Three-dimensional indirect boundary element method for deformation and gravity changes in volcanic areas: Application to Teide volcano (Tenerife, Canary Islands)

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[1] Most deformation models of volcanoes assume that the Earth is a linear, elastic, isotropic and homogeneous half-space, although some volcanic areas are associated with significant relief. We investigate the effects of topography on surface deformation and gravity changes caused by a magma intrusion in the Earth's crust. A three-dimensional (3-D) indirect boundary element method (IBEM) that incorporates realistic topographic features is developed in order to perform this analysis. Our results show that the topography alters both the magnitude and pattern of the deformation and gravity signal. As an example of realistic topography, we consider a spherical source of dilatation located at 4 km depth below Teide volcano summit (Tenerife, Canary Islands) in order to simulate the deformation and gravity changes that could be observed at Tenerife if a hypothetical intrusion occurred in the volcanic system. This approach gives a picture of the 3-D topographic effect at Teide that can provide insight in order to improve the geodetic monitoring of the volcano.


1. Introduction

[2] The analysis and interpretation of surface deformation and gravity change in volcanic areas is one of the most important tools for understanding the internal volcano processes and detecting precursors of volcanic activity. Consequently, geodetic techniques are being used extensively to monitor active volcanoes [see, e.g., Dzurisin, 2003; Poland et al., 2006]. Most deformation models used to interpret deformation and changes in gravity assume that the Earth is flat and employ half-space solutions. While such models have been quite successful at fitting surface measurements, it is clear that local topographic relief can produce significant variations in deformation and gravity changes and therefore misinterpretation of source parameter estimation.

[3] McTigue and Stein [1984] and McTigue and Segall [1988] studied the effect of topography by means of a two-dimensional model. Cayol and Cornet [1998] and Williams and Wadge [1998, 2000] calculated the effect of the volcanic relief caused by a dilatational source in a purely elastic medium. Folch et al. [2000] considered a viscoelastic medium in order to study the effect of topography. Charco et al. [2002] used an approximate methodology for the consideration of topography in the computation of thermo-viscoelastic displacements and gravity changes. Trasatti et al. [2003] considered topographic effects on the axial symmetric volcano shape applied to Mount Etna Lungarini et al. [2005] used a numerical finite element technique to model rigorously three-dimensional (3-D) topographic effects of ground displacement at Mount Etna. In general, the conclusion reached by these authors is that topography has a significant effect on the deformation field. Thus approximating the Earth by a half-space can lead to erroneous interpretation of observed ground deformation.

[4] Charco et al. [2007] performed a dimensional and scaling analysis to suggest a mathematical formulation that can be useful in the development of a numerical technique to accurately include topographic effects in the modeling of deformation and gravity changes, although it neglected the self-gravitation of the Earth. In this work, we employ this argument to compute displacements and gravity changes caused by volcanic loading (internal loading produced by pressure and mass changes). The displacement field perturbed by 3-D topographic features is represented by a single-layer boundary integral. The integral representation...
used in our technique is derived from Betti’s reciprocal theorem and the solution to Kelvin’s problem of a point load in an infinite body [see, e.g., Sánchez-Sesma and Luzón, 1995].

[5] One of the goals of this work is to study the topographic effect on gravity changes. Considering the volcanic loads mentioned above, the changes in gravity are due to the introduction of a new mass in the medium (density change) and to volume changes arising from the compressibility of the material. The topographic contributions to the gravitational potential gradient are implemented through the change in vertical displacement induced by chamber inflation/contraction due to a pressure change and by a point mass load within the medium. As well as the topographic effect due to vertical movements caused by pressure and mass changes, it includes the topographic variation produced by the attraction of the additional mass by varying the location of the source with the point elevation.

[6] Fernández et al. [1999] and Yu et al. [2000] studied the application of different geodetic techniques to volcanic activity monitoring the Teide volcano (Tenerife, Canary Islands) using a theoretical analysis. To account for one of the limitations of this work, namely, the approximate half-space solution, we extend it to model rigorously 3-D topographic effects on displacement and gravity change at Teide volcano and estimate the sensitivity of the permanent GPS network installed for volcano monitoring at Tenerife [Prieto et al., 2005; Fernández et al., 2006]. We further compute the theoretical deformation field by classical half-space models. Comparison between IBEM results and analytical solutions allows the effect of the real 3-D topography to be evaluated.

2. Integral Representation

[7] Formulation of the indirect boundary integral method (IBEM) is derived from Betti’s reciprocal theorem [see, e.g., Rizzo, 1967; Cruse, 1969]. In general, neglecting the body forces for an elastic body $\Omega$ with boundary $\mathcal{S}$ the displacement may be written

$$u_i(x) = \int_{\mathcal{S}} \phi_j(\xi) G_{ij}(x, \xi) d\mathcal{S}, \quad (1)$$

where $u_i(x)$ is the $i$th component of the displacement at point $x$ and $G_{ij}(x, \xi)$ is the Green’s function, i.e., the displacement in direction $i$ at field point $x$ of boundary $\mathcal{S}$ due to a unit point load acting along $j$ at point $\xi$ inside an infinite body:

$$G_{ij}(x, \xi) = \frac{1}{16\pi \mu (1 - \nu)} \left[ (3 - 4\nu) \delta_{ij} + R_d R_j \right], \quad (2)$$

with $R_d^2 = (x_1 - \xi_1)^2 + (x_2 - \xi_2)^2 + (x_3 - \xi_3)^2$, $R_d$ the partial derivative of $R$ with respect to field point $x$, $\mu$ is the shear modulus, $\nu$ is the Poisson ratio and $\delta_{ij}$ is the Kronecker’s delta. A system of Cartesian coordinates with the $x_3$ axis pointing down into the medium is assumed. $\phi_j(\xi)$ is the force density in direction $j$ at $\xi$ [see, e.g., Jaswon and Symm, 1977; Luzón et al., 1997]. Thus $\phi_j(\xi) d\mathcal{S}$ is a force distribution over the boundary $S$. Kupradze [1963] showed that if $\phi_j(\xi)$ is continuous on $S$, the displacement field is continuous across $S$.

