APPLYING THE JOINT WIGNER TIME-FREQUENCY DISTRIBUTION TO CHARACTERIZATION OF TRAIN-AVERAGE PARAMETERS INHERENT IN THE PULSED LIGHT RADIATION OF SEMICONDUCTOR HETEROLASERS.

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ABSTRACT.

We present an approach to the characterization of low-power bright picosecond optical pulses with an internal frequency modulation in both time and frequency domains in practically important cases of exploiting semiconductor heterolasers operating in near-infrared range in the active mode-locking regime. This approach uses the joint Wigner time-frequency distributions, which can be found for this regime due to involving the interferometric technique under consideration. In so doing, the modified scanning Michelson interferometers were chosen for shaping the field-strength auto-correlation functions peculiar to the above-mentioned types of infrared light radiation. We consider in principle the key features of a new experimental technique for accurate and reliable measurements of the coherence interval for continuous-wave light radiation as well as the train-average temporal width and the frequency chirp (including its value and sign) associated with picosecond optical pulses in high-repetition pulse trains. This technique is founded on an ingenious algorithm elaborated specially for the advanced optical metrology, which makes possible constructing the joint Wigner distributions for describing the parameters of light radiation. The InGaAsP/InP-heterolasers, operating at 1300 nm range, had been exploited during the experiments carried out. In particular, measuring the interval of coherence was initially performed under the continuous-wave regime of operation for heterolaser structures. These results of our experiments showed that typical auto-correlation function of the second order inherent in the continuous-wave semiconductor laser radiation had been characterized by a coherence temporal interval close to 10 ps. Then, when the optical signal consisted of contiguous pulses with the repetition frequency exceeding 1 GHz, due to operating semiconductor laser heterostructures in the active mode-locking regime, typical pulse train-average auto-correlation function had been characterized by a temporal width of about 2-20 ps. The accuracy of similar measurements increased with growth of the pulse repetition frequency due to rising in a number of samples
THE WIGNER TIME-FREQUENCY DISTRIBUTION FOR A GAUSSIAN PULSE.

The complex amplitude of one optical pulse with Gaussian shape of envelope can be written as

\[ A_c(t) = A_s \exp \left( \frac{i b t}{2 T^2} \right)^2, \]  

where \( A_s \) is the real-valued amplitude, \( T \) is the Gaussian pulse half-width measured at a level of \( 1/e \) for the intensity contour and \( b \) is the parameter of frequency modulation, i.e. the frequency chirp. In this case, the joint Wigner time-frequency distribution with \( A_s = 1 \), is given by

\[ W_c(t, \omega) = \frac{T}{\sqrt{\pi}} \exp \left( \frac{i b t}{T^2} - \frac{\omega}{T} \right) + \frac{b t}{T} \frac{\omega}{T}, \]  

(2)

When \( T=1 \) and \( b=0 \), Eq.(2) gives the distribution, which is symmetrical relative to repositioning the variables \( t \) and \( \omega \). With decreasing the parameter \( b \), the energy distribution concentrates in a bandwidth corresponding the chirp-free spectrum whose center lies along the line \( \omega = bt/T^2 \). Two examples of the time-frequency distribution \( W_c(t, \omega) = \pi^{-1/2} \exp \left( \frac{i b t}{T^2} - (\omega + b t)^2 \right) B^2 \) defined by Eq.(2) with \( T = 1 \) are presented in fig.1.

![Figure 1](image)

Figure 1. The Wigner time-frequency distributions for the Gaussian pulses with \( T = 1 \) and the varying parameter \( b \) : (a) \( b = 0 \) and (b) \( b = 2 \).

Integrations of Eq.(2) give the partial one-dimensional Wigner distributions for a Gaussian pulse over the time or frequency domains separately

\[ a) \left| A_c(t) \right|^2 = \frac{1}{\sqrt{2\pi}} \int W_c(t, \omega) d\omega = \exp \left( \frac{t^2}{2T^2} \right) \frac{1}{1+b^2}, \quad b) \left| S_c(\omega) \right|^2 = \frac{T^2}{\sqrt{1+b^2}} \exp \left( \frac{\omega^2}{2} \right) \frac{1}{1+b^2}. \]  

(3)
It is seen from Eq.(3b) that to reach a level of $1/e$ one need vary the variable $\omega$ from $-T^{-1}\sqrt{1+b^2}$ to $T^{-1}\sqrt{1+b^2}$, so that the variation $\Delta\omega = T^{-1}\sqrt{1+b^2}$ means actually the half-width of the spectral contour at a level of $e^{-1}$. In the particular case of $b = 0$ (i.e. in the absence of the frequency chirp or the phase modulation), one yields $\Delta\omega T = 1$ for the Gaussian pulse. Nevertheless, in the general case, $b \gg 1$, so that the product $\Delta\omega T$ can far exceed unity.

