Abstract.

The observation of gravimetric tides, which started in a systematic way from the sixties decade onwards, has come presently to cover a great part of the world surface, mainly due to the efforts of the group of Prof. Melchior (Melchior, 1982) and the realization of transcontinental profiles. The density of information existing in Europe and collected in the data bank of the International Center of Earth Tides in Brussels allows to make prediction tests on the main tides harmonics. A first work in this direction has been made for the Iberian Peninsula (Vieira, Camacho, 1988). In this communication there is an extension of such work given to a considerable part of the european continent through the application of the least square prediction method for the obtention of a model of gravimetric tides for Europe; likewise there is an study made on the security and reliability of the model functioning in the area, data density and quality of those data.

1. Introduction. European net of tidal stations.

The observed gravity tidal variations on a station can be expressed as:

$$\sum_{i} A_{i} \cos (\omega_{i} t + \varphi_{i})$$

where summation is extended along lunisolar frequencies $\omega_{i}$, with corresponding amplitudes $A_{i}$ and phases $\varphi_{i}$.

For a rigid and ocean less earth the gravity tidal variations would be:

$$\sum_{i} A_{0i} \cos (\omega_{i} t + \varphi_{0i})$$

obtained directly as derivatives of lunisolar potential along local vertical direction. (lunisolar potential being calculated determining the geodetic coefficients and using a development as Cartwright-Tayler ones).

Discrepancies $A_{i}/A_{0i} = \delta_{i}$ and $\varphi_{i} - \varphi_{0i} = \alpha_{i}$ are associated to non rigidity and ocean effects of the Earth. For each wave, values $A_{i}, \alpha_{i}$ can be graphically represented as a vector $A_{i}$ of polar coordinates $A_{i}$, $\alpha_{i}$ (figure 1).

The usual earth model (elastic with liquid core) of Molodensky supposes $\delta_{i} \approx 1.16$ and $\alpha_{i} \approx 0$ everywhere and for every waves $i$. The vector $M$ of Molodensky waves is $M : (M_{i} = 1.16 A_{0i}, \alpha_{i} = 0)$, see figure 1.
The existing difference vector $\mathbf{B}$ between observed values and global elastic model values is mainly owed to oceanic effects (loading, newtonian and indirect effects) and to regional rheologic properties.

\[ \mathbf{B} : (B, \beta) = (\delta A, \alpha) - \mathbf{M} : (1.16 A, 0) \]

Nowadays, oceanic effects can be calculated from adjusted models of oceanic tides as model of Schwidersky (1980). Let $\mathbf{L} : (L, \lambda)$ be the vector of externally calculated oceanic effects.

Vectors $\mathbf{B}$ and $\mathbf{L}$ will be very similar. The vector $\mathbf{X} : (X, \kappa)$ of difference $X = B - L$, figure 1, will be determined mainly by regional rheologic properties and deficiencies of oceanic effects model.

Finally vectors $\mathbf{M} + \mathbf{L} = \mathbf{A} - \mathbf{X}$ and $\mathbf{A} - \mathbf{L} = \mathbf{M} + \mathbf{X}$ represent: theoretic model with calculated ocean effect (as advanced model) and observation corrected with calculated ocean effect (as reduced observation).

The tidal data bank collected by the I.C.E.T. (Ducarme, 1983) let us investigate the magnitude and spatial distribution of tidal vectors. For a number of 133 stations on european area, the data file offers values of $A$, $\alpha$, $\delta$ (and corresponding observation square errors), $L$, $\lambda$, $B$, $\beta$, $X$, $\kappa$, ... for main tidal waves: $O1$, $P1$, $K1$, $N2$, $M2$, $S2$. Another interesting station values are collected: distance to sea, gravimeter identification, time of observation and useful readings, etc.

We have considered the area limited by latitudes $36^\circ$ and $72^\circ$ and longitudes $-12^\circ$ and $36^\circ$, containing 128 stations (figure 2). Any data mistake have been corrected, and also we recalculate values of $B, \beta, X, \kappa$.

