Communication

Can Dark Energy Emerge from a Varying *G* and Spacetime Geometry?

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Abstract: The accelerated expansion of the universe implies the existence of an energy contribution known as dark energy. Associated with the cosmological constant in the standard model of cosmology, the nature of this dark energy is still unknown. We will discuss an alternative gravity model in which this dark energy contribution emerges naturally, as a result of allowing for a time-dependence on the gravitational constant, *G*, in Einstein's field equations. With this modification, Bianchi's identities require an additional tensor field to be introduced so that the usual conservation equation for matter and radiation is satisfied. The equation of state of this tensor field is obtained using additional constraints, coming from the assumption that this tensor field represents the space-time response to the variation of *G*. We will also present the predictions of this model for the late-universe data, and show that the energy contribution of this new tensor is able to explain the accelerated expansion of the universe without the addition of a cosmological constant. Unlike many other alternative gravities with varying gravitational strength, the predicted *G* evolution is also consistent with local observations and therefore this model does not require screening. We will finish by discussing possible other implications this approach might have for cosmology and some future prospects.

Keywords: dark energy; varying constants; alternative gravity; cosmology

1. Introduction

Modern cosmology has increasingly become a precision science during the last 30 years, allowing the determination of many properties of the universe to a great degree of accuracy [1,2]. As it has happened many times in the history of physics, this new abundance of data has also created various tensions within the standard framework [3,4].

One of the major puzzles cosmologists are facing today is the so called cosmological constant problem [5,6]. A helpful way of looking at this problem is as the standard theory's inability to satisfactorily explain the nature of the "dark energy" that drives the observed accelerated expansion of the universe. In standard cosmology, dark energy is represented by a cosmological constant term, Λ, within Einstein's equations. A constant added this way to the equations behaves exactly as the vacuum energy, which was, historically, first noticed by Lemaître [7]. General relativity has no natural reason to produce this term, so that it has to be added ad hoc. Fortunately, quantum field theory expects that there should indeed be a vacuum energy. The problem is that, even only taking into account the energy scales for which we know quantum field theory works, without referring to anything unknown, the calculated value of vacuum energy density is many orders of magnitudes higher than what is measured with the expansion of the universe.

This large vacuum energy does not create problems in the quantum field theory, because quantum theory allows one to ignore the zero-point-energy, and only energy differences become important. For general relativity, on the other hand, all energy, including the zero-point-energy, should gravitate. The question of how the quantum vacuum should gravitate in itself highlights a deep incompatibility between quantum theory and general relativity.

It is possible to imagine that a theory of quantum gravity will have some sort of a mechanism allowing vacuum energy density to be set to a value that matches the cosmological observations.[1] However, even assuming this would be the case, the question "Why should the vacuum energy density be this far too small, but otherwise non-remarkable value instead of zero" still remains. To this, broadly, three types of response are possible:

1. There is nothing necessarily problematic about the value of the cosmological constant. Just like the gravitational constant, or fine structure constant, energy density of the vacuum is this value measured by the expansion rate of the universe. Quantum theory should simply incorporate this constant.

2. Dark energy is not really the vacuum energy, but some other particle field that looks like a cosmological constant at the first approximation. Cosmologists should then try to create models for this substance and understand its properties.

3. Gravity does not work exactly as general relativity envisions, and there should be a modification in such a way to explain away the accelerated expansion. In this view, dark energy is the artefact of a gravitational mechanism.

In the following, we will discuss the implications of an often overlooked possibility within the third option. We will investigate a model where the gravitational constant, G, changes with cosmological time, and we will argue that, in this picture, dark energy that drives the cosmological expansion emerges necessarily, as a response of the spacetime geometry. It can be noted that various modified gravity theories (Brans–Dicke being the prime example [11]) also involve an effective variation of the gravitational constant. In comparison to these, the $G(t)$ model we will discuss has the advantage of being more conservative, in the sense that it makes no additional assumptions about the nature of gravity beyond general relativity. In fact, as there is no fundamental reason that G should be a constant, this model can be said to lift that assumption. This way, instead of offering a new theory of gravity, we study a phenomenological extension to general relativity. Furthermore, we will show that the varying G model is not only consistent with the late-universe data, but also the expected fluctuation in G is small enough to be consistent with independent measurements of the gravitational constant such that no screening of gravitational interactions is needed.

