Voter heterogeneity and political corruption[*]

Enriqueta Aragonès

Institut d'Anàlisi Econòmica-CSIC and Barcelona GSE[†]

Javier Rivas

University of Bath[‡]

Áron Tóth

University of Bath[§]

6th November 2019

Abstract

We show that policies that eliminate corruption can depart from socially desirable policies and this inefficiency can be large enough to allow corruption to live on. Political competition between an honest (welfare maximiser) and corrupt politicians is studied. In our model the corrupt politician is at a distinct disadvantage: there is no asymmetric information, no voter bias and voters are fully rational. Yet, corruption cannot be eliminated when voters have heterogeneous preferences.

JEL: D72; Keywords: Political Corruption, Political Competition, Voting

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[†]IAE-CSIC, Campus UAB, 08193 Bellaterra (Spain); enriqueta.aragones@gmail.com; http://aragones.iae-csic.org/

[‡]University of Bath, Department of Economics, Claverton Down, Bath, BA2 7AY, UK; j.rivas@bath.ac.uk; http://people.bath.ac.uk/fjrr20/

[§]University of Bath, Department of Economics, Claverton Down, Bath, BA2 7AY, UK; a.toth@bath.ac.uk; http://www.bath.ac.uk/economics/staff/aron-toth/

1 Introduction

Corruption is the misuse of public office for private gains. It leads to the misallocation of talent, technology and capital and thereby hinders economic growth (e.g. Mauro 1995). The problem of corruption is enormous and its cost is estimated to be around 5% of the world GDP (UN, 2008).[1] Yet, so far the Political Economy literature has only partially uncovered its driving forces and some puzzling questions remain unanswered.

Why does corruption persist in many democracies in the developing world? Why does political competition often fail to eliminate corruption, why do voters not elect politicians who are not corrupt? In previous models, corruption often emerges through the introduction of voter bias (e.g. Dixit and Londregan (1996), Myerson (1993, 2006), Besley and Coate (1997), Pani (2011)) or asymmetric information and poor institutions (e.g. Ferejohn 1986, Tirole 1996, Persson et al 1997, Caselli and Morelli 2004, Besley and Smart 2007, and Besley 2006 for a survey and some independent results, Schwabe 2011). These modelling features naturally allow an opportunistic politician to extract rents. We study an environment where these features are absent, because we think they often appear insufficient to explain the observed cross-country variation of corruption.

Exogenous voter bias is meant to capture some dimension of social division (e.g. ideological, ethnic, religious, cultural, etc). However, social demarcation is a feature of most countries and therefore it is difficult to see how this could explain the different levels of corruption across countries. For instance, consider ethnic division, arguably one of the most pronounced and rigid social demarcations: while it is prevalent in many countries where corruption is rampant (e.g. India, Nigeria), it is also present in countries where corruption is barely detectable (e.g. Belgium, Canada or Switzerland are well-known examples from the developed world).[2] Moreover, poorly informed voters too (i.e. asymmetric information) could only provide partial explanation to the existence of corruption: there is ample evidence that in countries where corruption is virulent voters often elect / re-elect politicians who are known to be corrupt and criminal (Kurer 2001, Manzetti and Wilson 2007, Aidt et al 2011, Banerjee et al 2012).[3] Indeed, "unpopular corruption and popular corrupt politicians" is a widely observed paradox (Kurer 2001, pp. 63).

We show that as long as voter preferences are heterogeneous, corruption cannot be entirely eliminated. Furthermore, if income inequality is sufficiently high, the corrupt politician can win the majority of votes. These results hold despite the fact that we analyse a framework where the classic drivers of corruption discussed above (i.e. voter bias and asymmetric information) are absent. We analyse political competition between an honest (welfare maximising) and a corrupt politicians in a model of proportional representation *in government*,

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[1] Although widely cited in World Bank, IMF, OECD documents and by others, the accuracy of this figure has been questioned (see more on Matthew Stephenson's Global Anticorruption blog: https://globalanticorruptionblog.com/2016/01/05/its-time-to-abandon-the-2-6-trillion5-of-global-gdp-corruption-cost-estimate/).

[2] Perhaps more importantly, even when a society is deeply divided and hence voter bias plays a major role in elections, it is difficult to explain why political competition within the social (e.g. ethnic) group does not eliminate corruption: in principle, each group should be able to offer to voters a non-corrupt alternative with the same political platform.

[3] For instance, in India, a fourth of the members of the previous lower house of the national parliament faced pending criminal charges (Dutta and Gupta 2012, Chemin 2011).

where politicians share power and implement policies according to their vote shares.[4] The reason for our modelling choice of governing is twofold. First, in many cases the losing party can influence policy (for instance, in legislative-executive bargaining (USA), as part of the governing coalition, via supporting a minority government or supporting laws requiring supermajority). The extent and success of this influence is usually in proportion of the losing party's electoral support, so we believe our model describes most parliamentary systems well. Second, we wanted to avoid the workhorse model of probabilistic voting. Probabilistic voting models introduce exogenous popularity or voter bias (or both), which lends "market power" to corrupt politicians, and this naturally allows for rent extraction. Because our paper focuses on why corruption exists, we found the framework that avoids these biases more compelling.

In this paper, we show that corruption can never be eliminated as long as voter preferences are heterogeneous. There are two crucial driving forces behind our results. First, it is not always in the interest of the welfare maximising honest politician to fight corruption. In order to sway voters from the corrupt politician, the honest politician needs to offer a platform which matches the corrupt politician's offer and thereby deviate from the socially desirable (first best) political platform. The gain from eliminating corruption does not always justify the cost of choosing a socially suboptimal political platform. Second, even when the honest politician would prefer eliminating corruption, she cannot possibly please everyone and therefore in equilibrium there are always voters who vote for the corrupt politician. Interestingly, in both of these cases the corrupt politician can even win the majority: because the honest politician cares about voter welfare, she wants to adopt the preferred policies of the average voter and when the income distribution is sufficiently skewed (inequality is high enough and thus the minority (the rich) has a disproportional effect on total welfare), the honest politician will have a minority vote share.

While our model is static, we believe the mechanism we uncover could help explain the persistence of corruption over time. The history of fuel price subsidies is a case in point. For instance, in Nigeria fuel subsidies were introduced in the 1980s with the objective to help the poor. While fuel subsidies were ineffective to alleviate poverty, they had disastrous effect on the economy. The direct cost of these fuel subsidies has been large: the World Bank estimated that it varied between 1-5% of the GDP (depending on oil prices) and accounted for 10-25% of the federal budget (IMF 2013, Siddig et al 2015). But the real burden is the indirect cost: fuel subsidies were the major source of corruption, which cost 22% of GDP in 2014 and it was estimated that this cost could rise up to 37% of GDP by 2030 (PWC 2016). Yet, the subsidies were popular among the poor and over the last 30 years numerous governments made a number of attempts to remove them without success. This example highlights one of the major mechanisms in our model: a well-meaning government can be forced to implement (or preserve) suboptimal policies that may foster corruption in order to reduce the voter base and limit the power of corrupt politicians.

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[4]There are many articles where the implemented policy is the result of some compromise or power sharing. For instance, in Alesina and Rosenthal (2000) two parties reach a compromise on policy as a result of executive-legislative interaction, as is often the case in the USA. In coalitional bargaining games (e.g. Austen-Smith 2000, Baron and Diermeier 2001), one party is often selected to propose a policy with probability equal to vote share (for empirical evidence, see Diermeier and Merlo 2004). We adopt a simple reduced form of policy implementation similarly to e.g. Grossman and Helpman 1999, Saporiti 2014, Herrera et al 2014, Xefteris and Ziros 2017 to capture the idea that in practice the winner does not take it all.

2 Literature review

In many earlier studies corruption emerges in equilibrium because voters are assumed to have a bias towards particular candidates (see Persson and Tabellini 2000 for a detailed survey).[5] Much of this literature builds on the seminal article of Dixit and Londregan (1996) which identifies the basic trade-off between voters' affinity for politicians and economic benefits: a politician can afford to deliver less benefit to voters (e.g. shirk, be less smart or more corrupt) when voters are biased toward her. Higher levels of voter bias naturally results in more corruption and in the absence of a bias corruption disappears.[6] In seminal works, Myerson (1993, 2006) analyses the effectiveness of different electoral systems in preventing parties with known corruption levels to win legislative seats, when parties exogenously belong to two ideological camps (differentiated by a policy question). Myerson focuses on strategic voting and shows that the least corrupt party may not be selected due to a coordination failure among voters. Some studies use so-called citizen-candidate models, where a citizen, if elected, is not bound by electoral promises and implements his own preference as policy. A voter prefers the candidate whose preference is closer to hers, i.e. voters again have an exogenous bias for political candidates. In this setup, voters may vote for a less competent (e.g. Besley and Coate 1997) or corrupt (Pani 2011) politician despite their intrinsic preferences if politicians are unable to commit to the policy preferred by the majority. In our model, politicians can commit to their electoral promises, there is no voter bias, so our voters make their voting decisions based only on politicians' offers.

Other studies focus on the importance of institutional framework. For example, Persson et al 1997 argue that without checks and balances political constitutions are incomplete contracts and hence they leave scope for corruption. Following on this literature, Acemoglu et al 2013 argue that corruption can persist, because voters may dismantle checks and balances when checks and balances also allow the elite to influence politicians by non-electoral means. In contrast, in our model institutions play no role, voters accept some level of corruption because the corrupt politician can appeal to some voters with a proposal that the honest politician does not rationally want to match.

Another strand of literature introduced asymmetric information between voters and politicians, which naturally allows the politician to extract private rents (e.g. Ferejohn 1986, Tirole 1996, Persson et al 1997, Caselli and Morelli 2004, Besley and Smart 2007, and Besley 2006 for a survey and some independent results). These models suggest that low income may foster corruption because the asymmetric information problem is more severe due to the facts that poor people tend to be less educated and also have more limited access to information about candidates. However, the models of these studies are unsuitable to investigate why corruption exists when voters know (as opposed to know something about) the politicians.[7]

Previous work more related to our paper include papers where voters knowingly vote

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[5]There are two major strands of the literature, one on corrupt bureaucrats and one on politicians. The models on corrupt bureaucrats is very different from that of corrupt politicians, because bureaucrats are not elected (for a survey see e.g. Banerjee et al (2012)). In what follows, we focus on corrupt politicians.

[6]The studies of voter bias (e.g. ideological, ethnic or popularity) are akin to models of oligopolies where firms extract rents through their market power.

[7]In Caselli and Morelli (2004) corruption may persist even under perfect information because high quality candidates, whose opportunity cost of being a politician is higher, do not run for office if the payoffs from office are sufficiently low.

for corrupt politicians. For example, in Aghion et al 2010 voters willingly choose corrupt political intervention in countries where trust is scarce. The reason is that voters prefer to have an institution in place that allows for economic activities in the absence of trust, even if this institution extracts rents from voters. The setup of their paper is different to ours because we assume that voters can choose to elect non-corrupt politicians. Also related in this literature is Evrenk (2011), who considers a model with voter bias where a clean and a corrupt politician compete in an election and as part of their manifesto they can choose to eliminate corruption. In his study, although politicians could eradicate corrupt institutions, both the corrupt and non-corrupt politician have an incentive to preserve corrupt institutions, because these institutions provide rents to the corrupt, but also a competitive advantage to the clean politician. However, in Evrenk (2011), the clean politician is not welfare maximising as the honest politician is in our study.

