TENTH WORLD CONGRESS ON THE THEORY OF
MACHINE AND MECHANISMS
Oulu, Finland, June 20-24, 1999

Finding Infinitesimal Motions of Objects in Assemblies Using Grassmann-Cayley Algebra

E. Staffetti, L. Ros, and F. Thomas
Institut de Robòtica i Informàtica Industrial (UPC-CSIC)
Gran Capità 2-4, 08034 Barcelona, Spain

Keywords: Grassman-Caley algebra, kinematic constraints, geometric reasoning

1 Introduction

We present a method for deriving the set of allowed infinitesimal motions of a polyhedron in contact with a polyhedral assembly without breaking the established basic contacts. The result is obtained, under the frictionless assumption, by describing each basic contact by means of the Grassmann-Cayley algebra and using cycle conditions over closed kinematic chains between the polyhedron and the assembly. Although, in practice, subparts of assemblies need to be moved completely and not only infinitesimally, the obtained results constitute a very useful information for an assembly sequence planner [Thomas et al. 1992], [Staffetti et al. 1998]. We also apply the proposed technique to solve infinitesimal mobility analysis problems of general multiloop spatial mechanisms.

2 Background

In this section we give a general overview of the subset of the Grassmann-Caley algebra operations needed in subsequent developments without going into mathematical details. A deeper insight can be found in [White 94] or [White 95].

2.1 Projective Space and Plücker Coordinates

Let us consider the projective 3-space. A point q in this space is represented by a non-zero 4-tuple $q = (q_1, q_3, q_3, q_4)$ whose elements are called the homogeneous coordinates of the point. Two 4-tuples p and q represent the same projective point if, and only if, $p = \lambda q$ for some $\lambda \neq 0$. If $q_4 \neq 0$ we say the point is finite and it can be represented by the 4-tuple $p = (p_1, p_2, p_3, 1)$ where the first three components are the Euclidean coordinates of the same point indicated with $\mathbf{p}$. If $q_4 = 0$ the point lies on the plane at infinity.

Given two points a and b in homogeneous coordinates, a line L through them can be represented by the vector P_L formed by the six 2×2 minors of the following 2×4 matrix:

$$\begin{pmatrix} a_1 & a_2 & a_3 & 1 \\ b_1 & b_2 & b_3 & 1 \end{pmatrix}$$

called the Plücker coordinates of the line. It can be proven that:

$$P_L = \begin{pmatrix} b_1 - a_1 & b_2 - a_2 & b_3 - a_3 & a_2b_3 - a_3b_2 & a_3b_1 - a_1b_3 & a_1b_2 - a_2b_1 \end{pmatrix} = (\mathbf{s}, \mathbf{r} \times \mathbf{s}),$$

where $\mathbf{s} = (\mathbf{b} - \mathbf{a})$ and $\mathbf{r}$ is the Euclidean position of any point on L. In the Cayley algebra, a modern version of the Grassmann algebra, the subspace generated by a and b is called the 2-extensor of a and b and its symbolic Plücker coordinates are indicated by $a \vee b$ or $\vee\left(a, b\right)$ [Dubilet et al. 1974], [White 1995]. Thus the line L can be expressed as $L = a \vee b$.

The point at infinity on L is the vector $\begin{pmatrix} b_1 - a_1 & b_2 - a_2 & b_3 - a_3 & 0 \end{pmatrix}$. Each 4-tuple of the form $(t_1, t_2, t_3, 0) \neq (0, 0, 0, 0)$ represents a point at infinity. This point can be thought as infinitely far away in the direction given by $\mathbf{s}$. The same point at infinity lies on every line parallel to L but non-parallel lines have distinct points at infinity.