[8] Traction is determined from differentiation of equation (1) and Hooke’s law, except when $x$ is equal to $\xi$ on $S$ (boundary singularity). From a limiting process for the point $x$ in the boundary surrounded by a hemisphere with the radius tending to zero

$$t_i(x) = b\phi_i(x) + \int_{\mathcal{S}} \phi_j(\xi) T_{ij}(x, \xi) d\mathcal{S}, \quad (3)$$

where $t_i$ is the $i$th component of the traction at boundary and $b = 1/2$ or $b = -1/2$, depending on whether we approach $x$ from the inner or outer side of $\mathcal{S}$, and $T_{ij}(x, \xi)$ is the traction Green’s function:

$$T_{ij} = \frac{1}{8\pi(1 - \nu)R^2} \left[ (1 - 2\nu)\delta_{ij} + 3R_d R_j \right]$$

$$+ (1 - 2\nu) \left[ n_i R_j - n_j R_i \right], \quad (4)$$

with

$$\frac{\partial R}{\partial n} = \nabla R \cdot n = r_i n_i \quad (5)$$

and $n$ is the outward unit normal to the surface at $\xi$. The first term of the right-hand side of equation (3) is null if $x \notin \mathcal{S}$.

[9] Our problem is to find the perturbation of the elastic displacement by topographic features on the surface of an elastic half-space. Surface motion in and around topographic configuration comes from the interaction of the ground surface and perturbed displacement field [Luzón et al., 1997]. Therefore the total motion may be expressed as the superposition of the free-field displacement, $u_i^{(0)}$, which corresponds to the solution for volcanic loads (pressure and mass changes) in the full space and the field perturbed by ground topography, $u_i^{(0)}$:

$$u_i = u_i^{(0)} + u_i^{(1)}. \quad (6)$$

According to equation (1), the perturbed field may be written as

$$u_i^{(1)}(x) = \int_{\mathcal{S}} \phi_j(\xi) G_{ij}(x, \xi) d\mathcal{S}, \quad (7)$$

where the force density function, $\phi_j(\xi)$, is the unknown in the problem.

[10] We demand the boundary surface $\mathcal{S}$ be stress free, which implies null tractions. From equation (3) this boundary condition may be expressed by means of a Fredholm integral equation for the boundary sources that produce the perturbed field:

$$u_i^{(0)}(x) + \frac{1}{2} \phi_i(x) + \int_{\mathcal{S}} \phi_j(\xi) T_{ij}(x, \xi) d\mathcal{S} = 0. \quad (8)$$

Equation (8) is discretized over a finite portion of the surface $\mathcal{S}$ that includes the topographic relief and part of the lateral free surface in order to determine the unknowns of
the problem. Thus the field perturbed by topography is computed from the change in stresses of the reference surface, \( \sigma^{(0)} \), caused by mass and pressure sources within the medium.

[11] Numerical implementation of the problem is based on the approximation of the surface geometry by \( N \) circular planar elements. This choice is based on the fact that Green’s function integrals, that must be computed when \( \xi \rightarrow x \), can easily be obtained when the element is circular and flat. Through domain discretization, force density functions, \( \phi_j(\xi) \), are expressed in terms of their values at function nodes of each element. Considering that these are constant on the boundary element, and denoting the area of each element by \( n_j \) and flat. Through domain discretization, force density functions, \( \phi_j(\xi) \), are expressed in terms of their values at function nodes of each element. Considering that these are constant on the boundary element, and denoting the area of each element by \( n_j \). Equation (9) corresponds to a discretized form of equation (8). The integrals are computed numerically, except when \( x = \xi \). In this case:

\[
\phi_j(\xi) t_j(x, \xi) = -t^{(0)}(x),
\]

with

\[
t_j(x, \xi) = \frac{1}{2} \delta_{j(p)} + \int_{\Delta S_p} T_j(x, \xi) dS
\]

and \( q = 1, \ldots, N \). Equation (10) corresponds to a discretized form of equation (8). The integrals are computed numerically, except when \( x = \xi \). In this case:

\[
t_j(x, \xi) = \frac{1}{2} \delta_{j(p)},
\]

since the integrand \( T_j \) in equation (10) is a singular odd function on a circular and flat surface. Thus its Cauchy principal value is null. The perturbed field can be computed from the discretized form of the equation (7),

\[
u_j = \sum_{p=1}^{N} \phi_j(x) g_{pq}(x, x),
\]

where

\[
g_{pq}(x, x) = \int_{\Delta S_p} G_{pq}(x, \xi) dS.
\]

In this case, the integrals are also computed numerically, except when \( x = \xi \). Here we use analytical integration in polar coordinates. On the other hand, when \( x = \xi \), it is possible to show that

\[
\left( F + 1 \right) \delta_{j(p)} + \left( F - 1 \right) n_j n_j,
\]

with

\[
F = \frac{1}{4} \left( \frac{\lambda + 3 \mu}{\lambda + 2 \mu} \right),
\]

\( n_j \) is the \( j \)th component of the outward normal vector at the \( p \)th element, \( \lambda \) and \( \mu \) are the Lamé parameters and \( R_e \) is the element radius.

