Measurements of Charged Current Lepton Universality and $|V_{us}|$ using Tau Lepton Decays to $e^- \nu_e \nu_\tau$, $\mu^- \nu_\mu \nu_\tau$, $\pi^- \nu_\pi$, and $K^- \nu_\pi$

Using 467 fb$^{-1}$ of $e^+e^-$ annihilation data collected with the BABAR detector, we measure $\frac{B(e^+e^-\rightarrow\mu^+\mu^-)}{B(e^+e^-\rightarrow\eta\eta)} = (0.9796 \pm 0.0016 \pm 0.0036)$, $\frac{B(e^+e^-\rightarrow\tau^+\tau^-)}{B(\tau^+\tau^-)} = (0.5945 \pm 0.0014 \pm 0.0061)$, and $\frac{B(e^+e^-\rightarrow\pi^+\pi^-)}{B(\pi^+\pi^-)} = (0.03882 \pm 0.00032 \pm 0.00057)$, where the uncertainties are statistical and systematic, respectively. From these precision $\tau$ measurements, we test the Standard Model assumption of $\mu\tau$ and $\tau\mu$ charge current lepton universality and provide determinations of $|V_{us}|$ experimentally independent of the decay of a kaon.

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Decays of the $\tau$ lepton to a single charged particle and neutrino(s) probe the Standard Model (SM) predictions of charged current lepton universality and the unitarity relation of the first row of the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix. Previous measurements of universality \cite{2, 3, 4}, expressible in terms of the coupling strength $(g_\ell)$ of lepton of flavor $\ell$ to the charged gauge boson of the electroweak interaction are in agreement with the SM where $g_\mu/g_\pi = g_\mu/g_\pi = 1$. Similarly, kaon decay measurements \cite{3, 4} sensitive to $|V_{ud}|$, the relative weak coupling between up and strange quarks, yield a value consistent with unitarity $|\langle V_{ud}\rangle^2 + |V_{us}\rangle^2 + |V_{ub}\rangle^2 = 1$ where nuclear beta decays provide $|V_{ud}|$ \cite{3} and $|V_{us}|$ is negligible. \cite{4}. However, new physics that couples primarily to the third generation could be revealed through deviations from the SM in precision universality and $|V_{us}|$ measurements involving the $\tau$. Significant deviations of this nature are unambiguous signatures of new physics that provide crucial but complimentary information to the direct searches for Higgs bosons \cite{6} and other new physics models with e.g. lepto-quarks \cite{7}, heavy gauge $W'$ or $Z'$ bosons, heavy quarks or leptons, compositeness or extra dimensions \cite{8}.

Recent measurements of the sum of strange $\tau$ branching fractions interpreted in the framework of the Operator Product Expansion (OPE) and finite energy sum rules yield a value of $|V_{us}|$ that is approximately three standard deviations ($\sigma$) lower than expectations from CKM unitarity \cite{9}. This paper addresses both experimental and theoretical aspects of this question by providing the first precision measurements of $R_K \equiv \frac{B(\tau^+ \to K^+ \nu_\tau)}{B(\tau^+ \to \pi^+ \nu_\tau)}$ \cite{10} and $R_K/\pi \equiv \frac{B(\tau^+ \to K^+ \nu_\tau)}{B(\tau^+ \to \pi^+ \nu_\tau)}$ enabled by the unique combination of a very large $\tau$ sample with particle momenta amenable to particle identification using Cherenkov radiation. By using values of the meson decay constants from lattice QCD \cite{11}, we provide two precision determinations of $|V_{us}|$ from $\tau$ decays independent of the OPE framework. We also report on new measurements of $R_\pi \equiv \frac{B(\tau^- \to \pi^- \nu_\tau)}{B(\tau^- \to \pi^- \nu_\tau)}$ and $R_{\pi\nu} \equiv \frac{B(\tau^- \to \pi^- \mu_\tau\nu_\tau)}{B(\tau^- \to \pi^- \mu_\tau\nu_\tau)}$. $R_\pi$ provides an improved measurement of $g_\pi/g_\mu$ whereas $R_{\pi\nu}$ and $R_K$, when compared to the muonic branching fractions of the pion and kaon, yield improved measurements of $g_\pi/g_\mu$ involving pseudoscalar mesons.

