducing the beauty flavour from the decay \(K^{*+} \rightarrow K^+ \pi^+\). The analysis proceeds then as in the \(B^0\) case, but this time the two interfering amplitudes \((B^+_s \rightarrow D_{s0}^- K^{*-} + B^+_s \rightarrow D_{s0}^0 K^{*-})\) are likely to have comparable magnitudes with branching fractions of the order of \(10^{-5}\).

Another place for studying direct CP violation are decays of bottom baryons [27, 29], and a somewhat similar analysis may be performed [29]. CP violation can show up as a rate asymmetry and in various decay parameters. For instance, the decay \(1/2 \rightarrow 1/2 \pm 0\) (like \(A_4 \rightarrow A_{\bar{0}}D^0\)) involves, in addition to the partial decay rate \(\Gamma\), the three decay parameters \(a, \beta, \gamma\), that characterize the angular distribution in the rest frame of the decaying particle. If bare quantities describe the charge-conjugated process, then CP invariance requires \(\Gamma = \Gamma, \beta = -\beta, \gamma = -\gamma\) and \(\bar{\gamma} = \gamma\). Neither tagging nor time-dependences are required to observe CP violation with modes of baryons, in contrast to \(B^0\) modes that involve mixing. One has to be aware in mind that part of the mentioned observables need spin-polarization of the initial state in order to show the effect. Sizeable CP effects are expected in modes involving a \(D^0\), which is seen in a final state that can also be fed from a \(D^*\); six related processes, such as \(A_4 \rightarrow D^0, A_{\bar{0}} \rightarrow D^0, A_4 \rightarrow D^0, \bar{A}_4 \rightarrow D^0\), and their charge conjugate counterparts, could be used to extract the angle \(\gamma\), in [29], it is predicted that \(\text{Br}(A_4 \rightarrow D^0) \sim 10^{-5}\), thus \(\text{Br}(A_4 \rightarrow D^0) \sim 10^{-5}\). Under favourable circumstances CP asymmetries are estimated to occur at the few percent level. Table 1 gives a few examples of b-baryon decays with estimated branching ratios at the \(10^{-4}\) to \(10^{-6}\) level, with large detection efficiencies and potentially large CP-violating effects.

In flavour non-symmetric decays of the neutral \(B\) mesons (i.e. those final states which are common to \(B^+\) and \(B^-\) decays), one needs to know the flavour of the original \(B\) meson in order to study CP-violating rate asymmetries. However, there are some signals of CP violation which do not require flavour identification. These involve CP-odd decay asymmetries in the sum of all \(B^+\) and \(B^-\) events [29]. For instance, in the decay \(B^+ \rightarrow K_S \pi^+\pi^-\), the distribution in the \(\pi^+\pi^-\)-energy (the Dalitz plot distribution) need not be symmetric under exchange of \(\pi^+\) and \(\pi^-\). There will in general be an energy asymmetry of the form [29]

\[ \Gamma(B^+ \rightarrow K_S \pi^+\pi^-) = a + b(E_\pi - E_\pi - E_\pi - E_\pi), \]

(22)

where \(a, b\), and \(c\) are symmetric functions of the energy. CP invariance requires \(a = c, b = -b\), so that there is no energy asymmetry left if one sums together all \(K_S \pi^+\pi^-\) events (provided there is an equal number of \(B^+\) and \(B^-\) in the initial state). Therefore, the measurement of a net energy asymmetry in the sum of all \(B^+\) and \(B^-\) events would be a signal of CP violation. Such an effect could originate, in principle, from both direct and indirect CP violation. As discussed in the previous section, however, the mixing-induced asymmetry is expected to be very small in the Standard Model; thus, this Dalitz plot asymmetry should be a direct CP-violation effect, requiring two interfering amplitudes with different weak and strong phases. The decays \(B^0 \rightarrow K_S \pi^+\pi^-\) and \(B^+ \rightarrow \phi \pi^+\pi^-\) seem to be the most promising ones, since they proceed through the \(b \rightarrow \pi\pi\) mechanism, where the penguin amplitude is relatively enhanced with respect to the direct tree contribution.

2.3 Interplay between mixing and direct CP violation

There are quite a few non-leptonic final states which are reachable both from a \(B^+\) and a \(B^-\). For these flavour non-symmetric decays the \(B^0 (\text{or } B^-)\) can decay directly to the given final state \(f\), or do it after the

<table>
<thead>
<tr>
<th>Quark-subprocess</th>
<th>Examples for exclusive modes</th>
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<tbody>
<tr>
<td>( b \rightarrow d e f )</td>
<td>( \bar{\epsilon}_f \rightarrow \bar{\epsilon}_f )</td>
</tr>
<tr>
<td>( b \rightarrow s d m )</td>
<td>( A_{\bar{0}} \rightarrow p K_{s}^{-}, \bar{\epsilon}<em>s, \bar{\epsilon}</em>{p} K_{s}^{-} )</td>
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<td>( b \rightarrow s d m )</td>
<td>( \bar{\epsilon}<em>s \rightarrow A</em>{\bar{0}} p K_{s}^{-}, A_{\bar{0}} p K_{s}^{-} )</td>
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<td>( \bar{\epsilon}<em>s \rightarrow A</em>{\bar{0}} p K_{s}^{-}, A_{\bar{0}} p K_{s}^{-}, A_{\bar{0}} p K_{s}^{-}, A_{\bar{0}} p K_{s}^{-} )</td>
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Table 1: Examples of exclusive modes of bottom baryons with potentially large CP-violating effects. Column 1 shows the underlying quark-subprocess for the exclusive modes.

The 

\[ |A|/|\Delta M| < 1, \]

we will neglect the tiny \(\Delta M\) corrections in what follows.

The time-dependent decay probabilities for the decay of a neutral \(B\) meson created at the time \(t_0 = 0\) as
a pure $B^r$ ($B^f$) into the final state $f$ are 

$$
\Gamma(B^r \to f) \propto \frac{1}{2} \left| \alpha_f \right|^2 \left[ 1 + |\beta_f|^2 + |\phi_f|^2 \right] \cos(\Delta M t - 2\pi m_\beta / \rho) \sin(\Delta M t), \quad (26)
$$

$$
\Gamma(B^f \to f) \propto \frac{1}{2} \left| \alpha_f \right|^2 \left[ 1 + |\beta_f|^2 + |\phi_f|^2 \right] \cos(\Delta M t - 2\pi m_\beta / \rho) \sin(\Delta M t), \quad (26)
$$

where we have introduced the notation

$$
A_f \equiv A(B^r \to f), \quad \beta_f \equiv A(B^f \to f), \quad \rho_f \equiv A(B^r \to B^f), \quad \phi_f \equiv A(B^f \to B^r).
$$

(27)

CP invariance demands the probabilities of CP conjugate processes to be identical. Thus, CP conservation requires $A_f = \bar{A}_f$, $\beta_f = \bar{\beta}_f$, $\rho_f = \bar{\rho}_f$ and $m_\beta = m_\beta^\prime$. Violation of any of the first three equalities would be a signal of direct CP violation. The fourth equality tests CP violation generated by the interference of the direct decay $B^r \to f$ and the mixing-induced decay $B^f \to B^f \to f$. Note that in order to be able to observe any CP-violating asymmetry, one needs to distinguish between $B^r$ and $B^f$ decays. However, a final state $f$ that is common to both $B^r$ and $B^f$ decays cannot reveal by itself whether it came from a $B^r$ or a $B^f$. Therefore, one needs independent information on the flavour identity of the decaying neutral $B$ meson; this is referred to as “flavour tagging”. Since beauty hadrons are always produced in pairs, one can use for instance the flavour-specific decays of one $B$ to “tag” the flavour of the companion $B$.

### 2.3.1 Decays to CP self-conjugate final states

An obvious example of final states $f$ which can be reached both from the $B^r$ and the $B^f$ are CP eigenstates, i.e. states such that $f = \bar{f}$ ($\eta = \pm \eta$). The ratios $\beta_f$ and $\rho_f$ depend in general on the underlying strong dynamics. However, for CP self-conjugate final states, all dependence on the strong interaction disappears [22, 23] if only one weak amplitude contributes to the $B^r \to f$ and $B^f \to f$ transitions. In this case, we can write the decay amplitude as $A_f = M_f e^{i\delta_f} e^{i\phi_f}$, where $M_f$ is the phase of the weak decay amplitude and $\delta_f$ is the strong phase associated with final-state interactions. It is easy to check that the ratios $\beta_f$ and $\rho_f$ are then given by $(A_f = M_f e^{i\delta_f} e^{i\phi_f}, \rho_f = M_f e^{i\delta_f} e^{i\phi_f})$.

(28)

The unwanted effect of final-state interactions cancels out completely from these two ratios. Moreover, $\beta_f$ and $\rho_f$ simplify in this case to a single weak phase, associated with the underlying weak quark transition.

