Choosing the best geometries for the linear characterization of lossy piezoceramics: Study of the thickness-poled shear plate

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Resonance modes of a thickness-poled piezoceramic shear plate are studied. When determining material properties from impedance measurements, it is required to obtain an uncoupled shear resonance. The shear resonance, which is electrically excited, results to a series of plate resonances, which is mechanically excited by this, when frequencies match. Thus, the condition of a high aspect ratio of the plate is not valid to eliminate coupling. Instead, the thickness (t) and lateral dimensions \((L,w)\) of the plate must be tailored for each material. This approach was tested for a commercial piezoceramic and plates amenable for characterization obtained for aspect ratios below \(L/t=15:1\) \((L=w)\). © 2008 American Institute of Physics. [DOI: 10.1063/1.2911731]

Shear modes of vibration of piezoceramics, occurring when the driving voltage is applied in a direction perpendicular to the remanent polarization, are used in a number of devices.¹ Accurate determination of shear material parameters is relevant in the design of such devices.

The thickness-poled shear plates² are an alternative to the Standard³,⁴ length poled plates in determining material properties in the linear range from impedance measurements. The dynamic clamping at resonance of the Standard plate, which leads to underestimation of the piezoelectric coefficients of the ceramics, was recently discussed.⁵,⁶ Porous ceramics⁷ are an example of materials in which the Standard plate cannot be used for characterization due to the added problem of difficult poling in the long dimension of the sample.

The clear advantages of the thickness-poled shear plates with respect to the Standard one are not yet fully exploited due to the still open issue of the proper selection of the dimensional ratios needed to get an impedance versus frequency spectrum that will correspond to a single resonator. The first authors that faced this problem,² stated that a ratio between the lateral dimension \((w,L)\) and thickness \((t)\) higher than 5:1 is enough to obtain reliable parameters. Other authors⁸ could not find satisfactory experimental results with such aspect ratio. Even for samples with \(w:t=62:1\) and \(L:t=27:1\), a coupling of the main shear resonance mode with other modes was observed. Later on, other authors⁹ showed that even for a ratio of 20:1, extraneous resonances appear close to the main shear resonance mode but with reduced influence, so that reproducible measurements can take place.

In addition to this lack of agreement among the authors, to date, the discussion about the resonance modes of thickness-poled shear plate lacks experimental data that can provide a description of the shear and the unwanted spurious resonance modes. Laser interferometry provides a powerful tool in the analysis of the resonance modes of piezoceramics.¹⁰

Piezoceramics are lossy materials and the need to express their properties in a complex form¹¹ to account for all material losses has long been accepted. Iterative and fitting methods to calculate complex coefficients (elastic, dielectric, and piezoelectric) from the analysis of the complex impedance spectrum at resonance are currently used.¹²⁻¹⁸ The only method published to date for the determination of complex shear parameters from the resonance of non-Standard, thickness-poled shear plates is the automatic iterative method of Alemany et al.¹⁹

When aiming to determine complex shear coefficients from the impedance data, the discussion about resonance modes corresponding to given dimensional ratios of the thickness-poled shear plate becomes of primary importance. Such complex coefficients cannot be determined only by the position of the resonance and antiresonance frequencies, but their determination requires the accurate knowledge of values of impedance around these frequencies¹⁴,¹⁹ thus, mode coupling is undesired.

It is the purpose of this work to study thickness-poled shear piezoceramic plates by laser interferometry and complex impedance measurements aiming to determine the criteria to reduce the coupling of modes.

Square plates of a Navy-II-type commercial Piezoelectric ceramics (PZ27 of Ferroperm Piezoceramics A/S) with \(15 \times 15\) mm² and 2.0, 1.5, and 1.0 mm thicknesses, thus with aspect ratios of 7.5:1, 10:1, and 15:1, were thickness poled to saturation after attaching silver paste electrodes at the major faces. These electrodes were removed and new ones were attached at the perpendicular surfaces for the electrical measurements in a HP4192A low frequency impedance analyzer and the laser interferometric measurements, as explained elsewhere.⁵

Conductance \((G)\) and resistance \((R)\) peaks derived from the complex impedance measurements of the fundamental shear resonance of the three samples are shown in Fig. 1. The main peaks are always accompanied with lower peaks corresponding to additional resonance modes, whose nature is outside the scope of this work.

