Chaos-Based Optical Communications: Encryption Versus Nonlinear Filtering

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Abstract—Several chaos encoding schemes codify the message in such a way that the mean value of the transmitted signal (carrier with the message) is different for bits "0" and "1". We present a nonlinear filtering method that is able to detect very small changes in the mean value of a signal and therefore recover this kind of messages if its amplitude is larger than the chaotic fluctuations in the mean over the length of a bit. We also introduce a new codification method in which the mean value of the transmitted signal, over the length of each bit, is preserved and we show how it is able to beat the decryption scheme.

Index Terms—Chaos, dynamics, nonlinear optics, optical communications, semiconductor lasers.

I. INTRODUCTION

OPTICAL chaos-based communications have become popular in the past decade, evolving from a theoretical concept and an experimental demonstration [1] to an almost ready-to-use technique where successful field experiments have been reported [2]. Typically, the transmitted signal consists of a chaotic carrier generated by a semiconductor laser (SL) subject to feedback in which a message is encrypted. Then, a system similar to the emitter is necessary at the authorized receiver side to recover the message. Privacy relies on the difficulty to recover the message without the appropriate receiver. This depends not only on how strong the chaos is but also on the codification method. One of the most popular methods to encode information is chaos modulation (CM) [1], [3], [4] in which the message is encoded as a (small) modulation of the amplitude of the chaotic carrier.

Eventual eavesdroppers can attack in different ways. The simplest one is just trying direct detection of the modulated carrier. This attack usually fails except for messages codified with an extremely large modulation amplitude. A more sophisticated situation is such that the eavesdropper has a system similar to the authorized receiver although he may not know the parameters in which it operates. In this case, the degradation of the synchronization as a function of parameter mismatch [5]–[8] plays a critical role in determining how similar the eavesdropper system should be to succeed. Other possibilities include the reconstruction of the chaotic attractor using the chaotic time series. This is only practical in systems where the local dynamics is low dimensional [9], [10]. Finally, one can also use adaptive systems trained to synchronize to the chaotic carrier, such as, for example, neural networks [11].

In this work, we take a different approach exploiting eventual pitfalls in the codification technique. We do not attempt to match the receiver nor to reconstruct the chaotic carrier, rather we study the possibility of using a nonlinear filtering technique to break chaos encrypted messages in encoding schemes in which the mean value of the chaotic carrier is not preserved. The method that we apply to break the encryption is based in previous work on noise filtering and contrast enhancement by using nonlinear dynamics of extended systems [12] which can be used to detect sudden jumps that are masked by noise.

In Section II, we introduce the model of two SLs subject to feedback and the encoding scheme. Then, in Section III, we show how the messages encoded with this scheme can be recovered using a nonlinear filtering technique. Finally, we propose an alternative encryption method that requires a more sophisticated implementation but that avoids detection by the nonlinear filter.

II. CHAOS ENCODING

We model the dynamics of the SL subject to optical feedback in terms of the Lang–Kobayashi equations [4]. The equations for the slowly varying amplitude of the electric field \( E(t) \) and the carrier number \( N(t) \) (in single-mode operation and low to moderate feedback) are

\[
\dot{E}_{M,R}(t) = \frac{(1+\alpha)}{2} [G_{M,R}(t) - \gamma_p] E_{M,R}(t) + \gamma E_{M,R}(t - \tau) e^{-\Phi} + \kappa_r E_T(t) \tag{1}
\]

\[
\dot{N}_{M,R}(t) = \frac{I}{e} - \frac{N_{M,R}(t)}{\tau_N} - G_{M,R}(t) P_{M,R}(t) \tag{2}
\]