A line at infinity is determined by two distinct points at infinity:

$$\begin{pmatrix} s_1 & s_2 & s_3 & 0 \\ t_1 & t_2 & t_3 & 0 \end{pmatrix},$$

which has the following vector of Plücker coordinates

$$P_L = \begin{pmatrix} 0 & 0 & 0 & s_2t_3 - s_3t_2 & s_3t_1 - s_1t_3 & s_1t_2 - s_2t_1 \end{pmatrix}.$$

Likewise, the plane P determined by the three points a, b, and c is a 3-extensor indicated by $a \vee b \vee c$ whose Plücker coordinates are the four 3×3 minors of the following 3×4 matrix

$$\begin{pmatrix} a_1 & a_2 & a_3 & 1 \\ b_1 & b_2 & b_3 & 1 \\ c_1 & c_2 & c_3 & 1 \end{pmatrix}.$$

It can be easily proven that the Plücker coordinates of the plane P can be ordered such that $P_P = (\mathbf{n}, -\mathbf{r} \cdot \mathbf{n})$, where $\mathbf{n}$ is the normal vector to P and $\mathbf{r}$ is the Euclidean position vector of any point on P.

2.2 Projective Representation of Motions

Let $\mathbf{u}$ be the Euclidean velocity of an Euclidean point $\mathbf{p}$. The motion of the projective point p can be defined as $M(p) = (\mathbf{u}, -\mathbf{u} \cdot \mathbf{p})$, that is, the 3-extensor that represents the plane containing the point $\mathbf{p}$ whose normal is $\mathbf{u}$.

An instantaneous motion in projective 3-space, that is an assignment of motions $M(p_i)$ to the projective points p_i, is a rigid motion if the velocities preserve all distances between them. In projective terms rigid motions can be expressed in a simple and effective way [White 1994].

If r and s are projective points, for each point p in space we define $M(p) = r \vee s \vee p$. This assignment of motion preserves all distances and therefore it will correspond to a rigid motion in space determined by the 2-extensor $C = r \vee s$. This 2-extensor, that represents the line through r and s, is called the center of the motion. Since $M(r) = 0$ and $M(s) = 0$, it represents a rotation around the axis determined by r and s. For example, a counterclockwise rotation around the z axis is expressed by the following extensor $\vee \begin{pmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 2 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 & -1 & 0 & 0 & 0 \end{pmatrix}$ using two Euclidean points $\mathbf{r} = (0, 0, 1)$ and $\mathbf{s} = (0, 0, 2)$ on it.

A translation can be described as a rotation about an axis at infinity. Let $a = (a_1, a_2, a_3, 0)$ and $b = (b_1, b_2, b_3, 0)$ be two points at infinity. Then, the extensor $a \vee b$ can be used as the center of a motion $M(p) = a \vee b \vee p$. The corresponding velocity is $\mathbf{v} = (a_2 b_3 - a_3 b_2, a_3 b_1 - a_1 b_3, a_1 b_2 - a_2 b_1)$. Since it is independent from the point p, it will represent a translation. For example a translation along the positive direction of the z axis can be expressed by means of two Euclidean points $\mathbf{a} = (1, 0, 0)$ and $\mathbf{b} = (0, 1, 0)$ that lies on the plane xy. The corresponding extensor is $\vee \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$.

By composing translations and rotations represented by their centers S_i, a more general screw motion in space can be obtained. Instantaneously, this composition corresponds to a simple addition of the motion centers S_i, that is, the equivalent motion is $M(p) = \sum_i (S_i \vee p) = (\sum_i S_i) \vee p$.

3 GR Graphs

Let us consider two bodies hinged along the line $a \vee b$ then, for any instantaneous motion of the bodies with centers S_1 and S_2, there is a scalar λ such that $S_1 - S_2 = \lambda(a \vee b)$. This concept can be extended to any number of rigid bodies and hinges [Crapo et al. 1982].

