On the relative importance of self-gravitation and elasticity in modeling volcanic ground deformation and gravity changes

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[1] Elastic half-space models, widely used to interpret displacements and gravity data in active volcanic areas, usually compute the displacement response to dilatational sources that simulate a change in pressure of the magma chamber. Elastic-gravitational models allow the computation of gravity, deformation, and gravitational potential changes due to pressurized magma cavities and intruding masses together. This type of model takes into account the mass interaction with the self-gravitation of the Earth through coupling between model equations. We perform a dimensional analysis of the elastic-gravitational model estimating the magnitude of intrusion mass and coupling effects at the space scale associated with volcano monitoring. We show that the intrusion mass cannot be neglected in the interpretation of gravity changes while displacements are primarily caused by pressurization. Therefore the intrusion of mass, together with the associated pressurization of the magma chamber, produces distinctive changes in gravity that could be used to interpret gravity changes without ground deformation and vice versa, depending on what type of source plays the main role in the intrusion process. Theoretical experiments indicate that mass and self-gravitation could produce changes in the magnitude and pattern of predicted gravity that may be above microgravity accuracy. Application of the elastic-gravitational model to interpret geodetic precursors observed at Mayon volcano (Philippines) prior to the eruption of 2001 shows that inversions increase in precision by using this model. Therefore our elastic-gravitational model is a refinement of purely elastic models and can better interpret gravity and deformation changes in active volcanic zones.


1. Introduction

[2] Volcanic activity monitoring involves interpretation and analysis of ground deformation and gravity changes that may be eruption precursors. In this way, deformation models are essential tools. Current understanding of critical stages prior to a volcanic eruption is generally based on elastostatic analysis, developed from mechanical models of the overpressure in magma chamber and conduit. Within the elastic frame there are models that include spherical and ellipsoidal point sources, vertical and horizontal magma migration, finite sources, collapse structures, and fluid migration [e.g., Mogi, 1958; Rundle, 1980; Davis, 1986; McTigue, 1987; Bonafede, 1990; De Natale and Pingue, 1996]. Mogi [1958] was the first to apply a point source of pressure in an elastic flat half-space to interpret deformation in volcanic areas. This simple and basic model has been widely used to interpret geodetic and gravity data. It successfully reproduces displacement and gravity changes at many volcanoes during uplift or subsidence [e.g., Battaglia and Segall, 2004; Battaglia et al., 2003].

[3] Microgravity monitoring involves the measurement of small changes in the value of gravity with time that may contribute to assess subsurface processes related to volcanic centers. Gravity surveys are conventionally carried out to relate gravity changes with ground elevation changes as a means of inferring subsurface mass/volume changes [Gottsmann and Rymer, 2002]. The connection between the variation of gravity and elevation is usually interpreted in relation to two gradients. If gravity follows the theoretical free-air gradient, calculated by using the Mogi model and produced simply by elevation change [Rundle, 1978; Walsh and Rice, 1979], no subsurface change in mass has occurred. Data following the gradient after the standard Bouguer correction implies mass changes [e.g., Brown and Rymer, 1991; Rymer et al., 1993; Rymer, 1996]. Departures from these linear gradients are used to model
volcanic processes. Thus vertical gravity gradients can contribute to volcanic hazard assessment during unrest [Rymer and Williams-Jones, 2000; Williams-Jones and Rymer, 2002; Gottsmann and Rymer, 2002; Gottsmann et al., 2003].