The data sample corresponds to an integrated luminosity of $L = 467$ fb$^{-1}$ recorded at an $\epem$ center-of-mass (CM) energy ($\sqrt{s}$) near 10.58 GeV and was collected with the BABAR detector at the SLAC PEP-II $\epem$ storage rings. With a luminosity-weighted average cross-section of $\sigma_{\epem\to\tau^+\tau^-} = (0.919 \pm 0.003)$ nb \cite{12, 13}, this corresponds to the production of $4.29 \times 10^8 \tau$-pair events. The BABAR detector \cite{14} is composed of a silicon vertex tracker, drift chamber (DCH), ring-imaging Cherenkov detector (DIRC), and electromagnetic calorimeter (EMC), all contained in a 1.5-T solenoid. The iron flux return for the solenoid is instrumented (IFR) to identify muons.

Tau-pair events are simulated with the KK Monte Carlo (MC) generator \cite{14}, which includes higher-order radiation corrections. We simulate $\tau$ decays with Tauola \cite{15} and Photos \cite{16} using measured branching fractions \cite{3}. The detector response is simulated with Geant4 \cite{17}. Simulated events for signal as well as background processes \cite{11, 18} are reconstructed in the same manner as data. The MC samples are used for selection optimization, control sample studies, and systematic error studies. The number of simulated non-signal events is comparable to the number expected in the data, with the exception of Bhabha and two-photon events, which are not simulated but which data studies show to be negligible.

We study $e^+e^- \to \tau^+\tau^-$ events with the $\tau^-$ decaying via $\tau^- \to e^- \nu_e \nu_\tau$, $\tau^- \to \mu^- \nu_\mu \nu_\tau$, $\tau^- \to \pi^- \nu_\tau$ or $\tau^- \to K^- \nu_\tau$ modes and the $\tau^+$ decaying via a $\tau^+ \to \pi^+ \pi^- \nu_\tau$ tagging channel with the selection criteria optimized to minimize the combined statistical and systematic uncertainties \cite{19}. The number of signal events for decay modes $i = \{e, \mu, \pi, K\} = \{e^- \nu_e \nu_\tau, \mu^- \nu_\mu \nu_\tau, \pi^- \nu_\tau, K^- \nu_\tau\}$ are $N^D_i = \mathcal{E}_i (N^D_i - N^B_i)$ where $\mathcal{E}_i$ is the efficiency (including $B(\tau^- \to \pi^- \pi^- \nu_\tau) = (8.85 \pm 0.13)\%$ \cite{19}), $N^D_i$ the number of selected data events, and $N^B_i$ the estimated number of background events for the $i$th mode.

We measure the ratios $R_i = N^S_i / N^D_i$ which normalizes to the most precisely known relevant SM process available, and in which several common sources of systematic uncertainty cancel. $N^D_i$ are multiplied with reproducible random numbers until all efficiency and uncertainty estimates are finalized. Once unbinned, we use the values of the three branching ratios to update world averages of the branching fractions, which we then use to recalculate the backgrounds for our final results.

Events with a net charge of zero and with four well-reconstructed tracks not originating from the conversion of a photon in the detector material are selected. For good particle identification, each track is required to be within the acceptance of the DIRC and EMC, and have a transverse momentum greater than 0.25 GeV to ensure that it reaches the DIRC. The plane normal to the thrust axis divides the event into hemispheres in the CM frame. The “signal” hemisphere contains a single track and the “tag” hemisphere the other three tracks.

Each tag hemisphere track is required to be consistent with being a pion and the energy deposited in the EMC unassociated with any tracks in this hemisphere is required to be less than 0.20 GeV. Also, events that contain track pairs consistent with coming from a $K^0_S$ are vetoed.