In Section 2 we will begin by demonstrating how an additional term is required by the geometry of spacetime if G is not a constant. We will follow this up by outlining the derivation of the cosmological equations including the energy contribution from this additional term. In Section 3, we will discuss the cosmological data used in this research, paying particular attention to how supernovae should be affected by a change in the gravitational strength, and present the results of our analysis. We will then conclude with a discussion of the results, and what other impacts this model might have for cosmology. The discussion and results presented in the following are based on Ref. [12] by the present authors, and we refer the reader to this reference for further details.

2. Varying G in Einstein's Equations

Our starting point is Einstein's equations with G being a time-dependent variable, namely:

$$G^{\mu\nu} = 8\pi G(t, \vec{x}) T^{\mu\nu}, \tag{1}$$

with $G^{\mu\nu}$ and $T^{\mu\nu}$ being the Einstein and stress-energy tensors, respectively. According to the cosmological principle in the following, we take $G(t, \vec{x}) = G(t)$.

However, with equations in this form, Bianchi's identities imply that the energy density of the normal constituents, such as matter or radiation, become dependent on the gravitational constant:

$$D_\mu G^{\mu\nu} = 0 \quad \Rightarrow \quad D_\mu T^{\mu\nu} = -\frac{T^{\mu\nu}\partial_\mu G(t)}{G(t)} \neq 0 \quad \Rightarrow \quad \rho_{\text{matter}} \propto G^{-1}a^{-3}, \tag{2}$$

where ρ stands for the energy density and a represents the scale factor.

One simple way that preserves both the conservation of energy and Bianchi's identities is if there is an additional dynamical term in the equations. In general, this will be a rank-2 tensor which satisfies

$$D_\mu S^{\mu\nu} = -T^{\mu\nu}\partial_\mu G. \tag{3}$$

We see, then, if G is a time-dependent variable in Einstein's equations, $D_\mu G^{\mu\nu} = 0$ and $D_\mu T^{\mu\nu} = 0$ together necessitate a modification of the form

$$G^{\mu\nu} = 8\pi G(t)T^{\mu\nu} + 8\pi S^{\mu\nu}, \tag{4}$$

where the factor 8π is added for convenience.

With the usual cosmological assumptions, isotropy and homogeneity, this tensor $S^{\mu\nu}$ can be written in terms of only two scalar functions, $\Phi(t)$ and $\Psi(t)$,

$$S^{\mu\nu} = (\Phi(t) + \Psi(t))u^\mu u^\nu + \Psi(t)g^{\mu\nu}, \tag{5}$$

where $g^{\mu\nu}$ represents the metric. The 4-velocity, u^μ, is taken as the Hubble flow, as we take $S^{\mu\nu}$ to have no bulk motion with respect to space.

Note that the $\Phi(t)$ component works exactly as an energy contribution, which we need to determine in order to compare this model with cosmological observations. In fact, Equation (3) is a general expression that should also apply to a variety of dark energy fluid models, which would come with a specific Φ. In our case, we determine this from the argument that the tensor $S^{\mu\nu}$ does not represent a fluid, but instead represents the spacetime response to the varying G.

We can use Equation (3) above to find an expression between $\Phi(t)$ and $\Psi(t)$, but this is not easy to solve for Φ, even if $\Psi(t)$ is known. However, we can put the equations in a more useful form by defining an auxiliary function $\xi(t)$ such that, $\dot{\xi}/\xi = 3H(1 + \Phi/\Psi)$, with H being the Hubble rate. Then, in terms of this function we can integrate Equation (3) as

$$\Phi(t)\xi(t) = \lim_{\varepsilon \to 0} \left(\Phi(\varepsilon)\xi(\varepsilon) - \int_\varepsilon^t \dot{G}\rho\xi dt \right), \tag{6}$$

$$\xi(t) = \lim_{\varepsilon \to 0} \left[\xi(\varepsilon) \frac{a(t)^{3\left(1+\frac{\Phi(t)}{\Psi(t)}\right)}}{a(\varepsilon)^{3\left(1+\frac{\Phi(\varepsilon)}{\Psi(\varepsilon)}\right)}} \exp\left(-3\int_\varepsilon^t \frac{d(\Phi/\Psi)}{dt} \ln(a)dt \right) \right]. \tag{7}$$