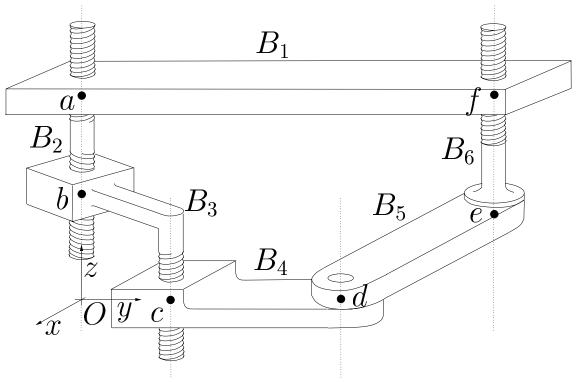
An kinematic chain is a set (B, A) where B is a finite collection of bodies $(B_1, \ldots, B_n)$, and $A = (\ldots, L_{i,j}, \ldots)$ is a set of hinges represented by non zero 2-extensors in projective space indexed by ordered pairs of indices with $L_{i,j} = -L_{j,i}$. An instantaneous motion of the kinematic chain (B, A) is an assignment of a center S_i to each body B_i such that for each hinge $L_{i,j} \in A$ and for some choice of scalars $\omega_{i,j}$ we obtain $S_i - S_j = \omega_{i,j} L_{i,j}$. The scalars $\omega_{i,j}$ being directly the rotational or translational velocities at the hinges $L_{i,j}$ if the involved extensors have unitary module. In what follows all the used extensors will be unitary.

Since in our case the same body may appear in different kinematic chains, it is better to represent the situation using a directed graph – called GR graph – whose nodes will represent the center of motion of the bodies and, if body B_i is restricted in its motion with reference to body B_j, there will be a directed arc going from node S_i to node S_j labeled with $L_{i,j}$.

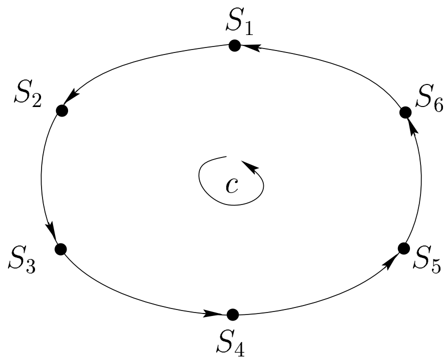
Now, let us assume that a GR graph has a cycle, for example S_0, $L_{0,1}$, S_1, $L_{1,2}$, S_2, $\ldots$, S_k, $L_{k,0}$, S_0. Since the net velocity around a cycle must be zero, we obtain the following loop equation:

$$\omega_{0,1} L_{0,1} + \omega_{1,2} L_{1,2} + \cdots + \omega_{k,0} L_{k,0} = 0, \tag{1}$$

which constrains the velocities $\omega_{0,1}$, $\omega_{1,2}$, $\ldots$, and $\omega_{k,0}$ to have compatible values. This can be done for any cycle in a GR graph but, in practice, we only need to consider the loop equations resulting from a complete set of basic cycles [Thomas 1992].



(a)



(b)

Figure 1. A single loop mechanism (a), and its associated GR graph (b).

Example 1. Let us consider the single loop mechanism shown in *fig. 1a*, where all the rotation axes are parallel to z axis of the reference frame, and the points $\mathbf{a} = (0, 0, 1)$, $\mathbf{b} = (0, 0, 0.5)$, $\mathbf{c} = (0.5, 0.5, 0)$, $\mathbf{d} = (0.5, 1.5, 0)$, $\mathbf{e} = (0, 2, 0)$, and $\mathbf{f} = (0, 2, 1)$ that lie on them as shown in the figure. The screw motion with pitch p of B_2 with respect to B_1 about the z axis can be described by means of the composition of a rotational motion around the axis represented by the extensor $\vee \begin{pmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 2 & 1 \end{pmatrix}$ and a translational motion, with velocity p times greater than the rotational one, in the direction represented by the extensor $\left[p \vee \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \right]$. In the same way, the complete set of extensors describing the motion of the mechanism can be obtained. So we have

$$L_{1,2} = \left[p \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} + \begin{pmatrix} 0 & 0 & -1 & 0 & 0 & 0 \end{pmatrix} \right] = \begin{pmatrix} 0 & 0 & -1 & 0 & 0 & p \end{pmatrix},$$

$$L_{2,3} = \left[-q \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} + \begin{pmatrix} 0 & 0 & -1 & 0 & 0 & 0 \end{pmatrix} \right] = \begin{pmatrix} 0 & 0 & -1 & 0 & 0 & -q \end{pmatrix},$$