Since $\rho_f = \beta_f \neq 1$, the time-dependent decay probabilities given in Eqs. (28) and (29) become much simpler. In particular, there is no longer any dependence on $\cos(\Delta M t)$. Moreover, for $B$ mesons $|\Gamma_\beta / \Gamma_M| < 1$, implying

$$
\frac{\gamma}{\rho} \approx \frac{\Gamma_{B^r} \Gamma_{B^f}^{\ast}}{\sqrt{\Gamma_{B^r}} \sqrt{\Gamma_{B^f}}} \approx e^{-\frac{\Delta M t}{\rho}}.
$$

(29)

Here $\rho \approx d$ stands for $B_s^r, B_s^f$. In deriving this relation we have used the fact that $M_{12}$ is dominated by the top contribution, due to the quadratic dependence with the mass of the quark running along the internal lines of the box diagram. Therefore, the mixing ratio $\gamma / \rho$ is also given by a known weak phase

$$
\phi_M = \beta_f (B_s^r), \quad 0 (B_s^f).
$$

(30)

and the coefficients of the sinusoidal terms in the time-dependent decay amplitudes are then fully known in terms of CKM mixing angles only

$$
\Im \left( \frac{\alpha_f}{\sqrt{2}} \right) \approx -\Im \left( \frac{\alpha_f}{\sqrt{2}} \right) \approx \eta_f \sin(4\phi_M + 2\phi_f) \approx \eta_f \sin(2\phi_f).
$$

(31)

The time-dependent decay rates (Fig. 5) are finally given by

$$
\Gamma(B^r \to f) = \Gamma(B^f \to f) e^{i\phi_f} (1 + \eta_f \sin(2\phi_f) \sin(\Delta M t)),
$$

(32)

$$
\Gamma(B^f \to f) = \Gamma(B^r \to f) e^{i\phi_f} (1 - \eta_f \sin(2\phi_f) \sin(\Delta M t)).
$$

(33)

In this ideal case, the time-dependent CP-violating decay asymmetry

$$
\Gamma(B^r \to f) - \Gamma(B^f \to f) = \eta_f \sin(2\phi_f) \sin(\Delta M t) \approx \eta_f \sin(2\phi_f) \sin(\Delta M t).
$$

(34)

provides a direct and clean measurement of the CKM parameters [30]. Integrating over all decay times yields

$$
\int_0^{\infty} dt \sin^{2}(\phi_f - \phi_f^0) \approx 1 \neq \eta_f \sin(2\phi_f) \sin(\Delta M t) \approx \eta_f \sin(2\phi_f) \sin(\Delta M t),
$$

(35)

where $\pm \Delta M t$ F, for $\gamma F$ mesons, $x_F = \frac{1}{2} \pm \frac{1}{2} \pi$ [31]; thus, the mixing term $x_F (1 + x_F^2)$ suppresses the observable asymmetry by a factor of about two. For $B_s^r$ mesons, one expects $x_s = x_s^0 [1 + (1 - 4x_s^0^2)] \sim x_s^0 [1 + (1 - 4x_s^0^2)] > x_s$, and therefore the large $B_s^r B_s^f$ mixing would lead to a large dilution of the CP asymmetry. The measurement of the time-dependence is then a crucial requirement for observing CP-violating asymmetries with $B_s$ mesons.

In $e^+ e^-$ machines, running near the $e^+ e^- \to B_s^r B_s^f$ production threshold, there is an additional complication coming from the fact that the $B$ mesons used to “tag” the flavour is also a neutral one, and therefore both mesons oscillate. Moreover, the $B_s^r B_s^f$ pair is produced in a coherent quantum state which is a $C$ eigenstate (C-odd in $e^+ e^- \to B_s^r B_s^f$). Taking that into account, the observable time-asymmetry takes the form

$$
\Gamma(B^r \to f) = \Gamma(B^f \to f) e^{i\phi_f} (1 + \eta_f \sin(2\phi_f) \sin(\Delta M t)),
$$

(36)

$$
\Gamma(B^f \to f) = \Gamma(B^r \to f) e^{i\phi_f} (1 - \eta_f \sin(2\phi_f) \sin(\Delta M t)).
$$

(37)

where the $B$ flavour has been assumed to be “tagged” through the semileptonic decay, and $t$ denotes the time of decay into $f$. Note that for $C = -1$ the asymmetry vanishes if $t$ and $t$ are treated asymmetrically. A measurement of at least the sign of $\Delta M t = \pm |\Delta M t|$ is necessary to detect CP violations in this case.

Unfortunately, we have assumed up to now that there is only one amplitude contributing to the given decay process. This is usually not the case. If several decay amplitudes with different weak and strong phases contribute, $\beta_f \neq 1$, and the interference term will depend both on the CKM mixing parameters and on the strong dynamics embedded in the ratio $\rho_f$.

The leading contributions to $\Delta M t$ are either “direct” (Fermi) or generated by gluon exchange ("penguin"). Although of higher order in the strong coupling constant, penguin amplitudes are logarithmically enhanced, due to the virtual $W$-loop, and are potentially competitive. Table 2 contains the CKM factors associated with the direct and penguin diagrams for different $B$-decay modes into CP-eigenstates. The $\Delta M t$ decay amplitudes are theoretically unambiguous [32]: the direct and penguin amplitudes have the same weak phase $\phi_M = \phi_f (0)$, for $B_s^r (B_s^f)$. Ditto for $\Delta M t$ and $\Delta M t$ decay, where only the penguin mechanism is possible. The name is true for the Cabibbo-suppressed $\Delta M t$ mode, which only gets contribution from the penguin diagram, the $B_s^r (B_s^f)$ phases are $\Delta M t$ in this case. The $\Delta M t$ and $\Delta M t$ moduli are not so simple, the two decay mechanisms have the same Cabibbo suppression ($\lambda^2$) and different weak phases, but the penguin amplitudes are down by $(\alpha_s / 2\pi) a_{\text{rel}} / (m_\gamma / m_\pi) \approx 3 \%$, these decay modes can be used as approximate measurements of the angles of the unitarity triangle. We have not considered doubly Cabibbo-suppressed decay amplitudes, such as $\Delta M t$, which for penguin effects can be important and spoil the simple estimates based on the direct decay mechanism.

Presumably the most realistic channels for the measurement of the angles $\theta = (\beta, \gamma, \eta) = B_s^r / J/\psi K_s, B_s^s \to \gamma e^+ e^-$ ($B_s^r / J/\psi K_s, B_s^s \to \gamma e^+ e^-$), using the unitarity of the CKM matrix and $B_s^r / J/\psi K_s, B_s^s \to \gamma e^+ e^-$, respectively. For all of these processes is no doubt the one with the clearest signature and the most tractable background. The last process has the disadvantage of requiring a $B_s^r$ meson and, moreover, its branching ratio is expected to be very small because the "direct" decay amplitude is colour suppressed, leading presumably to a much larger
penguin contamination; thus, the determination of $\gamma$ through this decay mode looks a quite formidable task. Alternative ways of measuring $\gamma$ will be discussed later.

The decay modes where $\Phi = 0$ are useless for making a determination of the angles of the unitarity triangle. However, they provide a very interesting test of the Standard Model mechanism of CP-violation, because the prediction that no CP-asymmetry should be seen for these modes is very clean. Any detected CP-violating signal would be a clear indication of new physics.

In Table 2 we have included final states with two spin-1 particles because their associated asymmetries do not provide directly any information on the weak CP phase. Although the decay mode $B^\pm \rightarrow J/\psi K^{*0}$, for instance, proceeds through the same quark diagrams as the $B^\pm \rightarrow J/\psi K^0$ one, the $K^0$, and hence $J/\psi K^*$, is not a CP-estate. Furthermore, the final state $J/\psi K^*$ contains several partial waves of varying CP parity, so that even if $K^{*0}$ were a CP-estate, this final state would still not be. Nevertheless, since spin-1 particles are unstable, one can obtain spin information - and therefore some handle on the CP properties of the final state - by looking to the angular distribution of their decay products [33]. One can select the CP = + component of the $K^{*0}$, $K^+$, through its decay into $K_{S}^{0}$; the helicity (0, 0) configuration of the $J/\psi K^*$ final state is a pure CP-estate, which can be disentangled from the others by looking to the angular distribution of the emerging $K_S^{*0}$ in the $K^*$ rest frame [33]. In this way, the decay modes $B^\pm \rightarrow J/\psi K^{*0}, K_S^{0}$ and $\psi' K^*$ could also become useful for increasing the statistical significance of the measurement of the angle $\beta$. Several angular-correlation techniques to separate the different CP contributions arising from different helicity terms have been discussed in ref. [34]. Nevertheless, to carry out an accurate angular analysis, one needs a much larger data sample than the one required for the true CP self-conjugate modes. Obviously such an analysis would not be necessary if the (0, 0) helicity state were to dominate; at present, there are some experimental evidence [35] indicating that the $J/\psi K^{*0}$ is indeed the case for $J/\psi K^{*0}$. A detailed study of $B \rightarrow VV$ modes can be found in refs. [36].

### Table 2: CKM factors and relevant angle $\Phi$ for some B-decays into CP-eigenstates.

<table>
<thead>
<tr>
<th>Decay mode</th>
<th>CKM factor (Direct decay)</th>
<th>CKM factor (Penguin)</th>
<th>Exclusive channels</th>
<th>$\Phi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B \rightarrow c\bar{s}$</td>
<td>$A_1^L$</td>
<td>$-A_1^L$</td>
<td>$B^\pm \rightarrow J/\psi K_S, J/\psi K_L$</td>
<td>$\beta$</td>
</tr>
<tr>
<td>$B \rightarrow d\bar{s}$</td>
<td>$-A_1^L$</td>
<td>$A_1^L$</td>
<td>$B^\pm \rightarrow K_S^0, K_L^0$</td>
<td>0</td>
</tr>
<tr>
<td>$B \rightarrow u\bar{s}$</td>
<td>$-A_1^L$</td>
<td>$A_1^L$</td>
<td>$B^\pm \rightarrow D^* \rightarrow J/\psi \pi^*$</td>
<td>$D^* \rightarrow J/\psi \pi^*$</td>
</tr>
<tr>
<td>$B \rightarrow \bar{d}\bar{s}$</td>
<td>$-A_1^L$</td>
<td>$A_1^L$</td>
<td>$B^\pm \rightarrow K_S^0, K_L^0$</td>
<td>0</td>
</tr>
<tr>
<td>$B \rightarrow \bar{d}\bar{s}$</td>
<td>$A_1^L$</td>
<td>$-A_1^L$</td>
<td>$B^\pm \rightarrow K_S^0, K_L^0$</td>
<td>$\beta$</td>
</tr>
<tr>
<td>$B \rightarrow s\bar{s}$</td>
<td>$A_1^L$</td>
<td>$-A_1^L$</td>
<td>$B^\pm \rightarrow K_S^0, K_L^0$</td>
<td>$\beta$</td>
</tr>
</tbody>
</table>

Let's consider as final state $f$ a pair of mesons $a_1$ and $a_2$, which differ in $J^{PC}$ or mass, so that $a_1a_2$ is a CP eigenstate, but such that at the quark level $a_1a_2 = \langle 0 | q \bar{q} | 0 \rangle$, so that $f$ is made up of a CP self-conjugate collection of quarks and antiquarks (for example $f = D^+ D^-, \bar{s} \bar{s}, \ldots$). It has been argued [37] that in this case $f$ is sufficiently close to being a CP eigenstate for CP-violating effects to be large. The CP-self-conjugacy of the quark content has the consequence that for each diagram contributing to $B^\pm \rightarrow a_1a_2$ there is a related one for $\overline{B}^\mp \rightarrow \overline{a}_1\overline{a}_2$, which is just the CP conjugate of the $B^+ \rightarrow a_1a_2$ diagram except for the way in which the final quarks are joined up to make hadrons. One could expect then both diagrams to be of comparable size. If in addition some single diagram dominates $A_1$, then its "almost-CP-conjugate" partner is expected to dominate $A_1$, and the amplitudes for these two decays should be of comparable size, as desired.