3   The environment

There are two politicians competing in an election: an honest politician denoted by h (henceforth referred to in the feminine form) who maximises voters' welfare and an opportunistic (corrupt) politician denoted by c (henceforth referred to in the masculine form) who maximises his own rent. There is no asymmetric information, voters know which politician is honest and which one is corrupt. Political competition can be characterised by two political dimensions b and G. We think of these two dimensions as two different public goods.

Let I denote the set of voters with cardinality $N = |I|$. Voters, who are indexed by i, can have two income levels $y \in \{0, \overline{y}\}$: $\alpha \in (0, 1)$ portion of the population is poor with income normalised to 0 and $1 - \alpha$ portion is rich with income $\overline{y} \in (1, \infty)$. To simplify the framework, we assume that the tax rate is 100%, so all income is collected and to be redistributed and/or invested in public good(s) by the state. This means that the budget is equal to $N\mu$, where $\mu = (1 - \alpha)\overline{y}$ is the average income. Importantly, voters are heterogeneous in their preferences over b and G. In particular, a voter with income y has utility $b + Gy$, where b and G represent the average spendings on the two public goods.[8] This utility function implies that rich voters prefer G over b, while the poor would rather have the state spend on b than on G.[9] Prior to election, the corrupt and the honest politicians make promises (b_c, G_c) and (b_h, G_h), respectively, where, for instance, b_c is the per voter spending on public good b promised by the corrupt politician. After election politicians receive a portion of the budget and consequently make good on their promises according to their power, i.e. equilibrium

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[8]A public good is both non-excludable and non-rivalrous. Most goods that governments provide, however, are rivalrous to some extent: for instance, the individual utility of a bridge is affected by congestions and larger population requires larger police force to provide the same level of protection. Therefore, to us it seems more appropriate to define individual utilities as a function of average, rather than total, spending. Note that if we were to adopt the notion of a pure public good, we would only need to multiply the individual utilities by N (so $Nb + NGy$), which would simply lead to our welfare function (see equation (2) below) being multiplied by N too. Clearly, this would not affect any of our results.

[9]One may wonder why income enters the voter's utility function when the tax rate is 100%. The reader can think of the 100% tax rate as a simplification of the concept of very high redistribution, where not all income is collected. Alternatively, income can also be substituted with wealth in the utility function. What matters for the analysis that follows is voter heterogeneity, the source of heterogeneity is of secondary importance.

vote shares.[10] That is, if the corrupt politician has a vote share $s \in [0,1]$ in equilibrium, then he receives s portion of the budget (i.e. $sN\mu$) and delivers only s portion of his total promises (i.e. sNb_c and sNG_c).[11] We do not allow for debt and hence a politician cannot promise more than the budget: $\mu \geq b_p + G_p$, $p \in \{c, h\}$. Thus, a voter with income y receives in equilibrium total utility $sb_c + (1 - s) b_h + (sG_c + (1 - s) G_h) y$.[12] If n_h and n_c other voters vote for the honest and corrupt politicians respectively (so $N = n_h + n_c + 1$), then a voter with income y votes for the corrupt politician if and only if

$$\frac{n_c + 1}{N} b_c + \frac{n_h}{N} b_h + \left(\frac{n_c + 1}{N} G_c + \frac{n_h}{N} G_h \right) y > \frac{n_c}{N} b_c + \frac{n_h + 1}{N} b_h + \left(\frac{n_c}{N} G_c + \frac{n_h + 1}{N} G_h \right) y \quad (1)$$

Simple rearrangement yields that a fully rational voter will always vote for the corrupt politician if he offers more: $b_c + G_c y > b_h + G_h y$. The intuition behind this result is simple: regardless of how other voters vote, it always pays to vote for the offer that the voter prefers, because by doing so the voter can always (marginally) increase the weight on this offer, which increases his utility in equilibrium.[13]

In what follows, we focus on the case where the average income $\mu > 1$ and hence the average voter will prefer G to b.[14] We assume that if voters are indifferent between the corrupt and honest politician, they then favour the honest politician, so the honest politician

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[10]Consider for instance the 2017 UK general election as a recent example of a party gaining access to the portion of the budget despite its minority share of votes: the Democratic Unionist Party won only 10 out of the 650 seats in the parliament and yet secured an extra £1bn from the budget for Northern Ireland for their support of the Conservative government (Northern Ireland had already received more than a fifth more public spending per head than the UK-wide average). In general, there are many reasons why a winning party would like to strike a deal with the (minority) opposition: laws requiring supermajority, rebels in the winning party, thin majority, or simply increasing legitimacy (e.g. in 1994 in Hungary while the post-communist socalist party (MSZP) won a comfortable majority with 54% of seats, it decided to form a coalition) to name a few.

[11]There is considerable evidence that in practice politicians keep their promises, see e.g. Thomson et al (2018).

[12]In principle, one could introduce uncertainty over voter preferences and recast our model of proportional representation in a majority voting system (probabilistic voting), where the probability of winning would be a function of politicians' strategies. This model would be akin to our proportional representation model with continuous income distribution, where vote shares depend on strategies. See more on the model with continuous income distribution in the Discussion section.

[13]Similarly to our model, in Alesina and Tabellini (1990) voters are heterogeneous and assign different weights to two public goods: in their model voter i's utility depend on $\gamma (i) b + (1 - \gamma (i)) G$, where $0 \leq \gamma (i) \leq 1$. Note that assuming $\gamma (y (i)) = 1/1 + y (i)$, our voters' utility depend on the two public goods in the same way as in Alesina and Tabellini (1990).

[14]$\mu > 1$ implies that the honest politician will prefer to spend the entire budget on G. It is, however, immaterial in our analysis whether the honest politician prefers a policy that favours the rich or the poor. What is important is that the honest politician has a clear policy preference and the corrupt politician (who doesn't) exploits this. We believe our choice of setup is a good representation of reality. First, the rich is often thought to be more productive than the poor (e.g. through their ownership of productive assets, scale of assets (e.g. land ownership), access to better education and healthcare, greater gender inequality among the poor etc). Therefore, a politician solely concerned with average utility (and thus productivity by implication) will want to invest public funds according to the preferences of the rich (for instance, by building motorways). Second, corrupt politicians often target the poor in practice because they are more numerous, easier to please and more responsive to vote buying. Third, as Pande (2008) points out, the poor often prefers candidates from their own rank, and poor politicians tend to be more susceptible to grafts. Lastly, the current setup closely corraborates our example of fuel subsidies in Nigeria, where the arguably optimal policy is to abolish fuel subsidies, despite the poor insisting on it.

only needs to match the offer of the corrupt politician to win all the votes. We formally define the objective functions of the politicians below: average voter welfare is the utility function of the honest politician and per voter corruption is the profit function of the corrupt politician, where corruption is equal to the budget allocated to the corrupt politician minus his spending on public goods, respectively:

$$
\begin{aligned}
W^V \left(b_h, G_h; b_c, G_c \right) &= \left(1 - s \left(b_h, G_h, b_c, G_c \right) \right) b_h + s \left(b_h, G_h, b_c, G_c \right) b_c \\
&\quad + \left(\left(1 - s \left(b_h, G_h, b_c, G_c \right) \right) G_h + s \left(b_h, G_h, b_c, G_c \right) G_c \right) \mu \quad (2) \\
\pi^c \left(b_c, G_c; b_h, G_h \right) &= s \left(b_h, G_h, b_c, G_c \right) \left(\mu - b_c - G_c \right) \quad (3)
\end{aligned}
$$

To summarise the timing of the game: first, politicians make their offers (b_c, G_c) and (b_h, G_h); second, voters vote for the politicians; third, voter incomes are taxed 100% and the budget $N\mu$ is divided between the corrupt and honest politicians according to their vote shares (i.e. $sN\mu$ and $(1-s) N\mu$, respectively); fourth, the corrupt and honest politicians make good on their promises in proportion of their vote shares, i.e. they deliver (sNb_c, sNG_c) and $((1-s) Nb_h, (1-s) NG_h)$, respectively.

Politicians can make the following promises: $b \in \left\{ 0, \underline{b}, \overline{b}, \mu \right\}$ and $G \in \left\{ 0, \mu - \overline{b}, \mu - \underline{b}, \mu \right\}$, where $0 < \underline{b} < \overline{b} < \mu$.[15] The set of G is defined such a way that no matter which b the honest politician chooses, she is able to exhaust her budget, i.e. spend the rest of her budget on G. We believe this simple discrete strategy space captures the most essential features of political competition, because political campaigns and the resulting budget commitments are organised around priorities and the nature of the public good often exogenously determines the possible levels of spendings.[16] One could think of our discrete strategy space as in a politician's manifesto a public good (e.g. b) may have 'no importance' ($b = 0$), 'low priority' ($b = \underline{b}$), 'high priority' ($b = \overline{b}$), or 'the highest priority possible' ($b = \mu$). See more on the strategy space in the Discussion section.

Therefore, we analyse a normal form game in which the set of available actions to both politicians is $S = \left\{ (b, G) \in \left\{ 0, \underline{b}, \overline{b}, \mu \right\} \times \left\{ 0, \mu - \overline{b}, \mu - \underline{b}, \mu \right\} | b + G \leq \mu \right\}$.[17] The game is formally defined as follows

$$
\begin{aligned}
\Gamma = &\langle I \cup \{c, h\}, \{\cup_{i \in I} \{c, h\}, S, S\}, \\
&\{\times_{i \in I} \{\times_{i \in I} \{c, h\} \times S \times S \to \mathbb{R}\}, \times_{i \in I} \{c, h\} \times S \times S \to \mathbb{R}, \times_{i \in I} \{c, h\} \times S \times S \to \mathbb{R}\} \rangle
\end{aligned}
$$

We make the following assumption that we maintain throughout the paper.

Assumption 1. *The difference between political platforms $\underline{b}$ and $\overline{b}$ is sufficiently large: in particular, $\overline{b} \left(\overline{y} - 1 \right) / \overline{y} > \underline{b}$.*

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[15] For other papers with discrete strategy space, see e.g. Aragones and Thomas (2000) or Chen and Eraslan (2017).

[16] For instance, in the 2015 UK electoral race, there were three possible levels of one of the key budgetary commitments, the renewal of Britain's only nuclear deterrent weapon system, the four Vanguard-class submarines armed with Trident II missiles: no renewal was represented by SNP, UKIP, and the Green Party; partial renewal (i.e. three submarines) by Labour and Liberal Democrats; and full renewal (i.e. four submarines) was promised by the Conservatives.

[17] That is, $S = \left\{ (0, 0), \left(0, \mu - \overline{b} \right), \left(0, \mu - \underline{b} \right), (0, \mu), \left(\underline{b}, 0 \right), \left(\underline{b}, \mu - \overline{b} \right), \left(\underline{b}, \mu - \underline{b} \right), \left(\overline{b}, 0 \right), \left(\overline{b}, \mu - \overline{b} \right), (\mu, 0) \right\}$.