$$L_{3,4} = \left[r \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} + \begin{pmatrix} 0 & 0 & -1 & 0.5 & -0.5 & 0 \end{pmatrix} \right] = \begin{pmatrix} 0 & 0 & -1 & 0.5 & -0.5 & r \end{pmatrix},$$

$$L_{4,5} = \begin{pmatrix} 0 & 0 & -1 & 1.5 & -0.5 & 0 \end{pmatrix}, \; L_{5,6} = \begin{pmatrix} 0 & 0 & -1 & 2 & 0 & 0 \end{pmatrix}, \text{ and}$$

$$L_{6,1} = \left[u \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} + \begin{pmatrix} 0 & 0 & -1 & 2 & 0 & 0 \end{pmatrix} \right] = \begin{pmatrix} 0 & 0 & -1 & 0 & 0 & u \end{pmatrix},$$

where p, q, r and u are the pitches of the four screws in the mechanism.

Moreover, we get the following loop equation for this mechanism:

$$\omega_{1,2} L_{1,2} + \omega_{2,3} L_{2,3} + \omega_{3,4} L_{3,4} + \omega_{5,6} L_{5,6} + \omega_{6,1} L_{6,1} = 0.$$

By substituting the extensor expressions in this loop equation, we obtain:

$$\begin{aligned} &- \omega_{1,2} - \omega_{2,3} - \omega_{3,4} - \omega_{4,5} - \omega_{5,6} - \omega_{6,1} = 0, \\ &0.5\, \omega_{3,4} + 1.5\, \omega_{4,5} + 2\, \omega_{5,6} + 2\, \omega_{6,1} = 0, \\ &- 0.5\, \omega_{3,4} - 0.5\, \omega_{4,5} = 0, \\ &p\, \omega_{1,2} - q\, \omega_{2,3} + r\, \omega_{3,4} + u\, \omega_{6,1} = 0. \end{aligned}$$

This system of equations has rank two and therefore the corresponding mechanism has two infinitesimal d.o.f.

4 Motion of Polyhedra in Contact

Any contact between polyhedra can be expressed as the composition of two basic contacts; namely: type-A and type-B contacts. A type-A contact occurs when a vertex v of a polyhedron touches a face of other polyhedron, and a type-B contact occurs when an edge of one polyhedron is in contact with an edge of other polyhedron (see, for example, [Thomas 1994]).

The constrained motion between two polyhedra, B_i and B_j, under a type-A contact can be thought as produced by a spherical and a planar joint between them. The spherical joint can be modelled as three revolute joints whose axes intersect in the contact point and the planar joint can be described with two prismatic joints that permit translations along two non-parallel axes on the face plane. In this case S_i and S_j, the centers of the motion of B_i and B_j, respectively, are related by the following expression:

$$S_i = S_j + \omega_{i,j}^{A,r_1} L_{i,j}^{A,r_1} + \omega_{i,j}^{A,r_2} L_{i,j}^{A,r_2} + \omega_{i,j}^{A,r_3} L_{i,j}^{A,r_3} + \omega_{i,j}^{A,t_1} L_{i,j}^{A,t_1} + \omega_{i,j}^{A,t_2} L_{i,j}^{A,t_2}, \qquad (2)$$

where $L_{i,j}^{A,r_1}$, $L_{i,j}^{A,r_2}$, $L_{i,j}^{A,r_3}$ are 2-extensors that define the axes of rotation and $L_{i,j}^{A,t_1}$ and $L_{i,j}^{A,t_2}$ the axes at infinity used to represent the translations on the face plane.

