

The radial BAO scale and Cosmic Shear, a new observable for Inhomogeneous Cosmologies

Juan García-Bellido¹, Troels Haugbølle^{2,1}

¹ Instituto de Física Teórica UAM-CSIC, Universidad Autónoma de Madrid, Cantoblanco, 28049 Madrid, Spain,

² Department of Physics and Astronomy, University of Aarhus, DK-8000 Aarhus C, Denmark

E-mail: juan.garciabellido@uam.es, haugboel@phys.au.dk

Abstract. As an alternative explanation of the dimming of distant supernovae it has recently been advocated that we live in a special place in the Universe near the centre of a large spherical void described by a Lemaître-Tolman-Bondi (LTB) metric. In this scenario, the Universe is no longer homogeneous and isotropic, and the apparent late time acceleration is actually a consequence of spatial gradients. We propose in this paper a new observable, the normalized cosmic shear, written in terms of directly observable quantities, and calculable in arbitrary inhomogeneous cosmologies. This will allow future surveys to determine whether we live in a homogeneous universe or not. In this paper we also update our previous observational constraints from geometrical measures of the background cosmology. We include the Union Supernovae data set of 307 Type Ia supernovae, the CMB acoustic scale and the first measurement of the radial baryon acoustic oscillation scale. Even though the new data sets are significantly more constraining, LTB models – albeit with slightly larger voids – are still in excellent agreement with observations, at $\chi^2/\text{d.o.f.} = 307.7/(310 - 4) = 1.005$. Together with the paper we also publish the updated `easyLTB` code ‡ used for calculating the models and for comparing them to the observations.

Preprint: IFT-UAM/CSIC-08-69

Submitted to: *JCAP*

‡ The code can be downloaded at <http://www.phys.au.dk/~haugboel/software.shtml>

1. Introduction

The nature of the matter and energy contents of the Universe is still a mystery. While many cosmological parameters are known to a few percents precision – which suggests we can start to speak of a Standard Cosmological Model – some of these parameters correspond to quantities for which we completely ignore its nature. According to this model, only 4% of the Universe is made of something whose nature we know. What is the rest made off? Moreover, our current model of the Universe is based on a set of symmetries we have never tested very thoroughly, like homogeneity and isotropy, and some assumptions we have taken for granted, perhaps too lightly, like the generalized Copernican Principle, also known as the Cosmological Principle. These symmetries and assumptions provide the theoretical basis for the nowadays predominant homogeneous and isotropic Friedmann-Robertson Walker (FRW) model of the Universe.

However, distant supernovae appear dimmer than expected in a matter-dominated FRW Universe. The currently favored explanation of this dimming is the late time acceleration of the Universe due to a mysterious energy component that acts like a repulsive force. Observations seem to suggest that this so-called Dark Energy is similar to Einstein’s cosmological constant, but there is inconclusive evidence. There has been a tremendous effort in the last few years to try to pin down deviations from a cosmological constant using deep galaxy catalogues like 2dFGRS [1] and SDSS [2], and extensive supernovae surveys like ESSENCE [3], SNLS [4], UNION [5] and SDSS-SN [6], and many more are planned for the near future e.g. DES [7], PAU [8, 9], BOSS [10] and LSST [11].

In the meantime, the realization that the universe around us is actually far from homogeneous and isotropic has triggered the study of alternatives to this mysterious dark energy. Since the end of the nineties it has been suggested by various groups [12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26] that an isotropic but inhomogeneous Lemaître-Tolman-Bondi (LTB) universe could also induce an apparent dimming of the light of distant supernovae, in this case due to local spatial gradients in the expansion rate and matter density, rather than due to late time acceleration. It is just a matter of interpretation which mechanism is responsible for the dimming of the light we receive from those supernovae. Certainly the homogeneous and isotropic FRW model is more appealing from a philosophical point of view (it satisfies the Cosmological Principle, i.e. it assumes spatial sections are maximally symmetric), but so was the static universe and we had to abandon it when the recession of galaxies was discovered at the beginning of last century.