[12] Validation of the IBEM is performed on a problem with a known analytical solution: the problem of a point of dilatation embedded in an elastic half-space, \( z = 0 \), (i.e., a flat topography), otherwise known as the Mogi model. Displacements calculated analytically are compared to numerical results found by using IBEM. The lateral extension of the mesh must be large enough to avoid boundary effects. After several tests, we conclude that the results are good enough when the mesh extension is between 3 and 4 times depth source. This is related to the fact that displacements vary quickly in a narrow region in which the thickness is of the same order of source depth, \( c \). Convergence analysis is performed to calculate the discretization errors by refining the mesh. As expected, the error decreases when discretization is increased until the results are not sensitive to discretization. In order to overcome the effects of the discretization scheme, the approximation is performed by using circles that cover the boundary surface with elements of radius between 10 and 15%. Furthermore, multidimensional integration of the Gaussian type is used for numerical integration in (10) and (12) by selecting 7 points on each circular element.

3. Changes in Gravity

[13] Gravity changes can be precursory phenomenon for volcanic eruptions. Furthermore, microgravity monitoring is becoming increasingly recognized as a valuable tool for mapping out the subsurface mass redistributions that are associated with volcanic activity. One of the goals of this work is to study whether the variations in topography significantly affect gravity changes.

[14] We develop a numerical technique for including topographic effects on changes in gravity that will be those observed or determined from observed data. Emplacement of a mass, \( M \), within the crust is meant to model the intrusion of mass into a shallow chamber that is a source of changes in gravity. Another process associated with mass injection is a pressurization of the chamber. Of course, the injection of mass cannot be accomplished without pressurization, but cavity overpressure can be produced by volatile saturation of magma or an increase in gas content. The pressure source is usually considered as a point-like source with radial expansion/contraction (center of dilatation) that is similar to the inflation/deflation of a spherical cavity that caused ground deformation. The close correlation between uplift and gravity change in some volcanic areas [see, e.g., Rymer, 1996; Rymer and Williams-Jones, 2000; Gottsmann and Rymer, 2002] suggests that the straining which produced the uplift can also alter local gravity. Thus a realistic model should include both sources, mass and pressurization, and its interaction with topographic relief.

[15] Rundle [1978] and Walsh and Rice [1979] pointed out that the surface gravity change due to spherical source of dilatation within an elastic half-space is the free-air effect due to uplift. Okubo [1997] considered the effect of the gravitational attraction of material which intrudes into dilatational cavities as well as the uplift of the observation point. Thus, including topographic effects, surface gravity change, \( g_s \), is given by

\[
g_s = \gamma_{\text{rad}} (u_{3p} + u_{3m}) + \frac{GM_e}{R_e^3},
\]
Figure 1. Schema of the problem. A volcano with height $H$; $c$ is the depth of the volcanic source. A system of Cartesian coordinates with the origin located at the projection of the intrusion at the half-space and the $X_3$ axis pointing down into the medium is assumed.

where $\gamma_{FA}$ is the free-air gradient (=0.3086 mGal m$^{-1}$), $u_{3\rho}$ is the vertical displacement caused by a dilatation source, $u_{tm}$ is the vertical displacement caused by a mass source, $c = c+f(x_1,x_2)$ and $R^2 = (x_1-s_1)^2 + (x_2-s_2)^2 + (x_3-c)^2$ with the source located at $(s_1, s_2, c)$. The term $f(x_1, x_2)$, which is a function of the computation point horizontal location, $(x_1, x_2)$, is the elevation above mean sea level. Therefore the higher the topography the deeper the source.

[16] Topographic contributions to the gravitational potential gradient are numerically implemented through the change in vertical displacement induced by chamber inflation/contraction due to pressurization and through the change in vertical displacement induced by mass addition within the medium. As well as the topographic effect on the vertical position caused by a mass change within the medium, the variation of the gravitational attraction of the mass caused by topography is taken into account. These changes are computed by varying the location of the source with the point elevation, $f(x_1, x_2)$. Since gravity is strongly height-dependent [Torge, 1989], the gravitational attraction primarily depends on the distance between the computation point and the intruded mass source, $R'$, rather than the local shape of the free surface.

4. Effects of Topography on Surface Deformation and Gravity Changes

[17] We perform numerical experiments that examine the effects of topography on ground deformation and gravity changes created at the surface of prominent relief. The modeling method used for our study is the three-dimensional indirect boundary element method described above. It is well known that deformation and gravity curves depend on source depths. Williams and Wadge [1998] pointed out that topography has a significant effect on predicted displacement, particularly for magma chambers at relatively shallow depths (depths of the order of topography height).

Scaling relations included by Charco et al. [2007] showed that these results can be generalized to changes in gravity. Thus we consider sources located at shallow depths compared to the topography. Mesh dimensions are selected according to depth source (distance between the free surface and source location when topography is included), as is discussed in section 2.

4.1. Example: Axisymmetrical Volcanoes

[18] First, we consider surface displacements and gravity changes due to a pressurized cavity with no mass change and to mass intrusion with no magma chamber overpressure located beneath axisymmetrical volcanoes. Note that a geologically meaningful solution is given by the superposition of both sources. We do not use a heterogeneous medium in the present study in order to single out topographic effects. The effect of topography is represented by volcanoes of altitude $H$ and average slope of the flanks of $\alpha = 0^\circ$, $15^\circ$, $20^\circ$ and $30^\circ$ (Figure 1). The height of the volcanoes is $H = 0$, 1340, 1820, and 2886 m, respectively; that is, the radius of the volcano edifice is 5 km in all the cases. The topography overlies a homogeneous half-space whose elastic behavior is represented by Lamé parameters $\lambda = \mu = 30$ GPa, i.e., Poisson ratio of 0.25. The topographic relief is characterized by the same set of parameters.