**EXPERIMENTAL STUDIES.**

Semiconductor lasers have a broad gain band (about $\Delta\nu \gg 10^{10}$ Hz), by this is meant that their operation in the regime of active mode-locking makes it possible to expect generating ultra-short optical pulses with a duration of about $\tau_\nu \gg 1/\Delta\nu$ lying in a picosecond time range. An external cavity was made of a single-mode silica optical fiber with the refractive index $n \gg 1.5$ and the length $L \gg 1$ meter with an additional mirror at its far end, providing the optical feedback. The corresponding feedback factor was estimated by 15% due to about 40% efficiency of exiting the light radiation in that optical fiber by semiconductor laser structure with the refractive index $n_s \gg 3.3$. The fiber cavity length $L$ corresponded to the frequency spacing about $f_\nu \gg 100$ MHz between its longitudinal optical modes because of $f = c/(2\pi)$, where $c$ is the light velocity. The scheme of our experiments is presented in Fig.2. Figures 3 illustrate profiles of light radiation spectra at the wavelength to $\lambda = 1320$ nm without any external modulation as well as with periodic RF-modulation applied the semiconductor heterolaser, i.e. in the active mode-locking regime. In the frequency domain, this estimation gives $\Delta\nu = (\Delta\lambda)c/\lambda^2 \gg 1.72$ THz.

![Figure 2. Schematic arrangement of the experimental set-up.](image-url)
A bit rugged profile inherent in the spectrum in Fig.3b is affected evidently by the presence of the laser diode cavity by itself and connected with residual reflections from the coated diode facet, which is facing the fiber cavity. Measuring the time-frequency parameters were carried out exploiting the modified interferometric technique described in ref. [1].

\[ a \]

\[ b \]

Figure 3. Radiation spectra inherent in semiconductor heterolaser operating at the wavelength \( \lambda = 1320 \) nm: (a) without an external modulation; (b) with an external sinusoidal modulation, i.e. in the active mode-locking regime.

A number of optical pulses circulating through a cavity can be estimated as \( \text{N} = \frac{2nfL}{c} \), and experimentally the cases with \( \text{N} = 1-8 \) had been successfully realized. It can be noted that the interferogram widths, measured on a level of \( 1/e \) for the intensity contour, were decreasing from 12.2 ps to 3.9 ps as the number \( \text{N} \) was growing from 1 to 8. The absolute frequency bandwidth, being available for the observation of mode-locking, was varying in the range 0.2-0.5 MHz, so that the relative frequency locking band was a little bit less than \( 10^{-3} \). Figure 4 represents the digitized interferogram of the second order auto-correlation function for a high-repetition-rate train of optical pulses; the width of this interferogram was estimated by 4.4 ps, while Fig.4b shows the digitized oscilloscope trace for a train of ultra-short pulses, which was identified as the most stable during the experiments performed. The parameter \( b \), related to the frequency chirp, was estimated with applying the above-mentioned technique by \( b \approx 1.46 \times 10^4 \). This is a train of picosecond pulses detected with the time resolution of about 300 ps, which is associated with the transfer function of a high-speed photodetector exploited. The off-duty ratio for optical pulses depicted in Fig.4b is in correspondence to the ratio between the repetition period \( 1/f \) and the above-mentioned time resolution of that high-speed photodetector.
Figure 4. The digitized oscilloscope traces related to a regular pulse train: (a) the train-average auto-correlation function; the pulse width of this interferogram, measured on a level of 1/e for the intensity contour, was estimated by 4.4 ps; (b) the output signal from a high-speed photodetector; a train of the same ultra-short optical pulses with the repetition frequency $f \gg 718$ MHz was detected with the time resolution of about 300 ps.

CHARACTERIZING OPTICAL PULSES.