where subindex \( M \) (\( R \)) refers to the emitter (receiver) laser and \( T \) to the transmitted signal (carrier with embedded message). The gain \( G_{M,R} = g(N_{M,R} - N_0)/(1 + s P_{M,R}) \). The laser intensity is \( P(t) = |E(t)|^2 \). For simplicity, we have assumed identical internal laser parameters and operating conditions and neglected noise effects in the lasers. Parameter \( \alpha = 5 \) is the linewidth enhancement factor, \( \gamma_p = 0.5 \text{ ps}^{-1} \) is the photon decay rate, \( \tau_N = 2 \text{ ns} \) is the carrier lifetime, \( g = 1.5 \times 10^{-8} \text{ ps}^{-1} \) is the differential gain coefficient, \( N_0 = 1.5 \times 10^9 \) is the carrier number at transparency, \( s = 5 \times 10^{-2} \) is the gain compression.
coefficient, $I = 2I_{th}$ is the injected current, $I_{th} = 14.7$ mA is the solitary laser threshold current, $\tau = 1$ ns is the feedback delay time, $\gamma = 25$ ns$^{-1}$ is the feedback strength, and $\Phi = 0$ is the optical feedback phase. The last term in (1) only appears in the equation for the receiver and it accounts for the injection of the emitter laser field into the receiver. Without loss of generality, we consider that this injection occurs instantaneously. An example of the generated chaotic carrier time trace and power spectrum is given in Fig. 1.

A high degree of synchronization between emitter and receiver is achieved by working in a closed loop configuration and for $\kappa_r > 60$ ns$^{-1}$ [6], [13].

We consider here a message encoded by modulating the emitter’s chaotic carrier power

$$P_T(t) = (1 - \epsilon m(t))P_M(t) \tag{3}$$

where $P_T(t)$ is the transmitted signal, $\epsilon$ is the message modulation amplitude, $m(t)$ is the message being transmitted, taking values 0.5 and $-0.5$, and $P_M(t)$ is the chaotic carrier. The message can be recovered by the authorized receiver as

$$m'(t) = \frac{P_R(t) - P_T(t)}{\epsilon P_R(t)} \tag{4}$$

where $P_R(t)$ is the power emitted by the receiver laser. The recovery strongly depends on the quality of the synchronization between the emitter and receiver lasers. For $P_R(t) = P_M(t)$ (ideal synchronization) the message can be perfectly recovered. In practice one applies a Butterworth filter after detection to further clean the message, as it is typically done in all-optical chaos-based communications.

In some instances, it may not be straightforward to implement the division in (4), then the message is recovered by using

$$m''(t) = P_R(t) - P_T(t). \tag{5}$$

This is later filtered using a Butterworth filter and then normalized to the interval $[-1/2, 1/2]$. In this work, we will compare both decoding techniques with the eavesdropper attack introduced in Section III.

To illustrate the quality of the recovered messages, we show in Fig. 2 (a.1) a sample of a sequence of recovered bits using (4) and in Fig. 2 (a.2) the corresponding eye diagram of this sequence. Fig. 2 (b.1) and (b.2) show the time trace and eye diagram of a sequence recovered using (5). To quantify the performance of the recovery process we use the quality factor

$$Q = \frac{\mu_1 - \mu_0}{\sigma_1 + \sigma_0} \tag{6}$$

where $\mu_1$ and $\mu_0$ are the average optical power of bits “1” and “0”, and $\sigma_1$ and $\sigma_0$ are the corresponding standard deviations. A larger value of $Q$ accounts for a better recovery of the message since the bit error rate is a monotonically decreasing function of $Q$ [14]. For a 5-Gb/s message, the authorized receiver shows a clean and open eye diagram with $Q = 10.66$ in the case of $m'(t)$ and $Q = 4.78$ for $m''(t)$. Note that, by not making the division in (5), the quality factor is largely reduced, but message recovery is still possible.

One property of CM is that the mean value of the transmitted power during the length of a bit “1′′ ($P_{T,1}$, where the bar stands for average over time) is different from a bit “0′′ ($P_{T,0}$). This difference is

