Thermodynamics of Gambling Demons: Theory and Experiment

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PRL **126**, 080603 (2021) [arXiv: 2008.01630]

UNIT OF EXCELLENCE MARÍA DE MAEZTU

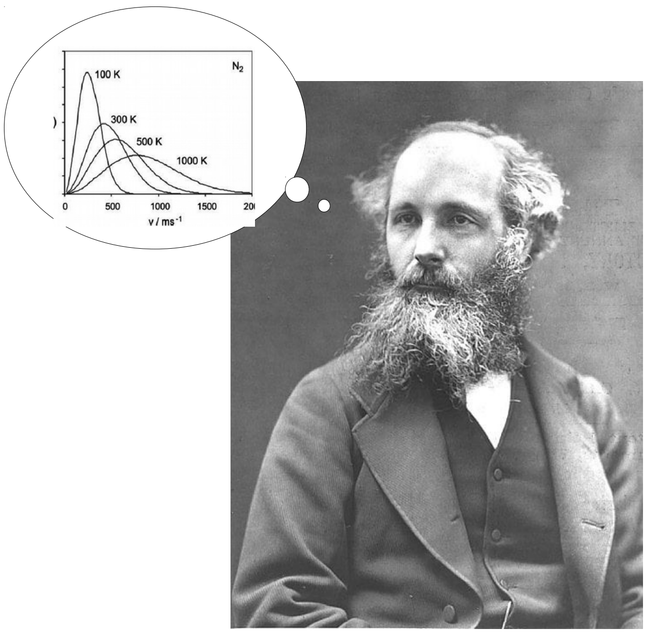
AGENCIA ESTATAL DE INVESTIGACIÓN

QTD 2021 (4-8 October 2021, Geneva)

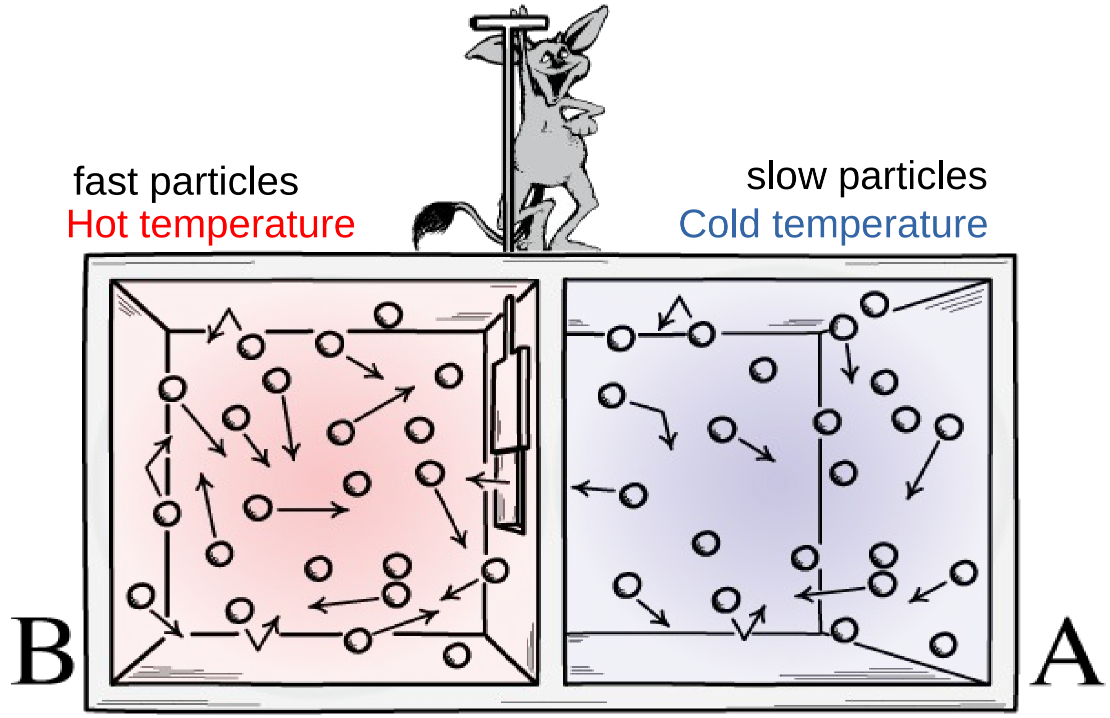
Outline:

- **Introduction and motivation**
- **Gambling demon theory**
- **Experiment in a single-electron box**
- **Quantum gambling**
- **Main conclusions**

Maxwell's demon 1867



James C. Maxwell (1831 – 1879)

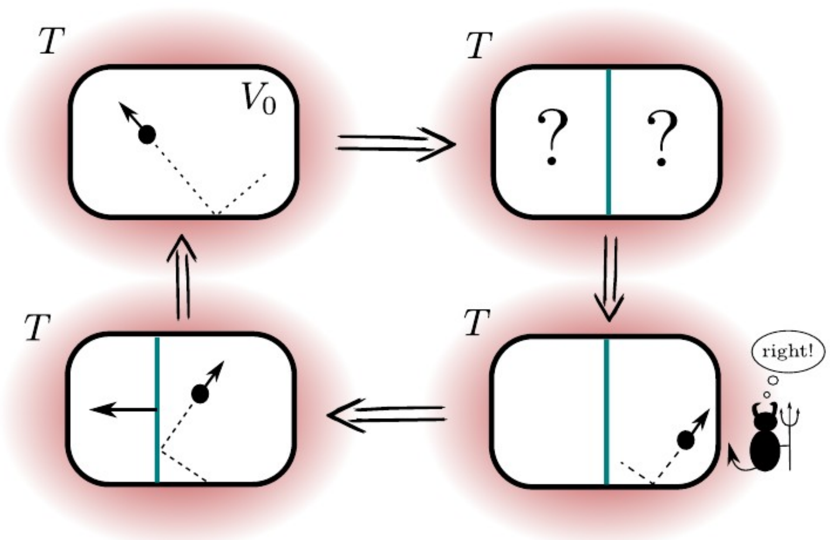


Drawing: Jonh D. Norton

Szilard's engine 1929:

Isothermal expansion:

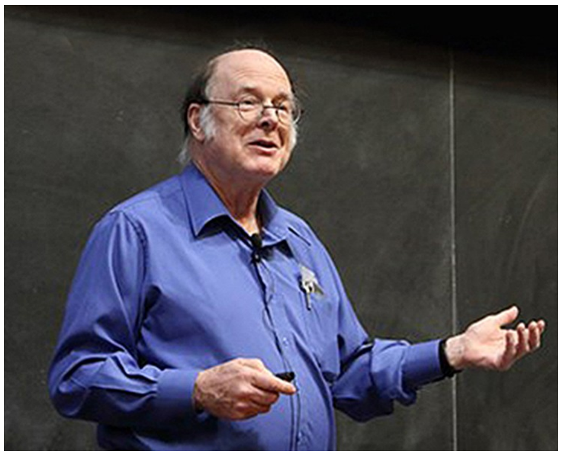
$$W_{\text{ext}} = k_B T \log 2$$



Leo Szilard (1898 - 1964)

Informational Exorcism

Only apparent “violations” of second law. Take into account information processing costs!