There is nothing wrong or inconsistent with the possibility that we live close to the centre of a gigaparsec-scale void. Such a void may indeed have been observed as the CMB cold spot [27, 28, 29] and smaller voids have been seen in the local galaxy distribution [30, 31]. The size and depth of the *distant* voids, i.e. $r_0 \sim 2$ Gpc and $\Omega_M \sim 0.2$, within a flat Einstein-de Sitter universe, seems to be consistent with that in which we may happen to live [32], and such a local void could account for the supernovae

dimming, together with the observed Baryon Acoustic Oscillations (BAO), the CMB acoustic scale, the age of the universe, the local rate of expansion, etc. [23].

Observations suggest that if there is such a large void, we should live close to the centre, otherwise our anisotropic position in the void would be seen as a large dipole in the CMB [33]. Of course, we do observe a dipole, but it is normally assumed to be due to the combined gravitational pull of the large scale structure in the local universe, such as the Virgo cluster, and the Shapley super cluster. There is always the possibility that we live off-centre and we are moving towards the centre of the void, so that the two effects are partially cancelled, giving rise to the observed dipole. However, such a coincidence could not happen for all galaxies in the void and, in general, clusters that are off-centred should see, in their frame of reference, a large CMB dipole. Such a dipole would manifest itself observationally for us as an apparent kinematic Sunyaev-Zeldovich (kSZ) effect for the given cluster. The still preliminary observations of the kSZ effect allowed us recently [24] to put some bounds on LTB models. It is expected that in the near future, improved measurements of this effect with the Atacama Cosmology Telescope (ACT) [34] and the South Pole Telescope (SPT) [35] will very strongly constrain the LTB model.

In fact, what we require are new data sets that could be used to constrain further the LTB models and eventually rule them out, if possible. We thus use the recently released compilation of 307 supernovae by the Supernova Cosmology Project - UNION collaboration [5]. Moreover, in this paper we make use of a new and very interesting observation of the baryon acoustic oscillation along the line of sight, otherwise known as radial BAO (RBAO) [36, 37]. It provides a direct measurement of the rate of expansion along the line of sight, $H_L(z)$, whose integral determines the luminosity distance and could in principle be different from that transverse to the line of sight, $H_T(z)$, which relates to the angular diameter distance. Both appear in the Einstein equations and it is possible to disentangle their respective effects by looking at a new observable, the ratio of shear to expansion in LTB models. In FRW universes, the shear of the background geometry is identically zero, while in LTB models it can be significantly different from zero, and becomes maximal at a certain redshift, corresponding to the size of the void. Therefore, by measuring the normalized shear, future galaxy surveys will be able not only to determine whether we live in a homogeneous universe or not, but to measure the size and depth of the void.

In section 2 we describe the general LTB void models, giving the corresponding Einstein-Friedmann equations, as well as parameterizations of their solutions. In a subsection we describe the GBH constrained model, where we assume the Big Bang is homogenous, and thus the model only depends on a single function, the inhomogeneous matter ratio $\Omega_M(r)$. In section 3 we introduce a new observable, the normalized cosmic shear, which could help distinguish observationally homogeneous FRW models from arbitrary inhomogeneous universes. In section 4 we show how to properly calculate the size of standard rulers in LTB models, constructing an effective Einstein de Sitter background, that can be used for calculating early universe quantities in the

corresponding LTB model using standard formulae. In section 5 we analyse present observations of BAO along the line of sight, as well as the recent UNION compilation of Supernovae, and give constraints on the model from current observations. Finally, in section 6 we give a discussion of future prospects and some conclusions.

2. Lemaitre-Tolman-Bondi void models

The LTB model describes general radially symmetric space-times and can be used as a toy model for describing voids in the universe [38, 39, 40]. The metric is

$$ds^2 = -dt^2 + X^2(r, t) dr^2 + A^2(r, t) d\Omega^2, \quad (1)$$

where $d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2$. Assuming a spherically symmetric matter source with negligible pressure,

$$T^\mu_\nu = -\rho_M(r, t) \delta_0^\mu \delta_\nu^0, \quad (2)$$

the $(0, r)$ component of the Einstein equations, $G^0_r = 0$, implies $X(r, t) = A'(r, t)/\sqrt{1 - k(r)}$, with an arbitrary function $k(r)$ playing the role of the spatial curvature parameter. The other components of the Einstein equations read [18, 19, 23]