In such situation, we would have $A_1 = M e^{i\phi_D}$, and $A_1 = M e^{-i\phi_D} e^{i\psi}$, with $\phi_D = \phi_D$ and $M'/M \sim O(1)$.

$$|\phi_D| = |\phi_D| = M' \sim O(1),$$

$$\text{Im}(\phi_D) = M' \sin(2\phi - \Delta t),$$

$$\text{Im}(\psi_D) = -M' \sin(2\phi - \Delta t),$$

where $\phi = \phi_D + \phi_D$ and $\Delta t = \frac{2M}{M'} - \frac{2M}{M'}$.

According to formula (37), the three unknown quantities $M'/M$, $S = \sin(2\phi + \Delta t)$, and $t = \sin(2\phi - \Delta t)$ can be experimentally extracted from the measured time distributions of the time-dependent decay rates (Fig. B). One could then determine the phase $\Phi$, up to a four-fold ambiguity, using:

$$\sin^2(2\Phi) = \frac{1}{2} \left[ 1 + S^2 \pm \sqrt{(1 - S^2)(1 - S^2)} \right].$$

Here, one of the signs on the right-hand side gives the true $\sin^2(2\Phi)$, while the other corresponds to $\cos^2(\Delta A)$. In order to solve this ambiguity one clearly needs some information about the final-state interaction phases. Nevertheless, one could always use other decay modes (such as CP eigenstates) which yield perhaps a less precise but unambiguous value for $\sin^2(2\Phi)$. The additional information provided by
the $B^- \to \alpha \bar{B}_2$ decay mode would allow them to improve the accuracy of the $\Phi$ determination. For instance, the final states $\pi^0 \pi^0$, $\eta \eta$, $\eta' \eta'$ and $\eta_1^0 \eta_1^0$ could be useful to improve the information on the angle $\alpha$, obtained through the $\pi^+ \pi^-$ mode. The method described above can be generalized to final states that are not made up of a CP self-conjugate collection of quarks and antiquarks. It is in fact sufficient to have final states in which the quark, antiquark can be reached either from a $B^0$ or a $\bar{B}^0$ [30]. The final states $D^{\pm} K^{0\pm}$ or $D^{0\pm} K^\mp$ made up of the quarks ($\bar{c} \bar{u}$) and ($c \bar{d}$) produced in the $B^0 \bar{B}^0$ are good examples. It is interesting to note that for these particular final states, there exist no contributions from penguin diagrams and that the dominant spectator diagrams involve $\delta = e$ and $\delta = u$ transitions. Furthermore, it has been estimated that $|P_{\delta}| = |P_{\delta}| \sim O(1/2)$ allowing one to extract $\sin \gamma$, since $\Im \left\{ P_{\delta} \right\} \approx |P_{\delta}| \sin \gamma$ (with $\delta = \epsilon_0$) and $\Im \left\{ P_{\delta} \right\} \approx -|P_{\delta}| \sin \gamma$ (with $\delta = \epsilon_0$).

2.3.3 Elimination of penguin uncertainties through isospin analysis

In principle, it should be possible to disentangle the effects of the "direct" and "penguin" contributions to the $B \to \pi \pi$ modes, by using a standard isospin analysis [30], in complete analogy with the textbook case of $K \to \pi \pi$. Due to Bose statistics, the final $\pi \pi$ state can have only $I = 0$ or $I = 2$. While the QCD-corrected left-handed four-fermion terms contribute to (essentially a linear combination of the direct coefficients). The $\pi^+ \pi^-$ final state has $I = 2$ and therefore does not suffer from any "penguin" contamination. Writing the amplitudes for $B^- \to \pi^+ \pi^-$, $B^- \to \pi^+ \pi^-$ and $B^- \to \pi^+ \pi^-$ ($A^{++}$, $A^{+-}$, and $A^{--}$, respectively) in terms of the two isospin transitions $A_0$ and $A_1 [A^{++} = \sqrt{2}(A_0 - A_1), A^{+-} = A_0 + A_1, A^{--} = A_1]$ one gets the complex triangle relation

$$\frac{1}{2} A^{++} + A^{--} = A^{+-}.$$  

(41)

There is a similar triangle relation for the charged conjugate processes:

$$\frac{1}{2} A^{+-} + A^{--} = A^{++},$$  

(42)

where $A^{++}, A^{--}$ and $A^{+-}$ denote the amplitudes for the processes $\bar{B}^- \to \pi^+ \pi^-$, $\bar{B}^- \to \pi^+ \pi^-$ and $\bar{B}^- \to \pi^+ \pi^-$, respectively. The $\bar{A}$ amplitudes are obtained from the $A$ amplitudes by simply changing the sign of the CKM phase (the strong phase remains the same). Obviously, $|A^{++}| = |A^{+-}|$, because there is only a single CKM phase (the one corresponding to the "direct" transitions) in the $I = 2$ amplitudes.

The magnitudes of the decay amplitudes are in principle obtained experimentally. The $B^- \to \pi^+ \pi^-$ branching ratio directly determines $A^{+-}$, while for the neutral-decay amplitudes it is necessary to measure the time dependence of the decays into $\pi^+ \pi^-$ and $\pi^+ \pi^-$ of both $B^\pm$ and $B_0$. From the coefficients of the constant and the $\cos(\Delta M_1 t)$ terms [see Eqs. (35) and (59)], it is possible to extract $|A^{++}|, |A^{+-}|, |A^{--}|$.

Due to the existence of two different isospin amplitudes, with different CKM phases, the coefficients of the $\sin(\Delta M_1 t)$ terms are no longer given by the simple relation (31). One has instead:

$$\Im \left\{ \frac{3}{2} P_{\delta} \right\} \approx -\frac{1}{2} \Im \left\{ P_{\delta} \right\} \approx \Im \left\{ \frac{3}{2} P_{\delta} \right\} \approx \Im \left\{ \frac{3}{2} P_{\delta} \right\} = \frac{1}{2} \Im \left\{ \frac{3}{2} P_{\delta} \right\},$$  

(43)

where $P_{\delta} = A_0 / A_1$ and $A_{1 \mp} = A_0 / A_1$. In the limit in which penguin effects are neglected, $x = \epsilon$, and Eq. (31) is recovered.

The magnitude of $|A_0| = |A_1|$ is obtained directly from $|A^{++}|$. Simple geometrical considerations allow one to obtain $|A_0|$ and $\cos \theta$, where $\theta$ is the relative phase between $A_0$ and $A_1$, from the triangle (41). $|A_{1 \mp}|$ and $\cos \phi$ are similarly obtained from the triangle (42). Therefore, both $x$ and $\epsilon$ can be determined up to a two-fold ambiguity in the sign of the phases. This leaves a four-fold ambiguity in the determination of $\sin 2\alpha$ from any of the two Eqs. (43) and (44). Combining both equations, however, $\sin 2\alpha$ is unambiguously determined, except in the very special cases in which one of the ambiguities overlap (in such cases a two-fold ambiguity would remain).

The experimental implementation of this kind of analysis is by no means trivial and is discussed in the next section. Indeed, if colour suppression holds, one expects the branching ratio of $B^- \to \pi^+ \pi^-$ to be about one order of magnitude smaller than that of $B^- \to \pi^+ \pi^-$, Nevertheless, if such a suppression is experimentally verified, it would go further support to the naive expectation that the penguin contribution is small.

The isospin analysis has also been applied [40] to other decay modes ($B \to \pi K, \eta, \ldots$). However, since the number of relevant isospin amplitudes is usually larger than three, one encounters too many ambiguities in relating an asymmetry to a weak phase. The usefulness of such analysis is then rather doubtful.