[19] Figure 2 displays displacement and gravity changes caused by a spherical pressurized point source and a point mass source located at 4 km depth in an elastic homogeneous medium. We consider a pressure increase of 10 MPa and a mass increment of 1 MU (1 MU = $10^{12}$ kg). We employ these pressure and mass values in order to ensure that both sources cause displacements on the same order of magnitude. In this way, we can isolate and compare topographic effects. Displacements and gravity changes perturbed by topography are obtained from the free-field displacement, $u_{(0)}$, and tractions, $t_{(0)}$, caused by a center of dilatation and by a single point force, respectively.

[20] The results of changing surface slope, $\alpha$, to include height variations, are compared with flat free-surface solution ($\alpha = 0^\circ$, $H = 0$ m). Topography has a significant effect on the magnitude of the predicted displacements and gravity changes. Near the symmetry axis, vertical displacements and gravity changes decrease in magnitude with increasing height. The reduction of the magnitude of the radial displacement is smaller than for the vertical displacement near the volcano summit. Thus the half-space solution gives a large overestimation or underestimation, depending on the source, of the changes in gravity and deformation.

[21] Cayol and Cornet [1998] pointed out that the interpretation of ground surface displacements without considering topography can lead to erroneous estimations. They found the steeper the volcano the flatter the vertical displacement field. This result can be also applied to gravity change interpretation as shown in Figure 2. Displacement and gravity changes vary in a narrow region with a half width similar in magnitude to source depth. Topography has a small but noticeable effect in changing the pattern of displacement and gravity change in this region. The curves corresponding to vertical displacement and surface gravity change are slightly flatter in the proximity of the symmetry axis. The location of radial displacement maximum/minimun value varies depending on the height model. Vertical
displacements and surface gravity changes caused by a center of dilatation vanish faster at large horizontal distances \((r > c)\) when topography is neglected.

[22] Pressurization of the magma chamber is responsible for the volcano edifice inflation while mass injection causes ground subsidence. Including the topographic effect decreases the ground subsidence caused by the mass source (Figure 2b). Therefore the addition of mass cancels some of the inflation caused by the pressure source, but by less than if the topographic effect is neglected (Figure 2c). For gravity changes we notice the inverse phenomena: gravity reduction due to volcano inflation is smaller when topographic relief is considered.

4.2. Example: Volcanoes With Ellipsoidal Base

[23] In the second step of our study into the topographic effect interpretation, we evaluate the effect of volcano with ellipsoidal cone shape: that is, the slope of their flanks depends on \(X_1 - X_2\) direction. Axisymmetrical volcanoes are also considered in order to study volcano shape influence. Numerical experiments are carried out considering elliptical volcanoes with minor radius \(b = 2.5\) km, major radius \(a = 5\) km and axisymmetrical volcanoes with radius \(a = b = 2.5, 5\) km. The height of the volcanoes is \(H = 2.5\) km in all the cases. Figures 3 and 4 show the displacement and gravity change caused by a mass point of 0.32 MU located at 3 km depth and by a center of dilatation of 10 MPa km\(^2\) strength at the same depth. Because of the symmetries of the elliptical cone, the solutions are described through two profiles corresponding to the cross section with planes \(X_1 = 0\) and \(X_2 = 0\). The profiles corresponding to \(X_2\) are shown in Figure 3, while Figure 4 displays displacement and gravity change profiles corresponding to \(X_1\). The influence of the elliptical volcano shape on displacement and gravity changes is midway between those of axisymmetrical volcanoes. The elliptical volcano solution is closer to that corresponding to an axisymmetrical volcano with \(a = b = 2.5\) km in the plane \(X_2\) (Figure 3). In the plane \(X_1 = 0\), however, it is closer to that corresponding to axisymmetrical volcano with \(a = b = 5\) km (Figure 4). This fact illustrates the effect of the slope of the volcano flanks and lateral extension of topography (horizontal distance from summit to the flank of topographic relief) on displacement and gravity changes since the volcanic cones have the same height. The differences between axisymmetrical and elliptical volcanoes are smaller for displacements and gravity changes caused by a mass point of loading. Gravity changes caused by the mass point for the elliptical topographic relief overlap the changes for axisymmetrical volcanoes in both Figures 3 and 4. Therefore mass effect on gravity changes can mask the effect of relief shape variations in some cases. In summary, the comparison points out how the results are rather similar in magnitude with the obvious difference that an axisymmetrical cone predicts axisymmetrical deformation and gravity change patterns.

[23] Charco et al. [2007] pointed out that the interaction of the source-induced gravity changes with topographic relief is significant only when source depth and lateral

![Figure 2](image-url)

**Figure 2.** Vertical \((u_z)\) and radial \((u_r)\) components of displacement field and surface gravity changes \((g_z)\) caused by (a) a center of dilatation of 10 MPa km\(^2\) strength, (b) a point of mass of 1 MU, and (c) superposition of both sources. \(H\) are the heights (m) of the axisymmetrical volcanoes shown in Figure 1. Sources are located at 4 km depth from zero altitude level.
Figure 3. Vertical ($u_z$) and radial ($u_r$) components of displacement field and surface gravity changes ($g_s$) caused by (a) a center of dilatation of 10 MPa km$^3$ strength and (b) a point of mass of 0.32 MU. The sources are located at 3 km depth under elliptical cones of 2.5 km height and elliptical bases described in the text. This figure corresponds to the profile $X_2 = 0$.