We show in the proof of Proposition 1 in the Appendix that the 10x10 game Γ can be reduced to a 3x3 game (see Figure 1). If Assumption 1 holds, then the reduction is through the elimination of strictly dominated strategies, so the equilibria that we identify and analyse in the 3x3 game are in fact unique in their respective parameter regions. However, if Assumption 1 does not hold, then some of the strategies that are eliminated would only be weakly dominated.

In what follows, we investigate three possible parameter regions:[18] $\tilde{b} < \underline{b} < \overline{b}$, $\underline{b} < \tilde{b} < \overline{b}$, and $\underline{b} < \overline{b} < \tilde{b}$, where

$$\tilde{b} \equiv \frac{\alpha \mu^2}{\mu - 1 + \alpha} = \frac{\alpha(1-\alpha)\bar{y}^2}{\bar{y} - 1} \tag{4}$$

It is easy to see that $\tilde{b}$ is always $0 < \tilde{b} < \mu$ when $\mu > 1$ and thus it divides the interval $[0, \mu]$ into two segments. The rationale for distinguishing these two segments is that $\tilde{b}$ is the trigger value for the honest politician to act against the corrupt politician. In particular, if the corrupt politician plays any action (b_c, G_c) such that $b_c > \tilde{b}$, then the honest politician does not try to match this offer, i.e. she does not try to challenge the corrupt politician and finds it optimal to play the socially desirable strategy $b_h = 0, G_h = \mu$. We summarise this observation in the following Lemma:

Lemma 1. *If the corrupt politician plays an action (b_c, G_c) such that $b_c > \tilde{b}$, then the best response of the honest politician is not to match this offer, i.e. the honest politician does not try to challenge the corrupt politician and plays $(0, \mu)$.*

To see why this is true, observe that in the absence of a rival, when $\mu > 1$ the honest politician would prefer in principle to spend the entire budget on G (i.e. she would set $b = 0$ and $G = \mu$), and thus would take a political platform which the rich prefer. This is because average welfare $b + \mu(\mu - b)$ is maximised at $b = 0$ when $\mu > 1$. The best response of the honest politician to the corrupt politician's strategy $(b_c, 0)$ is then[19]

$$BR_h(b_c, 0) = \begin{cases} (b_c, \mu - b_c) & for \quad 0 < b_c < \tilde{b} \\ (0, \mu) & for \quad \tilde{b} \le b_c \end{cases}$$

This is because, on the one hand, if the honest politician matches the offer of the corrupt politician, she wins all the votes and thus average welfare (her profit) is equal to $b_c + \mu(\mu - b_c)$. On the other hand, when the honest politician does not try to match the offer of the corrupt politician and consequently wins only the votes of the rich, average welfare is $\alpha b_c + (1-\alpha)\mu^2$. Clearly, she pursues the latter strategy as long as $b_c + \mu(\mu - b_c) \le \alpha b_c + (1-\alpha)\mu^2$ or

$$b_c \ge \frac{\alpha \mu^2}{\mu - 1 + \alpha} = \frac{\alpha(1-\alpha)\bar{y}^2}{\bar{y} - 1} \equiv \tilde{b}$$

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[18]We analysed the more general setting of continuous strategy space (i.e. when $b, G \in [0, \mu]$) with continuous income distribution and in that setup some of the basic features were the same as in our model: there was a pure strategy equilibrium and a region where only mixed strategy equilibrium existed, and we could also show that in both types of equilibria corruption was strictly positive. However, we could not characterise the mixed strategy equilibrium in this more general setup.

[19]Note that it is sufficient to investigate the honest politician's best response to the corrupt action $(b_c, 0)$ where $b_c > \tilde{b}$, because if the best response to this action is $(0, \mu)$, then $(0, \mu)$ remains the best response to all actions (b_c, G_c), where $b_c > \tilde{b}$ and $G_c > 0$.

		Honest		
		$(0, \mu)$	**$(\underline{b}, \mu - \underline{b})$**	$\left(\overline{b}, \mu - \overline{b}\right)$
Corrupt	**$(\underline{b}, 0)$**	$\alpha\left(\mu - \underline{b}\right),$ $\alpha\underline{b} + (1 - \alpha)\mu^2$	$0,$ $\underline{b} + (\mu - \underline{b})\mu$	$0,$ $\overline{b} + \left(\mu - \overline{b}\right)\mu$
	$(\overline{b}, 0)$	$\alpha\left(\mu - \overline{b}\right),$ $\alpha\overline{b} + (1 - \alpha)\mu^2$	$\alpha\left(\mu - \overline{b}\right),$ $\alpha\overline{b} + (1 - \alpha)\underline{b} +$ $(1 - \alpha)(\mu - \underline{b})\mu$	$0,$ $\overline{b} + \left(\mu - \overline{b}\right)\mu$
	$(0, \mu - \underline{b})$	$0,$ μ^2	$0,$ $\underline{b} + (\mu - \underline{b})\mu$	$(1 - \alpha)\underline{b},$ $\alpha\left(\overline{b} + \left(\mu - \overline{b}\right)\mu\right) +$ $(1 - \alpha)(\mu - \underline{b})\mu$

Figure 1: Reduced Game: actions constitute support of 3x3 MSE (in bold: 2x2 MSE)

4   Results

We show in the Appendix that the 10x10 game Γ can be reduced to a 3x3 game through the elimination of strictly dominated strategies (Reduced Game, see Figure 1). That is, in any equilibrium, each politician may use only three actions. In particular, the corrupt politician does not want to waste resources to please the rich ($G_c = 0$) when targeting the poor and may offer a high or low b to win the votes of the poor, playing $\left(\overline{b}, 0\right)$ or $(\underline{b}, 0)$. Furthermore, the corrupt politician may target the rich by playing $(0, \mu - \underline{b})$. The honest politician will naturally play the socially desirable platform $(0, \mu)$ and in order to sway poor voters she may also play two other actions that match the corrupt politician's high or low offer of b, while spending the rest of the budget on G (i.e. $\left(\overline{b}, \mu - \overline{b}\right)$ or $(\underline{b}, \mu - \underline{b})$, respectively).

We identify three possible equilibria: a pure strategy equilibrium, and two mixed strategy equilibria where the politicians mix over two or three strategies. In what follows, we call these mixed strategy equilibria 2x2 and 3x3 mixed strategy equilibrium, respectively. The three equilibria correspond to three different parameter regions (see Figure 2) As discussed above, we distinguish three possible scenarios along possible values of $\underline{b}$ and $\overline{b}$ and show that the type of equilibria depends on these parameter values.

Case when $\tilde{b} < \underline{b} < \overline{b}$: Pure Strategy Equilibrium

In Lemma 1 we established that for all (b_c, G_c) such that $b_c > \tilde{b}$ the honest politician's best response is $(0, \mu)$. The best response of the corrupt politician to the honest action $(0, \mu)$ is to offer $G_c = 0$ (because the honest politician's offer to the rich cannot be outbid) and offer the lowest possible strictly positive amount on b, which is $\underline{b} > \tilde{b}$, because any $b_c > 0$ would win over the poor. It then follows that the equilibrium in pure strategies is simply $b_c = \underline{b}, G_c = 0$ and $b_h = 0, G_h = \mu$.

Pure strategy equilibrium exists when $\tilde{b} < \underline{b} < \overline{b}$, because the lowest possible b_c that still
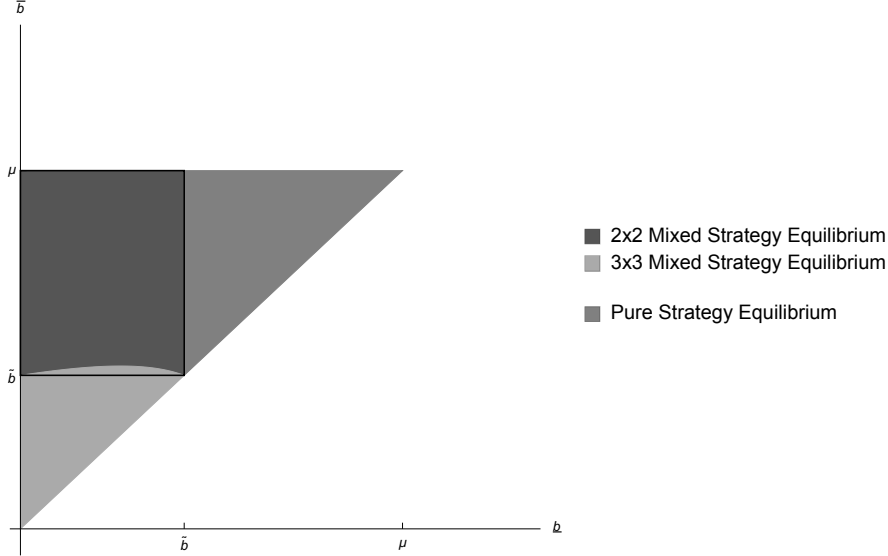
Figure 2: Parameter regions for equilibria, black rectangle: $\underline{b} < \tilde{b} < \overline{b}$ ($\alpha = 0.2, \mu = 1.5$)

yields positive profit for the corrupt politician is higher than the trigger value $\tilde{b}$. That is, $b_c = \underline{b} > \tilde{b}$ means that corruption is not too large and thus defeating corruption (i.e. matching the corrupt politician's offer and winning all votes) results in relatively little welfare gain compared to the welfare loss, which stems from deviating from the socially desirable political platform (i.e. $b_h = 0, G_h = \mu$).

It is instructive at this stage to think through why this pure strategy equilibrium exists. Consider the pure strategy equilibrium above. In general, why cannot the honest politician deviate and offer the equilibrium payoff to voters plus a bit more and get all the votes? That is, consider the offer $\hat{b}_h = sb_c + (1-s) b_h + \varepsilon, \hat{G}_h = sG_c + (1-s) G_h$, where s is the vote share of the corrupt politician and (b_c, G_c) and (b_h, G_h) are the politicians' original offers in the equilibrium outlined above. In this case all voters would be strictly better off if they voted for the honest politician and ε would be financed by the money that the corrupt politician would have stolen otherwise. However, this deviation would not be profitable: if the honest politician were to make this offer poor voters would still vote for the corrupt politician. Recall that all voters rationally choose the politician whose offer is better, simply because by voting for (and thereby increasing the weight on) the better offer, they increase their utility (see relation (1)). Indeed, when $b_c > b_h$, then $b_c > \hat{b} = sb_c + (1-s) b_h + \varepsilon$ for ε sufficiently small, so this offer would not sway poor voters. Poor voters would vote for the honest politician if and only if her offer of b is equal to (or higher than) b_c, because only then it would make sense for poor voters to put more weight on (i.e. vote for) the honest politician's offer. This offer, however, would not be optimal from a welfare perspective as we have shown above, hence the existence of the pure strategy equilibrium.[20]

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[20]In other words, when $b_c > b_h$ while it is true that all voters would be better off by voting for the honest politician who offers $(\hat{b}_h, \hat{G}_h)$, it is also true that given the offer $(\hat{b}_h, \hat{G}_h)$ poor voters are even more better off by voting for the corrupt politician. Given this voter behaviour the deviation $(\hat{b}_h, \hat{G}_h)$ does not pay.

