Model Predictive Control of Drinking Water Networks: A Hierarchical and Decentralized Approach

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Abstract—In this paper, a decentralized model predictive control (DMPC) strategy for drinking water networks (DWN) is proposed. The DWN is partitioned in a set of subnetworks using a partitioning algorithm that makes use of the topology of the network, the information about the actuator usage and heuristics. A suboptimal DMPC strategy was derived that allows the hierarchical solution of the set of MPC controllers used to control each partition. A comparative study between the centralized MPC (CMPC) and DMPC approaches is developed on the case study, which consists in an aggregate version of the Barcelona DWN. Results have shown the effectiveness of the proposed DMPC approach in terms of the computation time while an admissible level of suboptimality is obtained in all the considered scenarios.

I. INTRODUCTION

Optimization of drinking water networks (DWN) has gained much attention in the past few decades since water management in urban areas is a subject of increasing concern as cities grow. Limited water supplies, conservation and sustainability policies, infrastructure complexity as well as the satisfaction of water supply to the network users by appropriate flow, pressure and quality levels make water management a challenging control problem. Decision support systems provide useful guidance for human operators in complex networks, where resources management “best” actions are not intuitive. Optimization and optimal/predictive control techniques provide an important contribution to a smart management strategy computation for DWNs, see [1], [2], [3], among others.

Research in this field is spurred by the complexities associated with the connection management of multiple interconnected reservoirs in the case of large-scale networks, which still exceeds the capabilities of existing optimization tools in finding optimal actions in an appropriate computational time. Mathematical programming techniques are one of the many available tools and most widely used. Their main objective consists in generating control strategies ahead in time, using techniques such as model predictive control (MPC), to guarantee a competent network service and a certain degree of reliability in probability, while simultaneously achieving certain objectives as minimization of supply and pumping costs, maximisation of water quality and leak prevention, among others. This optimization problem is usually large and non-linear, because of pump, pipeline pressure and performances index characteristics. So far, the aforementioned control methods for water systems based on MPC have been implemented in a centralized manner over SCADA systems using a traditional hierarchical management architecture placed above the process instrumentation and basic regulatory control layers. However, such a centralized architecture leads to implementation problems because of dimensionality, multi-time scales and spatial distribution of DWNs. Complexity of the underlying optimization problem is not the only reason. The main hurdle for plant-wide centralized control is that it is not scalable: it requires a huge model, which needs to be rebuilt on every change of topological configuration. Subsequently, a model change would require re-tuning the complex controller. It can be seen that the cost of setting up and maintaining this monolithic solution is prohibitive. Moreover, any maintenance operation over even a single controlled element, which of course implies to turn of that element, would change the complex centralized scheme. Then, the possible choices are to use a control action who ignores the absence of the element under maintenance (or simply that is temporary unavailable), with all consequential implications, or to switch the whole control system considering the availability of several control configurations.

A way of circumventing these issues is to look into decentralized model predictive control (DMPC) techniques, where networked local MPC controllers are in charge of controlling the actuators related to a part of the whole network. In this line, this paper proposes a DMPC strategy for DWNs based on a hierarchical structure. DMPC control is still in its first infancy. References [4] and [5] present a review of the research in this topic. Some recent DMPC references are [6], [7] and [8], among others. The main contribution of this paper relies on the computation time reduction for finding the proper control actions when the proposed DMPC design is used, maintaining a convenient level of sub-optimality of the computed solutions with respect to a given set of control objectives associated to a centralized MPC (CMPC) controller design.

The paper is structured as follows. In Section II the modelling principles for DWNs are presented. Section III presents the basic ideas of the MPC strategy for the management of such networks. Section IV describes the algorithm for DWN partitioning as well as discusses the main issues of the hierarchical and decentralized control strategy proposed.

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in this paper. Section V describes the case study of the paper, discusses the implementation of the proposed hierarchical DMPC in that case study and presents the most relevant results. Finally, the main conclusions and further work close the paper in Section VI.