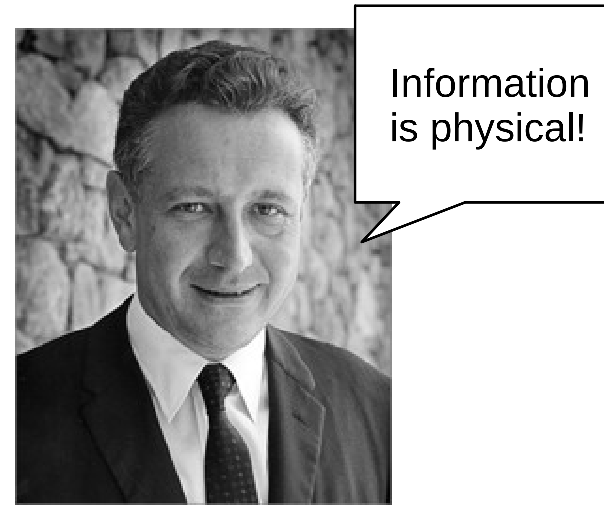
Charles H. Bennett

Landauer’s Erasure 1961

$$W \geq k_B T \log 2$$

Erasing (1 bit of) information has a minimum work cost even if doing it reversibly

NO FREE LUNCH!

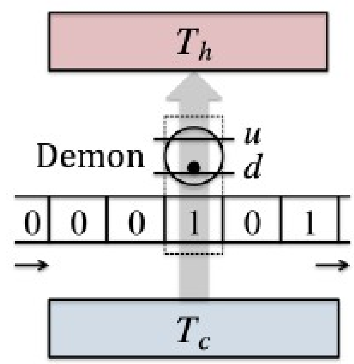


Rolf W. Landauer (1927 - 1999)

Thermodynamics of feedback control:

$$\Delta S - \frac{Q}{T} \geq -\mathcal{I}$$

(mutual) information acquired /stored



Information reservoirs (resource)

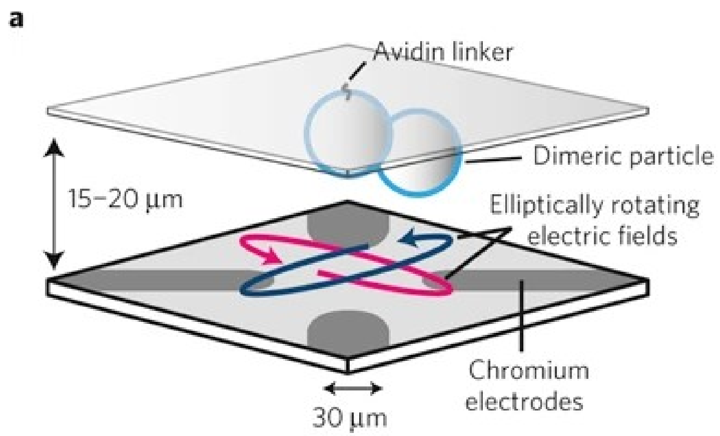
Mandal PRL (2013)

Theory Review: JMR Parrondo, JM Horowitz, and T Sagawa, Nat. Phys. (2015).

Experiments

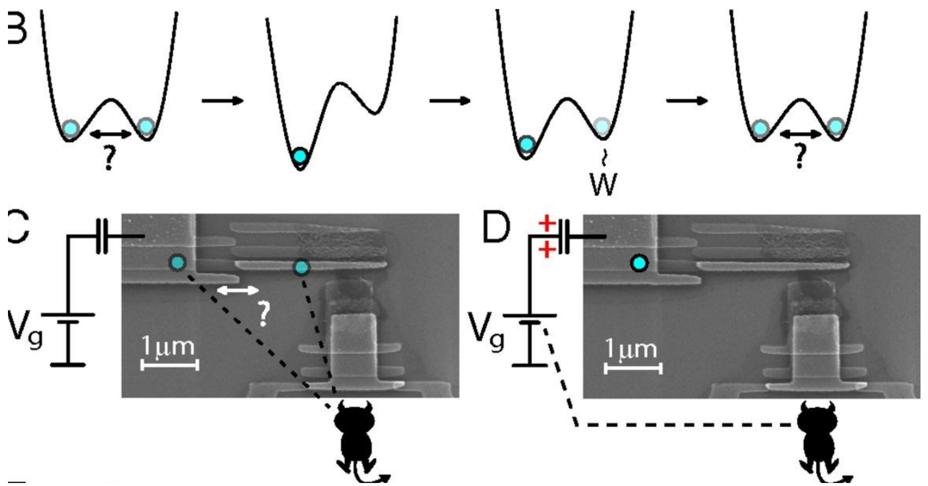
Colloidal particles in optical traps

S Toyabe *et al.* Nat. Phys. **6** (2010)



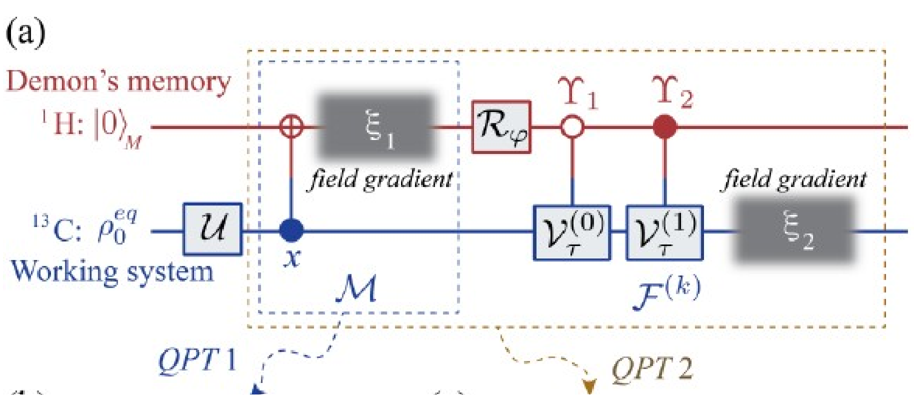
Electronic devices

JV Koski *et al.* PNAS **111** (2014)



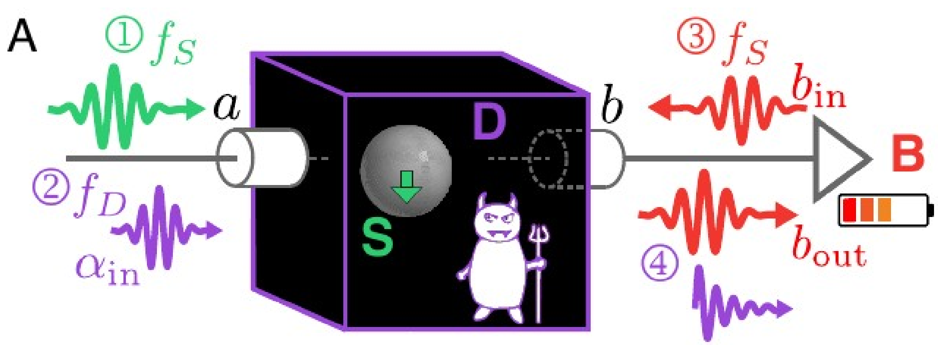
Nuclear spins and NMR spectrometry

PA Camati *et al.* PRL **117** (2016)



Circuit QED setups

N Cottet *et al.* PNAS **114** (2017)



Experiments Review: S Ciliberto, E. Lutz, Physics Today (2015)

Remark: Maxwell's demon has two basic ingredients

(I) gather information about the microscopic state

(II) feedback control over the system (trapdoor / piston)

Demon without feedback? → Gambling Demon

Can only stop (or not) the dynamics

Players in the casino can decide to play or quit a game, but cannot change the rules of the game (feedback).