[4] Nonlinear gravity-height relationships that cannot be interpreted with Mogi model have also been found at several volcanoes [e.g., Kisslinger, 1975; Rymer, 1996]. Also, evidence for ground deformation and seismicity in calderas and other volcanic areas indicate that these phenomena cannot be modeled by purely elastic effects [e.g., Bonafede, 1991; De Natale et al., 1997; Jentzsch et al., 2001]. Therefore discrepancies in data interpretation demand more complex calculations. Rundle [1980] solved the equations that represent the coupled elastic-gravitational problem for a stratified half-space of homogeneous layers. This type of model goes one step further than a purely elastic model, since it allows for numerical experiments considering jointly the effects of a pressurized chamber cavity and mass intrusion. Fernández et al. [1997] suggested, through the numerical comparison of an elastic-gravitational solution with that for a purely elastic medium, that the complete solution for deformation and gravity changes should include the coupling between elastic and gravitational effects. To determine if mass and coupling effects are important at distances and timescales associated with volcano monitoring we perform a dimensional analysis of the elastic-gravitational equations similar to that proposed by Battaglia and Segall [2004] or the one of Pollitz [1997] in the seismic context. Our results show that an elastic-gravitational model is able to interpret changes in gravity without ground inflation/deflation and vice versa, combining mass and pressure rates in a suitable way. Coupling between self-gravitation of the Earth and elastic response of the medium is a second-order effect that sometimes can produce changes on the order of 10's of µGal. Therefore intrusion mass and its interaction with the gravity field should be taken into account given the precision of modern microgravity instruments [see, e.g., Rymer, 1996].

2. Elastic-Gravitational Governing Equations

[5] We first review the elastic-gravitational model. The appropriate equations are given by Love [1911], in which the fully coupled gravity and displacement changes are obtained together. First, we approximate the Earth with a spherically symmetric, nonrotating, and isotropic sphere. Our problem is to determine a stress system by which disturbances to these fields can be taken into account given the precision of modern microgravity instruments [see, e.g., Rymer, 1996].

[6] We assume the Earth to be initially in hydrostatic equilibrium:

\[ \nabla \phi_0 = \frac{1}{\rho_0} \nabla P_0, \quad (1) \]

where the density \( \rho_0 \), the hydrostatic pressure \( P_0 \), and the potential \( \phi_0 \), define the initial stress on a spherically symmetric reference Earth model.

[7] To go one step further and give an analytic formulation for magma intrusions, the initial state of equilibrium is slightly disturbed by an external body force that will set up a displacement field accompanied by density, potential, and stress disturbances. For this purpose an additional stress field, \( \sigma \), is taken to be correlated with displacement, \( u \), as in the ordinary theory of elasticity:

\[ -\rho \nabla \phi + \nabla \cdot \tau = 0. \quad (2) \]

Here, \( \tau \) is the total stress at the undeformed points of the medium [Lanzano, 1982]:

\[ \tau = (P_0 - u \cdot \nabla P_0)I + \sigma, \quad (3) \]

where the additional stress field is added to initial hydrostatic stress and \( I \) is the identity. \( \rho \) is the mass density:

\[ \rho = \rho_0 + \rho_1, \quad (4) \]

with \( \rho_1 \), the change in density due to the displacement field which is established from the continuity equation:

\[ \rho_1 = -\rho_0 \nabla \cdot u; \quad (5) \]

\( \phi \) is the gravitational potential:

\[ \phi = \phi_0 + \phi_1, \quad (6) \]

with \( \phi_1 \), the potential of the elastically perturbed state that consists of the potential \( \phi_1^T \), generated by the mass redistribution due to the deformation and \( \phi_{int} \), which represents any gravitational field acting on Earth. This potential satisfies the Poisson equation:

\[ \nabla^2 \phi_1 = \nabla^2 \phi_1^T + \nabla^2 \phi_{int} = 4\pi G \rho_1 + 4\pi G \rho_{int} \delta(r - r_i), \quad (7) \]

where \( G \) is the gravitational constant, \( \Delta(r - r_i) \) is the three-dimensional delta function, that specifies the spatial location of the source at \( r_i \), and \( \rho_{int} \) is the density of the intrusion. In the following, we assume that disturbances to these fields are infinitesimal.