Presently known mechanisms of interacting optical pulses with semiconductors allow us to simplify the theoretical model of shaping an ultra-short pulse with the complex field amplitude $E(t) = A(t) \exp(i \omega_m t) + c. c.$ in a heterostructure. The pulse, grown during the process of active mode-locking, has a Gaussian shape and can be described by Eq.(1). The pulse width, measured on a level of 1/e for the intensity contour, is given by [2]

$$T = (g m)^{1/4} (\omega_m \omega_s)^{1/2},$$  \hspace{1cm} (3)

where $g$ is the maximal gain at $t = 0$, $m$ is the factor of external modulation of the losses in a cavity, $\omega_m$ is the external modulation frequency, and $\omega_s$ is the gain contour width. Finally, the frequency chirp can be expressed as [2]

a) $b = 2 T^2 \beta$ ,  
b) $\beta = \frac{L_p \omega_m \omega_s^2 \sqrt{m}}{4 \{ g \omega_s T_C / (2 Q) \}^{1/2} \delta k \delta}$ ,  \hspace{1cm} (4)

where $\beta$ is the dimensional factor of frequency chirp, $L_p$ is the length of high-dispersion components (for example, the laser crystal), $\omega_s$ is the central frequency of emission, $Q$ is the quality factor inherent in a cavity, $T_C$ is the transit time of a pulse through a cavity, and $k$ is the wave number. In fact, Eqs.(3) and (4) can be practically used to estimate the parameters of the optical pulses generated. Using the values characteristic of the experiments: $g = 3$, $m = 0.25$, $\omega_m = 2 \pi \times 718 \times 10^6$ rad/s, and $\omega_s = 2 \pi \times 10^{13}$ rad/s, one can
obtain $T \approx 2.73$ ps from Eq.(3), which can be considered as rather good agreement with the experimental data. The frequency chirp that arises within establishing the self-reproducing pulses can be estimated with Eq.(4). For $L_\omega \approx 0.5$ mm, $\omega_\omega = 2.1 \times 10^{15}$ rad/s (at $\lambda = 1320$ nm), $T_c = 10^6$ s, $Q = 10^3$, and $(d^2k/d\omega^2) = 3.7 \times 10^{-24}$ s$^2$/m, one can obtain $\beta = 7.3 \times 10^{18}$ s$^{-2}$ from Eq.(4b). Nevertheless, this dimensional magnitude of the estimated frequency chirp is relatively small, because one can find from Eq.(4a) in dimensionless values that $b \approx 0.84 \times 10^4 \ll 1$. In practically reasonable assumption that the envelopes of optical pulses under consideration can be described rather adequately by Gaussian functions, these estimations make it possible to create the corresponding theoretical version of Wigner time-frequency distribution with the above-calculated parameters $T$ and $b$. Together with this, the experimental version of similar time-frequency distribution can be designed with experimentally obtained parameters $T \approx 2.2$ ps and $b \approx 1.46 \times 10^4$ in the same approximation by Gaussian functions. The resulting plots of two Wigner distributions for the Gaussian-like optical pulses, obtained from estimations and from experiment, are shown in Fig.5.

![Figure 5. A pair of the Wigner time-frequency distributions for the Gaussian pulses obtained from the performed estimation with $T = 2.73$ ps and the $b = 0.84 \times 10^4$ as well as from the experiment with $T = 2.2$ ps and the $b = 1.46 \times 10^4$.](image)

CONCLUSION.

A novel approach to the characterization of low-power bright picosecond optical pulses with an internal frequency modulation in both time and frequency domains in practically important cases of exploiting semiconductor heterolasers operating in near-infrared range in the active mode-locking regime has been presented. This approach is oriented to using the joint Wigner time-frequency distributions. Similar distributions can be created for this regime within exploiting the progressed interferometric briefly described above. The modified scanning Michelson interferometer has been chosen for obtaining the field-
strength auto-correlation functions. In fact, we have presented the key features of a new experimental technique for accurate and reliable measurements of the train-average temporal width and the frequency chirp of picosecond optical pulses in high-repetition rate trains. This technique makes it possible to find the parameters needed for reconstructing the joint Wigner distributions inherent in optical pulses. The InGaAsP/InP-heterolasers, operating at 1320 nm wavelength range, have been used within the experiments. When the optical signal consists of contiguous pulses with the repetition frequency close to 1 GHz, conditioned by operating semiconductor lasers in the active mode-locking regime, typical requirements for measurements and operating with the Wigner distributions have been satisfied, so that the train-average pulse parameters have been successfully characterized.

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