Gambling Demon

Environment: isothermal $\beta = 1/T$

Driving: fixed protocol until final time

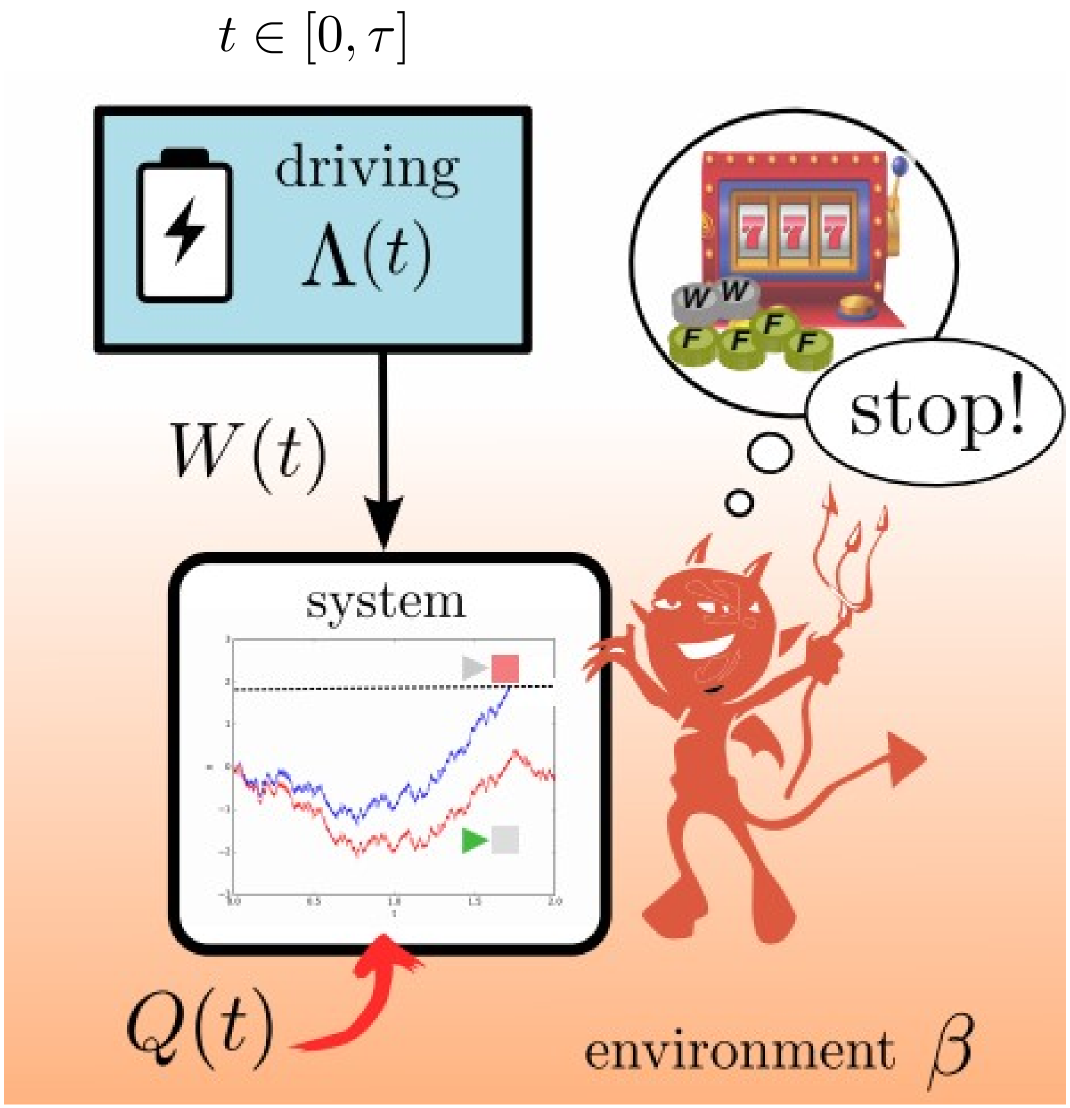
Measurement: continuous monitoring

$$\gamma_{[0,\tau]} = \{x(t)\}_{t=0}^{\tau}$$

Gambling strategy: When a condition is verified for the first time, the Demon stops the dynamics. Otherwise the process continues until the end.

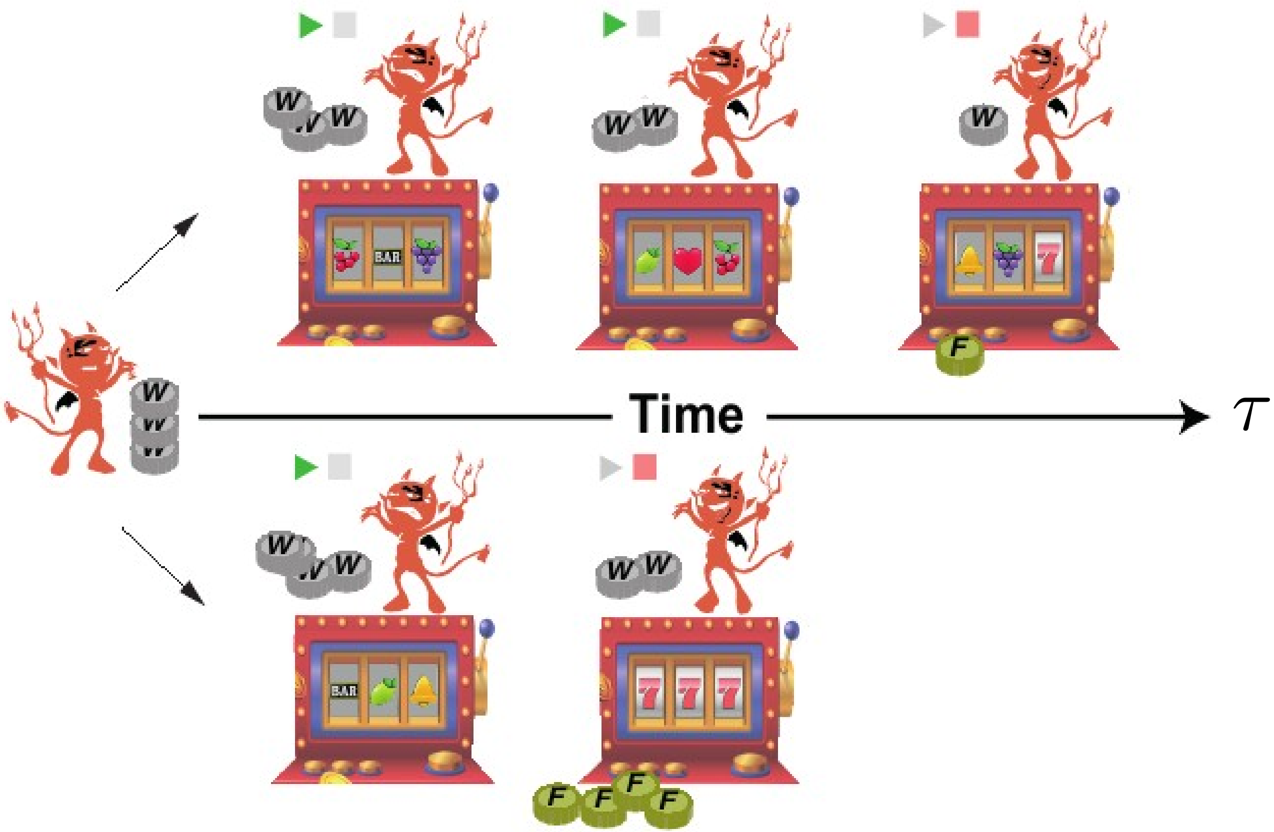
Stopping time: the time at which the demon stops