Likewise, the motion of two polyhedra, B_i and B_j, bound to keep a type-B contact, can be modelled by means of the composition of two cylindrical and one revolute joints whose axes intersect in the contact point. In this case, we obtain the following relation between the centers of motion:

$$S_i = S_j + \omega_{i,j}^{B,r_1} L_{i,j}^{B,r_1} + \omega_{i,j}^{B,t_1} L_{i,j}^{B,t_1} + \omega_{i,j}^{B,r_2} L_{i,j}^{B,r_2} + \omega_{i,j}^{B,t_2} L_{i,j}^{B,t_2} + \omega_{i,j}^{B,r_3} L_{i,j}^{B,r_3} \qquad (3)$$

where $L_{i,j}^{B,r_1}$ and $L_{i,j}^{B,r_2}$ are the 2-extensors that define the axes of rotation around the edges in contact, whereas $L_{i,j}^{B,t_1}$ and $L_{i,j}^{B,t_2}$ are 2-extensors used to describe the translations along directions parallel to each edge. Finally, $L_{i,j}^{B,r_3}$ represents a rotation axis normal to the plane that contains the edges in contact.

Example 2. Let us consider the two polyhedra in contact appearing in *fig. 2a.* They are in contact along an edge. This contact can be expressed in terms of four type-A basic contacts between the vertices v_1 and v_2 of B_1 and the faces f_1 and f_2 of B_2. This can be expressed using a GR graph as shown in *fig. 2b.* Then, the set of loop equation resulting from the set of basic cycles $\{c_1, c_2, c_3\}$ are:

$$\begin{aligned} -\,&\omega_{a_4,B_1}^{A,r_z} L_{a_4,B_1}^{A,r_z} - \omega_{a_3,a_4}^{A,r_y} L_{a_3,a_4}^{A,r_y} - \omega_{a_2,a_3}^{A,r_x} L_{a_2,a_3}^{A,r_x} - \omega_{a_1,a_2}^{A,t_y} L_{a_1,a_2}^{A,t_y} + \omega_{a_1,B_2}^{A,t_x} L_{a_1,B_2}^{A,t_x} \\ &- \omega_{b_1,B_2}^{A,t_x} L_{b_1,B_2}^{A,t_x} + \omega_{b_1,b_2}^{A,t_z} L_{b_1,b_2}^{A,t_z} + \omega_{b_2,b_3}^{A,r_x} L_{b_2,b_3}^{A,r_x} + \omega_{b_3,b_4}^{A,r_y} L_{b_3,b_4}^{A,r_y} + \omega_{b_4,B_1}^{A,r_z} L_{b_4,B_1}^{A,r_z} = 0, \end{aligned}$$

$$\begin{aligned} -\,&\omega_{b_4,B_1}^{A,r_z} L_{b_4,B_1}^{A,r_z} - \omega_{b_3,b_4}^{A,r_y} L_{b_3,b_4}^{A,r_y} - \omega_{b_2,b_3}^{A,r_x} L_{b_2,b_3}^{A,r_x} - \omega_{b_1,b_2}^{A,t_z} L_{b_1,b_2}^{A,t_z} + \omega_{b_1,B_2}^{A,t_x} L_{b_1,B_2}^{A,t_x} \\ &- \omega_{c_1,B_2}^{A,t_x} L_{c_1,B_2}^{A,t_x} + \omega_{c_1,c_2}^{A,t_z} L_{c_1,c_2}^{A,t_z} + \omega_{c_2,c_3}^{A,r_x} L_{c_2,c_3}^{A,r_x} + \omega_{c_3,c_4}^{A,r_y} L_{c_3,c_4}^{A,r_y} + \omega_{c_4,B_1}^{A,r_z} L_{c_4,B_1}^{A,r_z} = 0, \end{aligned}$$