[8] Volcanic loading is better represented as a near-field problem rather than the spherical Earth approach. In this way, elastic half-space models are widely used for volcanoes, where this approximation is an accurate representation of near-field displacements due to the intrusion process. The problem of coupling is more delicate in the half-space case since the initial stress system is not well defined. That is, balancing the pressure gradient with self-gravitation in expression (1) leads to infinite pressures at infinite depths and an infinite value for the gravity at the free surface. Following Rundle [1980], we circumvent this problem by coupling our perturbation problem to the initial...
stress appropriate to a spherical self-gravitating Earth. We assume

$$g = g_e \cdot e_z$$  \hspace{1cm} (8)

as constant surface gravitational acceleration and demand that any solution obtained agree with that derived from the spherical problem in the appropriate limit [Gilbert, 1976; Rundle, 1980]. A system of cylindrical coordinates $(r, \theta, z)$, with the origin located at the projection of the intrusion at the surface, is assumed. The $z$ axis points down into the half-space. Therefore [Rundle, 1980]

$$\rho_0 g \nabla (u \cdot e_z) - \rho_0 \nabla \phi_1^* - \rho_0 g e_z \cdot \nabla \cdot u + \nabla \cdot \sigma + F_m + F_p = 0 \hspace{1cm} (9)$$

$$\nabla^2 \phi_1 = -4 \pi \rho_0 G \nabla \cdot u + 4 \pi G \rho_m (r - r_c). \hspace{1cm} (10)$$

[8] Deformation and surface gravity changes can be signs of intruding mass anomalies and zones pressurized by magma intrusions, thermal expansions or changes in hydrothermal systems. Deformation of the Earth’s crust due to the inflation/deflation of a magma body has often been modeled as the response of an elastic half-space to a center of dilatation. However, elastic-gravitational models approach pressurization together with mass intrusion as two different sources that can be treated with the same formalism. In this way, we have approximated the intrusion of mass by the superposition of a point source of dilatation and a mass point source. So, we have added the term $F_p$ in equation (9) that represents the body force equivalent to a point source of dilatation:

$$F_p = \frac{M \nabla \cdot (\Delta (r-r_c))}{\Delta V}, \hspace{1cm} (11)$$

where $\Delta V$ is the volumetric change of the source and $M$ is a second-order tensor corresponding to the superposition of three mutually orthogonal dipoles (double force) of identical strength, $\Delta P$, that simulate a spherical cavity with radius $a$, undergoing a transformational expansion [Aki and Richards, 1980]:

$$M_y = \frac{\lambda + 2 \mu}{\mu} \frac{\Delta P a}{\rho_0}, \hspace{1cm} (12)$$

where $\lambda$ and $\mu$ are the Lamé elastic parameters. From equation (7), the force generated by the mass of intrusion, $F_m$, is given by

$$F_m = -\rho_0 \nabla \phi_m, \hspace{1cm} (13)$$

[10] Rundle [1980] solved these equations by using the propagator matrix technique [Thompson, 1950; Haskell, 1953] in a layered half-space to obtain surface gravity, deformation, and potential changes arising from volcanic loading. Rundle [1981] developed the numerical formulation for the case of a single layer in welded contact with an infinite half-space. Expressions for the case of two layers are given in Fernández and Rundle [1994], Fernández et al. [1997] gave the appropriate formulation for a media composed of up to four layers over a half-space. More recently, Charco et al. [2002] obtained the analytical expressions to compute vertical deflection and geoid changes.

3. Dimensional Analysis and Scaling

[11] Dimensional analysis and scaling are useful in developing and interpreting model equations. At this point, we can undertake a dimensional analysis of the problem and gain considerable insight without actually attempting a solution. We want to compare the magnitudes of the terms in the system described by (9) and (10) in order to neglect small terms.