II. DWN Control-Oriented Modelling Principles

Control-oriented modelling principles for DWNs have been widely presented in the literature, see [2], [9]. In order to obtain a control-oriented model of the DWN, the constitutive network elements as well as their basic relationships should be discussed. The reader is referred to the aforementioned references and to [10] for further details of DWN modelling and specific insights related to the case study of this paper.

Let us consider the main physical constraints of a DWN system given by the variables related to the tank volumes and manipulated flows. For the case of tank volumes, the physical constraint related to the range of volume capacities for the \( i \)-th tank is expressed as

\[
x_i^{\text{min}} \leq x_i(k) \leq x_i^{\text{max}},
\]

where \( x_i^{\text{min}} \) and \( x_i^{\text{max}} \) denote the minimum and the maximum volume capacity, respectively, given in m\(^3\). On the other hand, the physical constraints related to manipulated flows through the system actuators are expressed as

\[
u_i^{\text{min}} \leq u_i(k) \leq u_i^{\text{max}},
\]

where \( u_i^{\text{min}} \) and \( u_i^{\text{max}} \) denote the minimum and the maximum flow capacity, respectively, given in m\(^3\)/s.

By considering the mass balance in the tanks, the control-oriented model of a DWN in discrete-time state-space form can be written as

\[
x(k+1) = Ax(k) + Bu(k) + B_p d(k),
\]

where \( x \in \mathbb{R}^n \) is the state vector corresponding to the water volumes of the \( n \) tanks, \( u \in \mathbb{R}^m \) represents the vector of manipulated flows through the \( m \) actuators (pumps and valves), and \( d \in \mathbb{R}^p \) corresponds to the vector of the \( p \) water demands (sectors of consume). \( A, B, \) and \( B_p \) are system matrices of suitable dimensions. Since the demands can be forecasted and they are assumed to be known, \( d \) is a known vector containing the measured disturbances affecting the system. By also including static relations at network nodes, model (3) can be further rewritten as

\[
x(k+1) = Ax(k) + \Gamma v(k),
\]

\[
E_1 v(k) = E_2,
\]

where \( \Gamma = [B \ B_p] \), \( v(k) = [u(k)^T \ d(k)^T]^T \), and \( E_1, E_2 \) are matrices of suitable dimensions dictated by the network topology.

III. MPC Applied to DWN

Along the last few years, MPC has shown to be one of the most effective and accepted control strategies for large-scale complex systems [11]. The objective of using this technique for controlling DWNs is to compute, in a predictive way, the proper input actions in order to achieve the optimal performance of the network according to a given set of control objectives. MPC strategies have some important features to deal with complex systems (i.e., DWNs) such as the amenability to including disturbance (demand) prediction, physical constraints and multi-variable system dynamics and objectives in a relatively simple way. This section describes the main ideas of the DWN control within the MPC framework, in accordance with the following operational objectives:

1) Minimizing water production and transport cost:

The main economic costs associated with drinking water production (treatment) are due to chemicals, legal canons and electricity costs. Delivering this drinking water to appropriate pressure levels through the water transport network involves important electricity costs in pumping stations. For this study, this control objective is described by the expression

\[
f_1(k) = W_\alpha (\alpha_1 u(k) + \alpha_2(k)) u(k),
\]

where \( \alpha_1 \) corresponds to a known vector related to the economic costs of the water according to the selected source (treatment plant, dwell, etc.) and \( \alpha_2(k) \) is a vector of suitable dimensions associated to the economic cost of the flow through certain actuators (pumps only) and their control cost (pumping). Note the \( k \)-dependence of \( \alpha_2 \) since the pumping effort has different values according to the moment within the day (electricity costs). Weight matrix \( W_\alpha \) penalizes the control objective related to economic costs in the optimization problem behind the MPC controller design.