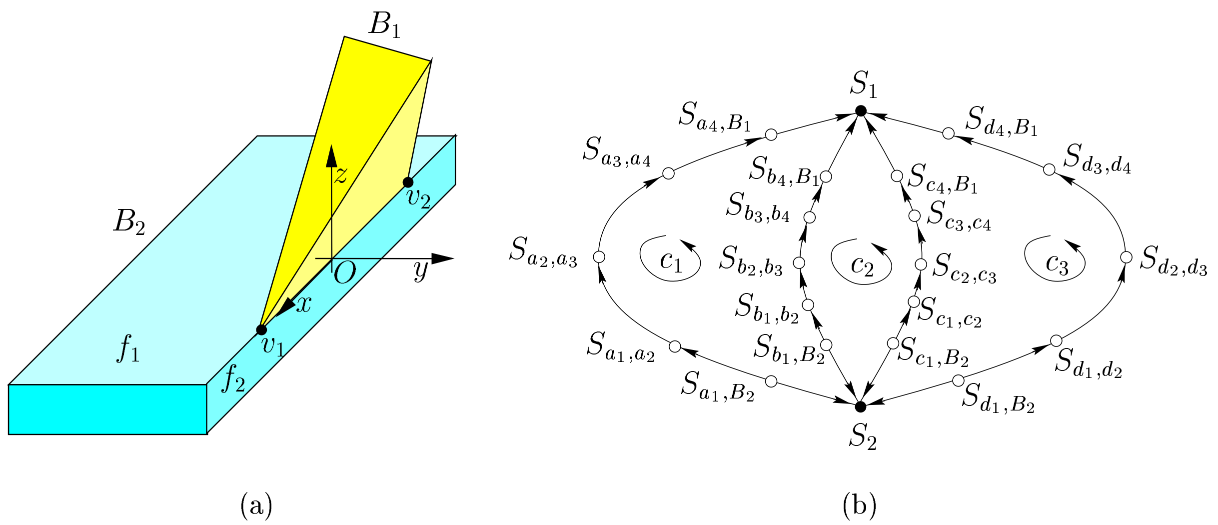


Figure 2. Two polyhedra in contact along an edge (a), and its representation in terms of four basic contacts as a GR graph (b).

$$- \omega^{A,r_z}_{c_4,B_1} L^{A,r_z}_{c_4,B_1} - \omega^{A,r_y}_{c_3,c_4} L^{A,r_y}_{c_3,c_4} - \omega^{A,r_x}_{c_2,c_3} L^{A,r_x}_{c_2,c_3} - \omega^{A,t_z}_{c_1,c_2} L^{A,t_z}_{c_1,c_2} + \omega^{A,t_x}_{c_1,B_2} L^{A,t_x}_{c_1,B_2}$$
$$- \omega^{A,t_x}_{d_1,B_2} L^{A,t_x}_{d_1,B_2} + \omega^{A,t_y}_{d_1,d_2} L^{A,t_y}_{d_1,d_2} + \omega^{A,r_x}_{d_2,d_3} L^{A,r_x}_{d_2,d_3} + \omega^{A,r_y}_{d_3,d_4} L^{A,r_y}_{d_3,d_4} + \omega^{A,r_z}_{d_4,B_1} L^{A,r_z}_{d_4,B_1} = 0.$$

If the Euclidean coordinates of the vertices v_1 and v_2 are $\mathbf{v_1} = (1, 0, 0)$ and $\mathbf{v_2} = (-1, 0, 0)$ we obtain:

$$L^{A,t_x}_{a_1,B_2} = L^{A,t_x}_{b_1,B_2} = L^{A,t_x}_{c_1,B_2} = L^{A,t_x}_{d_1,B_2} = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}, \; L^{A,t_y}_{a_1,a_2} = L^{A,t_y}_{d_1,d_2} = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix},$$

$$L^{A,t_z}_{b_1,b_2} = L^{A,t_z}_{c_1,c_2} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \; L^{A,r_x}_{a_2,a_3} = L^{A,r_x}_{b_2,b_3} = L^{A,r_x}_{c_2,c_3} = L^{A,r_x}_{d_2,d_3} = \begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

$$L^{A,r_y}_{a_3,a_4} = L^{A,r_y}_{b_3,b_4} = \begin{pmatrix} 0 & -1 & 0 & 0 & 0 & 1 \end{pmatrix}, \; L^{A,r_y}_{c_3,c_4} = L^{A,r_y}_{d_3,d_4} = \begin{pmatrix} 0 & -1 & 0 & 0 & 0 & -1 \end{pmatrix},$$

$$L^{A,r_z}_{a_4,B_1} = L^{A,r_z}_{b_4,B_1} = \begin{pmatrix} 0 & 0 & -1 & 0 & -1 & 0 \end{pmatrix}, \; L^{A,r_z}_{c_4,B_1} = L^{A,r_z}_{d_4,B_1} = \begin{pmatrix} 0 & 0 & -1 & 0 & 1 & 0 \end{pmatrix}.$$