The signal track momentum is required to lie between 1 and 4 GeV/c. Information from the five detector subsystems is combined in likelihood selectors which identify $e$, $\pi$, and $K$ particles and in a neural network which identifies muons. The $\pi$-$K$ separation is provided by the DIRC and DCH whereas $\pi$-$\mu$ separation is primarily ac-
accomplished with the IFR and EMC. The identification efficiencies are given in Table II and cross-contaminations are given below. We suppress di-muon and Bhabha backgrounds by requiring signal tracks identified as a lepton to have CM momentum less than 80% of √s/2c. To reduce cross-feed from e into the π and K channels, the ratio of deposited electromagnetic energy of a π or K candidate track to its measured momentum, E/pc, is required to be less than 0.85. A pion track also passing a loose muon selection is rejected. A similar veto is applied for a kaon track passing the loose muon selection if its measured momentum exceeds 3 GeV/c. Also, events with an EMC energy > {1.0, 0.5, 0.2, 0.2} GeV in the signal hemisphere unassociated with the {e, μ, π, K} track are removed.

Pion and kaon control samples from D^+ → π^+D^0, D^0 → π^+K^- decays are used to study and correct for small differences between MC and data. We cross-check these with independent π^± (K^-) control samples from τ^- → π^-ν^-π^±ν_π^- (τ^- → K^-π^-K^+π^-ν_π^-) decays using particle identification of two of the oppositely charged particles and the fact that the wrong sign τ^- → π^-ν^-K^+ν_τ decays are heavily suppressed. Samples of radiative Bhabha and radiative μ-pair events provide control samples of electrons and muons. The systematic uncertainty associated with charged particle identification is assessed from the control sample statistical errors, consistency between control samples, and the sensitivity of the control sample corrections to the number of particles near the track. The statistical errors in the more limited cross-check control samples dominate these errors. Because we use control samples to correct charge conjugate particles separately, charge-dependent detector responses are accounted for by construction.

To remove two-photon and Bhabha backgrounds, the event must have a missing CM energy between 10% and 70% of √s. The angle between the missing momentum and electron beam direction in the CM, θ^CM miss, is constrained to satisfy |cos(θ^CM miss)| < 0.7, the thrust of the event is required to be above 0.9, and the net missing transverse momentum in the CM greater than 0.009√s/c.

Each of the three tag-side tracks has an electron veto applied to further reduce the Bhabha contamination. This results in less than 0.03% contamination from two-photon events and less than 0.1% contamination from Bhabha events in the electron signal sample. These backgrounds were investigated by studying samples enriched in Bhaba and two-photon events by adjusting the requirements on the thrust, cos(θ^CM miss), and transverse momentum of the event. Potential background from Bhabha events were further probed by studying the number of events having a high signal track momentum as the electron veto was progressively lifted from one, then two, and finally all three tracks in the tag hemisphere.

To suppress backgrounds in the τ^- → π^-ν_τ and τ^- → K^-ν_τ channels from τ decays with undetected neutral particles other than the ν_τ (e.g. K^0_L mesons, ν_µ), we reconstruct the direction of the back-to-back τ^+τ^- system in the CM frame. The polar angle of the τ momentum with respect to the tag-side hadronic system is calculated assuming that the CM energy of the τ is √s/2, and the azimuthal angle of the τ momentum is fixed to a value that has been optimized to minimize the total error on Dτ/τ [20]. With this estimator for the τ momentum, we require the missing mass in the signal hemisphere to be less than 0.56 GeV/c^2.