Figure 4. Vertical ($u_z$) and radial ($u_r$) components of displacement field and surface gravity changes ($g_s$) caused by (a) a center of dilatation of 10 MPa km$^3$ strength and (b) a point of mass of 0.32 MU. The sources are located at 3 km depth under elliptical cones of 2.5 km height and elliptical bases described in the text. This figure corresponds to the profile $X_1 = 0$. 
extension of the relief are of the same order. This result can be extended to ground deformation since c controls the thickness of the region where displacements vary. In this way, the reduction of the solution magnitude that occurs for an axisymmetric volcano of \(a = b = 5\) km is less than the reduction that produces an axisymmetric volcano of \(a = b = 2.5\) km. The local maximum/minimum of the vertical displacement and gravity change located at the summit of the edifice becomes more pronounced. Nevertheless, the pattern of the radial displacements depends on whether the lateral extent of the relief is less than the pattern of the vertical displacement and gravity change.

5. Application: Teide Volcano (Tenerife, Canary Islands)

The Canary Islands (Figure 5) are an archipelago of volcanic origin located 60 km from the African coast. The archipelago is located within a plate where the volcanic activity has continued over the last 30 Myr. A total of 12 eruptions have been recorded in the last 500 years [Romero et al., 2003]. The longest-lasting eruption (1730–1736) took place on Lanzarote and the most recent one (1971) on La Palma. Today the main risks of eruption are located on the islands of Tenerife, La Palma and Lanzarote. A summary of the geodynamic framework is described in detailed by Araña and Ortiz [1991] and Anguita and Hernán [2000].

Teide stratovolcano dominates the eruptive system of Tenerife, the largest island of the Canary Archipelago [e.g., Albert-Füster et al., 1990; Martí et al., 1994; Ablay and Martí, 1995]. The island volcanic shield was produced by subaerial fissure eruptions (along NW–SE and SW–NE ridges of the island) that now outcrops in the corners of the island [Füster et al., 1968; Ancochea et al., 1990]. Volcanic activity migrated to the central part of Tenerife where shallow magma chambers developed and a central volcanic complex (Las Cañadas edifice) was constructed.

During the evolution of the Las Cañadas edifice, several periods of phonolitic volcanic activity separated by mafic volcanism have been identified [Martí et al., 1994, 1995]. The evolution of this complex culminates in a large elliptical depression known as Las Cañadas Caldera (0.2 Myr ago). It was on the northern edge of this depression that the Teide-Pico Viejo complex, which remains active today, began to form 150,000 yr ago. Teide and Pico Viejo are two large stratovolcanoes that overlap to form an elongated double edifice. The highest altitude corresponds to the youngest summit of Teide, at 3718 m. The main cone is approximately circular with a basal diameter of about 5 km. The last salic explosive eruption in the island occurred in Montaña Blanca, located on the Teide flanks, 2000 years ago. Furthermore, the presence of shallow magmatic system, included phonolitic chambers, is inferred from the products of the most recent eruptions [Ablay et al., 1995, 1998]. Therefore we consider the possibility of eruption in the area of this emission center. This section represents an attempt by three-dimensional modeling to estimate the deformation and gravity changes that could be observed at Tenerife island if a hypothetical magma intrusion occurred in a shallow magmatic system beneath Teide volcano.

One of the main tasks in studying an active volcanic zone is to define the most suitable monitoring system. Las Cañadas Caldera and Teide form the area where almost all volcanic research has been conducted in Tenerife Island. In particular, geodetic measurements have been performed in the southern part of the caldera, where a geodetic micro-network and a levelling network has been installed [Sevilla and Sánchez, 1996; Sevilla and Romero, 1991]. Prior to 2000, different authors have conducted seismic studies [Almendros et al., 2000], several gravimetric campaigns for structural studies [Vieira et al., 1986; Ablay and Keary, 2000].
Figure 6. Topography of Tenerife (Canary Islands). The blow up of the island shows the location of Teide stratovolcano, where the marks indicate the position of the permanent GPS network installed by Instituto Tecnológico y de Energías Renovables (ITER) for monitoring such volcano.

2000; Araña et al., 2000], temporary observations of gravimetric tides [Arnoso et al., 2000], thermal anomalies, fumarolic activity and different gasses at the top of the Teide [González et al., 2000; Salazar et al., 2000], and diffusive degassing in and around the caldera [Hernández et al., 2000]. Those observations did not detect any clear anomaly that can be regarded as indicating volcanic reactivation in the monitored areas [Fernández et al., 2005]. However, the historic volcanism of Tenerife (over the past 500 years) consists of a total of six eruptions [Solana, 1998] spread over the whole island [Carracedo et al., 2003]. This fact indicates that monitoring techniques capable of covering the whole island should be employed in order to detect possible anomalies associated with future eruptions.