For the spanish area we have substitute the file data by actualized values, including new stations. Two kind of oceanic effects values are available for iberian stations: those obtained from Schwidersky model and those obtained from an additional study of surrounding oceanic areas (Cantabrico, Mediterranean). For european comparisons we shall use Schwidersky data model.

A first view to data shows us a similarity between values on neighbour stations, that points a certain continuous spatial evolution. But also, we observe, specially for coastal stations, a local deviations or noise. To form spatial models we can only consider regional behavior, filtering punctual deviations. To make a signal-noise separation we use a covariance analysis, and then, by least square prediction, form the filtered and predicted values. The signal define spatial distribution and the noise can contain information about local or punctual circumstances that deviate the tidal reading.
2. Least square prediction. Example of application to $K(M2)$.

To can apply least square prediction, data values must offer a random distribution. Usually data values $d$ will contain a systematic part $p$:

$$d = p + v$$

We must calculate $p$ so that residual values $v$ give a random distribution.

For tidal values we can suppose a systematic component. For example, observed amplitudes have a clear N-S systematic effect, while residual values $X, \kappa$ probably have a very small systematic component.

A simple method to determine $p$ is by polynomial approximation (Mussio, 1987) of suitable degree (so that it adjusts systematic component but keeps a clear random signal).

Residual values $v$ (determined on points $P_i$, $i=1,\ldots,n$) will contain a correlated signal $s$ and an uncorrelated noise $n$:

$$v = s + n$$

Separation between $s$ and $n$ can be defined by means of covariance study of data, and then of signal. If we suppose an isotropic and homogeneous field, we can consider that covariance of signal $s$ between two points $P_i, P_j$ depends of the corresponding point distance $d$:

$$C_{ij} = \text{Cov.}(s(P_i), s(P_j)) = C(\text{dist.}(P_i, P_j))$$

The covariance $C(d)$ for signal can be obtained adjusting a typical covariance function to the empirical covariance distribution of data values (Barzaghi and Sanso, 1983). For that, we must make a correlation analysis of data values versus mutual point distances: The whole interval of distances is divided on several subintervals of suitable width $\Delta d$. If $d_k$ is the mean value of a subinterval, then, the corresponding covariance value can be calculated as:

$$c(d_k) = \frac{1}{n_k} \sum_{i,j} v(P_i)v(P_j)$$

with summation extended to the $n_k$ pairs of point $P_i, P_j$ so that:

$$d_k - \frac{\Delta d}{2} < \text{dist.}(P_i, P_j) \leq d_k + \frac{\Delta d}{2}$$

From calculated values $c(d_k)$, $k=1,\ldots,m$, ($d_k = \Delta d (2k-1)/2$), we can adjust them by an usual covariance function $C=C(d)$ (see Mussio, 1984).

The value $C(0)$ is the signal variance $\sigma^2_s$ and then:

$$\sigma^2_n = \sigma^2_v - \sigma^2_s$$

gives the noise variance from data variance.
The correlation step width \( \Delta d \) must be choice to obtain the best definition of signal. Practically, we can take several values of \( \Delta d \) and then select that to give a lesser resulting value of \( \sigma_s^2 \), (which is estimated by \( \sigma_s^2 \approx \sigma_v^2 - c(d) \)).

Using the obtained covariance function \( C=C(d) \), we can apply usual least square prediction formulae (Moritz, 1980) to obtain:

Predicted signal value \( \hat{s} \) on point \( P \):

\[
\hat{s} = C_{Ps} \left( C_{ss} + C_{nn} \right)^{-1} v
\]

where:

- n-vector \( C_{Ps} = (C(s(P_1), s(P)))_i = (C(\text{dist}(P_1, P)))_i \)
- n,n-matrix \( C_{ss} = (C(s(P_i), s(P_j)))_{ij} = (C(\text{dist}(P_i, P_j)))_{ij} \)
- n,n-matrix \( C_{nn} = I \cdot (\sigma_v^2 - C(0)) \)
- n-vector \( v = (v(P_1), \ldots, v(P_n))^T \)

For \( P = P_1 \) we obtained filtered values \( \hat{s}(P_1) \), and then adjusted noise \( \hat{n}(P_1) = v(P_1) - \hat{s}(P_1) \).