By solving these linear equations, we obtain that B_1 and B_2 keep in contact along the edge if, and only if, the motion of the moving body satisfies the following conditions:

$$\omega^{A,t_x}_{a_1,B_2} = \omega^{A,t_x}_{b_1,B_2} = \omega^{A,t_x}_{c_1,B_2} = \omega^{A,t_x}_{d_1,B_2},$$
$$\omega^{A,r_x}_{a_2,a_3} = \omega^{A,r_x}_{b_2,b_3} = \omega^{A,r_x}_{c_2,c_3} = \omega^{A,r_x}_{d_2,d_3},$$
$$\omega^{A,t_y}_{a_1,a_2} = \omega^{A,t_y}_{b_1,b_2} = \omega^{A,t_y}_{c_1,c_2} = \omega^{A,t_y}_{d_1,d_2} = 0,$$
$$\omega^{A,r_y}_{a_3,a_4} = \omega^{A,r_y}_{b_3,b_4} = \omega^{A,r_y}_{c_3,c_4} = \omega^{A,r_y}_{d_3,d_4} = 0, \text{ and}$$
$$\omega^{A,r_z}_{a_4,B_1} = \omega^{A,r_z}_{b_4,B_1} = \omega^{A,r_z}_{c_4,B_1} = \omega^{A,r_z}_{d_4,B_1} = 0.$$

In other words, a rotational and a translational d.o.f. remain between both polyhedra.

Example 3. Let us assume that we want to determine which is the movable subset of bodies in the assembly of the four workpieces in *fig. 3.* In this case we describe using Grassmann-Cayley algebra the permitted motions between the objects in the subassemblies where B_2

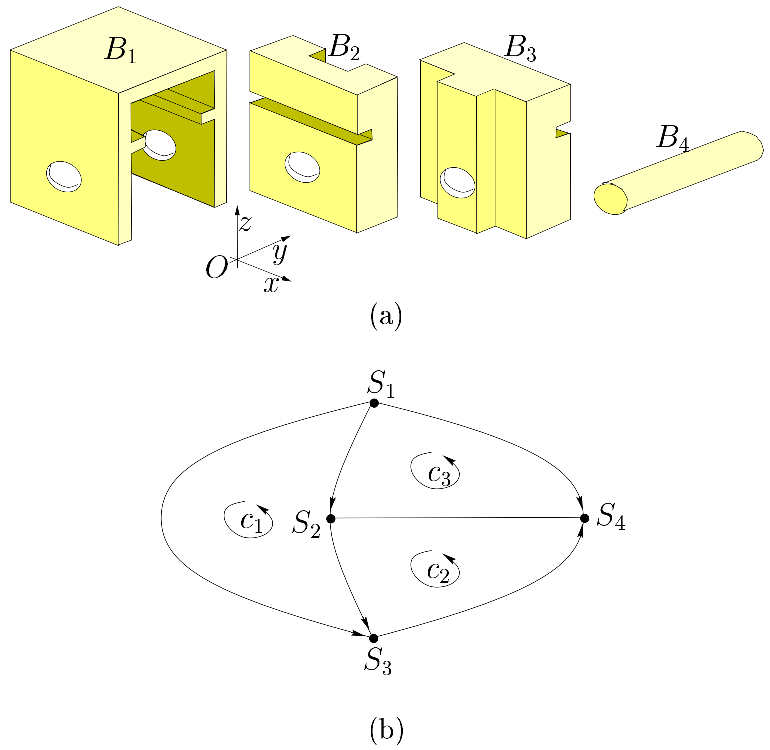


Figure 3. Four workpieces to be assembled (a), and the associated GR graph of the final assembly (b).