$$\bar{P}_{T,0} - \bar{P}_{T,1} = \epsilon \bar{P}_M. \tag{7}$$

This feature is shared by other encryption methods such as chaos shift keying [15] and chaos masking [4], [5], [16], which makes this discussion also relevant for these cases. If the message modulation amplitude $\epsilon$ is large, these differences on the mean value are easily detectable and the message can be recovered by using a linear filter, such as a low pass filter. One may also consider averaging the power over one bit. This requires the eavesdropper to know the exact bit rate as well as where the bit starts. As $\epsilon$ is reduced, usual filtering techniques fail to recover the message, nor does the average technique work. The use of a nonlinear filter can improve the ability to detect these deviations, and therefore push the range of secure operation to smaller amplitudes. The chaotic properties of the carrier set a limit for the message amplitudes that allow to operate in a secure regime. The average value of the chaotic carrier during a bit length fluctuates from one bit to another, and these fluctuations increase with the bit rate. We measure the standard deviation $\sigma$ of these fluctuations and define $\Delta$ as the ratio between $\sigma$ and the mean value of the chaotic carrier. For the chaotic carrier shown in Fig. 1(a), $\Delta = 0.02$ at 1 Gb/s, $\Delta = 0.05$ at 2.5 Gb/s, and $\Delta = 0.10$ at 5 Gb/s. Encoding a message with the CM method induces deviations from the mean value of the carrier. If these deviations are of the same order of magnitude or smaller than those of the chaotic signal, a filtering
technique will not be able to distinguish the intrinsic variations from the ones produced by the message, and therefore it will not be able to decrypt it. The drawback of working in this regime is that, to recover the message with enough quality, a very good synchronization between the emitter and the receiver is needed.

As we said, the variations of the chaotic signal at 5 Gb/s are about 10%, and we will show how, with a nonlinear filtering technique based on the forced Ginzburg Landau equation (GLE), we are able to decrypt signals up to these message modulating technique. The forced GLE second pass \((a = 1000, r = 0.5)\) and GLE second pass \((a = 2, r = 0.65)\).

**Fig. 2.** Recovered messages (left) and eye diagrams (right) for 256 bits encoded with CM at 5 Gb/s \((\epsilon = 0.2)\). (a) Authorized receiver \(m(t)\) \((\epsilon = 75 \text{ ns}^{-1})\), (b) authorized receiver \(m^0(t)\), (c) GLE first pass \((a = 1000, r = 0.5)\), and (d) GLE second pass \((a = 2, r = 0.65)\).

where \(\psi\) is the field, \(a\) is the diffusion strength, and \(b\) is the space-dependent forcing. For convenience, we set \(b = 1\).

Neglecting diffusion and \(h(x)\), (8) has two stable stationary solutions, \(\psi_+ = 1\) and \(\psi_- = -1\) (plus an unstable solution at \(\psi = 0\)). Starting from \(\psi(x, t = 0) = h(x)\), the points where \(h(x) > 0\) will evolve to \(\psi_+\) and those where \(h(x) < 0\) will evolve to \(\psi_-\). If we now consider the effect of diffusion, areas of a size of the diffusion length \((\text{given by} \frac{1}{a})\) that in average are larger than zero will approach to \(\psi_+\), and those areas that in average are lower than zero will approach to \(\psi_-\) as illustrated in Fig. 3. The diffusion will also wash out the fast frequencies of the chaotic signal. Finally, the effect of the forcing term \(h(x)\) is to change the basin of attraction of \(\psi_+\) and \(\psi_-\), favoring the separation mechanism and balancing the effect of diffusion, making \(\psi\) follow the variations of \(h(x)\). The GLE filtering method was initially intended to detect sudden jumps hidden by noise in ecological and biological experimental data\(^1\) developing on prior ideas of using nonlinear extended systems to filter noise in images and selectively enhance the contrast \([12], [18]\). To apply the GLE filtering method to our case, we identify

\[
h(x) = \frac{P_T(t)}{\text{MAX}(P_T(t))} - r
\]

with \(h(x)\), where \(r\) is a threshold parameter \((0 < r < 1)\). Starting from \(\psi(x, t = 0) = h(x)\), we let the equation evolve to the steady state, with a large value of \(a (a = 1000)\), this filters the fast frequencies of the chaos in a similar way that a low-pass filter would do. Once the field reaches the steady state, we set \(a = 2\), with this value of the parameter the nonlinear mechanism comes into play enhancing the contrast of the signal from the reference level given by \(r\), making the encoded message visible. The total effect of the process is to separate the zones (defined by the diffusion length) where the average of the signal is larger than \(r\) (\(\text{i.e., "1" bits}\)) and the zones where the average is less than \(r\) (\(\text{i.e., "0" bits}\)).