2.3.4 $B \to D_s^0 \pi^0$ decays

An analysis similar to the one discussed in Section 2.2 for the transition $B^+ \to D_s^0 K^+$ can be carried out using the decay $B_0 \to D_s^0 K^+$ [41]. Two different amplitudes of comparable magnitudes contribute to this process, corresponding to the transitions $B_0^* \to D_s^0 K^+$ and $B^- \to D_s^0 K^+$, with the same CKM factors than $B^+ \to D_s^0 K^+$ and $B^- \to D_s^+ K^0$. However a slight complication arises from the fact that both $B^0$ and $B^- \to D_s^0 K^+$ may decay into $D_s^0 K_0^+$. This difficulty is overcome by measuring the time-dependence of the corresponding $B_0^*$ and $B^- K_0^+$ decays. One can then determine the magnitudes of the six amplitudes $|A_0|, |A_0|, |A_1|, |A_1|, |A_1|$, and $|A_1|$, [see Eqs. (19) to (21)]. Thus, one could test the predicted absence of direct CP-violation in the $B^- \to D_s^0 K^+$ and $B_0 \to D_s^0 K^+$ decays, and try to get a direct CP-violation signal in the $B_0 \to D_s^0 K^+$ mode. Furthermore, these same measurements will also determine the coefficients of the $\sin(\Delta M_1 t)$ terms:

$$\Im \left\{ \frac{3}{2} P_{\delta} \right\} \approx \frac{|A_0|^2 \sin(2\beta + \gamma + \Delta)},$$  

(45)

$$\Im \left\{ \frac{3}{2} P_{\delta} \right\} \approx \frac{|A_0|^2 \sin(2\beta + \gamma + \Delta)},$$  

(46)

$$\Im \left\{ \frac{3}{2} P_{\delta} \right\} \approx \frac{|A_0|^2 \sin(2\beta + \gamma + \Delta)},$$  

(47)

where $\Delta = \epsilon - \epsilon_0$. An equation similar to (21) describes a complex triangle relation, from which the angle between $A_0$ and $A_1$ can be obtained: $\cos(\gamma + \Delta) = (2(A_0)^2 + (A_1)^2)/(A_0 A_1) \approx 0$. A similar triangle relation for the charged-conjugate processes determines $C \approx \cos(\gamma - \Delta)$. In addition, from Eqs. (45) and (46) one gets the two quantities $S \approx \sin(2\beta + \gamma + \Delta)$ and $S \approx \sin(2\beta + \gamma - \Delta)$. With this information, it is possible to obtain the values of $\sin 2\beta$ and $\sin(2\beta + \gamma)$:

$$\sin 2\beta = \frac{C_2 + S_2 - C_2 - S_2 + 2 CS - CS}{2(CS - CS)},$$  

(48)

$$\sin(2\beta + \gamma) = \frac{C_2 + S_2 - C_2 - S_2 + 2 CS - CS}{2(CS - CS)},$$  

(49)

From the unitarity triangle one has $\sin 2(\beta + \gamma) = -\sin 2\alpha$. Therefore this determines $\sin 2\alpha$ and $\sin 2\beta$. The additional information in (47) can be used as a consistency check, or to solve the possible ambiguities [41] arising in the unlikely cases when one or both of the denominators in Eqs. (48) and (49) vanish. The kind of study can be performed with the CP-odd state $D_s$. Although this does not provide any new information concerning the unitarity angle, the statistics could be increased. Furthermore, instead of a $K_0$ one could equally well use the state $K^- \pi^+$ ($K^- \pi^-$). In fact, the method could be used (in principle) inclusively, i.e., it could be applied to the decays $B \to D_s^0 \pi^-, B \to D_s^+ \pi^-, B \to D_s^0 \pi^+$, where $X$ is any state with the flavour quantum number of a $K^\pm$. 
The third angle $\gamma$ could be obtained by doing the same type of analysis for the decays $B^+_s \to D^*\ell^+\nu$, $B^0 \to D^0\ell^+$ and $B^0 \to D^0\ell^-$. The only difference stems from the mixing phase $\arg\delta$; thus, taking $\beta = 0$, all previous equations can be also applied to these processes. Both $\sin 2\gamma$ and $\cos 2\gamma$ can be, in principle, determined in this way.

Decays like $B^+_s \to D^*\ell^+\nu$, $D^*\ell^+$ are in principle similar to the $B^+ \to D^\ell K_S$ ones: there are two possible amplitudes, corresponding to the $B^+ \to D^*\ell^+$ and $B^+ \to D^\ell K_S$ transitions. However, the CKM factors associated with these two transitions are very different. While the first amplitude is doubly Cabbibo suppressed $|V_{ub}|m_u/|V_{cb}|m_c$, the second one is Cabibbo allowed $|V_{ub}|m_u/|V_{cb}|m_c$; thus, $|A[B^+_s \to D^*\ell^+]| < |A[B^+_s \to D^\ell K_S]|$. If the $B^+ \to D^*\ell^+$ amplitude is neglected, the transition $B^+_s \to D^\ell K_S$ is another example of decay into a CP-conjugate final state dominated by a single amplitude [42]. The formulae in Section 3.2.1 applies then to this process, with $\phi = \beta$.

3 Experimental possibilities at an asymmetric $e^+e^-$ B factory

3.1 Typical detector description

Many designs of detectors dedicated to $B$ physics have been studied during the past few years [43-46]. Although optimized for the observation of CP-violation, all proposed detectors are trying to be as universal as possible in order to explore a wide range of physics with $B$ mesons. In this paper we will essentially refer to the HELIX proposal [45], done in parallel with those ECPA-group studies. The general layout and the individual components of a detector for an $e^+e^-$ collider benefit from years of experience. In particular ARGUS [47] at DESY and CLEO [46] at Cornell are good examples of detectors designed for $B$ physics. They are now classical and their performances can be improved, thanks to the developments in technology. The fundamental difference at an asymmetric $B$ factory is the extension of the apparatus in the direction of the high energy beam (Fig. 7), to take into account the Lorentz boost of the center of mass system. For a $3.33$ GeV/$c$ by $3.0$ GeV/$c$ collider, the boost will be $\gamma = 0.6$ corresponding to an average flight path of about $230$ m for the mesons created from the $\Upsilon(4S)$ resonance. From the interaction region to the outside, the active components of the detector are a silicon microstrip device, a central drift chamber, a Ring Imaging Cherenkov (RICH) counter, an electromagnetic calorimeter and muon chambers. With the exception of the last, all components are immersed in a uniform magnetic field of $1T$.

- **Vertex detector.** The main goal of this device is to measure the $B$ flight distance by identifying the decay vertex position. A good vertex reconstruction will further constrain the momentum determination and improve the background rejection. The various vertexes are reconstructed with three layers of double-sided silicon strip detectors located at a radial distance from the interaction point ranging from $27$ to $80$ mm and covering polar angles $\theta$ from $12^\circ$ to $150^\circ$. The intrinsic resolution is $10$ mm in both the longitudinal and transverse directions. The silicon detector represents $0.86\sin^2\theta$ of radiation length in the hadron region.

- **Central tracking Chamber.** The requirement of a high momentum resolution in a $1$ Tesla magnetic field sets the radial dimension of the tracking device to $1m$. Its elliptical shape is such that $0.25\%$ on $\sigma_{x/y}$ can be achieved for high energy particles. The active volume filled with propylene is enclosed in two cylinders of $13$ and $104$ cm, closed by truncated cones, covering $94.5\%$ of the total solid angle in the cm frame. The chamber consists of $46$ layers of stereo cells. Tracks with $p_T > 50$ MeV/$c$ have an opening angle relative to the beam direction of $\theta > 200$ mrad and originating from a small volume around the interaction point will be detected with full efficiency. The spatial resolution is on average $120\mu m$ in the direction perpendicular to the beams and $1.5$ mm along the beam. Particle identification is made by the $dE/dx$ measurement with a resolution of $4.4\%$ in the worst case (hadron case). This corresponds to at least $4\sigma$ separation for $x/K < 800$ MeV/$c$ and greater than $2\sigma$ in the relativistic region. In the radial and forward direction, the material seen by particles amounts to $3.5\%$ to $5.5\%$ radiation length, depending on the polar angle. In the backward direction (23% of $\theta$), the material corresponds to $0.94 X_0$.

- **RICH.** For $B$ physics, the identification of the particles over a wide range of momentum is crucial. The aim is to separate pions, kaons and protons issued in the $B$ decays up to the kinematical limit (about $4.5$ GeV/$c$ for pions momentum). A good separation of electrons relative to pions is also requested at low momenta. The proposed RICH device will be located behind the tracking chamber, between $1m$ and $1.2m$. It uses a CsI(Tl) liquid radiator and CsI phototubes. The identification threshold will be $0.18, 0.62$ and $1.18$ GeV/$c$ for pions, kaons and protons respectively. More than $3\sigma$ separation is expected between high momentum hadrons and also between leptons and pions with $P < 1$ GeV/$c$. One may note that the RICH option is the more delicate part of the detector as experience with this technique is still very limited and needs more R&D. An alternative but less performing device is pursued using a TOF with $50$ ps of time resolution. The total amount of material of the RICH detector is $0.162 X_0$.

- **The electromagnetic calorimeter.** In $B$ decays $5$ photons are produced on average. The measurement of their energy and direction is necessary with comparable precision as for charged particles. The calorimeter will be made of CsI crystals with a cross section of $5 \times 5 \text{cm}^2$ and a depth of $39$ cm ($2X_0$), allowing a good spatial resolution. It will detect photons of more than $30$ MeV with a resolution

$$\frac{\sigma_E}{E} \approx \sqrt{0.004^2 + \frac{(0.01)^2}{0.25^4}}.$$  

The resolution for electrons will be poorer at an energy less than $1$ GeV and better at higher energy.

- **Muon chambers.** Muons will be detected using two layers (3 in the forward direction) of chambers radially positioned at $3.2$ m and $3.5$ m from the beam and behind the iron absorber used as the return yoke for the magnets. The forward chambers are placed from $4.0$ m to $4.7$ m. Each chamber consists of double layer drift tubes for which the spatial resolution will be about $15$ mm in the direction.
perpendicular to the wires and about 28 mm along the wire. Only muons of more than 1 GeV/c will reach the chambers.

3.2 Experimental techniques

The main goal of the B Factory is to measure CP violation and test the CKM anatomy of the Standard Model through the measurements of the angles of the unitarity triangle. One of the most promising ways to measure CP violation in the b sector is to study B mesons decaying into CP eigenstates fcp. This requires the measurement of the time-dependent decay probability and the identification of the nature of the decaying B. At the T(4S), tagging one B as a B* or B̄ will sign the other with certainty.