[12] The first three terms of (9) depend explicitly on either $g$ or $G$. The $g$-dependent terms scale as $g \rho_0 |u|/d$, whereas $G$ term scales as $4 \pi G \rho_0 |u|$ by applying Gauss' theorem to (10). $|u|$ and $d$ are the characteristic lengths scales for displacement field and domain, respectively. Characteristic quantities are formed by taking combinations of various dimensional constants and should be roughly the same order of magnitude as the quantity itself. Displacement and gravity changes are generally small, with the half width and half maximum similar in magnitude to source depth, $c$ [Rundle, 1981]. They change quickly in an interval near the origin that becomes narrower as $c \to 0$. Thus the domain characteristic distance is set by source depth, i.e., $d = c$. The elastic stress tensor, $\sigma$, scales as $\mu |u|/c$ and its gradient as $\mu |u|/c^2$. Therefore the relative importance of $G$ and the elastic stress terms in the force balance (9) is given by the dimensionless ratio:

$$\frac{g \rho_0 \nabla \phi_1^*}{\nabla \cdot \sigma} \sim \frac{4 \pi G \rho_0 c^2}{\mu}, \hspace{1cm} (14)$$

such that the $G$ term importance depends on $c$ and the shear modulus $\mu$. The relative importance of $g$ and the elastic terms is governed by the ratio

$$\frac{g \rho_0 \nabla (u \cdot e_z)}{\nabla \cdot \sigma} = \frac{g \rho_0 c}{\mu}, \hspace{1cm} (15)$$

that increases uniformly with $c$. Using typical parameter values for volcanic areas, Battaglia and Segall [2004] pointed out that $G$ and $g$ terms are negligible in (9). Neglecting $G$ and $g$ terms permits considerable simplification in mathematics.

[13] Displacement magnitude depends on source terms $F_p$ and $F_m$. $F_p$ simulates a spherical source with a transformed volume of expansion, and scales as

$$F_p = \frac{(\lambda + 2 \mu) \alpha^2 \Delta P}{\mu \rho_0 \Delta V}. \hspace{1cm} (16)$$

The volume change of the chamber caused by pressurization is [Delaney and McTigue, 1994]

$$\Delta V = 4 \pi a^2 \Delta a, \hspace{1cm} (17)$$

with $\Delta a$ being the change in radius of a spherical source region [McTigue, 1987]:

$$\Delta a = \frac{\Delta P}{4 \mu}. \hspace{1cm} (18)$$
which is of the order of $10^{-2}$ given by the dimensionless ratio:

$$F_m \sim \frac{\rho_m g}.$$

$F_m$ is attributable to influx of magma into the subvolcanic storage system. According to Newton’s law, the mass point source magnitude is given by [Zhong and Zuber, 2000]

$$F_m \sim \rho_m g.$$

Therefore the relative importance of body forces is given by the dimensionless ratio:

$$\frac{F_m}{F_p} \sim \frac{\rho_m ga}{\mu},$$

which is of the order of $10^{-2}$ for typical parameters in volcanic areas, a result that was also derived by Battaglia and Segall [2004]. Fernández et al. [1997] showed via numerical examples that gravitational effects are not significant for the displacements and tilt caused by magma intrusions. Tiampo et al. [2004a] performed a sensitivity analysis for the joint inversion of deformation and gravity to each of the elastic-gravitational model parameters. They noted that deformation measurements are very sensitive to the pressure and radius parameters. In fact, dilatation source strength depends on the effect of a change in inflation/deflation pressure or chamber wall displacements as can be seen in expression (16). In the same way, scaling analysis states that the contribution of the intrusion mass is almost null compared to the pressurization contribution in the displacement field calculations. Since the magnitude of $F_p$ is larger than $F_m$, neglecting $F_m$ is a close approximation to compute the displacement field caused by a magma intrusion. However, the emplacement of mass at some depth significant for changes in gravity?