can be translated along the z axis with respect to B_3, both B_2 and B_3 can be translated along the x axis with respect to B_1, and B_4 can be separately translated along the y axis with respect to all the other bodies (*fig. 4b*). We obtain in this case the following extensors:

$$L_{1,2}^{t_x} = L_{1,3}^{t_x} = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}, L_{2,4}^{t_y} = L_{3,4}^{t_y} = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}, L_{2,3}^{t_z} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix},$$

and the following loop equations:

$$\begin{aligned} &-\omega_{1,2}L_{1,2} - \omega_{2,3}L_{2,3} - \omega_{1,3}L_{1,3} = 0, \\ &\omega_{2,3}L_{2,3} - \omega_{3,4}L_{3,4} - \omega_{2,4}L_{2,4} = 0, \\ &\omega_{1,2}L_{1,2} - \omega_{2,4}L_{2,4} - \omega_{1,4}L_{1,4} = 0. \end{aligned}$$

This linear system has the solution $\omega_{1,2} = \omega_{1,3} = \omega_{2,3} = 0$, and $\omega_{1,4} = \omega_{2,4} = \omega_{3,4}$. This means that the only movable body in the final assembly is B_4 which can be translated in the direction of the y axis of the reference frame with an arbitrary instantaneous velocity.

5 Conclusions

It has been shown that the possible infinitesimal motions between polyhedra, bound to keep some set of basic contacts, can be obtained by solving a linear system of loop equations associated with a set of basic cycles in the corresponding graph of kinematic constraints. The projective representation of motions in the Grassmann-Cayley algebra allow us to straightforwardly get a tidy set of loop equations. Since the proposed technique is procedural and is not based on lookup tables, i.e. on a case-by-case analysis, it is more reliable and less cumbersome than symbolic techniques. We have also shown the usefulness of the proposed technique to infer the infinitesimal d.o.f. of arbitrary mechanisms.

5 Acknowledgments

This work has been partially supported by the Spanish CICYT under contract TAP 93-0451, and the Marie Curie RTG/TMR Program of the EC under contract ERBFM-BICT961666.

References:

Crapo, H., Whiteley, W.,"Statics of Frameworks and Motions of Panel Structures, a Projective Geometric Introduction," *Structural Topology*, vol. 6, pp. 43-82, 1982.

Dubilet, P., Rota, G.-C., Stein, J.,"On the Fundations of Combinatorial Theory: IX Combinatorial Methods in Invariant Theory," *Studies in Applied Mathematics*, vol. 53, no. 3, pp. 185-216, 1974.

Hirukawa, H., Matsui, T., Katase, K. "Automatic Determination of Possible Velocity and Applicable Force of Frictionless Objects in Contact from a Geometric Model," *IEEE Transactions on Robotics and Automation*, vol. RA-10, no.3, pp. 309-322, 1994.

Staffetti, E., Ros, L., Thomas, F., "A Simple Characterization of the Infinitesimal Motions Separating General Polyhedra in Contact," *submitted for publication.*

Thomas, F., Torras, C., "Inferring Feasible Assemblies from Spatial Constraints", *IEEE Transactions on Robotics and Automation*, Vol. 8, No. 2, pp. 228-239, April 1992.

Thomas, F., "Graphs of Kinematic Constraints," Chapter 4 in *Computer-Aided Mechanical Assembly Planning*, L.S. Homem de Mello y S. Lee (editors), pp. 81-111, Kluwer Academic Publishers, 1991.

Thomas, F., Torras, C., "Interference Detection Between Non-Convex Polyhedra Revisited with a Practical Aim", *Proceedings of the 1994 IEEE International Conference on Robotics and Automation*, Vol. I, pp. 587-594, May 1994.

N. L. White, "A Tutorial on Grassmann-Cayley Algebra," In N. L. White, editor, *Invariant Methods in Discrete and Computational Geometry*, pp. 93-106, Kluwer Academic Publisher, Dordrecht, 1995.

N. L. White, "Grassmann-Cayley Algebra and Robotics," *Journal of Intelligent Robotics Systems*, Vol. 11, pp. 91-107, 1994.