Case when $\underline{b} < \tilde{b} < \bar{b}$: a 2x2 and 3x3 Mixed Strategy Equilibrium

It also follows from the above discussion that when there is a political platform $\underline{b} \in (0, \tilde{b})$, then there is no equilibrium in pure strategies. To see this, consider the following: when $b_h = 0, G_h = \mu$, the corrupt politician can win the poor votes by any offer $b > 0$. Naturally, he wants to maximise his rents and thus offers the lowest $b > 0$, which is $\underline{b}$ and plays $(\underline{b}, 0)$. But then the honest politician wants to offer $b_h = \underline{b}, G_h = \mu - \underline{b}$ (note $\underline{b} < \tilde{b}$ and Lemma 1), to which the best response of the corrupt politician is $(\bar{b}, 0)$, because he would then win the poor votes $(\underline{b} < \bar{b})$ while he is still able to extract rent $(\bar{b} < \mu)$. But then the honest politician would of course rather play $b_h = 0, G_h = \mu$ (because $\tilde{b} < \bar{b}$ and Lemma 1).

Therefore, in what follows we look for equililibria in mixed strategies. As we show below, there can be two mixed strategy equilibria when $\underline{b} < \tilde{b} < \bar{b}$, depending on whether $\bar{b} \geq C(\underline{b})$, where

$$C(\underline{b}) \equiv \frac{\alpha \mu^3 (\mu - \underline{b}) - (1 - \alpha)(\mu - 1)\underline{b}^2}{\mu^2(\mu - 1 + \alpha) - (\mu^2 - 1 + \alpha)\underline{b}} \tag{5}$$

The function $C(\underline{b})$ determines two regions in the rectangle defined by the inequalities $\underline{b} < \tilde{b} < \bar{b}$ and depicted in Figure 2. We show that when $\bar{b} \geq C(\underline{b})$ there is 2x2 mixed strategy equilibrium, i.e. politicians mix over two actions, and when $\bar{b} < C(\underline{b})$ politicians mix over three actions, i.e. the mixed strategy equilibrium is 3x3. The next proposition identifies the two equilibria when $\underline{b} < \tilde{b} < \bar{b}$; Figure 1 depicts the actions played with positive probability in the 3x3 and the 2x2 (in bold) mixed strategy equilibria and the payoffs. (All proofs are relegated to the Appendix)

Proposition 1. _2x2 Mixed Strategy Equilibrium: If $\underline{b} < \tilde{b} < \bar{b}$ and $\bar{b} \geq C(\underline{b})$, then the unique Nash Equilibrium of the game is in mixed strategies. In equilibrium, the corrupt politician mixes between the two strategies $(\underline{b}, 0)$ and $(\bar{b}, 0)$ with probabilities p and $1 - p$, respectively, and the honest politician mixes between $(0, \mu)$ and $(\underline{b}, \mu - \underline{b})$ with probabilities q and $1 - q$, respectively, where_

$$p = \frac{(1 - \alpha)\underline{b}(\mu - 1)}{\alpha \mu (\mu - \underline{b})} \quad and \quad q = \frac{\mu - \bar{b}}{\mu - \underline{b}}$$

3x3 Mixed Strategy Equilibrium: If $\underline{b} < \tilde{b} < \bar{b}$ and $\bar{b} < C(\underline{b})$, then the unique Nash Equilibrium of the game is in mixed strategies. In equilibrium, the corrupt politician is mixing over $(\underline{b}, 0)$ with probability p_1, $(\bar{b}, 0)$ with probability p_2, and $(0, \mu - \underline{b})$ with probability $1 - p_1 - p_2$, where

$$p_1 = (1 - \alpha p_2)\frac{\underline{b}(\mu - 1)}{\alpha(\mu^2 - \underline{b})} \quad and \quad p_2 = \frac{\frac{(\mu - 1)(\bar{b} - \underline{b})}{(1 - \alpha)(\bar{b} - \mu(\bar{b} - \underline{b}))} + 1 - \frac{\underline{b}(\mu - 1)}{\alpha(\mu^2 - \underline{b})}}{\frac{\alpha[\mu(\mu - \underline{b}) - (\bar{b} - \underline{b})]}{(1 - \alpha)(\bar{b} - \mu(\bar{b} - \underline{b}))} + 1 - \frac{\underline{b}(\mu - 1)}{\mu^2 - \underline{b}}}$$

and the honest politician is mixing over $(0, \mu)$ with probability q_1, $(\underline{b}, \mu - \underline{b})$ with probability q_2, and $(\bar{b}, \mu - \bar{b})$ with probability $1 - q_1 - q_2$, where

$$q_1 = \frac{(1 - \alpha)(\mu - \bar{b})\underline{b}}{(\mu - \underline{b})(\alpha(\mu - \bar{b}) + (1 - \alpha)\underline{b})} \quad and \quad q_2 = \frac{(1 - \alpha)(\bar{b} - \underline{b})\underline{b}}{(\mu - \underline{b})(\alpha(\mu - \bar{b}) + (1 - \alpha)\underline{b})}$$

Case when $\underline{b} < \bar{b} < \tilde{b}$: 3x3 Mixed Strategy Equilibrium

We show in Lemma 2 in the Appendix that $C(\underline{b}) > \tilde{b}$, which then implies that $C(\underline{b}) > \bar{b}$ when $\tilde{b} > \bar{b}$. Therefore, in this parameter region we only have the 3x3 mixed strategy equilibrium identified in Proposition 1.

   Before we discuss the wider implications of the game, it is helpful to develop a deeper understanding of the basic driving forces and the underlying structure of these three equilibria.

4.1   Analysis of the equilibria structure

As discussed before, pure strategy equilibrium exists when the political platforms are such that even when offering the lowest available political platform $\underline{b}$ to the poor the corrupt politician is forced to pay out a considerable portion of his budget ($\tilde{\tilde{b}} < \underline{b}$), and thus corruption is limited. As a result the honest politician does not want to challenge the corrupt politician in this case, because welfare considerations dictate that she ignores corruption and represents the socially desirable platform $(0, \mu)$.

   However, once there is a platform $b < \tilde{b}$ where $b \in \{\underline{b}, \bar{b}\}$, then the corrupt politician who wants to minimise the payout to voters will play it. But this leads to too much corruption and thus prompts the honest politician to challenge the corrupt politician, i.e. to deviate from the socially desirable platform $(0, \mu)$ and offer $b_h > 0$ in order to sway poor voters from the corrupt politician. As argued above, this results in two possible mixed strategy equilibria.

   Next, we derive some comparative statics of the 2x2 mixed strategy equilibrium in order to shed some light on the mechanics of the two mixed strategy equilibria.

Proposition 2. *Comparative Statics on $\underline{b}$ and $\bar{b}$ in the 2x2 mixed strategy equilibrium:*

$$\frac{\partial \pi^c_{(2x2)}}{\partial \bar{b}} < 0, \quad \frac{\partial W^V_{(2x2)}}{\partial \bar{b}} > 0, \quad \frac{\partial p}{\partial \bar{b}} = 0, \quad \frac{\partial q}{\partial \bar{b}} < 0$$

$$\frac{\partial \pi^c_{(2x2)}}{\partial \underline{b}} = 0, \quad \frac{\partial W^V_{(2x2)}}{\partial \underline{b}} \lesseqgtr 0, \quad \frac{\partial p}{\partial \underline{b}} > 0, \quad \frac{\partial q}{\partial \underline{b}} > 0$$

   The simplicity of the 2x2 mixed strategy equilibrium stems from the fact that if $\bar{b}$ is high enough ($\bar{b} > C(\underline{b}) \geq \tilde{b}$), the honest politician does not try to match the corrupt politician's more generous offer $(\bar{b}, 0)$, hence the support of her mixed strategy consists only of the socially desirable action $(0, \mu)$ and the action $(\underline{b}, \mu - \underline{b})$ that matches and challenges the corrupt politician's lower offer $(\underline{b}, 0)$. The corrupt politician only ever plays $(\bar{b}, 0)$, because sometimes he can still win the poor when the honest politician tries to appease them by playing $(\underline{b}, \mu - \underline{b})$. However, note that in this case any $b_c > \underline{b}$ would sway voters to the corrupt politician, so higher $\bar{b}$ is essentially a redistribution from the corrupt politician ($\partial \pi^c_{(2x2)} / \partial \bar{b} < 0$) to the voters ($\partial W^V_{(2x2)} / \partial \bar{b} > 0$). As $\bar{b}$ decreases corruption is increasing, which in turn increases the incentive for the honest politician to try to defeat the corrupt politician even when he makes the higher offer $(\bar{b}, 0)$ to the poor. Indeed, when $\bar{b}$ is sufficiently low, i.e. $\bar{b} < C(\underline{b})$, the honest politician starts to play her third strategy $(\bar{b}, \mu - \bar{b})$ with positive probability to match the corrupt politician's higher offer $(\bar{b}, 0)$ too. But the honest politician's increasing effort to match the corrupt politician's offers and lure the poor comes at a price: she ends up playing

the socially desirable platform $(0, \mu)$ less often $(q_1 < q)$. This opens up the possibility for the corrupt politician to win over the rich and make a profit that way instead: the corrupt politician starts to play $(0, \mu - \underline{b})$ with positive probability. This results in a 3x3 mixed strategy equilibrium.

After discussing the equilibria structure and the basic mechanisms of the game, now we turn to the wider implications of our model.

4.2 Main results: properties of the equilibria

The most important result of our model is that corruption, i.e. the rent of the corrupt politician, can never be entirely eliminated:

Proposition 3. *Corruption is always strictly positive.*

The proposition suggests that as long as there is heterogeneity in voter preferences, there is always corruption, even when voters can vote for a benevolent politician. In our model, corruption is not a result of asymmetric information or voter bias, it is solely the result of voter heterogeneity. Consequently, when heterogeneity in voter preferences disappears, so does corruption. For instance, when the country is rich and all voters have income $\overline{y} > \underline{y} > 1$, then everyone prefers G to b and thus there is no corruption. Our model highlights the simple observation that a (well-meaning) politician cannot possibly make everyone happy and therefore there are always voters whose preferences can be effectively represented by an opportunistic politician.

Proposition 1 suggests that the honest politician's equilibrium strategy in the 2x2 mixed strategy equilibrium does not depend on α directly, while α does have a direct effect on the corrupt politician's equilibrium strategy. If inequality increases ceteris paribus (i.e. α and $\overline{y}$ increase such a way that μ does not change), then the corrupt politician will play $(\underline{b}, 0)$ less and $(\overline{b}, 0)$ more often in equilibrium, despite the fact that the honest politician would still mix with the same probabilities. This is because for larger α the welfare loss is bigger when the honest plays $(0, \mu)$, so in order to make the honest politician indifferent, the corrupt needs to reduce the honest politician's payoff from playing $(\underline{b}, \mu - \underline{b})$ by playing $(\underline{b}, 0)$ less often. This suggests that the corrupt politician benefits from increasing inequality.[21] We define an increase in inequality as a simultaneous increase in α and $\overline{y}$ such that average income $\mu = (1 - \alpha)\overline{y}$ remains fixed. Note that in our model this is the only way inequality can be varied.[22]

Proposition 4. *In the pure strategy and the 2x2 mixed strategy equilibrium, higher inequality ceteris paribus results in higher expected vote share of the corrupt politician, higher total corruption and lower voter welfare.*

In this model, the primary source of corruption is voter heterogeneity. Therefore, it is perhaps not too surprising that if voter heterogeneity is more pronounced, i.e. inequality

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[21]Empirical studies (e.g. Apergis et al 2010, Chong and Gradstein 2007, Glaeser and Saks 2006, You and Khagram 2005) often find that corruption and inequality are positively correlated.