2) Safety storage term: The satisfaction of water demands should be fulfilled at any time instant. However, some risk prevention mechanisms should be introduced in the tank management so that, additionally, the stored volume is preferably maintained around a given safety value for eventual emergency needs and to guarantee future water availability. A quadratic expression for this goal is used and written as follows:

\[
f_2(k) = (x(k) - \beta x^{\text{max}})^T W_x (x(k) - \beta x^{\text{max}}),
\]

where \( \beta \) is a term which determines the security volume to be considered for the control law computation and matrix \( W_x \) defines the weight of the objective in the cost function. This term might appear as unnecessary because of the guarantees of the MPC design but, since a trade off between the other costs and the volumes is present, the controller would tend to keep the lowest possible the tanks water volumes. This would reduce the robustness to demands forecasts mispredictions, hence maintaining a security volume makes sense considering such issue.

3) Smoothness of the control actions: Pumping stations must, in general, avoid excessive switching: valves should operate smoothly in order to avoid big transients in the pressurized pipes which can lead to poor pipe condition. Similarly, water flows requested from treatment plants must have a smooth profile due to the plants operational constraints.
Moreover, the proposed approach do not deal with pressure issues, hence a lower level controller to keep the desired flow is supposed. The use of a smooth reference surely helps the controller performance. To obtain such smoothing effect, the proposed MPC controller design includes a third term in the objective function to penalize control signal variation between consecutive time intervals, i.e., \( \Delta u(k) = u(k) - u(k - 1) \). This term is expressed as
\[
J(k) = \sum_{i=0}^{H_u-1} f_1(k+i) + \sum_{i=1}^{H_p} f_2(k+i) + \sum_{i=0}^{H_u-1} f_3(k+i),
\]
where \( W_u \) corresponds to a weight matrix of suitable dimensions. Therefore, the multi-objective performance function \( J(k) \), merging the aforementioned control objectives is defined as
\[
J(k) = \sum_{i=0}^{H_u-1} f_1(k+i) + \sum_{i=1}^{H_p} f_2(k+i) + \sum_{i=0}^{H_u-1} f_3(k+i),\tag{8}
\]
where \( H_p \) and \( H_u \) correspond to the prediction and control horizons, respectively. In (8), index \( k \) represents the current time instant while index \( i \) represents the predicted time along the horizons. The highest priority objective is the economic cost, which should be minimized while obtaining acceptable satisfaction of security and control signals smoothness objectives.

Collecting the parts described in previous subsections, the MPC design follows the traditional procedures presented for instance in [11], consisting in an optimization problem where a cost function (8) is minimized subject to (1), (2) and (4). Once the minimization is performed, a vector of control actions over a given horizon is obtained. Only the first component of that vector is considered and applied over the plant. The procedure is repeated for the next time instant taking into account the feedback measurements coming from the system.

IV. DWN PARTITIONING AND HIERARCHICAL DMPC APPROACH

The main idea of the DMPC is that the on-line optimization behind the MPC design for large-scale systems can be converted into a small-scale MPC controller, each one involving less computationally demanding optimization problems. The fact to apply any DMPC scheme requires partitioning the DWN in some way.

A. DWN Partitioning

In this paper, the partitioning of the DWN is carried out in two steps. First, the sensitivity-based partitioning algorithm is applied over the system [12], and then, in order to improve the resultant partitions, heuristic procedures are used. The partitioning algorithm needs the information explained below.

The topology of the network: Collected in the matrices
\[
A_{sp} = \begin{bmatrix} A & 0 \\ 0 & 0 \end{bmatrix} \quad \text{and} \quad B_{sp} = \begin{bmatrix} B \\ E \end{bmatrix},
\]
where \( A, B \) are the system matrices in (3), subscript \( sp \) identifies the matrices employed for the system decomposi-tion, and \( E \triangleq [E_1 \ E_2] \) is the matrix related to the equality constraints (4b).

The usage level of each actuator: This is an optional parameter but it is very useful since it can provide a more accurate partition. Unfortunately, despite its utility, this parameter has a drawback related to the requirement of a previously computed set of control signals. In this case study, to calculate the usage of the actuators, a previous simulation using a CMPC is needed, what allows to obtain the total amount of water flow through each actuator. This information offers the algorithm a criteria to evaluate how important is a single actuator.

A threshold of the actuator flow magnitude: This parameter, together with the actuator usage level, is used to neglect some actuators that have less effect in the entire system behaviour.