$0 \leq \mathcal{T} \leq \tau$ (random variable)



Demon without feedback? → Gambling Demon

Analogous to a gambler in a slot machine:



$$\mathcal{T} \leq \tau$$

At final time: τ

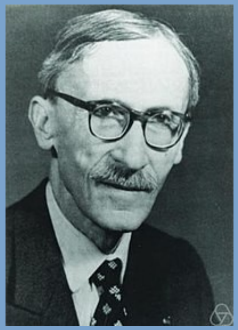
$$\langle W \rangle > \langle \Delta F \rangle$$

At stopping times: $\mathcal{T}$

$$\langle W \rangle_{\mathcal{T}} \overset{?}{<} \langle \Delta F \rangle_{\mathcal{T}}$$

Average over trajectories of different length !

Martingale theory:

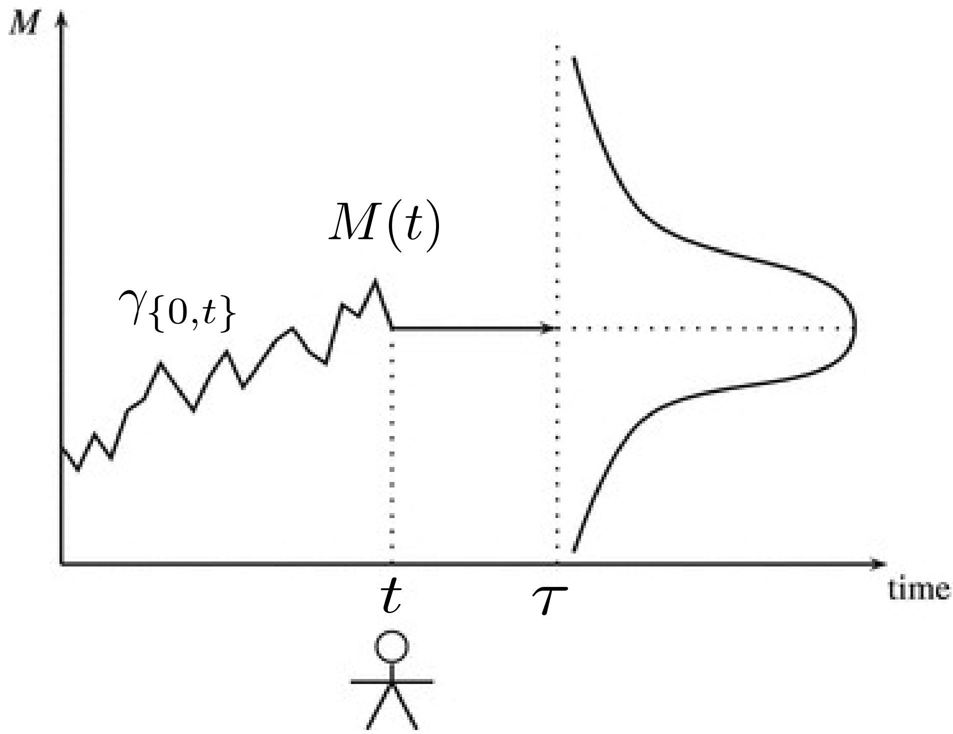


Martingale process

Introduced in prob. theory by Paul Lévy in 1934

$$\langle M(\tau)|\gamma_{[0,t]}\rangle = M(t)$$

$$\text{for} \quad 0 \leq t \leq \tau$$



Doob's optional stopping theorem:

$$\langle M(\mathcal{T})\rangle_{\mathcal{T}} = \langle M(0)\rangle$$

Useful in stochastic thermodynamics:

$$\langle e^{-\Delta S_{\rm tot}(\tau)}|\gamma_{[0,t]}\rangle = e^{-\Delta S_{\rm tot}(t)}$$

(Only in non-eq. stationary processes)

Stronger than fluctuation theorem! if $t=0$

$$\langle e^{-\Delta S_{\rm tot}(\tau)}\rangle = e^{-\Delta S_{\rm tot}(0)} = 1$$

I. Neri, É. Roldán, F. Julicher PRX **7** (2017), R. Chétrite *et al.* EPL **124** (2019), I. Neri *et al.* JSM (2019), ...

Stopping-times work fluctuation relation:

$$\langle e^{-\beta[W-\Delta F]-\delta} \rangle_{\mathcal{T}} = 1$$

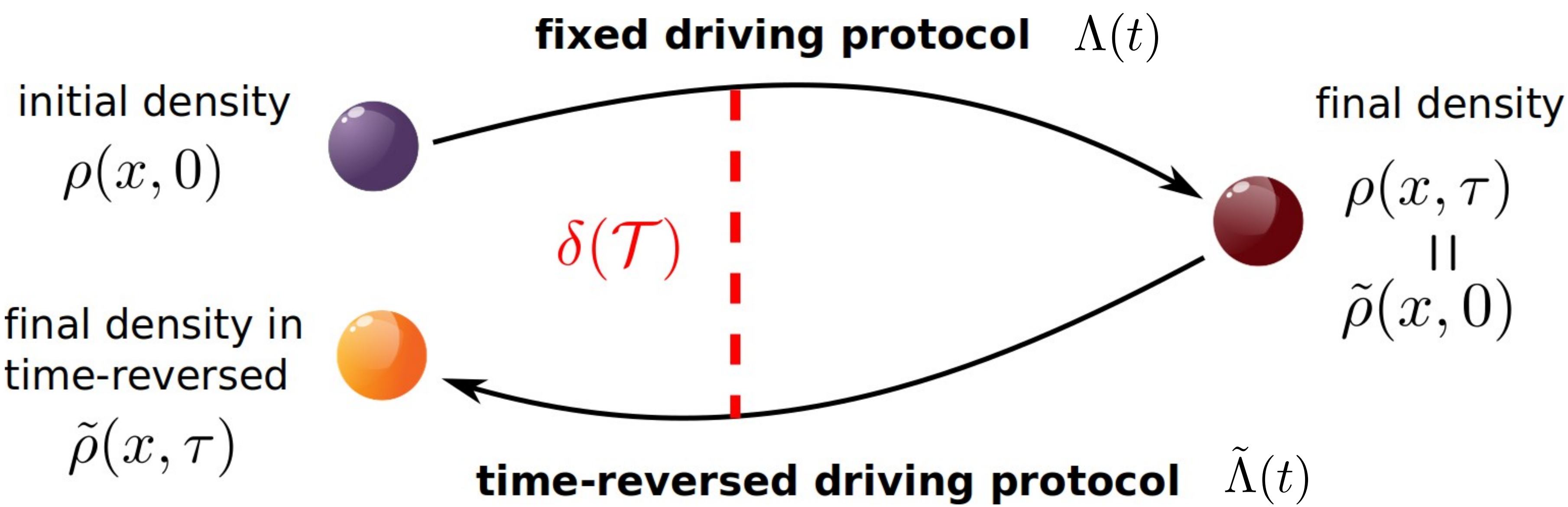
Entropy production — Extra term !