[22]If we allowed $\underline{y} > 0$, then there would be other ways to vary inequality. We thank the referee for pointing this out.

is higher, so is corruption. Proposition 4 implies that in two countries where the average incomes are equal, but in one of them the rich are fewer (but richer), then in that country the corrupt politician has a higher expected vote share and corruption is also higher on average. Voter welfare also decreases with inequality. However, inequality has an ambiguous effect in the 3x3 mixed strategy equilibrium. This is because in this equilibrium the corrupt politician wins the votes of the rich with probability $(1 - p_1 - p_2)(1 - q_1 - q_2)$, so he profits from the voter bases of both the poor and the rich.

The honest politician is solely concerned with voter welfare as opposed to power per se. Power (i.e. vote share) only concerns her to the extent that it affects welfare. This means that the honest politician lets the corrupt politician win the majority in all three equilibria under certain circumstances (i.e. parameter values). We give the intuition for this result for the case of the 2x2 mixed strategy equilibrium. In principle, the honest politician should be worried about the corrupt politician winning the majority: the higher α the higher share of the budget the corrupt politician has in equilibrium. However, this is counteracted by two other effects. First, when α is high, so is $\tilde{b}$, which means that $\bar{b}$ is relatively close to μ. Second, when α is high the corrupt politician is so keen to win the votes of the poor that he makes his more generous offer $(\bar{b}, 0)$ with higher probability (as $\alpha \to 1$, $p \to 0$). This in turn means that the welfare loss from corruption is relatively low, and it does not justify a strategy where the honest politician would play the socially optimal platform less often in order to win the majority (in expectation).[23]

5   Discussion

We employ a discrete strategy space with two interim points ($\underline{b}$ and $\bar{b}$). Two interim points are sufficient to understand the equilibria structure of the game. Fundamentally, the honest politician can either fight corruption (i.e. try to match the corrupt politician's offer in a mixed strategy equilibrium) or don't fight corruption and choose the socially optimal platform (pure strategy equilibrium). As we show above, this characterisation of the honest politician's behaviour is linked to two intervals in the action space $[0, \mu]$ (see Lemma 1 and the definition of $\tilde{b}$). Thus, two interim points are sufficient to explore politicians' behaviour within as well as across these two intervals and let us characterise some important features of the political game.[24]

For a general income distribution with cdf $F(y)$, everyone with income $y < 1$ will still prefer b to G, because for these voters b will have a higher marginal utility than G, and similarly voters with income $y > 1$ will again prefer G to b. So for all practical purposes there will be two income classes as before. However, with a more general income distribution vote shares (i.e. what is α in our model) would depend on strategies (in particular: $\underline{b}, \bar{b}$) and this complicates the analysis. The honest politician would still face the fundamental decision of fighting or not fighting corruption. That is, there would still be parameter values

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[23]Interestingly, in the 3x3 mixed strategy equilibrium, the corrupt politician can win the majority even when the majority is the rich (i.e. $\alpha < 1/2$): when $\bar{b} = 0.1$, $\underline{b} = 0.05$, $\mu = 1.5$ and $\alpha = 0.2$, then $C(\underline{b}) \approx 0.651$ and $\tilde{b} \approx 0.643$ (and so $\underline{b} < \bar{b} < \tilde{b} < C(\underline{b})$, thus we have the 3x3 mixed strategy equilibrium) and the corrupt politician's (expected) equilibrium vote share is 0.596.

[24]Adding more actions to the discrete strategy space or even allowing for a continuous strategy space will of course change the equilibrium mixing distributions, but will not change the equilibria structure.

(i.e. $\underline{b}, \overline{b}$) such that her best response is the socially optimal platform $(0, \mu)$. However, the case when she decides to fight corruption would be more complex, because depending on parameter values (i.e. $\underline{b}, \overline{b}$), and the vote shares these parameters imply (given $F(y)$), she may only choose to reduce the vote share of the corrupt politician.[25] But this just means that with continuous income distribution, we will probably find more instances when corruption is strictly positive. In sum, with continuous income distribution the corrupt politician will find it easier to maintain a strictly positive vote share.

Calculating the mixed strategy equilibria with continuous strategy spaces (i.e. $b, G \in [0, \mu]$) would be significantly more complicated. When strategy spaces are continuous, the expected profit functions are piecewise linear with multiple peaks and this suggests that the support of the mixed equilibria should consist of discrete points, similarly to our model. However, if one politician is mixing over, say, $\underline{b}_i$ and $\overline{b}_i$, then the other wins all votes with probability one by mixing over $\underline{b}_i + \varepsilon$ and $\overline{b}_i + \varepsilon$ (with $\varepsilon > 0$ infinitesimal). This instead suggests that the support of the mixed strategy equilibria contains continuous sections, which adds considerable complexity to the fact that the mixed equilibria are over two dimensions (b and G) and hence the mixing distribution is a joint distribution.[26]

We cast our model in the framework of proportional representation in a reduced form fashion (similarly to e.g. Grossman and Helpman 1999, Saporiti 2014, Herrera et al 2014, Xefteris and Ziros 2017), because in most political systems the winner does not take it all. However, one could introduce uncertainty over voter preferences and recast the model in a majority voting system, where the probability of winning would be a function of politicians' strategies.[27] This model would be akin to our proportional representation model with continuous income distribution, where vote shares depend on strategies. However, if the strategy space is continuous, then certain parameter regions would still only have mixed strategy equilibria and the complexity of calculating these equilibria discussed in the previous paragraph would prevail.

The model in this paper is laid out in the context of corruption. Rents, however, can also be understood more generally as some form of inefficient government spending (see the discussion in Persson and Tabellini 2000, section 4.2). For instance, the model could be presented in the context of "lazy politicians", where the opportunistic politician maximises leakage $\mu - b_c - G_c$ in order to reduce effort: the less of the budget needs to be spent productively (i.e. on b_c, G_c), the less effort is required from the politician.

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[25]For instance, in our model, if the corrupt politician offers, say, $(\overline{b}, 0)$, then the honest politician will never want to change from the socially optimal platform $(0, \mu)$ to $(\underline{b}, \mu - \underline{b})$, because it would not increase her vote share and would strictly decrease welfare (indeed, the only sensible alternative response is $(\overline{b}, \mu - \overline{b})$, in which case the corrupt politician would end up with zero vote share). With continuous income distribution, however, $(\underline{b}, \mu - \underline{b})$ can turn out to be an optimal response, if the associated increase in her vote share (which depends on the income distribution) is sufficiently large.

[26]Assuming continuous income distribution in addition would lead to continuous expected profit functions, but this would not solve the issue discussed above and not help characterising the mixed strategy equilibria. Results on the pure strategy equilibrium with continuous strategy spaces and continuous income distribution are available upon request.

[27]The results of this model would then be qualified in terms of the level of uncertainty, which we find unappealing. First, this uncertainity is hard to measure and consequently the qualified results of the model would be difficult to empirically verify. Second, we wanted to avoid models with voter bias for reasons discussed in the Introduction.

There is a large literature on fractionalization (see e.g. Pande 2008), where fractionalization is often incorporated in the model as exogenous voter bias. On the one hand, our model is distinct from this literature because our voters are not biased, they make their voting choices solely based on politicians' offers. On the other hand, fractionalization can of course be the source of voter heterogeneity and thus our findings could apply to the case of fractionalization too. In countries, such as India where the cast system prevented social mobility for centuries, incomes, preferences and also voter support of social classes have been entrenched. Our model speaks to this literature and highlights that this historical voter "bias" can be the result of voter heterogeneity, creating a vicious circle and leading to developmental trap: low income people support ineffective policies (directly voting for corrupt politicians and indirectly forcing honest politicians to adopt ineffective policies), which in turn preserves inequality and inefficient social structures.

6 Conclusion

We conclude that policies that eliminate corruption can depart from the socially desirable policies and this inefficiency can be large enough to allow corruption to live on. The present paper provides a mechanism that could help explain why corruption persists in some democracies even when voters are fully rational, show no bias for corrupt politicians, there is no asymmetric information and there is an honest, welfare maximizing politician who in principle could eliminate corruption by matching the corrupt politician's offer. We show that in the presence of voter heterogeneity, the corrupt politician can always find voters whose interests he can effectively represent and who will vote for him. Moreover, when inequality is high enough, i.e. heterogeneity is prominent, the corrupt politician can win the majority of votes as the honest finds it optimal to cater to the minority.

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Appendix: Proofs

Proof of Proposition 1

First, we show that the 10x10 game Γ can be reduced to a 4x5 game (see Figure 3) through the elimination of strictly dominated strategies. Next, we show that this 4x5 game can be further reduced to a 3x3 game (see Figure 1) through again the elimination of strictly dominated strategies. We then identify the equilibria in this 3x3 game.

It is immediate that the honest politician's actions when the total budget is not spent will be strictly dominated by actions when her budget is fully exhausted, because offering more (weakly) increases vote share and (strictly) increases voter welfare. Thus, her action set $S_h = \{(b, G) \in S | b + G = \mu\}$ consists of only 4 actions (see Figure 3) that she may play with positive probability in an equilibrium.

Furthermore, the corrupt politician's actions which exhaust his budget fully and thus leaves him with zero rent against every honest action are also strictly dominated. First, note that given Assumption 1, the second and the third conditions written in the last column of Figure 3 hold (Assumption 1 implies the second, and the second implies the third condition). This means that against all four honest actions in Figure 3 (i.e. all $(b, G) \in S_h$) there is at least one corrupt action in Figure 3 that yields strictly positive payoff. Therefore, a mixed strategy that mixes over the five corrupt actions in Figure 3 yields strictly positive payoff against all honest actions and thereby strictly dominates all corrupt actions that yield zero against all honest actions (i.e. $\left\{(0,0), (0,\mu), (\underline{b}, \mu - \underline{b}), (\overline{b}, \mu - \overline{b}), (\mu, 0)\right\}$). Thus, we will restrict our attention to the following five corrupt actions: $S_c = \{(b, G) \in S | b + G < \mu\})$.

The resulting game with action sets S_c, S_h we call the Original Game and it is depicted in Figure 3.