$$H_T^2 + 2H_T H_L + \frac{k}{A^2} + \frac{k'(r)}{AA'} = 8\pi G \rho_M, \quad (3)$$

$$2\dot{H}_T + 3H_T^2 + \frac{k}{A^2} = 0, \quad (4)$$

where $\dot{} \equiv \partial_t$ & $' \equiv \partial_r$, and we have defined the transverse and longitudinal Hubble rates as $H_T \equiv \dot{A}/A$, and $H_L \equiv \dot{A}'/A'$. Integrating the last equation, we get

$$H_T^2 = \frac{F(r)}{A^3} - \frac{k(r)}{A^2}, \quad (5)$$

with another arbitrary function $F(r)$, playing the role of effective matter content, which substituted into the first equation gives

$$\frac{F'(r)}{A'A^2(r, t)} = 8\pi G \rho_M(r, t), \quad (6)$$

where $\rho_M(r, t)$ is the physical matter density.

The boundary condition functions $F(r)$ and $k(r)$ are specified by the nature of the inhomogeneities through the local Hubble rate, the local total energy density and the local spatial curvature,

$$F(r) = H_0^2(r) \Omega_M(r) A_0^3(r), \quad (7)$$

$$k(r) = H_0^2(r) (\Omega_M(r) - 1) A_0^2(r), \quad (8)$$

where functions with subscripts 0 correspond to present day values, $A_0(r) = A(r, t_0)$ and $H_0(r) = H_T(r, t_0)$. With these definitions, the r -dependent transversal Hubble rate can be written as [18, 19]

$$H_T^2(r, t) = H_0^2(r) \left[\Omega_M(r) \left(\frac{A_0(r)}{A(r, t)} \right)^3 + (1 - \Omega_M(r)) \left(\frac{A_0(r)}{A(r, t)} \right)^2 \right], \quad (9)$$

and we fix the gauge by setting $A_0(r) = r$. For fixed r the above equation is equivalent to the Friedmann equation of an open universe, and have an exact parametric solution, while also very good approximate solutions can be found by Taylor expanding around an Einstein de Sitter solution (see [23] for details).

For light travelling along radial null geodesics, $ds^2 = d\Omega^2 = 0$, we have

$$\frac{dt}{dr} = \mp \frac{A'(r, t)}{\sqrt{1 - k(r)}} \quad (10)$$

which, together with the redshift equation,

$$\frac{d \log(1 + z)}{dr} = \pm \frac{\dot{A}'(r, t)}{\sqrt{1 - k(r)}} \quad (11)$$

can be written as a parametric set of differential equations, with $N = \log(1 + z)$ being the effective number of e-folds before the present time,

$$\frac{dt}{dN} = -\frac{A'(r, t)}{\dot{A}'(r, t)}, \quad (12)$$

$$\frac{dr}{dN} = \pm \frac{\sqrt{1 - k(r)}}{\dot{A}'(r, t)} \quad (13)$$

Notice, that while the angular diameter distance $d_A = A(r, t)$ only depends on the integral of the transversal Hubble rate $H_T(r, t)$, the comoving and the luminosity distances $d_L = (1 + z)d_C = (1 + z)^2 d_A$ depend through Eq. (12) on a mixture of H_T and H_L .

2.1. The constrained GBH model

In general LTB models are uniquely specified by the two functions $k(r)$ and $F(r)$ or equivalently by $H_0(r)$ and $\Omega_M(r)$, but to test them against data we have to parameterise the functions, to reduce the degrees of freedom to a discrete set of parameters. For simplicity in this paper we will use the constrained GBH model [23] to describe the void profile. First of all, it uses a minimum set of parameters to make a realistic void profile, and secondly, by construction, the time to the Big Bang is constant for spatial slices, as would be expected in a generic model of inflation. Moreover, void models with an inhomogeneous Big Bang contain a mixture of growing and decaying modes, and consequently the void does not disappear at high redshift making them incompatible with the Standard Big Bang scenario [41]. If we only consider constrained LTB models, then at high redshifts and/or at large distances the central void is reduced to an insignificant perturbation in an otherwise homogeneous universe described by an FRW metric, and physical results for the early universe derived for FRW space-times still hold, even though we are considering an LTB space-time.