These do not look much more promising, but they allow us further simplifications. First, notice that the scale factor, a, dependence appears exclusively through the function ξ. Second, the first equation above includes two constant terms coming from the lower limit of the integral. We want to argue that the energy contribution of $S^{\mu\nu}$, the term we added to preserve the geometrical consistency, is exclusively caused by varying G. Therefore, we should insist that Φ (and, by the virtue of Equation (3), Ψ) is zero if G is a constant. This implies that all the constants in Equation (6) should be zero. Furthermore, ξ should be a constant, otherwise Φ would have an additional scale factor dependence, beyond what is induced by G and ρ.

With these simplifications, we can obtain an expression for Φ in terms only of G and other cosmological parameters. For G, we use an ansatz in the form of a power series centred around the scale factor $a = 1$, of which we use only first few terms in the analysis,

$$G(a) = G_0 \left(1 + \sum_{n=1}^{\infty} b_n (1-a)^n \right). \tag{8}$$

Using the flat Friedmann–Lemaître–Robertson–Walker (FLRW) metric, we obtain the cosmological equations in the usual manner:

$$H^2 = \frac{8\pi G \rho}{3} + \frac{8\pi \Phi}{3}, \qquad \frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) - \frac{4\pi}{3}(\Phi + 3\Psi), \tag{9}$$

with p representing the pressure of the fluid.

3. Analysis and Results

As our model is concerned with dark energy, we use late-universe probes, namely type-Ia supernovae (SNIa) and baryon acoustic oscillations (BAO). For BAO, we use measurements from 6dFGS [13], SDSS-MGS [14], BOSS DR12 [15], and eBOSS DR14 [16], Ly-α autocorrelation function [17], and Ly-α-quasar cross-correlation [18] at $z = 2.4$. For SNIa, we use the JLA dataset [19].

As we do not assume screening, supernova luminosities should be modified as they would be affected by the variation in gravitational strength. A straightforward argument, popular in the cosmology literature, is to include a G dependence of luminosity through Chandrasekhar's mass. By assuming that a higher mass of explosion will lead to a higher nickel content and, therefore, higher luminosity, one has $L \propto M_{Ch} \propto G^{-1.5}$ [20]. On the other hand, the recent literature [21,22] on supernovae physics indicates a relationship on the opposite direction, $L \propto G^{1.46}$. In our study, we use both approaches, but as the predicted G variation is quite small we do not obtain a notable difference between the two. Therefore we only report the results from the first approach.

Using a χ^2 minimization analysis, we determine the cosmological parameters: matter density, Ω_m, radiation density, Ω_r, and the product of the Hubble constant times the standard ruler, $H_0 r_d$; as well as the parameters of G: b_1, b_2, and b_3. Of the latter, only two are independent due to flatness (we take the dependent parameter to be b_1). Higher order terms for G do not improve the fit, as they are less relevant for a close to one. The χ^2 values and best-fit values obtained for all parameters can be found in Equations (10)–(12). As Equation (10) shows, the present varying G model is consistent with cosmological data in a similar way to ΛCDM:

$$\chi^2_{\text{Varying G}}/\text{d.o.f.} = 697.73/747, \qquad \chi^2_{\Lambda CDM}/\text{d.o.f.} = 698.05/749. \tag{10}$$

$$b_1 = 0.07 \pm 0.15, \qquad b_2 = -0.51 \pm 0.33, \qquad b_3 = 0.679 \pm 0.094. \tag{11}$$

$$\Omega_m = 0.284 \pm 0.017, \Omega_r = (0.0 \pm 7.0) \times 10^{-3}, H_0 r_d = (101.7 \pm 1.3) \times 10^2 [\text{km s}^{-1}]. \tag{12}$$

When we look at the G evolution in Figure 1, we see that the expected variation of G is below 5% of the standard value in the epochs of interest. It appears that the change of G increases as we go to earlier times, but this is largely due to the function used as an ansatz for G. As we include no data from the early-universe, our present results have no predictive value from these times.