For the selected τ^- → μ^-ν_μτ events, the dominant backgrounds are τ^- → π^-ν_τ (1.84 ± 0.04)% and τ^- → π^-ν_τ (0.95 ± 0.01)%. For the τ^- → π^-ν_τ channel, the dominant backgrounds are τ^- → μ^-ν_ν (12.89 ± 0.7)%, τ^- → π^-ν_τ (5.87 ± 0.04), and non-τ backgrounds (0.04 ± 0.05). The major backgrounds in the τ^- → K^-ν_τ channel are from τ^- → π^-ν_τ (10.06 ± 0.13)% , τ^- → K^-K^0_Lν_τ (3.87 ± 0.41)%, τ^- → K^-π^-ν_τ (1.97 ± 0.14)% , τ^- → π^-ν_τ (1.06 ± 0.06)% , and non-τ backgrounds (2.58 ± 0.38)%. The uncertainties are from MC statistics, branching fractions and, for non-τ backgrounds, the systematic uncertainty on background rates. Fig. 1 shows the momentum distributions in the CM frame for each of the four decay modes for

FIG. 1: Data (points) and MC (histograms) distributions of CM momentum for (a) τ^- → e^-ν_τν_τ, (b) τ^- → μ^-ν_τν_τ, (c) τ^- → π^-ν_τ and (d) τ^- → K^-ν_τ modes. The small differences between MC and data are accounted for in the systematic errors.
data, along with the background MC contributions.

For the $\tau^- \rightarrow e^- \nu_e \nu_\tau$ channel, 884426 events are selected with an efficiency and purity of (0.589 ± 0.010)\% and (99.69±0.06)\%, respectively. The number of selected events, efficiency, purity and systematic uncertainties on $R_t$ of the $\tau^- \rightarrow \mu^- \nu_\mu \nu_\tau$, $\tau^- \rightarrow \pi^- \nu_\pi$, and $\tau^- \rightarrow K^- \nu_K$ selections are presented in Table I. These uncertainties include contributions from the particle identification, the sensitivity to detector response including the impact of changing the MC momentum scale and DCH resolution, modelling of hadronic and electromagnetic showers in the EMC, the EMC energy scale, and angular measurements made by these detectors within their modelling uncertainties, the backgrounds, initial- and final-state radiation, radiation in $\tau$ decays, rate and shape of $\tau^- \rightarrow \pi^- \pi^- \pi^+ \nu_\tau$, the trigger, and $\mathcal{L}_{\tau\nu}$. The systematic uncertainty on $R_\mu$ is dominated by uncertainties in particle identification. The $R_\pi$ and $R_K$ measurements have additional dominant contributions from the detector modelling and associated backgrounds, due to stronger cuts on the EMC energy necessary to reduce non-$\tau$ backgrounds. Presence of the $\sim 20\%$ backgrounds in these channels render them more sensitive to the modelling of the tag-side decays. The dominant background uncertainty in the $R_\pi$ measurement arises from the electron contamination in the $\pi$ sample investigated by measuring the number of events that fail the $E/p$ electron veto requirement in data and MC. In the $R_K$ event sample, the uncertainty arising from the $\tau$ decay branching fractions of background modes is 0.58\%, which is dominated by the uncertainty of the $\tau^- \rightarrow K_L^0 K^- \nu_\tau$ fraction. There is also a 0.49\% uncertainty assigned for $q\bar{q}$ backgrounds, which are studied using events with an invariant mass of the tracks in the tag hemisphere above the $\tau$-mass and cross-checked in regions of thrust and $\cos(\theta_{\text{miss}})$ enriched with these backgrounds.

The measured branching ratios and fractions are:

\[
R_\mu = (0.9796 \pm 0.0016 \pm 0.0036) \\
R_\pi = (0.5945 \pm 0.0014 \pm 0.00061) \\
R_K = (0.03882 \pm 0.00032 \pm 0.00057) \\
R_{hK} = R_\pi + R_K = (0.6333 \pm 0.0014 \pm 0.0061) \\
B(\tau^- \rightarrow \mu^- \nu_\mu \nu_\tau) = (17.46 \pm 0.03 \pm 0.08)\% \\
B(\tau^- \rightarrow \pi^- \nu_\pi) = (10.59 \pm 0.03 \pm 0.11)\% \\
B(\tau^- \rightarrow K^- \nu_K) = (0.692 \pm 0.006 \pm 0.010)\%
\]

where $h = \pi$ or $K$ and we use $B(\tau^- \rightarrow e^- \nu_e \nu_\tau) = (17.82 \pm 0.05)\%$. The off-diagonal elements of the correlation matrix for the measured ratios (branching fractions) are $\rho_{\mu\pi} = 0.25$ (0.34), $\rho_{\mu K} = 0.12$ (0.20), and $\rho_{\pi K} = 0.33$ (0.36). The $\mu$ and $\pi$ measurements are consistent with and of comparable precision as the world averages [3] whereas the $K$ measurement is consistent with but twice as precise as the world average [3].