$$\langle W \rangle_{\mathcal{T}} - \langle \Delta F \rangle_{\mathcal{T}} \geq -k_B T \langle \delta \rangle_{\mathcal{T}}$$

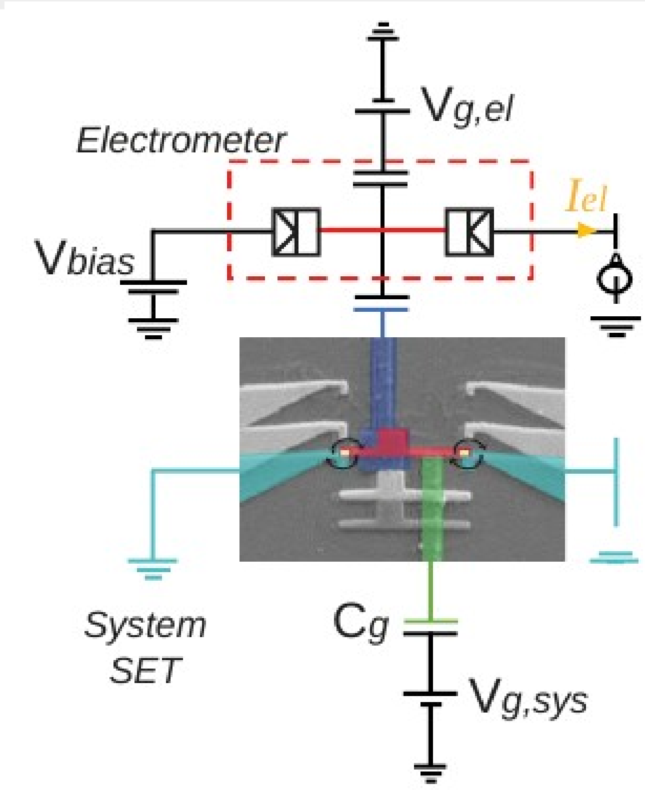
Second-law-like inequality

Stochastic distinguishability under time-reversal

$$\delta(\mathcal{T}) = \log \left(\frac{\rho(x_{\mathcal{T}}, \mathcal{T})}{\tilde{\rho}(x_{\mathcal{T}}, \tau - \mathcal{T})} \right)$$



Single Electron Box (SEB): [PICO group (Helsinki)]

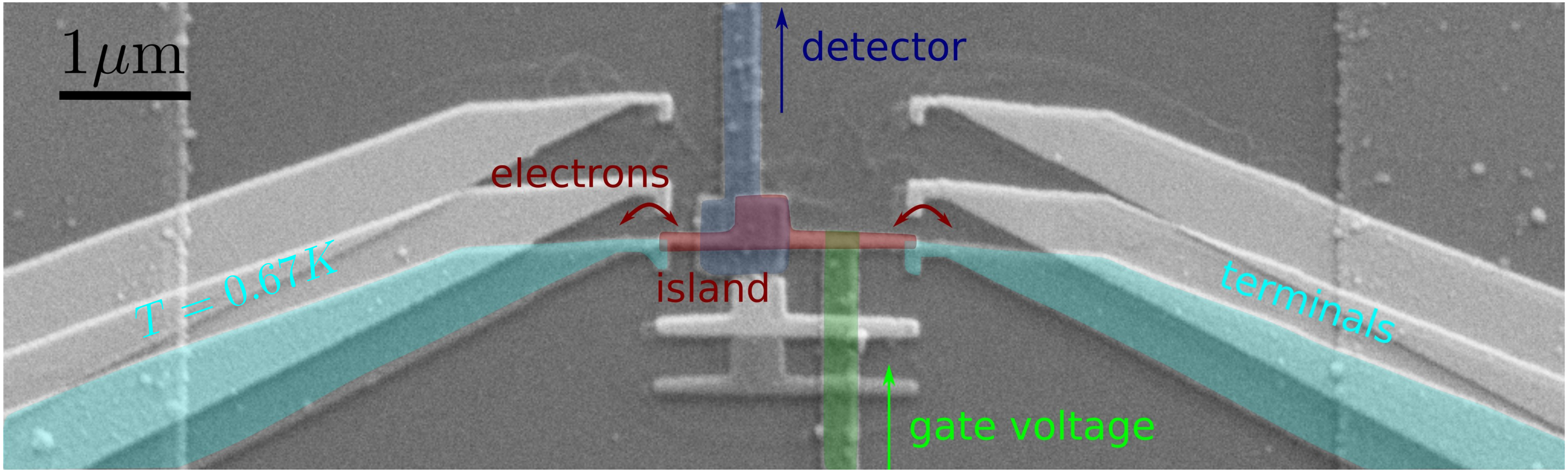


System: Cu island

Thermal reservoir: Al superconducting leads

Driving protocol: gate voltage following a linear ramp

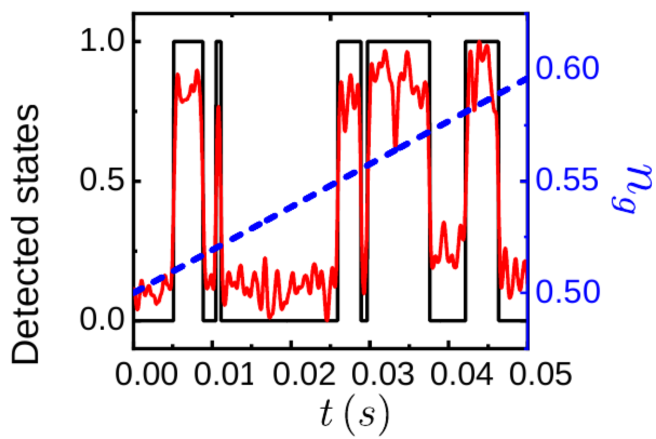
Detector: SET monitors tunneling events



scanning electron micrograph

Driven two-level system:

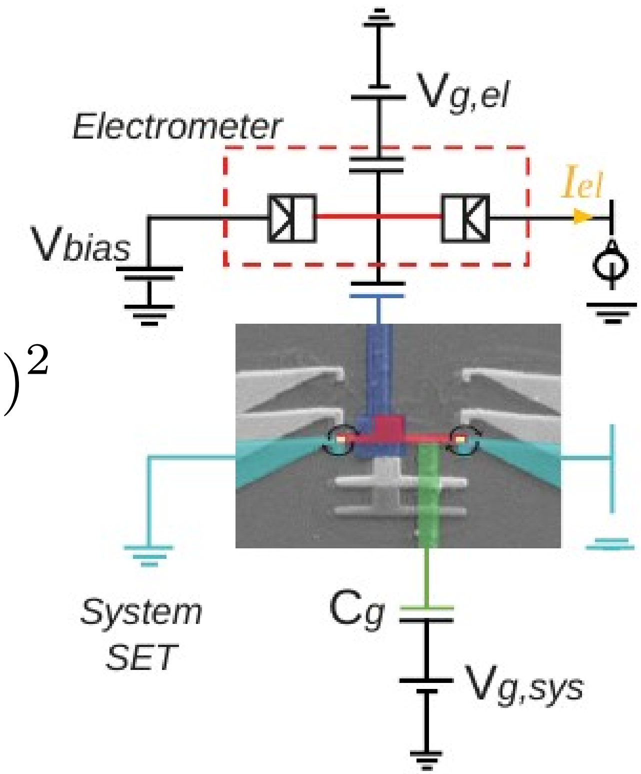
n=0 / n=1 (excess) electron in the box

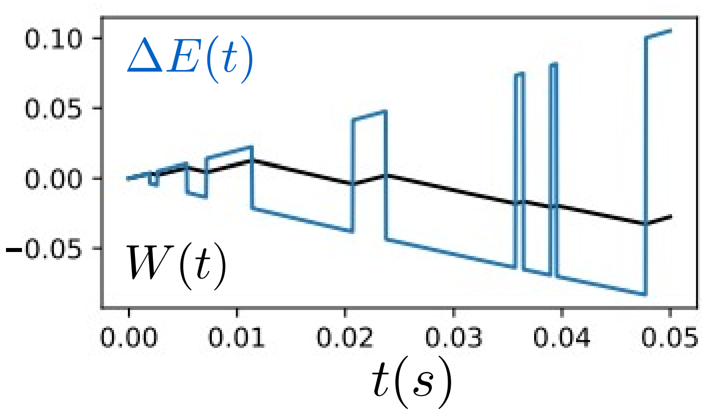


$$H(n) = E_{\rm c}(n - n_g)^2$$

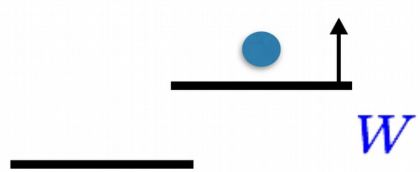
charge state

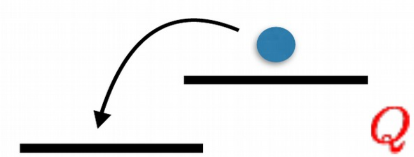
offset charge





$$E_c = 109\mu\text{eV} \simeq 1.9 k_B T$$



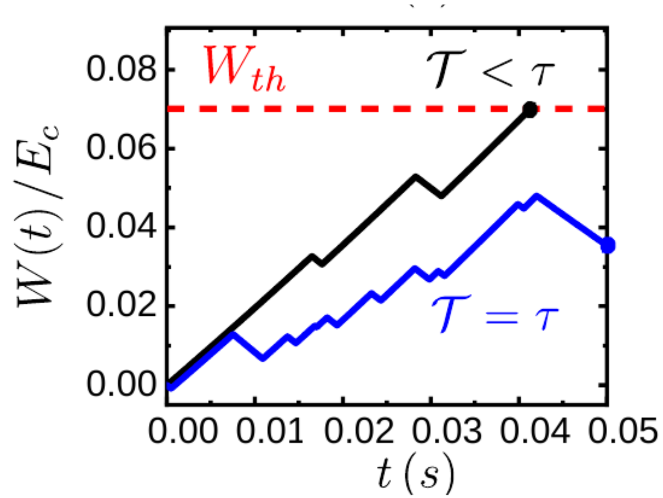


More details (setup): J.P. Pekola, Nat. Phys. **11** (2015) O. Maillet *et al*. PRL **122** (2019)

Strategy: *Finite-horizon work first-passage time*

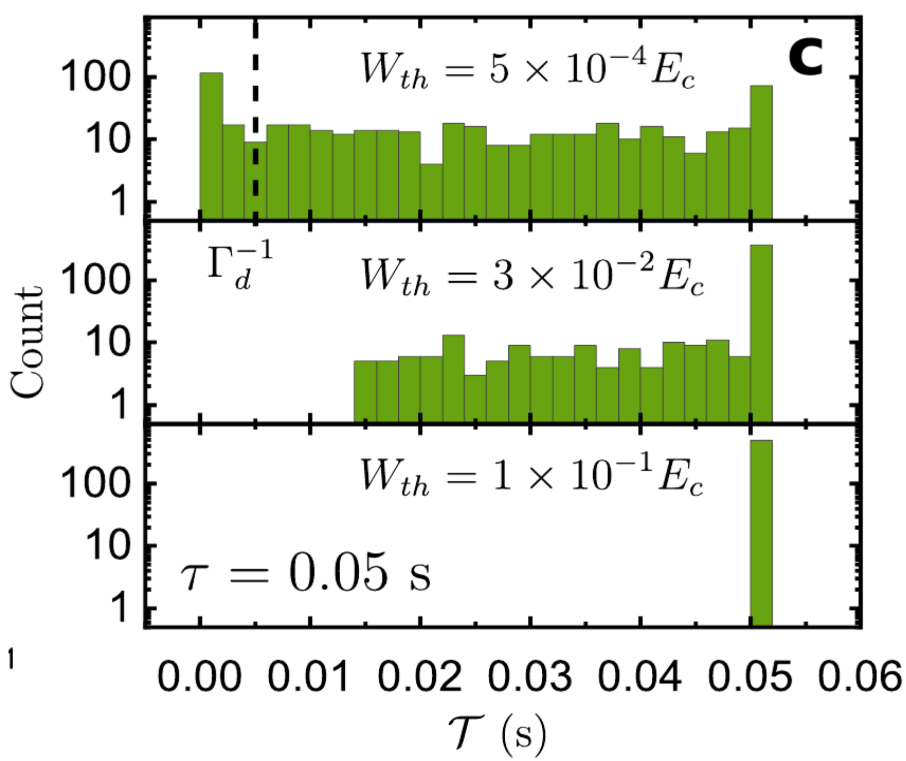
Stop trajectories when the work reaches a threshold

If threshold is not reached, we continue until the end.