Error matrices are also obtained

\[
E_{ss} = C_{PP} - C_{Ps} \left( C_{ss} + C_{nn} \right)^{-1} C_{sp}
\]

The final model is composed by the adjusted residual signal \( \hat{s} \) plus the previously adjusted systematic component \( p \).

As an example of application we consider the values of \( \kappa \) (phases of residual vector \( X \)) for M2 as data values.

First, the number of stations is not too large (moreover taking account the high level of noise), but it is enough to obtain several clear results. If we would have a very small number of stations, they only would show a general systematic component with high level noise and no appreciable signal (flat covariance function). A bigger number of stations let detect a correlated signal of a wave length smaller than of the global systematic component, obtaining a lesser noise level. As bigger the number of stations then smaller wave length of detected signal and bigger signal/noise relation.

In our case the number of stations let us obtain a sensible signal (see covariance functions later) with mean wave length (but with high noise level). If we would have a bigger number of stations a part of noise would appear as
correlated signal with lesser wave length.

Taking account the residual character of vector \( \mathbf{X} \), values \( \kappa \) will present a nearly random distribution. Nevertheless, we have study the possible covariance function using previous polynomial approximations of degree 0, 1, 2, 3.

First, the best correlator step width \( \Delta d \) have been investigated. For that, we study the resulting noise variance \( \sigma_n^2 = \sigma_v^2 - c(d_1) \) for several supposed step width (figure 3). We choice the value 2.8 degrees of spherical distance.

The direct correlation analysis of \( \kappa \) data is showed on figure 4.a. Correlation has been investigated taken as mutual distance between the stations the spherical distance \( \psi \):

\[
\cos \psi = \sin \varphi_1 \sin \varphi_2 + \cos \varphi_1 \cos \varphi_2 \cos (\lambda_1 - \lambda_2)
\]

Empirical covariance distribution (isolated points in figures 4) suggest us to adjust it by a exponential-Bessel function (continuous line):

\[
C(d) = a \exp(-b \cdot d) \cdot J_0(c \cdot d)
\]

where \( J_0(x) \) is the Bessel function of zero order, and \( a, b, c \) are parameters to adjust. \( a \) represent the value for origin, \( a=C(0), c \) is related to the "wave length" of function \( C(d) \), and \( b \) is related to dumping of oscillation. We take as width measure of the covariance function the distance for the first null \( C(d) \) value : \( d(0) \). In table 1 we give the numerical values for parameters of the adjusted function. The signal level is given by

\[
C(0)/\sigma_v^2 = a/\sigma_v^2 = 0.42
\]

We observe a covariance function with too small dumping. For a good random distribution, the covariance function must present a clear dumping with a flat behavior far from the origin. It suggest us to use a previous

<table>
<thead>
<tr>
<th>Pol. deg.</th>
<th>( \Delta d )</th>
<th>( \sigma_v )</th>
<th>( a/\sigma_v^2 )</th>
<th>( b )</th>
<th>( c )</th>
<th>( d(0) )</th>
<th>( \sigma_n )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2.8</td>
<td>98.4</td>
<td>0.42</td>
<td>0.1</td>
<td>10.3</td>
<td>6.6</td>
<td>75.0</td>
</tr>
<tr>
<td>1</td>
<td>2.8</td>
<td>97.8</td>
<td>0.39</td>
<td>0.0</td>
<td>10.5</td>
<td>6.4</td>
<td>76.4</td>
</tr>
<tr>
<td>2</td>
<td>2.8</td>
<td>93.8</td>
<td>0.37</td>
<td>0.5</td>
<td>10.9</td>
<td>6.2</td>
<td>74.5</td>
</tr>
<tr>
<td>3</td>
<td>2.8</td>
<td>87.4</td>
<td>0.31</td>
<td>0.3</td>
<td>11.0</td>
<td>6.1</td>
<td>72.6</td>
</tr>
</tbody>
</table>