Once all the input parameters are provided to the algorithm, a trial and error heuristic procedure is started, changing the threshold value of the actuator flow magnitude in order to find a reasonable amount of partitions for the considered DWN. In order to improve the quality of the partitions, some other indicators might be taken in to account. For further details regarding an automatic partitioning algorithm applied to DWNs, see [12].

B. Hierarchical DMPC Approach

In case that the obtained partitions do not have shared control variables (independent partitions), the DMPC approach proposed in [13] could be implemented. However, compositional elements in a DWN are in general highly cross-related, then interactions between the resultant subnetworks are always present. So, in order to control each one of the network partitions, a hierarchical DMPC control approach is proposed, which implies solving the MPC problems associated to the DWN partitions with a preestablished order.

The hierarchical-based approach consists in defining sets of shared variables (control inputs) depending on their connection direction, i.e., if the control flow goes from a Partition \( A \) to a Partition \( B \) or vice versa. Once these sets are defined, it is necessary to determine the partition with the higher amount of incoming and outgoing connections. This fact locates that partition at the top of the hierarchical pyramid. Next, other partitions with less connections with respect to this latter are defined and the criterion is again applied for the following partition. Notice that, from now on, two or more partitions can be located below the one in the top, fact that defines the hierarchical pyramid.

Figure 1 depicts a particular case where a DWN has been partitioned in three sub-networks. Here, Subsystem A is considered the most important in the hierarchy as well as Subsystems B and C have the same ranking below Subsystem A. Further, \( u_a, u_b \) and \( u_c \) determine three sets of control variables which are shared between the mentioned subsystems. As \( u_c \) corresponds to a vector of outgoing variables from Subsystem A, those variables are considered in time instant \( k \) as demands (measured disturbances) for the
MPC controller related to that partition. Their initial values are computed in the optimization problem solved in $k - 1$ behind the MPC controller associated to Subsystem C since $u_c$ is a set of incoming variables for this subsystem. Over a horizon $H_p$, the values of $u_c$ are set as constants for the MPC of Subsystem A.

![Diagram](image)

Fig. 1. Conceptual scheme for a DNW partitioned in three sub-networks.

On the other hand, sets of control variables $u_a$ and $u_b$ are taken as optimization variables in the optimization problem of the MPC for Subsystem A since they are incoming variables for that subsystem. This fact leads to consider that $u_a$ and $u_b$ are demands in time $k$ for Subsystems A and C, respectively. Notice that the values of $u_a$ and $u_b$ determined by the MPC controller of Subsystem A are only optimal for that subsystem. Thus, it induces suboptimal performances in Subsystems B and C. Also notice that for the first iteration of the control scheme, the values of $u_c$ are not defined. In this case, the corresponding values obtained from the implementation of a CMPC are used. An alternative way to solve this would be the computation of a set of feasible solution for the optimization problem related to Subsystem C defined in the first iteration and then building the initial vector $u_c$.

V. APPLICATION DESCRIPTION AND RESULTS

A. Case-study Description

The water transport network of Barcelona is used as the case study of this paper. This network covers a territorial extension of 425 km², with a total pipe length of 4470 km. Every year, it supplies 237,7 hm³ of drinking water to a population over 2.8 millions of inhabitants. The network has a centralized telecontrol system, organized in a two-level architecture. At the upper level, a supervisory control system installed in the control centre of AGBAR\(^1\) is in charge of managing the whole network by taking into account operational constraints and consumer demands. This upper level provides the set-points for the lower-level control system. The lower level optimizes the pressure profile to minimize losses due to leakage and to provide sufficient water pressure, e.g., for high-rise buildings.

This paper considers an aggregate version of the Barcelona DWN, which is a representative version of the entire network developed cooperatively by the AGBAR Company and the SAC research group. In the aggregate model, some consumer demand sectors of the network are concentrated in a single point. Similarly, some tanks are aggregated in a single element and the respective actuators are considered as a single pumping station or valve.

The control variables are required to compute the change in the state of the network produced by a control action. There, the model just considers the mass conservation law related to water flows, so the equations that describe the system dynamics are integrator-like, hence linear. A further extension of the model would include, for instance, the nonlinear relations between flow and pressure.