Surface gravity change arises from the potential generated by perturbed density, $\phi^g$, and from the potential due to a gravitational-source-mass distribution, $\phi_m$. The order of magnitude comparison of both potentials is governed by

$$\nabla \phi^g \sim \frac{|u|^2}{a^2 \Delta a}, \quad \nabla \phi_m \sim \frac{\rho_m g}{\mu}.$$  

Battaglia and Segall [2004] showed that potentials ($\phi^g$ and $\phi_m$) are of the same magnitude, although they are negligible compared to the elastic effect. However, since (22) is of unit order, the mass intrusion effect and its interaction with the gravity field, that is simulated by the coupling between elastic-gravitational model equations, could be important for surface gravity change modeling. Thus, although the gravitational contribution of the mass to deformation is negligible compared to that of pressure, this may not be the case for the surface gravity change.

A summary of scaling results is shown in Table 1. The description outlines the major forces or parameters to take into account for interpreting volcanic unrest by using the elastic-gravitational model. Theoretical results of section 4 will help us to explain the importance of gravity acceleration and self-gravitation for modeling changes in gravity.

<table>
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<th>Table 1. Description of Model Parameters</th>
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<td>Mass intrusion</td>
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Then, using expression (17), $F_p$ scales as

$$F_p \sim \frac{\mu}{a}.$$

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4. Mass and Coupling Effect on Changes in Gravity

We perform theoretical experiments that examine mass filling and its interaction with the ambient gravity field.

First, we investigate the source influence on understanding both the synthetic and the field measurements. Volume change of the chamber cavity is not directly equivalent to the quantity of magma recharged [Johnson et al., 2000]. Of course, the injection of mass cannot be accomplished without the pressurization of the cavity containing the magma, but cavity overpressure can be produced by volatile saturation of magma or an increase in gas content. The elastic-gravitational model allows the calculation of geologically meaningful solutions given by the superposition of a pressurized cavity and a mass intrusion. Figure 1 shows vertical displacements and surface gravity changes caused by two sources located at 3 km depth in a homogeneous medium with Lamé parameters $\lambda = \mu = 30$ GPa and density $\rho = 3000$ kg m$^{-3}$, i.e., Poisson’s ratio $\sigma = 0.25$. For the dilatational source we consider a constant pressure increment of 10 MPa and a radius of 1 km. Instead we consider 1, 0.5, and 0.1 MU (1 MU = $10^{12}$ kg) for the mass source. Maximum vertical displacement is of millimeter order while surface gravity change reaches a maximum magnitude value of around 740 $\mu$Gal (Figure 1a). This is due to the fact that gravity increases because of mass addition at constant volume. As added mass decreases, inflation caused by pressure increases (Figures 1b and 1c).

Nevertheless, total uplift is still negligible compared to gravity changes, which results in the mass source having the primary effect.

In the second step of our study into gravity and deformation interpretation, we evaluate the elastic-gravitational effect on changes in gravity to check the results of our dimensional and scaling analysis. Numerical experiments are carried out considering the effects on gravity changes
Figure 1. (a) Vertical displacement (cm) and surface gravity change (mGal) due to spherical mass intrusion of 1 MU (radius 1 km) and pressure change of 10 MPa in homogeneous elastic-gravitational medium with $\lambda = \mu = 30$ GPa. Mass of the intrusion replaced by (b) 0.5 and (c) 0.1 MU.