Table 1. Comparative values of covariance for several previous polynomial approximations.
polynomial adjust to absorb the possible systematic component. Approximations have been calculated (by least square adjustment for the coefficients and using $\varphi$ and $\lambda \cos(\varphi)$ as coordinates) for polynomial degrees 1, 2, 3. Figures 4.b, 4.c, 4.d and table 1 show the results.

With higher polynomial degree we observe a bigger dumping ($b$), a smaller wave length ($d(0)$), a smaller data variance ($\sigma_v^2$), etc. For degree 1 the dumping is not yet clear. We choose as initial values those of second degree polynomial approximation.

For a good application of covariance analysis a consideration about homogeneity and isotropy of the field can be made. About isotropy we have study the covariance distribution along two directions: N-S and E-W. For that we have consider correlations with points in the same N-S or E-W narrow band. Taking account the less number of related stations we obtain a bigger correlator step width, a smaller level of signal. The covariance function adjusted have been a normal-parabola (figure 5) defined by:

$$C(D) = a \left(1-c \, d^2\right) \exp \left(-b \, d^2\right)$$

Table 2. Comparative values of covariance for E-W and N-S directions.

<table>
<thead>
<tr>
<th>Cov direc</th>
<th>$\Delta d$</th>
<th>$\sigma_v$</th>
<th>$a/\sigma_v^2$</th>
<th>$d(0)$</th>
<th>$\sigma_n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>E - W</td>
<td>5.4</td>
<td>93.6</td>
<td>0.40</td>
<td>5.8</td>
<td>75.0</td>
</tr>
<tr>
<td>N - S</td>
<td>7.6</td>
<td>93.8</td>
<td>0.23</td>
<td>5.9</td>
<td>76.4</td>
</tr>
</tbody>
</table>

Values obtained (Table 2) for both directions show a similar distance of null covariance and a better signal/noise relation for E-W direction. Nevertheless, the number of correlation subintervals is too small for assure any conclusion.

To check the homogeneity we have consider the covariance study for two different areas: the quadrants N-W (64 points) and S-E (43 points). Obtained

Table 3. Comparative values of covariance for S-E and N-W areas.

<table>
<thead>
<tr>
<th>Area</th>
<th>$\Delta d$</th>
<th>$\sigma_v$</th>
<th>$a/\sigma_v^2$</th>
<th>b</th>
<th>c</th>
<th>$d(0)$</th>
<th>$\sigma_n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>N - W</td>
<td>1.6</td>
<td>98.9</td>
<td>0.38</td>
<td>0.2</td>
<td>6.1</td>
<td>6.3</td>
<td>77.9</td>
</tr>
<tr>
<td>S - E</td>
<td>1.7</td>
<td>78.4</td>
<td>0.46</td>
<td>0.0</td>
<td>8.5</td>
<td>4.9</td>
<td>57.6</td>
</tr>
</tbody>
</table>
covariances are showed in figure 6 and corresponding values on table 3. They show a better signal/noise relation for S-E area with smaller null covariance distance.

Figure 7 shows the feature of polynomial approximation. Applying the least square formulae to residual values \( v \), we obtain as predicted residual distribution that represented on figure 8. Adding both fields we obtain the final model for adjusted \( \kappa \) values (on degrees), figure 17.

The root of diagonal elements of \( \mathbf{E}_{ss} \) give us the root mean square error for predicted values. They have been represented on figure 9. We observe that the best quality of model is obtained on central Europe, central Iberia and Finland area.