In what follows, we show that through the iterated elimination of strictly dominated strategies the 4x5 in Figure 3 can always be reduced to the 3x3 game in Figure 1. In particular, the following three actions can be eliminated in the Original Game above:

1. The honest strategy $(\mu, 0)$ is strictly dominated by the strategy $(\overline{b}, \mu - \overline{b})$ and hence it can be eliminated. This is because $(\overline{b}, \mu - \overline{b})$ pays better than $(\mu, 0)$ against the following strategies of the corrupt politician:
   i) $(0, \mu - \overline{b})$: $\overline{b} + (\mu - \overline{b}) \mu > \alpha\mu + (1 - \alpha)(\mu - \overline{b})\mu \leftrightarrow (1 - \alpha)\overline{b} + \alpha(\mu - \overline{b})\mu > \alpha(\mu - \overline{b})$, which holds since $\mu > 1$ (note also that even if 1) in the table fails to hold $(\overline{b}, \mu - \overline{b})$ still pays better for the honest than $(\mu, 0)$ against $(0, \mu - \overline{b})$);
   ii) $(0, \mu - \underline{b})$: By Assumption 1, conditions 2) and 3) hold (condition 3) is implied by 2)), so: $\alpha\overline{b} + [\alpha(\mu - \overline{b}) + (1 - \alpha)(\mu - \underline{b})]\mu > \alpha\mu + (1 - \alpha)(\mu - \underline{b})\mu \leftrightarrow \alpha(\mu - \overline{b})\mu > \alpha(\mu - \overline{b})$, which holds since $\mu > 1$;
   iii) $(\underline{b}, 0)$: $\overline{b} + (\mu - \overline{b})\mu > \mu \leftrightarrow (\mu - \overline{b})\mu > \mu - \overline{b}$, which holds since $\mu > 1$;
   iv) $(\underline{b}, \mu - \overline{b})$: $\overline{b} + (\mu - \overline{b})\mu > \alpha\mu + (1 - \alpha)\underline{b} + (1 - \alpha)(\mu - \overline{b})\mu \leftrightarrow \alpha(\mu - \overline{b})\mu > \alpha(\mu - \overline{b}) - (1 - \alpha)(\overline{b} - \underline{b})$ (note that even if 4) in the table fails to hold $(\overline{b}, \mu - \overline{b})$ still pays better for the honest than $(\mu, 0)$ against $(\underline{b}, \mu - \overline{b})$);
   v) $(\overline{b}, 0)$: see iii) above.

Corrupt	Honest	π^c	W^V	Provided:
$\left(0, \mu-\bar b\right)$	$(0,\mu)$	0	μ^2	
	$(\underline b, \mu-\underline b)$	0	$\underline b + (\mu-\underline b)\,\mu$	
	$(\bar b, \mu-\bar b)$	0	$\bar b + (\mu-\bar b)\,\mu$	
	$(\mu,0)$	$(1-\alpha)\,\bar b$	$\alpha\mu + (1-\alpha)\left(\mu-\bar b\right)\mu$	1) $(\mu-\bar b)\bar y > \mu$
$(0, \mu-\underline b)$	$(0,\mu)$	0	μ^2	
	$(\underline b, \mu-\underline b)$	0	$\underline b + (\mu-\underline b)\,\mu$	
	$(\bar b, \mu-\bar b)$	$(1-\alpha)\,\underline b$	$\alpha\bar b + \left[\alpha\left(\mu-\bar b\right) + (1-\alpha)\left(\mu-\underline b\right)\right]\mu$	2) $(\bar b - \underline b)\bar y > \bar b$
	$(\mu,0)$	$(1-\alpha)\,\underline b$	$\alpha\mu + (1-\alpha)\left(\mu-\underline b\right)\mu$	3) $(\mu-\underline b)\bar y > \mu$
$(\underline b, 0)$	$(0,\mu)$	$\alpha\left(\mu-\underline b\right)$	$\alpha\underline b + (1-\alpha)\,\mu^2$	
	$(\underline b, \mu-\underline b)$	0	$\underline b + (\mu-\underline b)\,\mu$	
	$(\bar b, \mu-\bar b)$	0	$\bar b + (\mu-\bar b)\,\mu$	
	$(\mu,0)$	0	μ	
$\left(\underline b, \mu-\bar b\right)$	$(0,\mu)$	$\alpha(\bar b - \underline b)$	$\alpha\underline b + \left[\alpha\left(\mu-\bar b\right) + (1-\alpha)\mu\right]\mu$	
	$(\underline b, \mu-\underline b)$	0	$\underline b + (\mu-\underline b)\,\mu$	
	$(\bar b, \mu-\bar b)$	0	$\bar b + (\mu-\bar b)\,\mu$	
	$(\mu,0)$	$(1-\alpha)\left(\bar b - \underline b\right)$	$\alpha\mu + (1-\alpha)\,\underline b + (1-\alpha)\left(\mu-\bar b\right)\mu$	4) $(\mu-\bar b)\bar y > \mu - \underline b$
$(\bar b, 0)$	$(0,\mu)$	$\alpha\left(\mu-\bar b\right)$	$\alpha\bar b + (1-\alpha)\,\mu^2$	
	$(\underline b, \mu-\underline b)$	$\alpha\left(\mu-\bar b\right)$	$\alpha\bar b + (1-\alpha)\,\underline b + (1-\alpha)\left(\mu-\underline b\right)\mu$	
	$(\bar b, \mu-\bar b)$	0	$\bar b + (\mu-\bar b)\,\mu$	
	$(\mu,0)$	0	μ	

Figure 3: Original Game (π^c: payoff to corrupt politician; W^V: payoff to honest politician)

2. Then the corrupt strategy $\left(0, \mu - \bar{b}\right)$ is strictly dominated by a strategy mixing (with any probabilities) between $(0, \mu - \underline{b})$, $\left(\bar{b}, 0\right)$ and hence it can be eliminated.

3. Then, the corrupt strategy $\left(\underline{b}, \mu - \bar{b}\right)$ is strictly dominated by a strategy mixing between $(0, \mu - \underline{b})$ with an infinitesimal probability ϵ, $(\underline{b}, 0)$ with probability p and $\left(\bar{b}, 0\right)$ with probability $1 - p - \epsilon$ and hence it can be eliminated. Clearly, this mixed strategy generates a strictly positive (expected) payoff and hence pays better against the honest strategies $\left(\bar{b}, \mu - \bar{b}\right)$ and $(\underline{b}, \mu - \underline{b})$. It remains to show that this mixed strategy also pays better against the honest strategy $(0, \mu)$. In particular, we need to show that there exists a p such that $p\left(\mu - \underline{b}\right) + (1 - p - \epsilon)\left(\mu - \bar{b}\right) > \bar{b} - \underline{b} = \left(\mu - \underline{b}\right) - (\mu - \bar{b})$ or $(2 - p - \epsilon)\left(\mu - \bar{b}\right) > (1 - p)\left(\mu - \underline{b}\right)$. Note that for any $\mu > \bar{b}$ there exists a sufficiently small $\kappa > 0$ such that $(2 - p - \epsilon)\left(\mu - \bar{b}\right) > (2 - p - \epsilon)\kappa$ and also clearly $(1 - p)\mu > (1 - p)\left(\mu - \underline{b}\right)$. To complete the proof observe that there always exists a high enough p such that $(2 - p - \epsilon)\kappa > (1 - p)\mu$, because rearranging yields $p > \left(\mu - (2 - \epsilon)\kappa\right) / (\mu - \kappa)$ and $\left(\mu - (2 - \epsilon)\kappa\right) / (\mu - \kappa) < 1$.

Thus, a 3x3 game results, see Figure 1. In what follows, we show that there can be two mixed strategy equilibria in this 3x3 game when $\underline{b} < \tilde{b} < \bar{b}$: one when the politicians mix over two actions and one where they mix over 3 actions (henceforth 2x2 and 3x3 mixed strategy equilibrium, respectively). Before we turn to the derivations of these mixed strategy equilibria, we prove that the two parameter regions that the two mixed strategy equilibria correspond to, always exist, i.e. the 2x2 or 3x3 region alone never covers entirely the black rectangle in Figure 2 defined by the inequalities $\underline{b} < \tilde{b} < \bar{b}$.

Lemma 2. When $\underline{b} < \tilde{b} < \bar{b}$, there are always two parameter regions, one where $\bar{b} \geq C\left(\underline{b}\right)$ and one where $\bar{b} < C\left(\underline{b}\right)$, where $C\left(\underline{b}\right)$ is defined in (5).

Proof. There is always a region where $\bar{b} \geq C\left(\underline{b}\right)$, because for e.g. $\underline{b} = 0$ or $\underline{b} = \tilde{b}$, $C(0) = \tilde{b} < \bar{b}$ and $C(\tilde{b}) = \tilde{b} < \bar{b}$, respectively.

To prove that there can be $\tilde{b} < \bar{b}$ such that $\bar{b} < C\left(\underline{b}\right)$, it suffices to show that $\tilde{b} < C\left(\underline{b}\right)$:

$$\tilde{b} \equiv \frac{\alpha\mu^2}{\mu - 1 + \alpha} < \frac{\alpha\mu^4 - \alpha\mu^3\underline{b} - (1 - \alpha)(\mu - 1)\underline{b}^2}{\mu^2(\mu - 1 + \alpha) - (\mu^2 - 1 + \alpha)\underline{b}} \equiv C\left(\underline{b}\right)$$

First, observe that the denominator on the RHS is always positive: $\mu^2(\mu - 1 + \alpha) > (\mu^2 - 1 + \alpha)\underline{b}$ certainly holds if $\mu^2(\mu - 1 + \alpha) > (\mu^2 - 1 + \alpha)\tilde{b}$ or $(\mu - 1 + \alpha)^2 > \alpha(\mu^2 - 1 + \alpha)$; that is, $\mu^2 - 2\mu(1 - \alpha) + (1 - \alpha)^2 - \alpha\mu^2 + \alpha - \alpha^2 > 0 \Leftrightarrow (1 - \alpha)(\mu - 1)^2 > 0$. Therefore, we can crossmultiply with the denominators to get

$$\alpha\mu^2\left(\mu^2 - 1 + \alpha\right) > \alpha\mu^3(\mu - 1 + \alpha) + (1 - \alpha)(\mu - 1)\underline{b}(\mu - 1 + \alpha)$$

After some rearrangement, this yields

$$\alpha\mu^2(1 - \alpha)(\mu - 1) > (1 - \alpha)(\mu - 1)\underline{b}(\mu - 1 + \alpha) \quad \Leftrightarrow \quad \tilde{b} > \underline{b}$$

which holds by definition. $\qquad\square$

2x2 mixed strategy equilibrium

Next we show that when $\bar{b} \geq C(\underline{b})$, where $C(\underline{b})$ is defined in (5), the equilibrium is 2x2. First, let's calculate the mixed strategy equilibrium in the 2x2 subgame, when in equilibrium the corrupt politician mixes between the two strategies $(\underline{b}, 0)$ and $(\bar{b}, 0)$ with probabilities p and $1 - p$, respectively, and the honest politician mixes between $(0, \mu)$ and $(\underline{b}, \mu - \underline{b})$ with probabilities q and $1 - q$, respectively (See actions in bold and the corresponding payoffs in Figure 1). The honest politician mixes to make the corrupt politician indifferent, that is,

$$q\alpha(\mu - \underline{b}) = \alpha(\mu - \bar{b}) \quad \rightarrow \quad q = \frac{\mu - \bar{b}}{\mu - \underline{b}}$$

Clearly, $0 < q < 1$, where the last inequality follows from $\bar{b} > \underline{b}$. The corrupt politician mixes to make the honest politician indifferent, that is,

$$p\left(\alpha\underline{b} + (1 - \alpha)\mu^2\right) + (1 - p)\left(\alpha\bar{b} + (1 - \alpha)\mu^2\right) =$$
$$p\left(\underline{b} + (\mu - \underline{b})\mu\right) + (1 - p)\left(\alpha\bar{b} + (1 - \alpha)\underline{b} + (1 - \alpha)(\mu - \underline{b})\mu\right)$$

which after some rearrangement yields

$$p = \frac{(1 - \alpha)\underline{b}(\mu - 1)}{\alpha\mu(\mu - \underline{b})}$$

Clearly, $0 < p < 1$, where the last inequality follows from $(1 - \alpha)\underline{b}(\mu - 1) < \alpha\mu(\mu - \underline{b}) \leftrightarrow \underline{b} < \alpha\mu^2/((1 - \alpha)(\mu - 1) + \alpha\mu) \equiv \tilde{b}$, which holds by definition.