Figure 2. (a) Surface gravity change (mGal) due to a center of dilatation with pressure increase of 300 MPa. (b) Surface gravity change (mGal) for a spherical mass source of 0.35 MU in homogeneous medium with $\lambda = \mu = 30$ GPa.
due to a pressurized magma chamber and an intruded mass. Figure 2 displays surface gravity changes produced by a mass point of 0.35 MU located at 3 km depth and the effect caused by a center of dilatation of 300 MPa km$^{-3}$ strength at the same depth. In this example, both source terms have the same influence on equation (10), i.e., predicted gravity changes are similar quantitatively and qualitatively. Individually, these effects are equivalent to those obtained by a purely elastic model. In fact, adding both effects produces null changes in gravity (Figure 3a, solid line). However, the elastic-gravitational model includes self-gravitation as one of the medium characteristics through coupling between the model equations. The terms responsible for coupling are those dependent on the potential due to perturbed density, $<\rho>$. That is, displacement and potential disturbances are connected as can be seen in (9)-(10). We have computed superposition effect of the sources described above to consider self-gravitation on gravity changes (Figure 3a). The results of numerical computations are compared with purely elastic ones (solid line). The coupling effect (dashed line) is shown at a horizontal distance of the order of source depth. This effect is diminished by reducing the magnitude of the mass, since potentials $<\rho>$ and $\phi_m$ are not of the same magnitude in this case (Figure 3b). Mass interaction with the gravity field decreases as the free-air effect due to uplift is corrected (Figure 4). Variations in pattern and magnitude of the gravity change, corrected of free-air and Bouguer effect, prove that the intrusion of mass is not negligible as it is for displacement calculations. Figures 3 and 4 illustrate how the errors due to neglecting $g$ and $G$ terms are propagated through the gravity change calculations.

[20] We produce synthetic gravity data for a magma chamber located at 3 km depth in order to illustrate theoretically the coupling behavior as pressure increases, i.e., as $\Delta P \to \infty$. Figure 5 shows the results for dilatational sources of 50, 100, 300, and 500 MPa km$^{-3}$ strength. The corresponding mass for the sources has the same effect on gravity changes. The fact that the coupling effect increases with increasing pressure is consistent with the idea that the effect is due to disturbed potential generated by density changes.

[21] The results of varying source depth on the coupling effect are demonstrated in Figure 6, where we show the results for chambers at 2, 3, 5, 8, and 10 km depth. All models assume a center of dilatation of 300 MPa km$^{-3}$ and 0.35 MU. Coupling has a significant effect on the magnitude of the predicted gravity changes, particularly for magma chambers at shallow depths. As the magnitude of $<\rho>$ depends on $|\mathbf{u}|/c$ and magnitude of $\phi_m$ depends on $a^2\Delta \rho/c^2$, the mass effect decreases faster than the dilatation effect with depth.

[22] As we pointed out, Battaglia and Segall [2004] stated that potential magnitudes are negligible compared with the elastic term in the elastic-gravitational model, and thus coupling is a second-order effect. They inferred that coupling between gravity and elasticity is negligible in the space scale associated with volcano monitoring. The main advantage in using the elastic-gravitational model is that it...
Figure 4. (a) Free-air gravity change (μGal) and (b) Bouguer gravity change (μGal) due to a center of dilatation with pressure increase of 300 MPa and a spherical mass source of 0.35 MU in an elastic- gravitational homogeneous medium with $\lambda = \mu = 30$ GPa.

Figure 5. Predicted surface gravity change (μGal) for various models. Model 1 represents a dilatation center with pressure increase ($\Delta P$) of 50 MPa and a spherical point mass source ($M$) of 0.06 MU; in model 2, $\Delta P = 100$ MPa and $M = 0.115$ MU; in model 3, $\Delta P = 300$ MPa and $M = 0.35$ MU; and in model 4, $\Delta P = 500$ MPa and $M = 0.6$ MU.
Figure 6. Surface gravity change (μGal) for various source depths. Results are for dilatation center with pressure increase of 300 MPa and for spherical mass of 0.35 MU at $c = 2$ km (model 5); $c = 3$ km (model 6); $c = 5$ km (model 7); $c = 8$ km (model 8); $c = 10$ km (model 9).