5.1. Geodetic Studies

30] Geodetic techniques have provided useful eruption precursors at active volcanoes [e.g., Newhall and Dzurisin, 1988; Dvorak and Dzurisin, 1997; Dzurisin, 2003]. A summary of previous geodetic studies performed in Tenerife island can be found by Fernández et al. [2005]. In summary, interferometric synthetic aperture radar (InSAR) studies detected two areas of subsidence, located in the northwest part of the island (Pinar de Chio and Garachico), that none of the other geodetic techniques had observed before. These results emphasized the need to define a geodetic network covering the whole island in order to validate the revealed deformations, to give additional information to radar images, and to monitor deformations on the island. In this way, a GPS network formed by 18 geodetic benchmarks was designed [see, e.g., Fernández et al., 2004, 2005]. In deformation areas revealed by earlier InSAR studies, this main GPS network was densified. The network was observed in 2000, and the densification areas were observed in 2001 and 2002, allowing for detection of rebound in Pinar de Chio, and additional subsidence in Garachico zone. Considering previous results, the related volcanic hazard, and the anomalous seismic activity registered at Tenerife since 2004 in places where no activity was revealed before in former studies, a permanent GPS network has also been installed [González et al., 2005; Prieto et al., 2005; Fernández et al., 2006; http://www.iter.es]. This network consists of seven stations: TEIT, ITER, BOCA, LANO, NORD, ICOD, and PORT (Figure 6). High-quality and GPS instruments were installed in these locations. The equipment possesses similar characteristics to those used by International Global Navigation Satellite System Service (IGS) stations. The spatial distribution joins logistical, volcanic and geodetic requirements in order to achieve the necessary accuracy to perform a good surveillance around Teide stratovolcano and the N–W rift zone. The observations using this network and Bernesse 5.0 software allow for the determination of horizontal coordinates with mean square error (MSE) of 2–6 mm and vertical coordinates with MSE of 5–10 mm.

[30] Fernández et al. [1999] and Yu et al. [2000] performed a theoretical study of sensitivity to ground deformation and gravity changes to define the most suitable geodetic monitoring system. They considered a flat surface in their methodology, although they pointed out that the results were preliminary and needed corrections for topography. We employ the IBEM numerical technique in order to perform a theoretical study of sensitivity of the permanent GPS network installed in the vicinity of Teide volcano. In particular, Teide volcano is characterized by significant topography. IBEM numerical modeling allows us to define the real topographic features of this volcano. Therefore this theoretical application will provide additional insights for improvement of the geodetic monitoring system. Of course,
predictions of the model should be considered simply as an approach to reality, since some parameters involved in the modeling are approximated.

5.2. Theoretical Studies
5.2.1. Numerical Modeling
[31] A mesh of the ground surface (Figure 7) is constructed from a digital elevation model (DEM) provided by the Centro Nacional de Información Geográfica (CNIG) of Spain (Figure 6). This mesh is made of 2902 circular elements. To minimize edge effects, it is extended far enough from the source projection over a free surface. This mesh completely encloses Tenerife Island. A system of Cartesian coordinates with the origin located at the sea level, just below Teide summit, is assumed. In this case, $X_1$ and $X_2$ axis are oriented along WE and SN directions, respectively, while the $X_3$ axis points up out the medium.

An important aspect concerns the crustal properties. The elastic parameters of the area have been assumed to be constant spatially, with a rigidity of $\mu = 3.8$ GPa and a Poisson ratio of 0.23. This choice is based on the elastic Lamé parameters and the thickness of the layers overlaying the mantle given by Fernández et al. [1999] for the central part of Tenerife Island.

[32] The volume change caused by overpressure in a magma chamber may not be equal to the volume of mass recharged, it can be caused by other factors [Delaney and McTigue, 1994]. However, it is standard to assume the opposite hypothesis. Thus, taking into account the phonolitic density and a hypothetical pressure of 50 MPa, the mass of a magma intrusion can be estimated. After several
tests, we conclude that gravitational mass effects can be neglected compared to pressurization effects in this case. Therefore the magma chamber is represented as a spherical source of dilatation of 50 MPa km$^{-3}$ strength. This source is located at 4 km depth below Teide volcano summit, i.e., approximately 300 m below sea level. The depth of the source is taken in order to simulate the shallow magmatic system mentioned above. The results are illustrated in Figure 8.

[33] Most of the deformation and surface gravity change is restricted to the vicinity of the volcano; that is, at distances greater than 5–7 km from the volcano summit the related deformation and surface gravity change might be indiscernible from background noise. Thus predicted deformation and gravity change lie inside the caldera walls. The greatest component of the displacement is the vertical one that takes a maximum value of 8.83 cm at the volcano summit. Thus the maximum vertical displacement predicted could be detected by the station TEIT of the permanent GPS network considering the precision attainable nowadays [see, e.g., Dixon et al., 1997; Sagiya et al., 2000; Segall and Davis, 1997; Bartel et al., 2003] and the MSE of the coordinates provided by this network. The GPS network should detect the magnitude of the horizontal displacement as well. In fact, considering the sensitivity of the permanent GPS network mentioned above, it should detect pressure changes at 4 km depth below Teide volcano summit lower than 10 MPa. Nevertheless, it is proposed that the stations PORT and PARA should be relocated on the flanks of the volcano closer to its summit or the emplacement of some complementary GPS stations on the flanks of the volcano between NORD and TEIT stations in order to properly measure the deformation field caused by a shallow source.