3.2.1 Tagging methods

Two methods have been used in our studies. The first one relies on the semi-leptonic B meson decays, the second uses charged kaons from the main cascade K → e → μ. The positive charged particles tag B*, whereas the negative ones tag B̄.

- High momentum lepton tagging. Here we exploit the 21% branching fraction of the B mesons decaying in an electron or a muon. The following criteria are required:
  1. In the T(4S) rest frame, the track momentum must be greater than 1.4 GeV/c.
  2. The track seen in the central chamber has to be associated with an electromagnetic cluster (electrons) or hits in the muon chambers (muons). The electron (muon) is also identified whenever possible by its energy loss in the central chamber.
  3. The event is kept if there is only one lepton candidate.

Such a procedure has been followed in a study, the aim of which was to probe penguin contribution in B→K*γ [49]. A 11% tagging efficiency with a wrong sign-tag of ~ 4% has been found. This result is in agreement with other studies [44, 45].

One may note other attempts using low momentum leptons [44], in combination with missing momentum. However a higher wrong sign-tag of about 20% is found. Such a method could be used in the detector discussed above since the performance of the HCAL is expected to discriminate electrons from pions with P < 1.6 GeV/c and from muons with P < 1.2 GeV/c with 3σ separation. Muons of less than 1 GeV/c momentum could also be separated from pions.

- Kaon tag. Charged kaons are identified using dE/dx alone for low momentum particles (P < 0.4 GeV/c). For higher momentum dE/dx is used in combination with the HCAL. Events with either one or two like sign identified kaons are used.

In the study mentioned above, a 28% efficiency with 10% wrong tag is found. This result agrees with similar studies [44]. All tagging efficiencies are detailed in Table 3 together with the the wrong tagging probability. These results are extracted from [49].

Currently one cannot envisage to use fully reconstructed B mesons for tagging purpose due to the very small efficiency (~ 0.1%). However partial reconstruction can be used in a study made for the CESR B project [44]. Final states of the kind (D)π/K→μ for which the missing particle is a D* or a D** meson and where the detected particles are pions have been considered. Despite the lack of knowledge on most of the B decay modes, a sizeable efficiency of about 5% may be achieved with a very low contamination from wrong tag (1%). This tagging method has to be further investigated.

3.2.2 Δt measurement

The B̄B system issued from the T(4S) decay is produced in a coherent state. The B decay probabilities depend on the time difference between the decays of the two B's. Since B mesons are almost produced as rest in the center of mass frame they will essentially have no transverse momentum in the laboratory. Therefore, the time information is obtained by observing the B vertices and measuring their spatial separation along the z direction:

\[ Δt = γ' c Δt + γ' B̄ c \cos(θ)(t_1 - t_2), \]

where Δt = t1 - t2 and c is the velocity of light.

γ', B̄ define the boost of the T(4S) in the laboratory frame.

γ, B define the boost of the B mesons in the T rest frame, with γ = 0.065, B = 1.

B is the polar angle of the B in the T rest frame.

This measurement relies on the tracking detector performances but is also part of the characteristics of the collider as we will see.

- Boost optimization. In an ideal world a large boost would lead to an easier measurement for Δt since r(Δt) < Δt > will be negligible. However besides the technical difficulties for the collider, the acceptance will be decreased significantly for a given solid angle because particles will move closer to the beam direction. Fig. 8 shows the degradation of the resolution on sin(2θ) as function of the high energy beam Ebeam assuming different values for the resolution on Δt [50]. From this figure one sees that Ebeam ranging from 8 to 9 GeV is safe enough, with little influence of the tracking resolution. This range covers effectively the different proposals of B Factories [43-45]. Similarly a small increase (≤ 5%) of the error on sin(2θ) is expected with a beam pipe radius of 4 cm instead of 2.5 cm [50].

- Δt measurement. For each event in which one B meson has been fully reconstructed, we measure the B vertex position with a fitting procedure using all charged tracks belonging to its decay. As noticed earlier this vertex lies near the z axis, so that the z coordinate can be deduced to a good approximation from the vertex position. The basic problem concerns the determination of the position of the second B vertex. Here one may follow a method independent of the tagging [49]. The fitted vertex of the reconstructed B candidate defines the origin of a "B" track. A vertex finder algorithm is then applied with the remaining tracks and the "B" track. In 80% of the events a unique vertex is found, while 10% have two vertices. For the single vertex sample, the corresponding coordinates are identified to the second B vertex position. When several vertices are found we use the one which lies on the "B" track if there is no ambiguity, otherwise vertices are ordered along the direction of the high energy beam and the first one along that direction is taken. The difference between the measured and generated Δt distribution (reconstructed vertex - tagging vertex) is shown in Fig. 9. The resolution obtained is 65 μm.
3.3 Asymmetry measurements and sensitivity to CP violation

It has been shown in Section 2 that the B-decay modes useful to observe CP violation can be classified in two categories: those for which the final states are flavour specific to the decaying B and those which are not.

3.3.1 Flavour-specific final state

For this category of events one does not need any tagging as the observed mode is self-tagging. The time measurement is not necessary for the class of events revealing Direct CP violation. For the other class which involves B*B mixing, a time-dependent measurement may be done but CP violation could also be observed from integrated decay rates.

- **CP violation due to mixing.** As seen in Section 2.1 these CP violating effects can be observed by a simple comparison between the number of positive and negative like-sign lepton pairs [Eq. (11)]. We can estimate the number of B*B events required to measure an asymmetry at the 3σ level. The sample size is N = N_{BG} Br^2 c^2 σ/2, where Br ≈ 20% is the semi-leptonic branching ratio. Taking c = 50% for the high-energy lepton efficiency and the mixing ratio r = (2/\sqrt{3}) ≈ 0.18, we get N_{BG} ≈ 10^3/A^2, where A is the asymmetry to be measured. Unfortunately this asymmetry is predicted to be very small, at the 10^{-9} level [Eq. (10)], N_{BG} ≈ 10^1 is needed.

- **Direct CP violation.** For a given number of produced B*B at the Υ(4S), CP violation is established by measuring an asymmetry between the B → f and B → f' decay rates, where f is the CP conjugate of the final state f. Charged B mesons are the most practical candidates but B*B may be used as well (e.g. B*B → K^+π⁻). We show in Table 4 some final states with their expected branching fraction [21,51], the range of the estimated asymmetry and the corresponding experimental sensitivity to a CP violation at 50 fb⁻¹-integrated luminosity.

<table>
<thead>
<tr>
<th>Modes</th>
<th>Branching fraction</th>
<th>Expected asymmetry</th>
<th>A &gt; 3σ (100 fb⁻¹)</th>
</tr>
</thead>
<tbody>
<tr>
<td>B^+ → K^+π⁻</td>
<td>~10⁻⁵</td>
<td>(0.1-1.1)%</td>
<td>≥30%</td>
</tr>
<tr>
<td>B^+ → K^-π⁺</td>
<td>~10⁻⁵</td>
<td>(0.1-1.1)%</td>
<td>≥30%</td>
</tr>
<tr>
<td>B^- → K^-π⁺</td>
<td>~10⁻⁵</td>
<td>(0.1-1.1)%</td>
<td>≥8%</td>
</tr>
<tr>
<td>B^- → K^-π⁺</td>
<td>~10⁻⁵</td>
<td>(1.0-1.3)%</td>
<td>≥8%</td>
</tr>
</tbody>
</table>

Table 4: Examples of final states sensitive to direct CP violation.

B*B events are necessary. Nevertheless probing CP non-conservation with these final states is very important as it can give evidence for direct CP violation and/or could indicate new physics. Finally one should remember that measurements of direct CP violation will not translate easily to the measurement of the angles α, β or γ because of the final-state interactions.

3.3.2 Non-flavour-specific final state

As discussed in Section 2.3, for this category of events CP-violation can be established by computing the decay rates of B and B*B mesons to a common final state f once the nature of the second B is tagged by its decay into a flavour-specific state. The most obvious candidates are those where f are CP eigenstates (fCP). CP-violating effects can be determined following two methods:

1. by measuring an asymmetry between the time integrated number of events n_f = n_1 + n_2 and n_f = n_1 - n_2 as given in Table 5.

<table>
<thead>
<tr>
<th>Δt = t_2 - t_1</th>
<th>n_1</th>
<th>n_2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Δt = 0</td>
<td>n_1</td>
<td>n_2</td>
</tr>
</tbody>
</table>

Table 5: Event rates for fCP final states.

The integrated asymmetry is then [see also (33)]:

\[ A \equiv \frac{n_+ - n_-}{n_+ + n_-} \approx 2\eta \sin(2\phi), \]

where \[ \eta \equiv \frac{\sin(2\phi)}{1 + \sin(2\phi)} \approx 0.47 \] is the dilution factor for B_s mesons.

2. by simultaneously fitting the Δτ distributions for the two samples n_1 and n_2 [52]. A 20% smaller measurement error is achieved for sin(2φ) with this method.