Figure 7. (a) Vertical, (b) radial displacements (cm), and (c) surface gravity change (mGal) due to a center of dilatation with pressure increase of 3 MPa and a spherical mass source of 0.35 MU in elastic (solid line) and elastic-gravitational (dashed line) homogeneous medium with $\mu = 0.3$ GPa.
will yield accurate results for all elastic structures, even exotic ones that might occasionally exist in volcanic regimes. As it is known, the presence of incoherent materials and high temperatures produce a lower effective viscosity of the Earth’s crust in the vicinity of active volcanoes making it necessary to consider anelastic properties [e.g., Bonafede et al., 1986; Fernández et al., 2001a; Newman et al., 2001]. When $\mu$ is small the medium could represent the behavior of thermal metamorphic rocks that normally surround a magma chamber [Dragoni and Magnanensi, 1989]. Furthermore, small values of $\mu$ might indicate that the relaxation was produced a significant time after intrusion. In such cases, smaller pressure increases are required to model displacement and gravity changes. Figure 7 shows displacement and changes in gravity caused by the superposition of a center of dilatation of 3 MPa km$^{-3}$ and a point of mass of 0.35 MU located at 3 km depth in a homogeneous medium with a shear modulus $\mu = 0.3$ GPa. Variations in displacements and surface gravity changes are due to absolute effects of elastic-gravitational coupling and gravity acceleration. Both effects become more important when $\mu$ relaxes to a small value, as shown by (14)–(15). In such cases, magma mass effect could not mask the coupling effect, although the pressurization decreases.

[23] Microgravity monitoring involves the measurements of small changes with time in the sensitivity and magnitude of gravity. Theoretical results show that the error due to neglecting $g$ and $G$ terms in equation (9) is inherited by equation (10) when the gravitational source (intrusion mass) is taken into account, as the disturbed potential depends on both the displacement field and the continuity of the Poisson equation. Therefore we should take into account intrusion mass and its interaction with the gravity field since both effects produce changes in the gravity pattern and magnitude at the accuracy attainable nowadays in microgravity surveys [e.g., Rymer, 1996].

5. Application

[24] We use the elastic-gravitational model to interpret geodetic observations made at Mayon, a classic stratovolcano cone with an altitude of 2462 m. Mayon is the most active volcano in the Philippines, located in the Bicol volcanic chain southeast of the island of Luzon, Philippines (Figure 8a), part of the Legaspi Lineament of the central Philippine fault system, which runs NW-SE across Legaspi City [Jentzsch et al., 2001]. Since 1616, this volcano has erupted 47 times and nearly every 10 years during the last century. The last strong eruption was in 1984, the youngest one in 2001 [Jentzsch et al., 2004]. Because of the population density in the area, it has been monitored more closely in recent years [Völksen and Seeber, 1995; Jahr et al., 1998; Jentzsch et al., 2001; R. S. Punongbayan et al., Operation Mayon, Philippine Institute of Volcanology and Seismology, unpublished report, 1990].

Figure 8. (a) Location of Mayon volcano, Philippines. (b) Gravity and GPS networks around Mayon crater. (c) Gravity variations over the horizontal distance from the crater, for the Tumpa-Lahar-Channel profile, and epochs 2-1, 3-1, 4-1, and 5-1 (modified from Tiampo et al. [2004a] with permission from Birkhäuser Verlag, Basel, Switzerland).
Figure 9. Measured values in the epoch 5-1 for Tumpa-Lahar-Channel profile (solid line), inversion results using elastic-gravitational model (dashed line) and the volcanic conduit model (dotted line) (modified from Fernández et al. [2001b]).
place, i.e., re injection into existing cavities instead of deflation. Fernández et al. [2001b], Tiampo et al. [2004a], and Tiampo et al. [2004b] tested this hypothesis by modeling the changes in gravity without resolvable deformation using the Genetic Algorithm (GA) inversion technique [e.g., Michalewicz, 1992; Tiampo et al., 2000, 2004a, 2004b]. The elastic-gravitational model has several advantages for this particular inversion. First, the numerical formulation allows for the joint inversion of deformation and gravity data. Second, the incorporation of elastic-gravitational coupling that simulates mass interaction with the gravity field introduces a long-wavelength effect to the predicted gravity signal that might prove important in data interpretation. Finally, the elastic-gravitational model allows us to combine mass and pressurization effects to interpret the unusual measurements that lead to surprisingly high or low gravity gradients.