Finally residual noise are represented on figures 10 a,b. Highest noise corresponds mainly to coastal stations.

This process have been applied to the several tidal parameters. The automatic programs of covariance adjustment and least square calculus have been developed on our center.

3. Models of tidal vectors \( A, \delta, \alpha, B, \beta, \ldots \) for \( M2 \).

Applying the former process, final model for the several tidal vectors and parameters for main component \( M2 \) are given by figures 11 to 26. Error maps and noise location are similar to those of \( \kappa \) (\( M2 \)).

Table 4 shows several comparative parameters from covariance analysis of direct data (without previous approximation).

We can make several remarks:

Observed amplitudes show a zonal distribution (given by geodetic coefficient expressions), with clear perturbations connected with atlantic influence. The map corresponding to A-L vector (reduced observations) has a more regular zonal distribution, remaining perturbations are around Mediterranean, Artic and North seas.

Maps of amplitude factor and phase show a different form of response to oceanic effect.

A close relation can be observed between graphical representations of \( L \) (calculated oceanic effect) and \( B \) (observed values minus simple elastic model). It points that the main part of perturbations of gravimetric tidal recordings are due to oceanic effects. (Phases of vectors \( B \) and \( L \) show a bigger discrepancy).
Table 4. Comparative values of covariance for tidal vectors of M2.

Residual vector X offers a good signal level (at least for $M_2$) of 40% (see Tables) in variance of total residual data variance. Perhaps the more significant map is that of $X \cos \kappa$ (cosine component). That picture shows a strong perturbation focused on the North Atlantic - Artic area, prolongated on the North Sea area. A minor perturbations associated to mediterranean area. (Perspective view of figure 27 shows these circumstances). It is difficult to investigate further non oceanic effects without removing those strong oceanic effects.

Vector $M + L$ (corrected model) offers an amplitude factor and phase distributions very similar to that of observed values (but something smoother).

Finally, vector $A-L$ offers a very similar pictures for amplitude factor and phase to pictures of $X \cos \kappa$ and $X \sin \kappa$ (with another kind of magnitudes). As immediate example of application a values of coefficients for variation of $\delta$ along latitude have been obtained using filtered stations:

<table>
<thead>
<tr>
<th>Value</th>
<th>$M_2$</th>
<th>$\Delta \delta$</th>
<th>Data mean</th>
<th>$\sigma_v$</th>
<th>$a/\sigma_v^2$</th>
<th>$\sigma_n$</th>
<th>$d(0)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Obs ampl</td>
<td>4.7</td>
<td>34.44</td>
<td>11.44</td>
<td>0.92</td>
<td>3.23</td>
<td>16.66</td>
<td></td>
</tr>
<tr>
<td>Amp fac $\delta$</td>
<td>4.7</td>
<td>1.165</td>
<td>0.092</td>
<td>0.53</td>
<td>0.063</td>
<td>12.00</td>
<td></td>
</tr>
<tr>
<td>Obs des $\alpha$</td>
<td>4.3</td>
<td>2.579</td>
<td>3.463</td>
<td>0.39</td>
<td>2.705</td>
<td>13.45</td>
<td></td>
</tr>
<tr>
<td>Amplit $L$</td>
<td>2.8</td>
<td>2.129</td>
<td>2.016</td>
<td>0.70</td>
<td>1.104</td>
<td>16.14</td>
<td></td>
</tr>
<tr>
<td>Phase $L$</td>
<td>3.6</td>
<td>61.02</td>
<td>43.27</td>
<td>0.54</td>
<td>29.34</td>
<td>10.72</td>
<td></td>
</tr>
<tr>
<td>Amplit $B$</td>
<td>2.8</td>
<td>2.212</td>
<td>2.118</td>
<td>0.57</td>
<td>1.389</td>
<td>16.45</td>
<td></td>
</tr>
<tr>
<td>Phase $B$</td>
<td>4.9</td>
<td>51.96</td>
<td>60.41</td>
<td>0.15</td>
<td>55.69</td>
<td>12.71</td>
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<tr>
<td>Amplit $X$</td>
<td>4.7</td>
<td>0.669</td>
<td>0.774</td>
<td>0.27</td>
<td>0.661</td>
<td>7.22</td>
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<tr>
<td>Phase $X$</td>
<td>2.8</td>
<td>-25.91</td>
<td>98.43</td>
<td>0.42</td>
<td>74.97</td>
<td>6.57</td>
<td></td>
</tr>
<tr>
<td>$X \cos \kappa$</td>
<td>4.7</td>
<td>0.003</td>
<td>0.830</td>
<td>0.28</td>
<td>0.704</td>
<td>13.39</td>
<td></td>
</tr>
<tr>
<td>$X \sin \kappa$</td>
<td>1.5</td>
<td>-0.225</td>
<td>0.559</td>
<td>0.18</td>
<td>0.506</td>
<td>5.19</td>
<td></td>
</tr>
<tr>
<td>Ampl $A-L$</td>
<td>4.7</td>
<td>33.99</td>
<td>11.39</td>
<td>0.91</td>
<td>3.467</td>
<td>16.89</td>
<td></td>
</tr>
<tr>
<td>Fact $A-L$</td>
<td>4.8</td>
<td>1.152</td>
<td>0.049</td>
<td>0.34</td>
<td>0.040</td>
<td>12.42</td>
<td></td>
</tr>
<tr>
<td>Phase $A-L$</td>
<td>4.5</td>
<td>-0.505</td>
<td>1.367</td>
<td>0.20</td>
<td>1.223</td>
<td>6.82</td>
<td></td>
</tr>
<tr>
<td>Ampl $M+L$</td>
<td>4.7</td>
<td>34.44</td>
<td>11.10</td>
<td>0.92</td>
<td>3.139</td>
<td>16.66</td>
<td></td>
</tr>
<tr>
<td>Fact $M+L$</td>
<td>2.7</td>
<td>1.173</td>
<td>0.057</td>
<td>0.63</td>
<td>0.035</td>
<td>9.34</td>
<td></td>
</tr>
<tr>
<td>Phase $M+L$</td>
<td>2.8</td>
<td>2.893</td>
<td>2.604</td>
<td>0.60</td>
<td>1.647</td>
<td>14.78</td>
<td></td>
</tr>
</tbody>
</table>
Theoretic model of Wahr (1981) gives:

\[ \delta = \delta_0 + \delta_1 \frac{\sqrt{3}}{2} (7 \sin^2 \varphi - 1) \]

with \( \delta_0 = 1.1599 \) and \( \delta_1 = -0.0045 \).

Melchior and De Becker (1983) obtained the empirical values:

\[ \delta_0 = 1.175 (\pm 0.0021) \quad \delta_1 = -0.0046 (\pm 0.0010) \]

Here, considering only the sub-diagonal area of figure 27, results:

- for non filtered data: \( \delta_0 = 1.179 (\pm 0.0093) \quad \delta_1 = -0.0041 (\pm 0.0027) \)
- for filtered data: \( \delta_0 = 1.184 (\pm 0.0036) \quad \delta_1 = -0.0058 (\pm 0.0011) \)

Nevertheless to obtain a definitive values, data must be reduced with better adjusted oceanic effects (taking account the effect of local oceanic influences).

4. Any values for S2 and O1 components.

For comparison, certain vectors and values associated to S2 and O1 have been also studied. Table 5

For O1, residual vector X gives a signal level (in variance respect to total variance) of 10 % for amplitude and 12 % for phase. Distances of (first) null covariance are of 3.4 and 7.2 respectively. Nevertheless, for so small signal level conclusions are doubtful. The maps corresponding to O1 do not show a strong North Atlantic effect. So, the picture of X cos \( \kappa \) shows a nearly flat surface (corresponding to small detected signal with high noise) with distributed rugosities (see figures 28,29,30,31,32).