An interesting parameter to note is b_1, which is equal to $-\frac{1}{H_0}\frac{\dot{G}}{G}$ at $a = 1$. As independent observations of G measure $\frac{\dot{G}}{G}$ (for instance Lunar Laser Ranging [23]), this parameter can be used to estimate if the present measurements of G are consistent with our model (alternatively, it is possible to constrain b_1 using $\frac{\dot{G}}{G}$ measurements). We see that b_1 is consistent with zero, meaning that, under the present model, no small-scale modifications are required to the gravitational interactions for consistency with local observations.
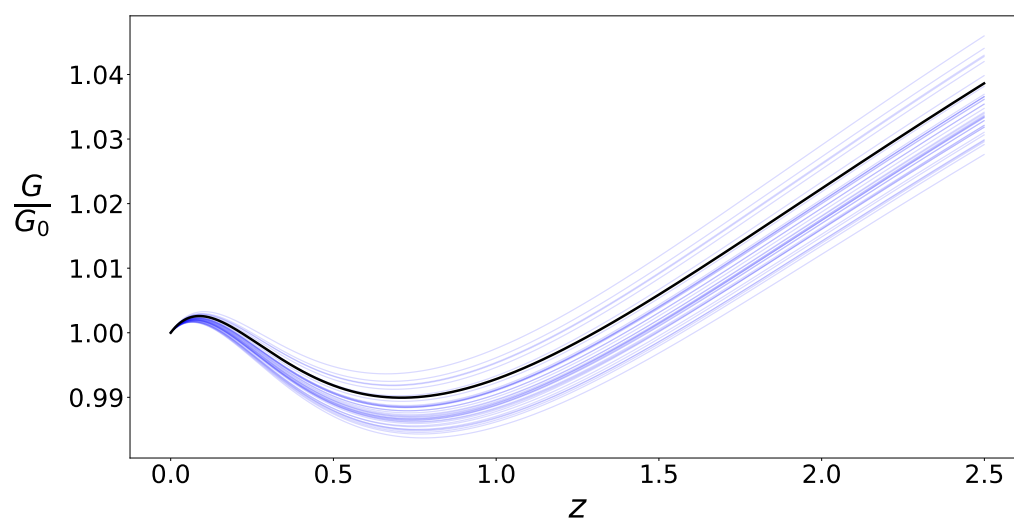
Figure 1. Evolution of *G* vs redshift. The line is drawn using the best fit values for b_1, b_2, and b_3 while the blue lines are some sample error lines with one χ^2 difference ($\Delta\chi^2 = 1$) from the black line.

4. Conclusions and Outlook

In the preceding discussion we have considered the possibility that the observed "dark energy" is emergent as a result of a variation of *G*, due to the geometry of spacetime in Einstein's equations. This allows the interpretation that dark energy can be formulated as the response of the spacetime to the varying *G*. Comparing our model with cosmological observations from the late universe, we have found that the considered model is consistent with the data in a similar way to the standard cosmology. This indicates that this model can adequately explain the accelerated expansion of the universe without a cosmological constant, avoiding one part of the cosmological constant problem.

Furthermore, we have found that the necessary variation of *G* is relatively small, which means that the small-scale interactions do not need to be ignored (or screened), in contrast to many other modified gravity approaches. As this model is also very conservative in terms of assumptions, it can be a good basis for studying possible changes of gravitational strength on astrophysical scales.

Of course, we do not claim that this model can compete the standard model in terms of its completeness, and this investigation can be extended in various ways. From a cosmological perspective, an immediately interesting question is whether a varying *G* model is compatible with the early-universe data. For this, the most important indicator is cosmic microwave background (CMB) constraints on the variation of *G*. However, this analysis is not straightforward and requires a treatment of the perturbations. Along with CMB, constraints from structure growth and big bang nucleosynthesis can also be included in the analysis to have a fuller understanding of the constraints on *G* beyond the late-universe we have focused on. Such an analysis may also have the effect of decreasing other tensions within modern cosmology, an example being the H_0 value. A cursory look at the cosmological equations suggests that, at the background level, a smaller *G* at CMB epoch can accommodate a larger H_0 value without changing the expansion history. While it is possible that such a change in *G* will not be compatible with the observations, a further analysis of the present model may bring insights into the H_0 problem as well.

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Note

[1]   It is possible that this may be achieved through a more careful treatment of quantum vacuum within the current semi-classical paradigm as proposed by [8,9]. This approach, however, is subject to ongoing debate [10].

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