Figure 9 compares the results for the elastic-gravitational inversion (dashed line) with Tumpa-Lahar-Channel profile measurements (solid line) as well as that for the model that redistributes magma down the volcanic conduit (dotted line). Neither the Mogi model [Jahr et al., 1998; Jentzsch et al., 2001] nor the vent magma model provide a satisfactory solution for the observed data. The characteristics obtained for the intrusion through GA inversion are a depth of 1.82 km, 31 MPa pressure increase, a radius of 1.71 km, and 0.841 MU mass increase for changes in gravity observed during December 1992 and December 1996. Then we have both pressurization together with mass injection. These parameters correspond to low-density value estimated; that is, if we compute chamber volumetric expansion assuming that it is equal to the volume of magma that enters or leaves the cavity, it yields a low density for the intruding mass. However, we have to take into account that the increase of the chamber volume caused by overpressure would not be equal to the volume of magma injection. Furthermore, since displacements are mainly caused by pressurization, the error in volume can be quite large if deformation measurements cannot constrain pressure sufficiently in the inversion. Choosing the right model to invert deformation and gravity data is a critical step in the interpretation of volcanic processes. In this application, the elastic-gravitational model provides a better match, emphasizing the importance of considering gravitational effects in modeling magmatic sources. Neglecting the interaction between pressure and mass effect (coupling) could be an error source in the parameter estimation by using flat half-space models, since such an interaction could cause gravity signals larger than the accuracy that is attainable presently.

6. Summary and Conclusions

This paper reviews the governing equations of the elastic-gravitational model in order to investigate its properties. This model provides a complete solution of the problem of calculating gravity, deformation, and potential changes arising from volcanic crustal loading that includes the directly coupled effects of gravity and displacement changes. Deformation and gravity changes are obtained as a part of one solution. Through dimensional analysis and scaling we have shown that while displacements are mainly caused by overpressure, we cannot neglect the mass intrusion effect when modeling gravity changes. Thus, with two sources of loading, a point mass and a pressurized cavity, it is theoretically possible to interpret changes in gravity without any significant deformation or vice versa (Figure 1). The right combination between mass and pressurization can be used to interpret unusual geodetic measurements.

Coupling between gravity and elasticity, that is caused by the superposition of both sources, is negligible for displacements in the spatial scale associated with volcano monitoring. However, as the rigidity decreases in magnitude the absolute effects of gravity become important as we have proven using a dimensionless analysis. Rundle [1981] pointed out that this particular property implies that for time-dependent displacements which are a result of a source embedded in a viscoelastic medium, the effects of gravity will become important at sufficiently long times, e.g., when μ has relaxed to some small value. Furthermore, we have shown that the second-order effect of coupling cannot be ignored in flat half-space models when the mass source term represents the emplacement of mass at some depth. The error generated when coupling is neglected in displacement calculations is propagated to gravity since the disturbed potential depends on the displacement field. Theoretical results show how mass and its interaction with the medium could vary the gravity pattern and produce measurable gravity changes. Therefore it is necessary to take into account both the mass of the intrusion and its interaction with gravity field of the Earth in order to interpret gravity changes in active volcanic areas.

We have studied the gravity changes observed in Mayon volcano (Philippines) as an example of the practical application of the elastic-gravitational model. The results demonstrate that the accuracy of a volcanic source parameter search can be increased by applying this type of model since the error due to neglecting gravitational sources can be propagated to the joint inversion of deformation and gravity data.

The main advantage in using the elastic-gravitational model is that it will yield accurate results for all elastic structures, even exotic ones that might occasionally exist in volcanic regimes since it allows the explanation of changes due to unusual elastic properties. Thus the elastic-gravitational model is a refinement of the elastic models that can provide a better description of reality and more accurate results.

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References


Kisslinger, C. (1975), Processes during the Matsushiro, Japan, earthquake swarm as revealed by leveling, gravity, and spring-flow observations, Geology, 3, 57–62.