Decays to CP-eigenstates are not the only class of interesting events. Non CP-eigenstates can be used with a slightly more complicated analysis. Those events can be separated in two classes. In the first class, the final states are made of a CP self-conjugate collection of quarks (see Section 2.5.2); as an example is given by the B*B → f final states [51]. As for fCP, a global fit of the time-dependent distributions allows one to extract sin(2φ) [Eq. (40)] up to a four-fold ambiguity due to final-state interactions. The second class of events contains the modes in which different partial waves contribute with different CP parity, like B_s → J/ψK^*⁺. Here, one has to carry out an angular analysis in order to sort out the various CP contributions. Details on the measurement procedure can be found in [53,54].
All these measurements rely on a good tagging efficiency (~40%) and a precise vertex detection as expected with the proposed detector (< $\Delta x > / \Delta L_x > c > 0$). In some cases, a clean measurement of the angles of the unitary triangle requires to separate the contributions from different decay amplitudes. For example in $B \rightarrow \pi x$, the measurement of $\alpha$ may be spoiled by the contribution of the penguin amplitude. Theoretically, this problem can be overcome by doing an isospin analysis with the use of the decays $B^+ \rightarrow \pi^+ \pi^-$, $B^0 \rightarrow \pi^+ \pi^-$, $B^+ \rightarrow \pi^+ x^+$ and their CP conjugates (see Section 2.3.3). In [46], such an analysis has been performed to study its feasibility. Assuming an integrated luminosity of 100 $fb^{-1}$ it is found that, if the penguin diagrams dominate in the $\pi^+ x^+$ mode (the spectator diagram is colour-suppressed), an upper limit of 0.22 can be set on the ratio of the penguin amplitude to the spectator one. This corresponds to a limit on the branching fraction $Br(B \rightarrow \pi^+ \pi^-) < 10^{-4}$ at 95% confidence level. The uncertainty on $\sin 2\alpha$ will suffer from the limited measurement accuracy on the isospin amplitudes $A_1$ and $A_2$ and their relative phase, which reflects by a 60% increase on the error $\sigma(\sin 2\alpha)$ with a value of 0.16.

In Table 6, the expected number of events belonging to the classes mentioned above are given for an integrated luminosity of 100 $fb^{-1}$.

<table>
<thead>
<tr>
<th>Modes</th>
<th>Branching fraction</th>
<th>Overall efficiency</th>
<th>N events</th>
<th>$B/S$ measured</th>
</tr>
</thead>
<tbody>
<tr>
<td>$J/\psi K_S$</td>
<td>$4 \times 10^{-4}$</td>
<td>3.0%</td>
<td>$\sim 1180$</td>
<td>$10^{-2}$</td>
</tr>
<tr>
<td>$\omega K_S$</td>
<td>$1 \times 10^{-3}$</td>
<td>0.6%</td>
<td>$\sim 520$</td>
<td>$10^{-3}$</td>
</tr>
<tr>
<td>$\phi K_S$</td>
<td>$9 \times 10^{-4}$</td>
<td>0.7%</td>
<td>$\sim 400$</td>
<td>0.1</td>
</tr>
<tr>
<td>$D^q \bar{D}^q$</td>
<td>$5 \times 10^{-4}$</td>
<td>4.0%</td>
<td>$\sim 200$</td>
<td>0.1</td>
</tr>
<tr>
<td>$\pi^+ \pi^-$</td>
<td>$2 \times 10^{-3}$</td>
<td>29.0%</td>
<td>$\sim 640$</td>
<td>0.1</td>
</tr>
<tr>
<td>$K^+ K^-$</td>
<td>$6 \times 10^{-4}$</td>
<td>19.0%</td>
<td>$\sim 1160$</td>
<td>0.2</td>
</tr>
<tr>
<td>$a_1^0 a_1^0$</td>
<td>$6 \times 10^{-4}$</td>
<td>8.8%</td>
<td>$\sim 520$</td>
<td>0.5</td>
</tr>
<tr>
<td>$D^{*0} \bar{D}^{*0}$</td>
<td>$2 \times 10^{-3}$</td>
<td>0.2%</td>
<td>$\sim 420$</td>
<td>0.3</td>
</tr>
<tr>
<td>$J/\psi K_S \pi^0$</td>
<td>$4 \times 10^{-4}$</td>
<td>1.3%</td>
<td>$\sim 500$</td>
<td>$10^{-2}$</td>
</tr>
<tr>
<td>$\omega K_S \pi^0$</td>
<td>$2 \times 10^{-4}$</td>
<td>0.3%</td>
<td>$\sim 60$</td>
<td>0.1</td>
</tr>
<tr>
<td>$D^{*0} \bar{D}^{*0}$</td>
<td>$2 \times 10^{-3}$</td>
<td>0.2%</td>
<td>$\sim 420$</td>
<td>0.3</td>
</tr>
<tr>
<td>$\rho^0 \rho^0$</td>
<td>$5 \times 10^{-4}$</td>
<td>8.4%</td>
<td>$\sim 420$</td>
<td>0.2</td>
</tr>
</tbody>
</table>

Table 6: Expected number of reconstructed and tagged events for measuring $\alpha$ and $\beta$. The overall efficiency includes the reconstruction efficiency, the tagging efficiency as well as the branching fraction for the studied mode. $B/S$ is the ratio of the background over the signal.

### 3.3.3 Luminosity requirement

The resolution on $\sin(2\alpha)$ obtained from the fit of the time dependent distributions is given by [37]:

$$
\sigma(\sin 2\alpha) \simeq \frac{C}{\mathcal{M}(1-\mathcal{M})} \frac{1 + \eta B/S}{N} \exp\left[\frac{1}{2} \left(1 + \frac{1}{\Delta x^2}\right) \exp\left(\frac{1 + \frac{1}{2}}{2}\right) \right] (1 + O(\sin^2(2\alpha))),
$$

where $C = (1 + 4\mathcal{M})/2\mathcal{M}$ [3 for $B_1$], $\mathcal{M}$ is a dilution factor for non CP-eigenstates, $\omega$ is the fraction of $\phi B$ with a wrong tagged $B$ meson. The rejection factor $\eta$, is due to the shape difference between the $\Delta$ distribution of the background events ($B$) and the signal events ($S$). It depends on the $\Delta$ resolution and is $\approx 0.20$ for the continuum $\phi B$ background. Finally $N$ is the total number of selected $B \bar{B}$ and the exponential factor accounts for loss in resolution due to finite vertex resolution, expressed in unit of $L_{	ext{int}}$:

$$
\sigma(\sin 2\alpha) \approx \sigma(\Delta x)/L_{	ext{int}} \approx 0.3.\text{ This factor corresponds to } \approx 4.6\text{% correction. We give in Table 7 and 8 the expected errors on } \sin 2\alpha \text{ and } \sin 2\beta \text{ for an integrated luminosity of } 100 \text{ }fb^{-1} \text{[46].}
$$

<table>
<thead>
<tr>
<th>Modes</th>
<th>$J/\psi K_S$</th>
<th>$D^q \bar{D}^q$</th>
<th>$J/\psi K_S \pi^0$</th>
<th>$\omega K_S$</th>
<th>$\phi K_S$</th>
<th>$D^q \bar{D}^q$</th>
<th>$\rho^0 \rho^0$</th>
<th>All</th>
</tr>
</thead>
<tbody>
<tr>
<td>d</td>
<td>1</td>
<td>1</td>
<td>0.7</td>
<td>0.045</td>
<td>0.122</td>
<td>0.106</td>
<td>0.049</td>
<td></td>
</tr>
<tr>
<td>$\sin 2\phi$</td>
<td>29.0</td>
<td>3.6</td>
<td>4.8</td>
<td>34.4</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 7: Expected errors on $\sin 2\phi$.

<table>
<thead>
<tr>
<th>Modes</th>
<th>$\pi^+ \pi^-$</th>
<th>$\rho^0 \rho^0$</th>
<th>$\omega K_S$</th>
<th>$a_1^0 a_1^0$</th>
<th>All</th>
</tr>
</thead>
<tbody>
<tr>
<td>d</td>
<td>0.001</td>
<td>0.002</td>
<td>0.161</td>
<td>0.049</td>
<td></td>
</tr>
<tr>
<td>$\sin 2\alpha$</td>
<td>63.4</td>
<td>14.1</td>
<td>21.6</td>
<td>22.6</td>
<td></td>
</tr>
</tbody>
</table>

Table 8: Expected errors on $\sin 2\alpha$.

Let us define an effective number of events $N_{eff}$, for each decay channel and per $fb^{-1}$ (Tables 7, 8):

$$
N_{eff} = N \frac{d}{1 + \eta B/S}.
$$

The minimum luminosity needed in order to observe CP violation at the level of $\sigma$ standard deviations is

$$
L \simeq \frac{C}{(1-\mathcal{M})^2} \frac{1}{N_{eff} fb^{-1}} \sin^2(2\alpha),
$$

With a top mass of 140 GeV, the combined fit of the CKM parameters obtained by Ali and London [33], gives $\sin 2\beta = 0.27$ (0.30) and $\sin 2\alpha = 0.67 (0.76)$ for $fb/\mathcal{M} = 130 \text{ (200)}$ MeV. Taking these values, an integrated luminosity of 20 $fb^{-1}$ (about 6 months of running at an asymmetric $B$ factory with a peak luminosity of $3 \times 10^{32} cm^{-2} s^{-1}$) would be needed to measure $\sin 2\beta \neq 0$ at the 3rd level if $\sin 2\beta = 0.27$, while a rather modest luminosity of $4 \text{ to } 5 \text{ }fb^{-1}$ would be sufficient to probe all the allowed fitted values. However, being very conservative, the measurement of the minimum possible value for $\sin 2\beta (\geq 0.1)$ would require an integrated luminosity of 140 $fb^{-1}$ using all the decay channels studied in Tables 7 and 8.