Next, we show that when $\bar{b} \geq C(\underline{b})$ neither politician wants to play their third strategies with positive probabilities in equilibrium in Figure 1. In particular, the honest politician's equilibrium payoff from the 2x2 mixed strategy equilibrium calculated above is (see the details of the derivation in the proof of Proposition 2):

$$W_{(2x2)}^V = \alpha\bar{b} + (1 - \alpha)\mu^2 - \frac{(1 - \alpha)\underline{b}(\bar{b} - \underline{b})(\mu - 1)}{\mu(\mu - \underline{b})}$$

The payoff from playing $(\bar{b}, \mu - \bar{b})$ against the corrupt politician's equilibrium mixed strategy is simply: $\bar{b} + (\mu - \bar{b})\mu$. Thus, the necessary condition for the honest never wanting to play $(\bar{b}, \mu - \bar{b})$ in equilibrium is $W_{(2x2)}^V > \bar{b} + (\mu - \bar{b})\mu$ or:

$$-\alpha\mu^2 + (\mu - 1 + \alpha)\bar{b} - \frac{(1 - \alpha)\underline{b}(\bar{b} - \underline{b})(\mu - 1)}{\mu(\mu - \underline{b})} > 0$$

which after some further rearrangement yields:

$$\frac{-\alpha\mu^4 + \alpha\mu^3\underline{b} + (1 - \alpha)(\mu - 1)\underline{b}^2 + \bar{b}\left(\mu^2(\mu - 1 + \alpha) - (\mu^2 - 1 + \alpha)\underline{b}\right)}{\mu(\mu - \underline{b})} > 0$$

The denominator is clearly positive ($\mu > \underline{b}$) and so is the numerator when $\bar{b} > C(\underline{b})$.

Lastly, observe that if the honest politician never plays $(\bar{b}, \mu - \bar{b})$, the corrupt politician never wants to play $(0, \mu - \underline{b})$, because it yields zero, i.e. it is strictly dominated.

3x3 mixed strategy equilibrium

When $\bar{b} < C(\underline{b})$, then the two politicians are mixing over all three of their respective strategies in Figure 1 with strictly positive probabilities. For brevity, we omit the derivations of the mixing probabilities, but we prove they are all strictly between 0 and 1. The corrupt politician is mixing over $(\underline{b}, 0)$ with probability p_1, and $(\bar{b}, 0)$ with probability p_2, and $(0, \mu - \underline{b})$ with probability $1 - p_1 - p_2$, where

$$p_1 \;=\; (1 - \alpha p_2)\,\frac{\underline{b}(\mu - 1)}{\alpha(\mu^2 - \underline{b})} \tag{6}$$

and

$$p_2 \;=\; \frac{\dfrac{\underline{b} - \alpha(\bar{b} - \mu(\bar{b} - \underline{b}))}{(1 - \alpha)(\bar{b} - \mu(\bar{b} - \underline{b}))} - \dfrac{\underline{b}(\mu - 1)}{\alpha(\mu^2 - \underline{b})}}{\dfrac{(1 - \alpha)(\bar{b} - \mu(\bar{b} - \underline{b})) + \alpha[\mu(\mu - \underline{b}) - (\bar{b} - \underline{b})]}{(1 - \alpha)(\bar{b} - \mu(\bar{b} - \underline{b}))} - \dfrac{\underline{b}(\mu - 1)}{\mu^2 - \underline{b}}} \tag{7}$$

$$= \frac{\dfrac{(\mu - 1)(\bar{b} - \underline{b})}{(1 - \alpha)(\bar{b} - \mu(\bar{b} - \underline{b}))} + 1 - \dfrac{\underline{b}(\mu - 1)}{\alpha(\mu^2 - \underline{b})}}{\dfrac{\alpha[\mu(\mu - \underline{b}) - (\bar{b} - \underline{b})]}{(1 - \alpha)(\bar{b} - \mu(\bar{b} - \underline{b}))} + 1 - \dfrac{\underline{b}(\mu - 1)}{\mu^2 - \underline{b}}}, \tag{8}$$

The proof consists of a sequence of steps organised in a particular way for convenience of exposition: step 1: $p_2 > 0$, step 2: $p_1 + p_2 < 1$, step 3: $p_2 < 1$, step 4: $0 < p_1 < 1$, step 5: $p_1 + p_2 > 0$.

Step 1: $p_2 > 0$. We proceed by dividing this step into two parts. First, assume that $\bar{b} - \mu(\bar{b} - \underline{b}) > 0$. Using the (second) expression of p_2 (8), note that $\alpha \in (0, 1)$ implies $1 - \frac{\underline{b}(\mu - 1)}{\mu^2 - \underline{b}} > 1 - \frac{\underline{b}(\mu - 1)}{\alpha(\mu^2 - \underline{b})} > 0$, where the first inequality is implied by $\alpha \in (0, 1)$ and the second inequality also holds because $\alpha(\mu^2 - \underline{b}) > \underline{b}(\mu - 1) \leftrightarrow \tilde{b} > \underline{b}$, which holds by assumption. Moreover, $\mu > 1$ and $\mu > \bar{b} > \underline{b}$ imply $(\mu - 1)(\bar{b} - \underline{b}) > 0$ and $\mu(\mu - \underline{b}) - (\bar{b} - \underline{b}) > 0$ and thus $p_2 > 0$ as required. Second, assume that $\bar{b} - \mu(\bar{b} - \underline{b}) < 0$. In order to show that $p_2 > 0$ we proceed by showing that both the numerator and the denominator of the expression of p_2 are negative. In the (first) expression of p_2 (7), the numerator is clearly negative when $\bar{b} - \mu(\bar{b} - \underline{b}) < 0$. Now, to prove that the denominator in the (first) expression of p_2 (7) is also negative we need to show that

$$\frac{(1 - \alpha)(\bar{b} - \mu(\bar{b} - \underline{b})) + \alpha[\mu(\mu - \underline{b}) - (\bar{b} - \underline{b})]}{(1 - \alpha)(\bar{b} - \mu(\bar{b} - \underline{b}))} < \frac{\underline{b}(\mu - 1)}{\mu^2 - \underline{b}}$$

On the LHS, multiply both the denominator and numerator by minus one and then cross-multiply to get

$$(1 - \alpha)\mu(\mu - \underline{b})(\mu(\bar{b} - \underline{b}) - \bar{b}) \;<\; \alpha(\mu^2 - \underline{b})(\mu(\mu - \underline{b}) - (\bar{b} - \underline{b}),$$

and then rearrange

$$\bar{b} \;<\; \frac{(1 - \alpha)\mu^2(\mu - \underline{b})\underline{b} + \alpha(\mu^2 - \underline{b})(\mu(\mu - \underline{b}) + \underline{b})}{(1 - \alpha)\mu(\mu - \underline{b})(\mu - 1) + \alpha(\mu^2 - \underline{b})}.$$

Since $\bar{b} < C(\underline{b})$, the inequality above clearly holds if the RHS is larger than $C(\underline{b})$; this is what we show next, that is:

$$\frac{(1-\alpha)\mu^2(\mu-\underline{b})\underline{b}+\alpha(\mu^2-\underline{b})(\mu(\mu-\underline{b})+\underline{b})}{(1-\alpha)\mu(\mu-\underline{b})(\mu-1)+\alpha(\mu^2-\underline{b})} >$$

$$\frac{\mu(\mu-\underline{b})\left(\alpha-\frac{\underline{b}(\mu-1)}{\mu^2-\underline{b}}\right)}{(\mu-1)\left(1-\frac{\underline{b}(\mu-1)}{\mu^2-\underline{b}}\right)+\alpha-\frac{\underline{b}(\mu-1)}{\mu^2-\underline{b}}}+\underline{b} = C(\underline{b})$$

Cross-multiply, rearrange and simplify by dividing through $\mu(\mu-\underline{b})$ to get:

$$\left[(1-\alpha)\underline{b}+\alpha(\mu^2-\underline{b})\right]\left[(\mu-1)\left(1-\frac{\underline{b}(\mu-1)}{\mu^2-\underline{b}}\right)+\alpha-\frac{\underline{b}(\mu-1)}{\mu^2-\underline{b}}\right] >$$

$$\left(\alpha-\frac{\underline{b}(\mu-1)}{\mu^2-\underline{b}}\right)\left[(1-\alpha)\mu(\mu-\underline{b})(\mu-1)+\alpha(\mu^2-\underline{b})\right]$$

Then,

$$\left[(1-\alpha)\underline{b}+\alpha(\mu^2-\underline{b})\right](\mu-1)\left(1-\frac{\underline{b}(\mu-1)}{\mu^2-\underline{b}}\right) >$$

$$\left(\alpha-\frac{\underline{b}(\mu-1)}{\mu^2-\underline{b}}\right)(1-\alpha)\left[\mu(\mu-\underline{b})(\mu-1)-\underline{b}\right]$$

which after multiplying through by $\mu^2-\underline{b}$ yields

$$\left((1-\alpha)\underline{b}+\alpha(\mu^2-\underline{b})\right)(\mu-1)\mu(\mu-\underline{b}) > (1-\alpha)\left(\mu(\mu-\underline{b})(\mu-1)-\underline{b}\right)\left(\alpha\mu^2-\underline{b}(\mu-1+\alpha)\right)$$

and then

$$\left(\alpha^2(\mu^2-\underline{b})+(1-\alpha)\underline{b}\mu\right)(\mu-1)\mu(\mu-\underline{b}) > -(1-\alpha)\underline{b}(\alpha\mu^2-\underline{b}(\mu-1+\alpha)).$$

Notice that on the RHS $\alpha\mu^2-\underline{b}(\mu-1+\alpha) > 0$ because $\tilde{b} > \underline{b}$ and thus the RHS of the inequality above is negative while the LHS is positive. This means that whenever the numerator is negative, so is the denominator, proving that $p_2 > 0$ always.