Results:

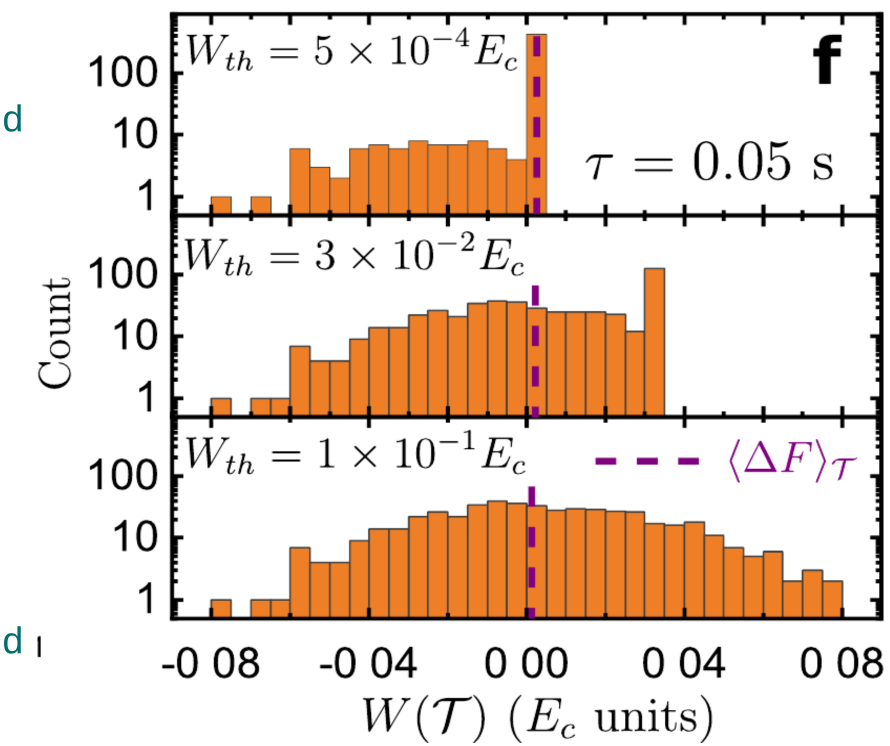
Stopping times histogram

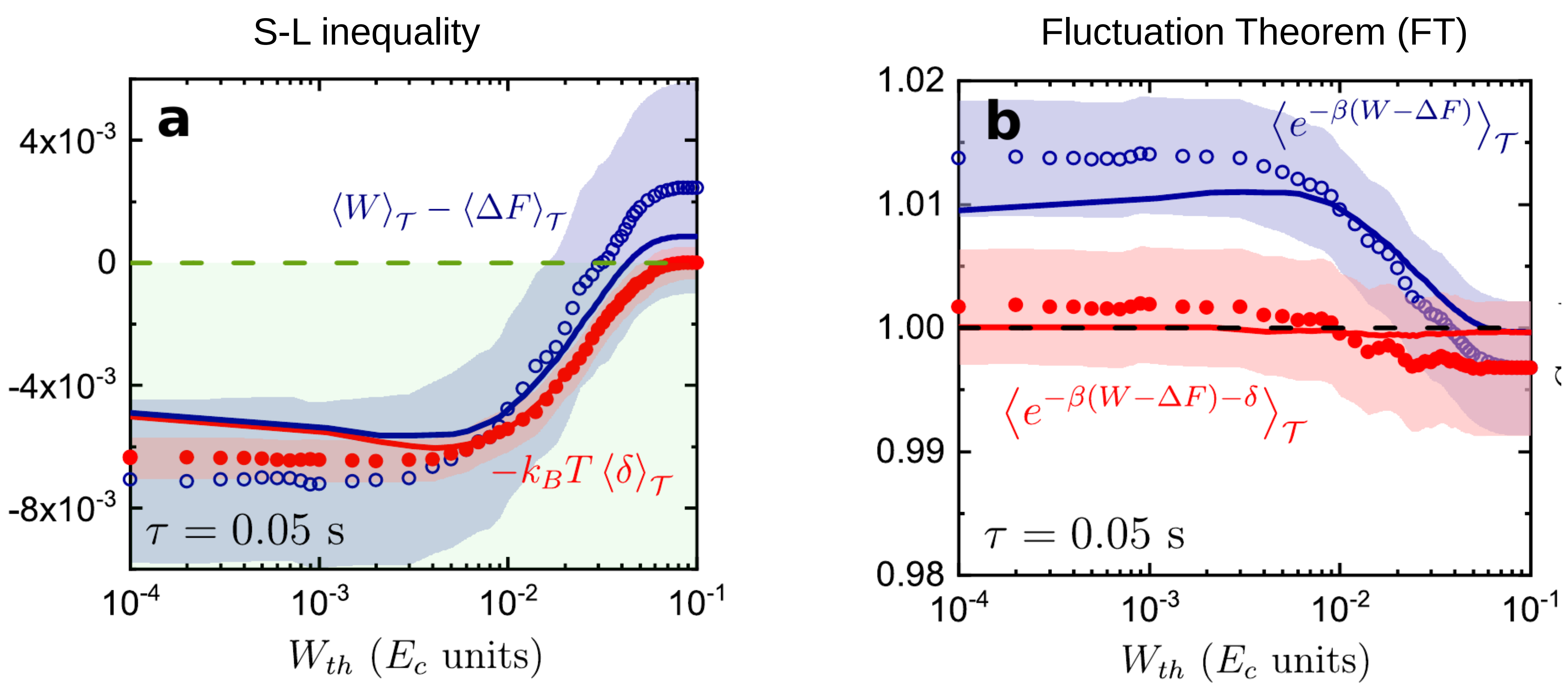


small threshold

big threshold

Work prob. distribution





We can extract work by stopping at small thresholds (on average) !

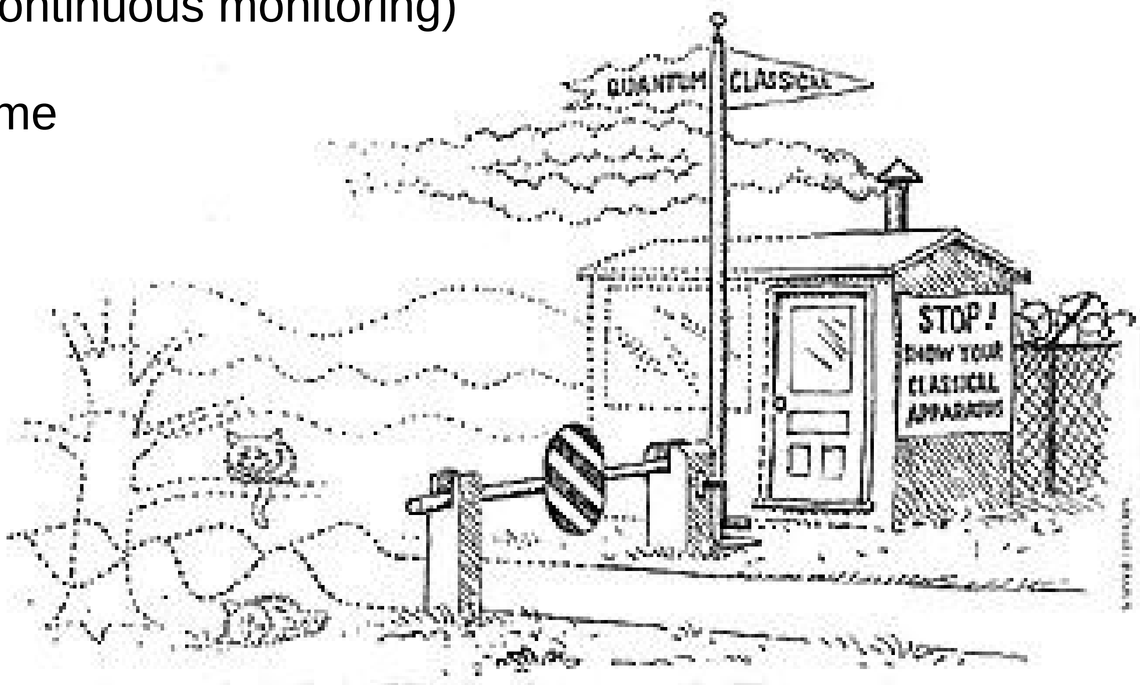
S-L inequality is tight for small thresholds in noneq. situation.