To illustrate the procedure, we start by processing the message using an encoding amplitude of \(\epsilon = 0.2\). In Fig. 2 (c.1)

\(^1\)An implementation of the GLE method for filtering data series can be found online at http://ifisc.uib-csic.es/users/jacobo/
and setting, and therefore we. This means that we is smaller. Considering these results, respectively. It can be seen that, in this
approaches 0.1 (which is of the order of
factor of the re-

at 1 Gb/s. Since for this bit rate

one of the eye diagram is very closed
case, we are able to recover some features of the message but
will monotonically decrease function of

statistics and computation time than computing the bit error rate.

will use it to characterize the accuracy of the recovered message.
We choose to use the quality factor because it requires much less
statistics and computation time than computing the bit error rate.

In Fig. 2 (d.1) and (d.2), we show the effect of a second pass
using \( \alpha = 2 \) and setting \( r \) to maximize the \( Q \) factor of the re-
covered message. Here, it can be seen that the message is nearly
recovered (ten errors over 256 bits of the message), and the eye
diagram is quite open (\( Q = 3.85 \)). Considering these results,
it would not be secure to encode messages with this amplitude
using CM. In Fig. 4, we show the quality factor as a function of
the message modulation amplitude \( \epsilon \). We can see that, applying
the GLE twice, the results are much better than with a single pass
with large diffusion (which is basically a linear filter). While the
performance is not as good as the authorized receiver, it is still
enough to decode the message if the value of \( \epsilon \) is not too small.
Performance decreases with decreasing \( \epsilon \). At 5 Gb/s [Fig. 4(a)],
when \( \epsilon \) approaches 0.1 (which is of the order of \( \Delta \)), the nonlinear
filtering method recovers only part of the message. However, it
should be kept in mind that reproducing part of the message is
already a security threat.

Decreasing the bit rate the performance of the GLE filter im-

proves. Fig. 5(a) shows the eye diagram for a message encoded
at 1 Gb/s with \( \epsilon = 0.04 \) using CM and decoded by an authorized
receiver using (5). Fig. 5(b) shows the message decoded by an
eavesdropper using the GLE filter with \( \alpha = 1 \) in the second
pass. Fig. 4(b) shows the quality factor as function of the mes-

age amplitude \( \epsilon \) at 1 Gb/s. Since for this bit rate \( \Delta \) is smaller
than for 5 Gb/s, the GLE filter provides a better performance

for lower values of \( \epsilon \). In general, to prevent decoding using a
nonlinear filter like the one considered here, the change in bit
average power induced by message encoding should be of the
order or smaller than the chaos fluctuations of the bit mean.

At this point, a few remarks on noise effects are in order since
in practice the transmission scheme will always be subject to
noise. As stated before, the GLE filtering method was initially
designed to detect sudden jumps hidden by noise in data. As
for the method, the chaotic carrier is considered as a kind of
noise (whose origin is the deterministic chaos) that hides the
message. If some amount of noise is added to the transmitted
signal, within reasonable limits, the GLE will filter this noise
along with the chaotic variations and the message will be re-
covered. The presence of noise in the system will also affect the
ability of the authorized receiver to synchronize. The larger the
noise, the worst the synchronization, requiring a larger message
amplitude to keep the same quality factor at the authorized
receiver. The larger the amplitude of the message, the better the
performance of the GLE.

One can also compare the nonlinear method with the much
simpler approach of taking the mean value of the time series
during the length of one bit. In order to compute the mean value
of one bit, we need to know both the transmission bit rate and
the bit starting time. This is already one advantage of the GLE
filtering method, for which this information is not needed (only
the order of magnitude of the bit rate is needed, in order to ad-
just the value of \( \alpha \) for the second pass). But the main advantage

of the nonlinear filtering can be seen in Fig. 4, where the dotted line shows the quality factor obtained by computing the average value over one bit. Here is clearly seen that due to its contrast enhancement capabilities the GLE filtering provides a much better quality factor than averaging over the bit length, and therefore the message can be decoded for smaller values of $\epsilon$.

### IV. NEW CODIFICATION SCHEME

The GLE filter acts by detecting changes in the mean value of the signal, therefore it is useless if the message is encoded in such a way that the mean value is the same for bits "1" and "0". As an example of an encoding method which cannot be broken by the GLE filter, we introduce the mean preserving chaos modulation (MPCM) as

$$P_T(t) = (1 - em(t))P_M(t) + em(t)\bar{P}_M$$  \hspace{1cm} (10)

where $\bar{P}_M$ is the mean of the chaotic carrier and $m(t)$ is now a binary message taking values 0 and 1. This method, in which the message is encoded in the variations around the mean value, is an extension of CM in which the average power for bits one and zero is the same for any modulation amplitude, albeit more difficult to implement experimentally. As a possible implementation, one could consider dividing the output of the emitter in two beams. The power of one of the beams is modulated as $(1 - em(t))P_M(t)$. The other beam goes through a low-pass frequency filter to obtain the power average which then is modulated as $em(t)\bar{P}_M$. Finally, both beams are recombined.