Amongst the three angles of the unitarity triangle, the angle $\gamma$ is the most difficult to measure at an asymmetric $B$ factory. A promising method involves the decays of the $D^0$ meson into $D_s^0 K^{(*)0}$ or $D_s^{*0} \phi^0$. Since the energy for the $T(45)$ is below the threshold of $B \bar{B}$ production, one needs to run at the $T(58)$. Unfortunately the cross section for the $B \bar{B}$ production is about 0.1 fb (~10 times smaller than the $B \bar{B}$ production rate at the $T(45)$). Therefore one is led to study the decays $B^+ \rightarrow D^0 K_S$ or $B^0 \rightarrow D^* K_S$ following

1. A big $fb$ value around 100 - 200 MeV is obtained in nearly all modern calculations. Thus, the second solution, which leads to bigger CP asymmetry, should be preferred.
the methods discussed in Sections 2.2 and 2.3.4. Because the branching fractions for these decay modes are expected to be of the order of $10^{-9}$ and since one needs to observe the $D^*$ decay into CP eigenstates to extract $\gamma$, the statistics for the interesting events will be rather limited with an integrated luminosity of $10^{36}$ $b^{-1}$. A study has been carried out in more detail for the KEK project [46]. Their conclusion is that the angle $\gamma$ can be determined with an accuracy of $25^\circ$ over a wide range of values of $\gamma$ if the difference $\Delta$ in the strong phases (Eqs. (46) to (47)) is relatively high ($\simeq 5^\circ$) and if the ratio of the amplitudes $|A(B^+ \to D^+K^+)|/|A(B^\to D^+K^0)|$ is larger than $0.25$. A better precision of about $15^\circ$ or less can be achieved by studying the charged $B$ meson decays to the extend that $|A(B^+ \to D^+K^+)|/|A(B^\to D^+K^0)|$ is sufficiently large ($\geq 0.25$). However, this last condition may not be fulfilled because the amplitude $B^+ \to D^+K^+K^+$ is colour suppressed whereas the $B^+ \to D^+K^+K^0$ amplitude is not. Therefore the ratio of the amplitudes is presumably $\leq 0.1$ and the uncertainty on the angle $\gamma$ may well be twice as large.

4 CP violation beyond the Standard Model

The study of CP violation in the $B$ meson system provides an important tool to search for physics beyond the SM. Indeed, some of the extensions of the SM contain new sources of CP violation thus leading to predictions for CP asymmetries in $B$ decays which differ from those of the SM [54].

4.1 Introducing new quarks

We consider a simple extension [55] of the SM where, in addition to the three standard generations of quarks, there are charge $\frac{1}{2}$ SU(2)$_L$-singlet quarks. The addition of isosinglet quarks to the SM provides a possible connection [56] between CP breaking at a high-energy scale and the observed CP violation at low energies and, furthermore, it gives a simple solution to the strong-CP problem [57,58]. The most salient implications of this class of models for $B$ physics stem from the fact that, with the introduction of isosinglet quarks, there are Z-mediated flavour-changing neutral currents (FCNC) and deviations from unitarity in the CKM matrix. For simplicity, let us assume that there is only one charge $\frac{1}{2}$ isosinglet quark and choose the weak basis where the up quarks are diagonal. Then the charged-current mixing matrix is just a $3 \times 4$ submatrix of the 4-dimensional unitary matrix $U$ which diagonalizes the down quarks. The charged-current interactions can be written:

$$L_W = \frac{g}{\sqrt{2}} [\bar{Q}_L V_{eq}^T d_L W_R^e + h.c.],$$

where $V_{e} = U_{\alpha q}$ with $i = 1, \ldots, 3$ and $\alpha = 1, \ldots, 4$.

The neutral-current interactions are given by:

$$L_Z = \frac{g}{2 \cos \theta_W} [\bar{Q}_L \gamma^\mu u_L - Z_{d\alpha} L \bar{d}_L \gamma^\mu d_L - \sin^2 \theta_W Z_{u\alpha} L \bar{u}_L \gamma^\mu u_L] \not{e}_\mu,$$

where $Z_{\alpha \beta} = \langle V^T V \rangle_{\alpha \beta} = \delta_{\alpha \beta} - U_{\alpha d} U_{\beta d}$.

It is clear from (55) that FCNC are closely connected to deviations from unitarity in the CKM matrix. More explicitly, one has:

$$V^\alpha_L V_{e\alpha} + V^\alpha_L V_{d\alpha} + \bar{V}^\alpha_L V_{d\alpha} = Z_{\alpha e},$$
$$V^\alpha_L V_{\alpha} + V^\alpha_L V_{d\alpha} + \bar{V}^\alpha_L V_{d\alpha} = Z_{\alpha d},$$
$$V^\alpha_L V_{e\alpha} + \bar{V}^\alpha_L V_{e\alpha} = Z_{\alpha e}.$$

4.1.1 Experimental restrictions on FCNC

It is well known that FCNC couplings are strongly constrained by experiment. For example, Z-exchange contributes at tree level to the $K^*-K^0$ transition, and thus the size of the $K_s-K_s$ mass difference puts a limit on $|\text{Re}(Z_{d\alpha})|$ while the CP-violating parameter $\epsilon_K$ leads to a limit on $|\text{Im}(Z_{d\alpha})|$. Altogether one has [59]:

$$|\text{Re}(Z_{d\alpha})| \leq 2.4 \times 10^{-5},$$
$$|\text{Im}(Z_{d\alpha})| \leq \min\{6.4 \times 10^{-4}, 1.3 \times 10^{-3}/|\text{Re}(Z_{d\alpha})|\}.$$

On the other hand, $Z_{\alpha d}$ and $Z_{\alpha e}$ are constrained by a recent experimental search [60] for the decays $B \to X_s \mu^+\mu^-$ by the UA1 collaboration, which has led to the upper bound $BR(B \to X_s \mu^+\mu^-) \leq 6 \times 10^{-5}$. In the SM, this ratio is predicted [61] to be $BR(B \to X_s \mu^+\mu^-) = (6-9) \times 10^{-4}$, thus one order of magnitude smaller than the present limit. From the UA1 upper bound one derives the limits [62]:

$$|Z_{\alpha d}/V_{ud}| \leq 0.029,$$
$$|Z_{\alpha e}/V_{td}| \leq 0.029.$$

We will see next that, in spite of the stringent bounds of (61), FCNC can contribute significantly to $B^\pm\bar{B}^\mp$ mixing, and thus lead to predictions for CP asymmetries in $B$ decays which are significantly different from those of the SM.

4.1.2 CP asymmetries in $B^\pm\bar{B}^\mp$ decays

Let us assume that the off diagonal element of the $B^\pm\bar{B}^\mp$ system is modified by a factor $\Delta_{s\alpha}$ as a result of a new contribution from physics beyond the SM:

$$M_{12} = M^{(S)}_{12} \Delta_{s\alpha},$$

where $M^{(S)}_{12}$ is the SM box-diagram contribution. Obviously, the new contribution will affect the predicted CP asymmetries. We will restrict the discussion to decays into CP self-conjugate final states and, moreover, we will assume that all amplitudes contributing to the decay have the same CKM phase. In this case, the CP-odd asymmetry is given by:

$$\frac{\Gamma(B^\pm \to f) - \Gamma(B^\mp \to f)}{\Gamma(B^\pm \to f) + \Gamma(B^\mp \to f)} = \eta \sin \Delta M t \sin \left(\Phi + \arg \Delta_{s\alpha} \right) \approx \eta \sin \Delta M t \sin \phi,$$

where $\Phi$ is the SM phase defined in (31).

There are two possible sources which may change the SM prediction:

(i) The presence of the phase $\arg \Delta_{s\alpha}$, which determines the deviation from the box-diagram contribution. It is possible to incorporate different new-physics contributions for $B^\pm$ and $B^\mp$, if $\arg \Delta_{s\alpha} \neq \arg \Delta_{s\alpha}$.

(ii) Although the expression for $\Phi$ is the one given by the SM, its actual numerical value may differ from the SM prediction. This is due to the fact that models beyond the SM allow in general for a different range of CKM matrix elements.

In the model under consideration the new contribution to the $\Delta B = 2$ effective Hamiltonian arises from $Z$-exchange tree graphs. One readily obtains [63]:

$$\Delta_{s\alpha} = 1 + i \epsilon \frac{\alpha_{s\alpha}}{4 \pi \sin^2 \theta_W},$$

where

$$\epsilon = \frac{1}{\nu E(x_\ell)} \left| \frac{Z_{s\alpha}}{V_{td} V_{tq}^*} \right|^2, \quad \epsilon_{s\alpha} = \arg \left( \frac{Z_{s\alpha}}{V_{td} V_{tq}^*} \right).$$

Here, $\nu = \alpha/4\pi \sin^2 \theta_W$ and $E(x_\ell)$ is the Inami-Lim [19] function for the SM top-quark box diagram (see Eq. (12)). For $m_t = 140$ GeV, one has $\nu E(x_\ell) = -0.0045$. 

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The experimental value of $B_d^- \rightarrow \phi K^0$ mixing fixes the allowed range for the product $|V_{td} V_{td}^*| \Delta m_d$, through the relation

$$|V_{td} V_{td}^*| \Delta m_d = \frac{G_F^2 m_d}{4 \pi^2} \left( \frac{1}{r} \right) \frac{1}{|\Delta m_d|},$$

(66)

where we have followed the standard notation. For a given value of $r$, $\Delta m_d$, one obtains $|V_{td} V_{td}^*| \Delta m_d$ from (64), and then (66) fixes the allowed range of $|V_{td} V_{td}^*| \Delta m_d$.

It is convenient to consider two distinct cases of interest in the study of CP asymmetries:

(a) $r_0 > 1$. Z-exchange gives the dominant contribution to $B_d^- \rightarrow \phi K^0$ mixing.

(b) $r_0 = 1$. Z-exchange and box diagrams give comparable contributions to $B_d^- \rightarrow \phi K^0$ mixing.

Figure 10: The unitarity quadrangle in the $B_d^-$ sector.

At this stage it should be pointed out that the bounds of (61), together with the quadrangular relations of Eqs. (60), imply (62) that in the case of $B_d^-$ the dominant contribution to the mixing may arise from Z-exchange while in the case of $B_s^-$, Z-exchange can at most compete with the box-diagram contribution.

Case (c) was studied in ref. [60], where it was shown that the CP asymmetries $\alpha_{\phi K_S^0}$, $\alpha_{\phi \bar{K}_S^0}$ do not longer measure the angles $\beta$ and $\alpha$, as it happens in the SM, but the angles $\beta'$ and $\alpha'$ defined by Fig. 10, where a geometrical interpretation of (60) is given. Case (d) is more involved and it was studied in ref. [62].