A convenient description of the model of a DWN is obtained by considering the set of flows through the actuator elements as the vector of control variables and the set of reservoir volumes as a vector of observable state variables.

The amount of water demand from the network users is known at each time instant so it is considered as measured disturbances. Nevertheless, at each time over the prediction horizon this magnitude should be estimated, what implies the employment of the appropriated demand forecasts to be used with the prediction model of the system.

The aggregate network (Figure 2) is comprised of 17 tanks (state variables), 61 actuators (26 pumping stations and 35 valves), 11 nodes and 25 main sectors of water demand (model disturbances). The model has been simulated and compared against real behaviour assessing its validity. The detailed information about physical parameters and other system values are reported in [10].

B. Simulation Scenarios and MPC Tuning

The model parameters and measured disturbances (demands) have been supplied by AGBAR. Demands data correspond to the consume of drinking water of the city of Barcelona during the year 2007. Using this information, some scenarios are considered by modifying some controller parameters presented in Section III. They are the safety volume, denoted as $\beta$, and the weight matrices in the cost function (8). Regarding $\beta$, this parameter has been set to the following values:

- the 80% of $x_{\text{max}}$, that is denoted as $\mu = 0.8 \ x_{\text{max}}$. This value is purely illustrative to show the effectiveness of the MPC controller;
- the minimum tank volumes requested to satisfy the demands (except for tanks $x_5$, $x_6$ and $x_8$ in Figure 2, since they are considered as sources due to their strategic management requirements and network location). This second vector of safety volumes, denoted as $\eta$, is more convenient since it keeps the volumes of the tanks as low as possible, satisfying the demands at each time instant. These minimum volumes are taken from previous studies reported in [14].

\(^1\)AGBAR: Aguas de Barcelona, S.A. Company which manages the Barcelona DWN.
Fig. 2. Aggregated case of the Barcelona Drinking Water Network

About the second set-up parameter, note that the MPC controllers designed for the case study of this paper do not consider the inclusion of the economic costs as a control objective. This fact is mainly due to data availability when this paper was prepared. However, this issue is currently underway. Let \((\omega_x, \omega_{\triangle u})\) be the couple of weights associated to the weight matrices \(W_x = \omega_x I\) and \(W_{\triangle u} = \omega_{\triangle u} I\) used in (6) and in (7), respectively. According to this, in this case study are used two couple of weights that are \((1, 1)\) and \((1, 0.1)\). These particular values of the weights are carefully selected, according to a previous study based on trial and error tuning procedure [14] and corresponding with two different prioritization scenarios of the control objectives for the particular case study. Hence, the following scenarios have been defined:

- **Scenario 1**: \(\beta = \mu\) and \((\omega_x, \omega_{\triangle u}) = (1, 1);\)
- **Scenario 2**: \(\beta = \mu\) and \((\omega_x, \omega_{\triangle u}) = (1, 0.1);\)
- **Scenario 3**: \(\beta = \eta\) and \((\omega_x, \omega_{\triangle u}) = (1, 1);\)
- **Scenario 4**: \(\beta = \eta\) and \((\omega_x, \omega_{\triangle u}) = (1, 0.1).\)

C. Barcelona DWN Partitioning

Using the partitioning algorithm presented in the previous section, the Barcelona DWN is partitioned in three subsystems, as depicted in Figure 2 in different colours. The partition follows the scheme shown in Figure 1. The subsystems are defined by the following elements:

- **Subsystem 1**: Composed by the tanks \(x_i, i \in \{1, 2\}\), inputs \(u_j, j \in \{1 : 5\}\), demands \(d_l, l \in \{1, 2, 3\}\), and nodes \(n_q, q \in \{1, 2\}\). It is represented in Figure 2 with red colour and corresponds to Subsystem B in Figure 1.