The automatic iterative method at Alemany et al. was
used to directly calculate the complex coefficients involved in the shear mode of resonance and, from these, the spectra were recalculated using the one-dimensional analytical solution of the wave equation for the shear resonance mode of motion when the dimensional ratio given by \( w \gg t \) is fulfilled.

\[
Y = G + iB = i \frac{2 \pi f i L e_1^i}{w} + \frac{2 \pi e_1^2}{w} \sqrt{\frac{E}{\rho}} \tan(\pi f L \sqrt{\rho E}),
\]

\( \rho \)

The regression factors \( R^2 \) of the recalculated to the experimental spectra for the samples are 0.920, 0.828, and 0.959 as the aspect ratio increases. These values give a quantitative measure on how close the measured spectrum is to the single resonator one, the higher the \( R^2 \) value, the closer.

Figure 1 also shows the out-of-plane displacement scans obtained by laser interferometry for the largest surfaces of the samples and at the frequencies of the maximum conductance \( G \) for each sample: 430, 560, and 869 kHz, respectively. For the interpretation of the main features of these patterns, finite element modeling was carried out, as explained elsewhere. The modeled patterns for the modes of movement were compared to the experimental ones.

Figure 1(b) shows an interference pattern. Two standing plate waves, which are perpendicular to each other, are excited for this frequency in addition to the shear movement, which is not revealed by this technique. The pattern obtained for 420 kHz, which is the frequency of the secondary maximum of the \( G \) curve of this sample [Fig. 1(a)], is quite similar.

Figure 1(d) shows ten wide bands parallel to the electrodes—in and out of phase maxima of displacement—separated by nine lines, which are the nodes, of an uncoupled shear wave of five full wavelengths. The displacement perpendicular to the scanned surface measured here is as an average smaller than that in Fig. 1(b) since for this mode, we are only detecting the normal to the surface component of the shear movement here. As soon as we move up from the frequency of the peak of the \( G \) curve, other modes are also excited and interfere with the shear one, resulting to a blurred scan at 590 kHz, which is the frequency of the secondary maximum [Fig. 1(e)].

Figure 1(f), which shows the maximum surface displacements, is consistent with the previous result and shows 16 bands with maximum displacements separated by 15 nodes, corresponding to eight full wavelengths. The bands are cut into rows of islands by a standing wave between the two nonelectroded minor surfaces. These features of the scans in Figs. 1(d) and 1(f) follow reasonably well the ratio between the thicknesses of the two samples, indicating that for both samples, this is a thickness driven shear mode of motion.

Even for the highest aspect ratio (15:1), mode coupling is found, which means that inherent to this resonator geometry, the thickness of the sample is simultaneously driving two types of modes of resonance. In order to test this, the 7.5:1 aspect ratio sample was reduced from 2.00 to 1.00 mm thickness in steps of 0.02 mm, smoothly varying the ratio from 7.5:1 to 15:1. The impedance spectra were recorded at each step and the automatic iterative method of Alemany et al. was applied to determine the complex coefficients that allow the reproduction of the spectra and the calculation of the \( R^2 \) factor. This is plotted in Fig. 2(a), which clearly shows the cyclic nature of the phenomena with the aspect ratio variation. Figures 2(b) and 2(c) show the evolution of the \( G \) and \( R \) peaks, respectively, in one of the cycles, corresponding...
Summarizing, we must speak about the optimum length and thickness of the shear plate for each material to obtain uncoupled modes. The minimum length should be the one where the distance between two plate resonances is the same as the distance between the maximum of the conductance and the maximum of the resistance for the thickness-shear resonance. Such distance is determined by the shear electromechanical coupling factor of the material. Optimum thicknesses are the ones that place both maxima between given overtones of the plate resonances.

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FIG. 2. (Color online) (a) Regression factor of the recalculated spectra to the experimental one in non-Standard thickness-poled shear plates (15×15 mm²) with aspect ratios from 7.5:1 to 15:1; evolution (b) of the G curve and (c) of the R curve for a full cycle between two spectra with a low mode coupling (corresponding $R^2$ values are marked in Fig. 2(a)) obtained for samples with aspect ratios between 11.2:1 and 13.2:1.