Step 2: $p_1+p_2 < 1$. We have that

$$p_1+p_2 = p_2+(1-\alpha p_2)\frac{\underline{b}(\mu-1)}{\alpha(\mu^2-\underline{b})}$$

$$= \frac{\underline{b}(\mu-1)}{\alpha(\mu^2-\underline{b})}+\left(1-\frac{\underline{b}(\mu-1)}{\mu^2-\underline{b}}\right)p_2.$$

Then $p_1+p_2 < 1$ if

$$p_2 < \frac{1-\frac{\underline{b}(\mu-1)}{\alpha(\mu^2-\underline{b})}}{1-\frac{\underline{b}(\mu-1)}{\mu^2-\underline{b}}}.$$

Similarly to Step 1 we consider two cases. First, assume that $\bar{b} - \mu(\bar{b} - \underline{b}) > 0$. Then the denominator of p_2 is positive as we showed in Step 1 and so substituting the (second) expression for p_2 (8) we can cross-multiply and get

$$(\mu - 1)(\bar{b} - \underline{b})\left(1 - \frac{\underline{b}(\mu - 1)}{\mu^2 - \underline{b}}\right) < \alpha[\mu(\mu - \underline{b}) - (\bar{b} - \underline{b})]\left(1 - \frac{\underline{b}(\mu - 1)}{\alpha(\mu^2 - \underline{b})}\right).$$

The expression above can be rewritten as

$$(\bar{b} - \underline{b})\left[(\mu - 1)\left(1 - \frac{\underline{b}(\mu - 1)}{\mu^2 - \underline{b}}\right) + \alpha - \frac{\underline{b}(\mu - 1)}{\mu^2 - \underline{b}}\right] < \mu(\mu - \underline{b})\left(\alpha - \frac{\underline{b}(\mu - 1)}{\mu^2 - \underline{b}}\right)$$

which can then be rearranged as

$$\bar{b} < \frac{\mu(\mu - \underline{b})\left(\alpha - \frac{\underline{b}(\mu-1)}{\mu^2-\underline{b}}\right)}{(\mu - 1)\left(1 - \frac{\underline{b}(\mu-1)}{\mu^2-\underline{b}}\right) + \alpha - \frac{\underline{b}(\mu-1)}{\mu^2-\underline{b}}} + \underline{b} = C\left(\underline{b}\right)$$

which holds by assumption. Second, assume that $\bar{b} - \mu(\bar{b} - \underline{b}) < 0$. Now, both the numerator and denominator of p_2 are negative as we showed in Step 1. Using the second expression for p_2 again (8), multiply both the denominator and numerator with minus one and then cross-multiply to get the same inequality as above. Thus, $p_1 + p_2 < 1$ as required.

Step 3: $p_2 < 1$. In Step 2 we showed that

$$p_2 < \frac{1 - \frac{\underline{b}(\mu-1)}{\alpha(\mu^2-\underline{b})}}{1 - \frac{\underline{b}(\mu-1)}{\mu^2-\underline{b}}}.$$

Note that since $\alpha \in (0, 1)$, $1 - \frac{\underline{b}(\mu-1)}{\alpha(\mu^2-\underline{b})} < 1 - \frac{\underline{b}(\mu-1)}{\mu^2-\underline{b}}$, and this proves the step.

Step 4: $0 < p_1 < 1$. The first part of the product in the expression of p_1 (6) is between 0 and 1 because $0 < p_2 < 1$. The second part of the expression in (6) is also positive and clearly smaller than 1 because $\alpha(\mu^2 - \underline{b}) > \underline{b}(\mu - 1) \leftrightarrow \tilde{b} > \underline{b}$. This proves the step.

Step 5: $p_1 + p_2 > 0$. This follows from Step 1 and 4.

Lastly, it remains to show that the honest politician's mixing probabilities too all belong to $[0, 1]$. The honest politician is mixing over $(0, \mu)$ with probability q_1, $(\underline{b}, \mu - \underline{b})$ with probability q_2, and $(\bar{b}, \mu - \bar{b})$ with probability $1 - q_1 - q_2$. It is convenient to rearrange the mixing probabilities of the honest politician using $\bar{\lambda} \equiv (\mu - \bar{b})/\mu$ and $\underline{\lambda} \equiv (\mu - \underline{b})/\mu$ (thus $0 < \bar{\lambda} < \underline{\lambda} < 1$) and verify the claim by inspection:

$$q_1 = \frac{\bar{\lambda}}{\underline{\lambda}} \cdot \frac{(1 - \alpha)(1 - \underline{\lambda})}{(1 - \alpha)(1 - \underline{\lambda}) + \alpha\bar{\lambda}}, \quad q_2 = \left(1 - \frac{\bar{\lambda}}{\underline{\lambda}}\right)\frac{(1 - \alpha)(1 - \underline{\lambda})}{(1 - \alpha)(1 - \underline{\lambda}) + \alpha\bar{\lambda}},$$

$$1 - q_1 - q_2 = \frac{\alpha\bar{\lambda}}{(1 - \alpha)(1 - \underline{\lambda}) + \alpha\bar{\lambda}}$$

Proof of Proposition 2

Before we derive the comparative static results we note that in equilibrium, the corrupt politician obtains an (expected) profit:

$$\pi^c_{(2x2)} = \alpha \left(pq \left(\mu - \underline{b} \right) + (1-p) \left(\mu - \bar{b} \right) \right) = \alpha \left(\mu - \bar{b} \right)$$

and the equilibrium (expected) average voter welfare is equal to:

$$
\begin{aligned}
W^V_{(2x2)} =&\, p \left(1 - q \right) \left(\underline{b} + \mu \left(\mu - \underline{b} \right) \right) + pq \left(\alpha \underline{b} + (1-\alpha) \mu^2 \right) \\
&+ (1-p)(1-q) \left(\alpha \bar{b} + (1-\alpha) \underline{b} + (1-\alpha) \mu \left(\mu - \underline{b} \right) \right) + (1-p) q \left(\alpha \bar{b} + (1-\alpha) \mu^2 \right) \\
=&\, \alpha p \left(\underline{b} - \bar{b} + (1-q) \mu \left(\mu - \underline{b} \right) \right) + \alpha \bar{b} - (1-q)(1-\alpha) \underline{b} \left(\mu - 1 \right) + (1-\alpha) \mu^2 \\
=&\, \alpha \bar{b} + (1-\alpha) \mu^2 - \frac{(1-\alpha) \underline{b} \left(\bar{b} - \underline{b} \right) \left(\mu - 1 \right)}{\mu \left(\mu - \underline{b} \right)}
\end{aligned}
$$

Most of the signs of the derivatives in the proposition can be verified by inspection. Furthermore, the second inequality follows because

$$\frac{\partial W^V_{(2x2)}}{\partial \bar{b}} = \alpha - \frac{(1-\alpha) \underline{b} \left(\mu - 1 \right)}{\mu \left(\mu - \underline{b} \right)} > 0 \quad \leftrightarrow \quad \alpha \mu \left(\mu - \underline{b} \right) > (1-\alpha) \underline{b} \left(\mu - 1 \right) \quad \leftrightarrow \quad \tilde{b} > \underline{b}$$

Lastly, the seventh inequality holds, because

$$\frac{\partial p}{\partial \underline{b}} = \frac{(1-\alpha)(\mu - 1)}{\alpha \left(\mu - \underline{b} \right)^2} > 0$$

Proof of Proposition 3

It is easy to verify that in the pure strategy equilibrium (when $\tilde{b} < \underline{b} < \bar{b}$) per voter corruption is $\pi^c_{PS} = \alpha \left(\mu - \underline{b} \right)$ and in the 2x2 mixed strategy equilibrium it is $\pi^c_{(2x2)} = \alpha \left(\mu - \bar{b} \right)$, both strictly positive. In the 3x3 mixed strategy equilibrium, per voter corruption is equal to

$$\pi^c_{(3x3)} = (1 - p_1 - p_2)(1 - q_1 - q_2)(1-\alpha)\underline{b} + p_1 q_1 \alpha(\mu - \underline{b}) + p_2(q_1 + q_2)\alpha(\mu - \bar{b})$$

which after substitution and some tedious derivation simplifies to

$$\pi^c_{(3x3)} = \frac{\alpha(\mu - \bar{b})(1-\alpha)\underline{b}}{\alpha(\mu - \bar{b}) + (1-\alpha)\underline{b}}$$

This expression is strictly positive too.

Proof of Proposition 4

In the pure strategy equilibrium, the vote share of the corrupt politician is α, so his vote share increases when inequality increases (i.e. α and $\overline{y}$ increase such that $\mu = (1-\alpha)\overline{y}$ is unchanged). Similarly, per voter corruption $\pi^c_{PS} = \alpha\left(\mu - \underline{b}\right)$ increases and welfare $W^V_{PS} = \alpha\underline{b} + (1-\alpha)\mu^2$ decreases with α, ceteris paribus.

In the 2x2 mixed strategy equilibrium, the corrupt politician wins the votes of the poor in all states, except when the corrupt politician plays $(\underline{b}, 0)$ (with probability p) and the honest politician plays $(\underline{b}, \mu - \underline{b})$ (with probability $1 - q$). Thus the expected vote share of the corrupt politician in the 2x2 mixed strategy equilibrium is

$$\alpha\left(1 - p\left(1 - q\right)\right) = \alpha - \frac{(1 - \alpha)\underline{b}\left(\mu - 1\right)\left(\overline{b} - \underline{b}\right)}{\mu\left(\mu - \underline{b}\right)^2} \tag{9}$$

This expression is increasing in α for any fixed μ, so the higher the inequality (i.e. the higher α and $\overline{y}$ such that $\mu = (1 - \alpha)\overline{y}$ is unchanged), the higher the expected vote share of the corrupt politician is.

When increasing α and $\overline{y}$ such that $\mu = (1 - \alpha)\overline{y}$ is unchanged the expected profit of the corrupt politician $\pi^c_{(2x2)} = \alpha\left(\mu - \overline{b}\right)$ clearly increases and voters are worse off too:

$$\left.\frac{\partial W^V_{(2x2)}}{\partial \alpha}\right|_{\mu\ held\ constant} = \frac{\underline{b}\left(\overline{b} - \underline{b}\right)\left(\mu - 1\right)}{\mu\left(\mu - \underline{b}\right)} - \mu^2 + \overline{b} < 0$$

This is negative if and only if $\overline{b} < \mu\left(\mu - \underline{b}\right) + \underline{b}$. To see this observe that $\overline{b} < \mu\left(\mu - \underline{b}\right) + \underline{b} \leftrightarrow \left(\overline{b} - \underline{b}\right) / \mu\left(\mu - \underline{b}\right) < 1$, and then $\partial W^V_{(2x2)}/\partial\alpha < \underline{b}\left(\mu - 1\right) - \mu^2 + \overline{b} < 0$, where the last inequality follows when again $\overline{b} < \mu\left(\mu - \underline{b}\right) + \underline{b}$. Lastly, note that $\overline{b} < \mu\left(\mu - \underline{b}\right) + \underline{b}$ always holds, because $\overline{b} - \underline{b} < \mu - \underline{b} < \mu\left(\mu - \underline{b}\right)$. Thus more inequality yields more overall corruption and lower voter welfare.