Tests of $\mu - e$ universality can be expressed as

\[
\left( \frac{g_{\mu}}{g_{e}} \right)_\tau^2 = \frac{B(\tau^- \rightarrow h \nu_{\tau})}{B(h \rightarrow \mu \nu_{\mu})} \left( \frac{2 m_h m_{\mu}^2 \tau_h}{(1 + \delta_{\mu}) m_{\tau}^2 \tau_{\tau}} \right) \left( \frac{1 - m_{\mu}^2 / m_{\tau}^2}{1 - m_{h}^2 / m_{\tau}^2} \right)^2,
\]

where the radiative corrections are $\delta_{\mu} = (0.16 \pm 0.14)%$ and $\delta_{\tau} = (0.90 \pm 0.22)\%$ [22]. Using the world averaged mass and lifetime values and meson decay rates [2], we determine \(\left( \frac{g_{\mu}}{g_{e}} \right)_h = 0.9856 \pm 0.0057 (0.9827 \pm 0.0086)\) and \(\left( \frac{g_{\mu}}{g_{e}} \right)_h = 0.9850 \pm 0.0054\) when combining these results; this is 2.8σ below the SM expectation and within 2σ of the world average.

We use the kaon decay constant $f_K = 157 \pm 2$ MeV [11], and our value of

\[
B(\tau^- \rightarrow K^- \nu_K) = \frac{G_F^2 f_K^2 |V_{us}|^2 m_{\tau}^2 \tau_{\tau}}{16 \pi^3} \left( 1 - \frac{m_{\mu}^2}{m_{\tau}^2} \right)^2 S_{EW},
\]

where $S_{EW} = 1.0201 \pm 0.0003$ [24], to determine $|V_{us}| = 0.2193 \pm 0.0032$. This measurement is within 2σ of the value of 0.2255 ± 0.0010 predicted by CKM unitarity and is also consistent with the value of $|V_{us}| = 0.2165 \pm 0.0027$ derived from the inclusive sum of strange $\tau$ decays [9].

Both of our measured $|V_{us}|$ values depend on absolute strange decay rates. Our value of $R_{K/\pi} = (0.06531 \pm 0.00065 \pm 0.00093)$, however, provides a $|V_{us}|$ value driven

| Table I: Number of selected events, purity, total efficiency, component of the efficiency from particle identification, and systematic uncertainties (in %) on $R_t$ for each decay mode. |
|---|---|---|---|
| $N^\tau$ | $\mu$ | $\pi$ | $K$ |
| 731102 | 369091 | 25123 |
| Purity | 97.3% | 78.7% | 76.6% |
| Total Efficiency | 0.485% | 0.324% | 0.330% |
| Particle ID Efficiency | 74.5% | 74.6% | 84.6% |

**Systematic uncertainties:**

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<th>Value</th>
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<tr>
<td>Total [%]</td>
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by the ratio between strange and non-strange decays. We use $f_K/f_\pi = 1.189 \pm 0.007$, $|V_{us}| = 0.2255 \pm 0.0024$ where short-distance electro-weak corrections cancel in this ratio. This value is consistent with CKM unitarity $|V_{ud}|^2$ and $2.5\sigma$ higher than $|V_{us}|$ from the inclusive sum of strange $\tau$ decays.

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[4] F. Ambrosino et al. (KLOE Collaboration), JHEP 0804, 059 (2008); |V_{us}| = 0.2237 ± 0.0013; M. Antonelli et al. (FlaviaNet Kaon Group), arXiv:0801.1817 (2008).


[10] Charge conjugate $\tau$ decays are implied throughout.