- **Subsystem 2**: Composed by the tanks \(x_i, i \in \{3, 4, 5, 12, 17\}\), inputs \(u_j, j \in \{7 : 16, 18, 19, 25, 26, 32, 34, 40, 41, 47, 48, 56, 60\}\), demands \(d_l, l \in \{4 : 7, 15, 18, 22\}\), and nodes \(n_q, q \in \{3, 4, 7\}\). It is represented in Figure 2 with green colour and corresponds to Subsystem A in Figure 1.

- **Subsystem 3**: Composed by the tanks \(x_i, i \in \{6 : 11, 13 : 16\}\), the inputs \(u_j, j \in \{6, 17, 20 : 24, 27 : 31, 33, 35 : 39, 42 : 46, 49 : 55, 57, 58, 59, 61\}\), demands \(d_l, l \in \{8 : 14, 16, 17, 19, 20, 21, 23, 24, 25\}\), and nodes \(n_q, q \in \{5, 6, 8 : 11\}\). It is represented in Figure 2 with blue colour and corresponds to Subsystem C in Figure 1.

According also to the scheme in Figure 1, vectors \(u_a, u_b\) and \(u_c\) with the shared control variables are defined as

\[
u_a = u_6, \quad u_b = [u_{20}, u_{21}]^T, \quad u_c = [u_{18}, u_{32}, u_{34}, u_{40}, u_{47}, u_{56}, u_{60}]^T.
\]

D. Application of the Hierarchical DMPC Approach

Since the obtained Barcelona DWN partitions share some control variables, the hierarchical DMPC approach described in previous section may be suitable. This approach implies solving an MPC problem for each of the DWN partitions with a pre-established order, which is given as follows:

**MPC Subsystem 3**: It needs the values for the shared elements \(u_{18}, u_{32}, u_{34}, u_{40}, u_{47}, u_{56}, u_{60}\), which are considered as demands, values that in the next iterations will be provided...
by the optimal inputs calculated at Subsystem 2. For the first step, these values are available from a previous simulation of a CMPC. The MPC of this subsystem at each step generates the optimal inputs that will represent the value of known disturbances $u_{20}, u_{21}, u_{6}$ for the Subsystems I and 2.

MPC Subsystem 2: It considers the elements $u_{20}, u_{21}$ as demands. At each step, the value of these elements are provided by the previous execution of the MPC of the Subsystem 3. Moreover, this MPC provides for the next step the values for the actuators $u_{18}, u_{32}, u_{40}, u_{47}, u_{56}, u_{60}$, that are considered as demands in the Subsystem 3.

MPC Subsystem 1: It considers the element $u_{6}$ as a demand. At each step, the value of this element is provided by previous computations from the MPC related to the Subsystem 3.

E. Results Discussion

A hierarchical DMPC controller is compared with a CMPC in the considered scenarios. The control objectives values obtained using both controllers as well as the computational times are presented in Tables I and II. Moreover, the economical cost has been evaluated even if both controllers do not optimize this term. This cost has been evaluated employing a water network simulation tool developed in MATLAB/SIMULINK® [14]. Table I shows that the lost of performance is not so big for all the scenarios. Moreover, it can be noticed from Table II that the DMPC controller requires half of computational time than the CMPC controller to solve one iteration in the worst-case. Thus, despite the DMPC approach inevitably leads to a small loss of performance, the benefits in terms of time and computational load are remarkable. It is important to notice that in Tables I and II, the economical cost is given in economical units (e.u. in tables) and not in the real values (in Euro) because of confidentiality reasons.

VI. Concluding Remarks

In this paper, a DMPC strategy for DWN has been proposed. The DWN is partitioned in a set of subnetworks using a partitioning algorithm that makes use of the topology of the network, the information about the actuator usage and heuristics. A suboptimal DMPC strategy was derived that allows the hierarchical solution of the set of MPC controllers used to control each partition. The proposed DMPC approach is compared against a CMPC controller in an aggregate version of the Barcelona DWN. Results have shown the effectiveness of the proposed DMPC approach in terms of the computation time while the lost of performance is small in all the considered scenarios. As further work, an improvement of the partitioning algorithm used in this paper should be done using results from graph theory. Finally, particular issues related to the the possibility of allowing the subsystems overlapping will be deeply studied.

REFERENCES