[34] The presence of thermal systems in volcanic areas would induce changes in groundwater density and location and in the hydrothermal circulation that could produce deformation and gravity changes. Dvorak and Dzurisin [1997] argue that the addition of fluids and the transfer of heat into a confined aquifer within a volcano are diffusive processes. Thus the resulting pressure change within the hydrothermal system should be gradual, with the transition times in deformation patterns limited by the ability of fluids to change circulation patterns within the crust. Transitions may occur over timescales of months to years. The initiation of magma movement, on the other hand, is known to occur quite rapidly. Thus permanent GPS networks such as the one installed in the Teide volcano should readily resolve these types of eruption precursors. Nevertheless, in order to

Figure 9. Analytical vertical ($u_z$) and horizontal ($u_x, u_y$) components of displacement field and surface gravity changes ($g_s$) caused by a center of dilatation of 50 MPa km$^{-3}$ strength within a half-space in which the top is considered at sea level (model A). Color appears in back of the print issue.
understand the geometry of the magmatic system, the influence of regional tectonics and the effects induced by changes in ground water reservoirs and hydrothermal circulation, we propose here a network densification. Spatial densification of GPS networks has been limited to instrumental costs. To overcome this obstacle a low cost single-frequency (L1) GPS system was developed by UNAVCO [Meerlend et al., 1999]. This system, combined with the dual frequency sites showed at Figure 6, could provide high-precision measurements [e.g., Bartel et al., 2003] spatial and temporally arranged to overcome the nonuniqueness of the deformation data.

Three scenarios are worth considering when assessing causative processes of gravity changes: (1) arrival of new magma at depth, (2) migration of hydrothermal fluids, and (3) a hybrid of both. Surface gravity change computed by IBEM takes a maximum value of around 25 μGal (pressurization causes gravity decrease at constant mass) at the volcano summit. It is frequently possible to achieve microgravity measurements at high precision at volcanoes (±10–15 μGal) when following strict survey procedures [Rymer and Brown, 1989; Williams-Jones et al., 2003]. Thus this value also would be detected considering the microgravimetry accuracy attainable nowadays. Migration of the hydrothermal fluids through a permeable medium causes little surface deformation, but filling of pore space increases the bulk density of the material resulting in a gravity increase at the ground surface. In fact, it could produce effects of the order of the surface gravity changes predicted by the IBEM [see, e.g., Battaglia et al., 2006; Gottsmann et al., 2006]. Considering that microgravity could help in better understanding whether the behavior leading to eruptions differs from that during migration of hydrothermal fluid or routine magma recharge events, installation of continuous gravimeter stations at some of the GPS locations in the flanks of the volcano is proposed. In particular, we suggest the installation of a continuous recording gravimeter at the station TEIT where an intrusion located beneath the volcanic summit could caused the maximum gravity change. At a minimum, periodic microgravity surveys should be developed in the neighborhood of the volcano in order to complement the GPS information and acquire insight into the volcano internal processes.
Figure 11. Analytical vertical ($u_z$) and horizontal ($u_x, u_y$) components of displacement field and surface gravity changes ($g_s$) caused by a center of dilatation of 50 MPa km$^3$ strength within a half-space in which the top is considered at 3718 m height that corresponds to Teide volcano height (model C). Color appears in back of the print issue.

5.2.2. Analytical Modeling

[36] The deformation due to the expansion/contraction of a magma chamber has frequently been modeled as a source of dilatation in an elastic half-space [e.g., Mogi, 1958]. Furthermore, a commonly used method to account for topographic effects is to choose a reference surface at a constant elevation above sea level. We use three half-space models to compute surface displacements and gravity changes at Teide volcano. The comparison of these models with the IBEM numerical model including accurate topography allows for the quantification of the implicit error due to applying half-space solutions in areas of prominent relief.

[37] Our results for the analytical models are shown in Figures 9, 10, and 11. The sources and elastic parameters involved are equal to the IBEM model. Figure 9 shows the results for a Mogi model (model A), in which the free surface, $X_3 = 0$, is considered at the mean sea level. Thus topography of Tenerife island is completely neglected. Maximum uplift and surface gravity change are one hundred times larger than the IBEM ones. Deformation and surface gravity change are concentrated in a small area. This is due to the fact that the amplitude of deformation and gravity is a function of the source depth and the magma chamber is assumed to be located at 4 km depth below the volcano summit, i.e., the source is located at 300 m below sea level (bsl) ($X_3 = 0$) in this case. The magnitude of displacements and surface gravity changes depending on the source strength, which is too large (50 MPa km$^{-3}$) for such a depth. Thus this model provides a large overestimation of predicted displacements and changes in gravity.

[38] Figure 10 displays the results assuming a reference elevation corresponding to a “representative elevation” for the region (model B). The reference elevation should be somewhere between the summit and the mean sea level [Williams and Wadge, 2000]. Model B adds a constant value accounting for the average topography of the island to the source depth. Predicted displacements and surface gravity change overestimation is lower in this case (twice for model B versus 100 times for model A) due to the fact that $X_3$ is equal to the sum of source depth below sea level and the average height above sea level in the computations. Model C (Figure 11), which assumes a reference elevation of 3718 m corresponding to Teide volcano height, underestimates the IBEM displacement and surface gravity change for most of the station points.
Figure 12. Vertical ($u_z$) and horizontal ($u_x$, $u_y$) components of displacement field and surface gravity changes ($g_s$) caused by a center of dilatation of 50 MPa km$^2$ strength located under axisymmetrical cone that simulates Teide topography. The Teide is assumed to be a cone with a height equal to that of the volcano and with average slope of the flanks of 16.5°. Color appears in back of the print issue.

[39] The change in displacement and surface gravity change pattern caused by topography can be explained because the ground material is removed when the real relief (numerical model) instead of a flat surface is modeled. This is the reason that varying depth elevation models works well for vertical displacement [Williams and Wadge, 1998, 2000] and for surface gravity change. We can notice a considerable amplification of this pattern in the horizontal displacements (Figure 8). Therefore, while the topographic effect on vertical displacement and surface gravity changes are mainly due to the change in source depth produced by height of the station points, horizontal displacements are more sensitive to local features of the relief.