The recovery of the message at the authorized receiver side needs to account for the mean of the transmitted signal

$$m(t) = \frac{P_M(t) - P_T(t)}{e(P_h(t) - \bar{P}_M)}.$$  \hspace{1cm} (11)

In order to recover the message with a good quality, the authorized receiver must neglect the data points in which the power of the receiver is close to the mean value $\bar{P}_M$. Specifically, we discard the data points that satisfy $|P_T(t)/\bar{P}_M - 1| < K$, where $K$ is an arbitrary number larger than 0. Fig. 6 shows samples of the recovered message for different values of $K$ and its corresponding eye diagrams for $\epsilon = 0.2$. In Fig. 6(a)–(c), the largest quality factor ($Q = 5.2$) is obtained for $K = 0.05$. In the latter case, 9% of the points are neglected. Notice that the neglected points do not imply the loss of bits in the message since the discarded portion of the time trace is much shorter than the length of one bit.

Fig. 7 (a.1) and (a.2) show the message recovered using (11) and the eye diagram for $\epsilon = 0.2$ and $K = 0.05$. The eye diagram shown in Fig. 7 (a.2) is clearly open. The eye diagrams have been obtained with the same synchronization degree between the emitter and receiver lasers than in the CM case ($\kappa_6 = 75 \text{ nm}^{-1}$). The MPCM method enhances the security of the communications at the expense of sacrificing part of the quality of the recovered message. In Fig. 8, we show the quality factor of the recovered message by an authorized receiver for different values of $\epsilon$. We can see that while the quality of the
recovered message is lower compared to Fig. 4, even though we are modulating at a slower bit rate, the message is still well recovered.

As stated in Section II, the division in (11) may not be straightforward to implement experimentally, therefore we will also consider a second message recovery method, given by

\[ m'(t) = |P_N(t) - P_E(t)|. \]  

(12)

Fig. 7 (b.1) and (b.2) show the same message as Fig. 7 (a.1) and (a.2) but recovered using (12), when the division is not available. In this case, the eye diagram is barely open and the quality factor is reduced, as shown in Fig. 8. Given these results, it is not clear that this method could be implemented experimentally if the division is not feasible.

Now we apply the GLE method to a message encoded using this scheme. The result of the decoding operation is shown in Fig. 7(c) for \( \epsilon = 0.2 \). As can be seen, nothing can be recovered from the encoded message but random bits. Independently of the value of \( \epsilon \) used to encode the message the GLE method is unable to extract it. Due to the way in which the GLE filter works in this case, it creates bits given by the variations of the chaotic signal only, and not by the variations in the mean value produced by the encoded message. We do not estimate the \( Q \)-factor since in this situation its value is basically meaningless.

V. CONCLUSION

In this work, we applied a nonlinear filtering method based on the GLE. Since this method is able to detect changes on the mean value of a data series, by properly tuning the parameters of the GLE, we were able to break communications schemes in which the change in bit average power induced by message encoding is larger than the one induced by the chaotic fluctuations. This nonlinear filtering method outperforms linear filters, due to the fact that besides filtering the fast frequencies of the chaotic signal as the linear filter does, the nonlinear one enhances the contrast of the bits “1” and “0”, improving the quality factor of the recovered message.

Therefore, if the codification is done without preserving the average power for bits “1” and “0”, the codification amplitude should be of the same order or smaller than the variations of the mean value of the chaotic signal over the length of one bit. Amplitude modulations larger than that pose a security threat since the message could be eventually detected by an eavesdropper using the method proposed here or a similar one.

Finally, we have introduced a new codification method which, although its implementation is more cumbersome, constitutes an example of encoding that preserves the average power for bits “1” and “0”. In this way, the nonlinear filtering method described here or similar methods aiming at detecting variations in the mean become ineffective.

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REFERENCES

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