The main point is that even for relatively small values of $r_0$ (i.e., non-dominant contribution from the Z-exchange diagrams) the predictions for the CP asymmetries differ substantially from those of the SM. This is illustrated in Fig. 11 where we have plotted the regions in $r_0$, $\Delta m_d$ space where the model predicts the sign of the asymmetry $\alpha_{\phi K_S^0}$ to be opposite to the one predicted in the SM.

Recently, it has been pointed out [63, 64] that in the SM there is a strong correlation between the CP-violating asymmetries in the decays $B_d^- \rightarrow \phi K_S^0$ and $B_s^- \rightarrow \pi^-\pi^0$, leading to a very restricted allowed region for these asymmetries. The study of these correlations is thus a powerful tool to test for new physics, since models beyond the SM may give drastically different correlations. This is illustrated in Fig. 12 where the correlations for $\alpha_{\phi K_S^0}$ and $\alpha_{\phi \bar{K}_S^0}$ are given for the SM, the superweak model, and the model with an isosinglet quark.

Although we have considered the appearance of FCNC within the context of a specific extension of the SM, it is clear from our discussion that most of our analysis is model-independent and illustrates the great impact that FCNC may have on B physics. The measurement of CP asymmetries in B-meson decays can either provide experimental evidence for the couplings $Z_{td}$, $Z_{ts}$, or put stringent bounds on them.

4.2 Introducing more Higgs doublets

If one extends the SM by introducing more than one Higgs doublet, it is possible to have CP violation mediated by scalar exchange [65]. With more than one Higgs doublet, scalar couplings to fermions in general no longer conserve flavour. If one implements natural flavour conservation (NFC) in the Higgs sector through an extra symmetry of the Lagrangian [66], then one needs three Higgs doublets in order to have spontaneous-C.P. violation [67]. In this case, the CKM matrix is real and CP violation arises only from Higgs exchange [67]. On the other hand, if one does not require NFC in the Higgs sector, one can have spontaneous-C.P. violation with just two Higgs doublets. In this class of models there will be in general two sources of CP violation: the usual CKM mechanism, and flavour-changing neutral Higgs interactions contributing at tree level to superweak-type $\Delta S = 2$ and $\Delta S = 1$ transitions. This is a simple realization of a general scenario, where one assumes that the decay amplitudes are correctly given by the SM, but there may be new physics contributing to the off-diagonal matrix elements in the $K^+\pi^-$ and $B^0\bar{B}^0$ mass matrices.

The implications of this hypothesis for B physics were recently analyzed by Soares and Wolfenstein [68], who have considered various possibilities for adding the SM and superweak-type contributions. One possibility is the one proposed in the original superweak interaction, where there is a new CP-violating contribution to $M_{13}$ in the $B_s$ sector, but no superweak contribution in the $B^+$ system. In this case, the main impact of the new contribution is to relax the constraint on $\eta$ derived from the experimental value of $\Delta m_{K_S^0\bar{K}_S^0}$. This in turn has an important effect on correlations between the $\alpha_{\phi K_S^0}$ and $\alpha_{\phi \bar{K}_S^0}$ asymmetries, which can differ drastically [63] from those predicted by the SM. Another possibility which is considered in [63], is having a new contribution to the real part of $M_{13}$ in the $B_s^+$ sector. This is analogous to the situation considered in Section 4.1, within the context of models with isosinglet quarks. Again important departures from the SM predictions can arise [63].

4.3 Supersymmetry

In the minimal supersymmetric (SUSY) model [69, 70] (i.e., the one which emerges from the low-energy limit of the minimal $N = 1$ supergravity theory) the superbox diagrams give new contributions to $B^0\bar{B}^0$ mixing, but the weak couplings contain the same CKM factors, namely $V_{td}^2 V_{td}^* V_{td} V_{td}^*$. As a result, the SM predictions for CP asymmetries are not modified. However, if one considers non-minimal SUSY extensions, then significant departures from the SM predictions can arise [70].
Figure 12: $a(x^2 r^2)$ versus $a(J/u K_a)$ for $r_2 = 0.6$. Several values of $a_2$ are explicitly indicated. The solid line gives the predictions of the model with FCNC with the choice of central values for the parameters and $m_t = 140$ GeV. The dashed line gives the superweak prediction and the dotted line encloses the allowed region in the SM, as obtained in [63], when the penguin contribution to $a(x^2 r^2)$ is considered.

5 Summary

The SM naturally incorporates a mechanism to generate CP violation. With three fermion generations, the most general form of the CKM quark-mixing matrix contains a single phase, which should be at the origin of all CP-violation phenomena in electroweak interactions. So far, this explanation is in agreement with the known experimental observations. The SM makes very specific predictions for CP asymmetries in $B$ mesons, which are directly related to the angles of the unitarity triangle whose sides are already constrained by independent data. However, the correctness of the CKM mechanism cannot be established with present data. We still don’t know whether the observed violation of the CP symmetry in the decay of neutral kaons has its origin in the $\Delta S = 1$ charged-current vertex, as expected in the SM, or if it is a specific property of the neutral kaons due to some unknown $\Delta S = 2$ interaction (superweak scenario). Like fermion masses and quark-mixing angles, the origin of the CKM phase lies in the most obscure part of the SM Lagrangian: the scalar sector. Obviously, CP violation could well be a sensitive probe for new physics beyond the SM. Indeed, there are simple extensions of the SM which may have drastically different values for the CP asymmetries and their correlations than those predicted by the minimal SM. If the experimental value of these CP asymmetries were to show that there is need for physics beyond the SM, then the identification of its source would require some further experimental evidence like the discovery of non-minimal SUSY or the detection of either $Z$-mediated or Higgs-mediated flavour-changing neutral currents. Needless to say, either one of the above possibilities is extremely interesting.

With only two generations, the quark-mixing mechanism cannot give rise to CP violation. This implies strong constraints on the SM predictions: for CP violation to occur in a particular process, all three generations are required to play an active role. Moreover, all CKM-matrix elements must be non-zero and the quarks of a given charge must be non-degenerate in mass. If any of these conditions were not satisfied, the CKM phase could be rotated away by a redefinition of the quark fields; therefore, CP-violation effects are necessarily proportional to the product of all CKM angles, and should vanish in the limit where any two (equal-charge) quark-masses are taken to be equal [71].

According to the SM mechanism, the decays of $S$ quarks should be an ideal ground to study CP violation. In contrast to the known kaon system, where the necessary presence of the third quark generation requires higher-order loops, $S$ decays involve the three quark families already at tree level (nevertheless, loop effects are still needed to generate $B^+\bar{B}^0$ mixing or final-state-interaction phases). In both cases ($S$ and $B$), differences of rates that signal CP violation are proportional to the small product $A^2 S_1$, but the corresponding asymmetries (difference $J$ sum) are much bigger in $S$ decay because the decay width involves smaller CKM elements ($|V_{us}|^2$ or $|V_{ub}|^2 << |V_{ub}|^2$).

The SM mechanism of CP violation predicts many (and sizeable) $CP$-odd effects in the $B$ meson system. Similarly to the kaon sector, the quantitative predictions are usually uncertain, owing to the not so-well understood long-distance strong-interaction dynamics. This is certainly the case in flavour-specific decays. Nevertheless, the experimental observation of a non-zero CP-violating asymmetry in any self-tagging decay mode (in $B^\pm$ decays for instance) would be a major achievement, as it would clearly establish the existence of a direct CP-violation mechanism in the decay amplitudes.

The $B^\pm\bar{B}^0$ system offers a much better situation for a quantitative analysis. CP violation effects associated with $\Delta B = 2$ transitions are predicted to be very tiny in the SM. Thus, the observation of an asymmetry (at the percent level) between the number of positive and negative like-sign lepton pairs produced in the process $e^+e^- \rightarrow B^+\bar{B}^0$ or $\mu^+\mu^- \rightarrow B^+\bar{B}^0$ would be a clear indication for new physics [72]. Moreover, assuming the SM to be correct, the interplay between the $B^\pm\bar{B}^0$ mixing and direct CP-violation amplitudes results in sizeable CP-violating asymmetries, which are amenable to a fair theoretical treatment. Decays to CP self-conjugate states, for instance, generate CP-odd asymmetries which are free of final-state-strong-interactions effects, if only one weak amplitude is relevant. Thus, in some decay modes the CP-odd signals can be directly translated into a clear measurement of the CKM parameters.

The unitarity of the CKM matrix implies strong constraints in the different quark-mixing matrix elements. This is better visualized through the so-called unitarity triangle shown in Fig. 1. In the absence of CP violation, $s = 0$, and the triangle would squeeze to a line along the real axis. The existence of a complex phase in the CKM matrix can be inferred from pure CP-conserving measurements (assuming they are accurate enough, which is not the case at present) of the three triangle sides. As extensively shown in this report, the observation of CP-violating asymmetries with neutral $B^0\bar{B}^0$ mesons, would allow to independently measure the three angles of the triangle, providing an unconstrained determination of the CKM matrix. If the measured sides and angles turn out to be consistent with a geometrical triangle, we would have a beautiful test of the CKM unitarity, providing strong support to the SM mechanism of CP violation. On the contrary, any deviation from a triangle shape would be a clear proof that new physics is needed in order to understand the CP-violating phenomena.

A high luminosity asymmetric $B$ factory would allow to explore in detail the sector of the quark mixing and CP violation. Besides the precise measurements of the CKM matrix elements, the very large number of detected $B$ mesons will provide the opportunity to observe CP violation with many different $B$ decay modes allowing thus to measure consistently the various angles of the unitarity triangle. The Standard Model could then be tested in its most unsatisfactory sector - namely the sector of the quark mass generation and mixing. Large surprises may well be discovered, possibly giving the first hints of new physics. The prospects to explore this outstanding physics program at an asymmetric $B$ factory has been studied in this report and we conclude that such a facility would clearly be a very adequate and useful tool for covering very widely this exciting physics program.
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