For large thresholds we recover the standard version of S-L and FT. $\mathcal{T} \simeq \tau$

Thermodynamics along quantum trajectories:

+ Quantum jump trajectories (indirect continuous monitoring)

+ Two Point Measurement (TPM) scheme



- $\gamma_{\{0,\tau\}} \equiv \{ n(0), \mathcal{R}_0^\tau, n(\tau) \}$
- $|\psi(t)\rangle$ conditioned on $\mathcal{R}_0^t$ (SSE)
- Average over trajectories:

  $\rho(t)$ (master eq. GKLS form)

Stochastic entropy production:

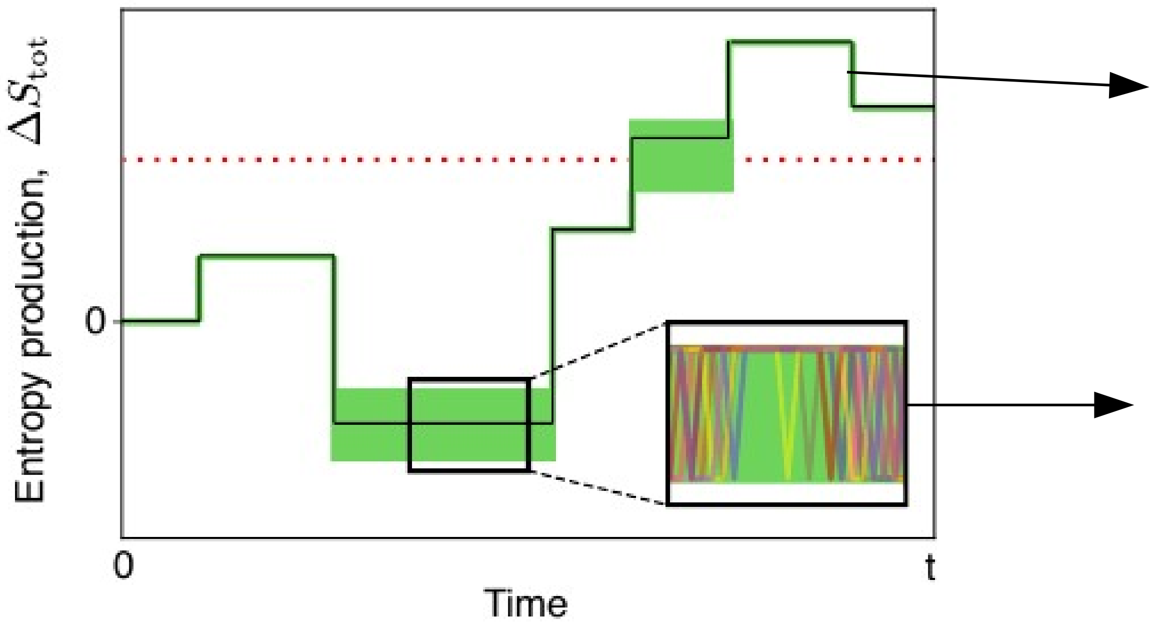
$$\Delta S_{\text{tot}}(\tau) = \log \left(\frac{p_{n(0)}(0)}{p_{n(\tau)}(\tau)} \right) - \beta Q[\mathcal{R}_0^\tau] = \beta \left[W(\tau) - \Delta F(\tau) \right]$$

system entropy change (TPM) stochastic heat (jumps record) $W(\tau) = \Delta E(\tau) - Q[\mathcal{R}_0^\tau]$

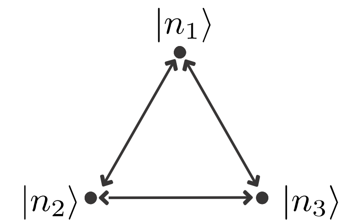
JM Horowitz PRE **85** (2012), JM Horowitz and JMR Parrondo, NJP (2013), G. Manzano *et al.* PRX **8** (2018) ...

Thermodynamics along quantum trajectories:

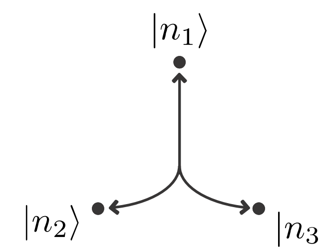
We extended our results for quantum systems indirectly and continuously monitored



classical jumps



superposition $|\psi(t)\rangle$



How it affects the bounds to gambling?

- Beneficial if $\langle \Delta S_{\rm unc}\rangle_\mathcal{T} < 0$
- Detrimental if $\langle \Delta S_{\rm unc}\rangle_\mathcal{T} > 0$

Quantum term:

$$\langle W\rangle_\mathcal{T} - \langle \Delta F\rangle_\mathcal{T} \geq -k_B T[\langle \delta_{\rm q}\rangle_\mathcal{T} - \langle \Delta S_{\rm unc}\rangle_\mathcal{T}]$$

$$\langle e^{-\beta[W-\Delta F]-\delta_{\rm q}+\Delta S_{\rm unc}}\rangle_\mathcal{T} = 1$$

split EP (quantum fluctuations)

G. Manzano, R. Fazio, É. Roldán PRL **122** (2019)

Main conclusions

- We introduced a “gambling demon” which stops a driven nonequilibrium thermodynamic process at stochastic times according to some gambling strategy.

  + Since it does not require feedback control, its applicability should be easier.

- We derived classical and quantum universal fluctuation relations at stopping times and second-law-like inequalities. We tested them in a SEB experiment.

  + Results can be extended to more general situations e.g. several baths, etc.

  + Use for small heat engines? Optimal gambling strategies?

  + Autonomous Gambling Demon?

- In the quantum case, coherent dynamics introduces an extra uncertainty term that can be, in principle, either beneficial or detrimental.

  + How to design beneficial quantum strategies?

THANK YOU

for your attention

MORE INFORMATION:

Gambling demons:	Phys. Rev. Lett. **126**, 080603 (2021) [arXiv: 2008.01630]
Quantum Martingales:	Phys. Rev. Lett. **122**, 220602 (2019) [arXiv: 1903.02925]
Recent related work (stochastic thermo):	G. Manzano and É. Roldán, arXiv: 2109.03260 (2021)