[40] A 3-D axisymmetrical cone with an approximate topography has also been studied (Figure 12). The island is assumed to be an axisymmetrical cone with a height equal to that of the Teide and with average slope of the flanks of 16.5°. The reference system chosen in this case has similar characteristics to the one we use when considering the real topography of the island. The source is located under the top of the cone. In this example, the mesh is made up of 2561 circular elements. Thus the CPU time is slightly reduced. The comparison between Figure 12 and the results obtained by using a 3-D realistic topography (Figure 8) illustrates how both are similar for the peak magnitude. However, the real topography alters the axisymmetrical pattern of displacements and gravity changes caused by point sources located under axisymmetrical volcanoes. Comparing both Figures 8 and 12, we can see that the uplift pattern is elongated through the WE direction and consequently, quasi-elliptical in shape when considering a realistic topography. The symmetry of the horizontal displacements is broken as well, appearing as a change of sign in the horizontal component $u_y$.

[41] The vertical component of deformation, when combined with the horizontal components, can be used to address whether the increased rate of surface deformation reflects movement of magma significantly closer to the
2.8 cm LOS Displacement

Figure 13. Synthetic interferograms computed from (left) model C that corresponds to a half-space with reference surface located at 3718 m height and (right) IBEM results of considering the real topography of the Tenerife island. Color appears in back of the print issue.

surface [e.g., Newman et al., 2001]. Furthermore, displacements and gravity changes must be interpreted together whenever possible because this would allow for discrimination between the role played by pressure and mass movements [e.g., Fernández et al., 2001]. Since surface deformation and gravity changes can be ascribed to a wide variety of tectonic, magmatic, hydrothermal and shallow processes, the differences between our 3-D (IBEM model) and axisymmetric results point out the necessity of including realistic topography in order to properly relate changes in gravity and deformation to magma sources at depth.

5.3. Application to Radar Interferometry

Finally, we show synthetic interferograms performed by using the IBEM model and model C described above (Figure 13). Interferometric results are typically displayed as a set of fringes, each fringe representing a displacement of half the radar wavelength in the direction of the satellite; that is, for C-band radar, a displacement of 28 cm projected onto that direction. The results of Williams and Wadge [1998, 2000] and Beauducel et al. [2004] indicate that topography can have a significant effect on both the number of fringes and the geometry of the fringes. It is therefore important to model interferograms by taking into account topographic effects. Figure 13 illustrates this fact. We can notice a geometry change when the realistic topography of Teide volcano is taken into account. Both synthetic interferograms display 2 fringes located over the Teide volcano. Thus, considering the coherence characteristics of the considering area [see, e.g., Fernández et al., 2005], such displacements should be detectable by using radar images in C band [see, e.g., Massonnet and Feigl, 1995].

6. Conclusions

[43] We have developed a 3-D numerical technique (IBEM) to model topographic effects on displacement and gravity change predicted by intruded masses and pressurized magma reservoirs. The influence of topography introduces some variations in theoretical modeling. Both the magnitude and pattern of the geodetic signals are significantly different compared to half-space solutions. We have determined the topographic effects on deformation and gravity changes created at the surface of axisymmetric and ellipsoidal base volcanoes. Topography reduce the magnitude of displacement and surface gravity change because source depth increases with the height of the relief and modifies the shape of the curves. Thus both source depth and topographic relief affect the magnitude and patterns of deformation and gravity changes. Consequently, neglecting real topography may introduce considerable errors in estimates of source depth through data interpretation.

[44] We have focused on the volcanic context because volcanic activity produces deformation and gravity changes that can be precursors of future eruptions, and volcanoes are often associated with prominent relief. A theoretical study is provided in order to estimate the topographic effects at Teide volcano (Tenerife, Canary Islands) and to test the sensitivity of the permanent GPS network installed there. We expect that changes within the magmatic system leading
to eruption will result in precursory deformation measurable by the current GPS network. The study assumes that the displacements and gravity changes are caused by the presence of a shallow magmatic system. We use IBEM to model rigorously 3-D topographic effects on deformation and changes in gravity. In view of the results, the displacements could be detected with the permanent GPS network if a hypothetical shallow intrusion should take place under the volcano summit, given the precision attainable nowadays and the MSE of the coordinates determined by using this network and Bernese 5.0 software. However, we propose some improvements in order to discriminate between different geometries and processes of the magma system considering the related volcanic hazard. It is also shown that microgravity techniques would be a suitable method for monitoring the Teide stratovolcano. The shallow intrusion we consider could produce surface gravity changes of the order of 25 μGal, and it is frequently possible to achieve some improvements in order to discriminate between different half-space and axisymmetrical approaches.

[45] Interferometric observations have become an increasingly used method for studying surface deformations due to both the availability of the data and the areal coverage provided. The synthetic interferogram produced by topographic variations shows a deformation magnitude of around 6 cm and also is detectable by using radar images.

[46] We have also studied the variations in results considering different half-space and axisymmetrical approaches. In this study, the source is located below the higher island elevation point at a very shallow depth within the medium. Depending on the analytical model we choose to interpret deformation and gravity changes, the depth of the source may be significantly over/underestimated. The results show that a proper selection of the representative elevation of the area is a critical step for matching the magnitude of uplift and surface gravity changes without attempting changes in deformation and gravity change pattern. In this particular case, it appears that the optimal reference level lies somewhere between the average elevation for the region and the Teide volcano height. The errors induce on the 3-D axisymmetrical cone solution would be of the same order as that of the analytical solution with an optimal reference elevation. Therefore, although a numerical solution should always give more realistic results, analytical methods that consider topography in an approximate way provide a simple way to interpret surface deformation and gravity changes when local topography features introduce minor errors in the solutions.

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