For S2 detected signal levels are about 18 % (in variance).

5. Conclusions.

It is possible to detect a correlated signal on the residual vector X, at least for the main tidal components.

That residual model can be interpreted mainly as small deficiencies of calculated ocean effects. We think that to obtain further reologhic conclusions a better knowledgement of oceanic effect must be applied. For components of minor oceanic sensibility, but enough signal level, other effects can be analyzed.

The error maps show us possible areas for install new stations to obtain
a more homogeneous quality map.

Noise maps point the stations of irregular behavior. They are located mostly on the Atlantic coast. Then, local irregular oceanic effects can be suspected. (Any data mistake would be also possible).

The predicted models of amplitude factors and observed phases for every main tidal waves can be automatically used to obtain empirical tidal correction for the gravimetric survey. (For Spain, we use a calculus program that, first, from coordinates, geodetic coefficients and $\delta, \alpha$ are calculated and, second, using the Cartwright-Tayler development, gravimetric tidal correction $\Delta g$ are calculated for desired date). (Vieira et al.).

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Fig. 1. Tidal vectors.

Fig. 2. European net of tidal stations.

Fig. 3. Determination of the best correlator step width (spherical distance).
Fig. 4. Correlation study for several initial polynomial approximations.

Fig. 5. Comparative study of correlation along N-S and E-W directions.

Fig. 6. Comparative correlation study for two extreme areas.
Fig. 7. Polynomial approximation for $K(m2)$. Eq. $= 20^\circ$.

Fig. 8. Predicted residual values for $(m2)$. Eq. $= 20^\circ$.

Fig. 9. R.m.s.e. of prediction. Quality of model for $k(m2)$. Eq. $= 5^\circ$.

Fig. 10. a. Positive noise. b. Negative noise. Anomalous stations.
Fig. 11. Observed amplitude M2.
Eq. = 2μgal.

Fig. 12. Amplitude of A-L vector (reduced observation).
Eq. = 2μgal.

Fig. 13. Amplitude factor of observed values M2.
Eq. = 0.02.

Fig. 14. Phase of observed values M2. Eq. = 1°.

Fig. 15. Amplitude factor of M + L vector (corrected model). Eq. = 0.02.

Fig. 16. Phase of M + L vector (corrected model). Eq. = 1°.
Fig. 17. Phase of residual vector X for M2. Eq. = 30°.

Fig. 18. Amplitude of residual X for M2. Eq. = 0.2 μgal.

Fig. 19. Cos component of residual vector X of M2. Eq. = 0.2 μgal.

Fig. 20. Sin. component of residual vector X of M2. Eq. = 0.2 μgal.

Fig. 21. Amplitude factor of A-L vector (reduced observations), Eq. = 0.01.

Fig. 22. Phase of A-L vector (reduced observations), M2. Eq. = 0.4°.
Fig. 23. Amplitude of vector L (oceanic effects), M2.
Eq. = 0.5 µgal.

Fig. 24. Amplitude of vector B, M2.
Eq. = 0.5 µgal.

Fig. 25. Phase of vector L (oceanic effects), M2.
Eq. = 20°.

Fig. 26. Phase of vector B, M2
Eq. = 20°.

Fig. 27. Variation of $\delta$(M2) (reduced) along latitude.
Fig. 28. Amplitude factor of observed values $S_2$. Eq. = 0.02.

Fig. 29. Phase of observed values $S_2$. Eq. = 1°.

Fig. 30. Amplitude of residual vector $X$ for 01. Eq. = 0.1 μgal.

Fig. 31. Phase of residual vector $X$ for 01. Eq. = 023.

Fig. 32. Cos. component of residual vector $X$ for 01. Eq. = 0.1 μgal