The second condition gives a relation between $H_0(r)$ and $\Omega_M(r)$, and hence constrain the models to one free function, and a proportionality constant describing

the overall expansion rate. Our chosen model is thus given by

$$\Omega_M(r) = \Omega_{\text{out}} + \left(\Omega_{\text{in}} - \Omega_{\text{out}} \right) \left(\frac{1 - \tanh[(r - r_0)/2\Delta r]}{1 + \tanh[r_0/2\Delta r]} \right) \quad (14)$$

$$H_0(r) = H_0 \left[\frac{1}{\Omega_K(r)} - \frac{\Omega_M(r)}{\sqrt{\Omega_K^3(r)}} \sinh^{-1} \sqrt{\frac{\Omega_K(r)}{\Omega_M(r)}} \right] = H_0 \sum_{n=0}^{\infty} \frac{2[\Omega_K(r)]^n}{(2n+1)(2n+3)}, \quad (15)$$

where $\Omega_K(r) = 1 - \Omega_M(r)$, and the second equation follows from the requirement of a constant time to a homogeneous Big Bang. We use an ‘‘inflationary prior’’, and assume that space is asymptotically flat, i.e. in the following we set $\Omega_{\text{out}} = 1$. The model has then only four free parameters: The overall expansion rate H_0 , the underdensity at the centre of the void Ω_{in} , the size of the void r_0 , and the transition width of the void profile Δr . For more details on the model see Ref. [23].

3. The Raychaudhuri equation and the normalized shear

Now that cosmological surveys are beginning to provide detailed maps of the universe up to significant distances we can start to consider measuring deviations from homogeneous FRW models. The main difference between FRW and LTB models is that the latter introduces two different components to the rate of expansion: There is both a transverse (perpendicular to the line of sight) and longitudinal (along the line of sight) components. This induces a differential growth of the local volume of the universe. Let us see how to quantify this geometrical distortion in the expansion of the universe.

Suppose we start with a set of comoving observers localized on a sphere of volume V in a LTB universe. Each observer has a unit vector n^μ tangent to its trajectory, and follows a geodesic of the metric, i.e. $n^\mu n^\nu_{;\mu} = 0$. If we now consider a congruence of observers (i.e. geodesics) we can follow the change in positions of the observers with time. The covariant derivative of the congruence can be split into three distinct components,

$$\Theta_{\mu\nu} = n_{\mu;\nu} = \frac{1}{3}\Theta P_{\mu\nu} + \sigma_{\mu\nu} + \omega_{\mu\nu}, \quad (16)$$

where $P^\mu_\nu = \delta^\mu_\nu + n^\mu n_\nu$ projects out any tensor into a plane orthogonal to the congruence n^μ . The tensor $\Theta_{\mu\nu}$ measures the extent to which neighbouring trajectories deviate from remaining parallel. The three components of $\Theta_{\mu\nu}$ in Eq. (16) have different physical meanings. The trace gives the expansion rate of the congruence, $\Theta = n^\mu_{;\mu}$, and characterizes the growth of the overall volume V of the sphere of observers. The traceless symmetric part is the *shear* tensor of the congruence,

$$\sigma_{\mu\nu} = \frac{1}{2}P^\alpha_\mu P^\beta_\nu (n_{\alpha;\beta} + n_{\beta;\alpha}) - \frac{1}{3}\Theta P_{\mu\nu}, \quad (17)$$

and represents the distortion in the shape of the sphere of test particles (observers), giving rise to an ellipsoid with possibly different axes’ lengths but same volume V . Finally, the antisymmetric part is the *vorticity* tensor,

$$\omega_{\mu\nu} = \frac{1}{2}P^\alpha_\mu P^\beta_\nu (n_{\alpha;\beta} - n_{\beta;\alpha}), \quad (18)$$

which describes the rotation of the sphere of test particles around the center, again without changing the volume of the sphere. Note that these are purely geometrical quantities and are independent of the dynamics responsible for time evolution in this metric. By computing the covariant derivative of $\Theta_{\mu\nu}$ along the worldlines of observers and taking the trace, one finds the famous Raychaudhuri equation,

$$\frac{d\Theta}{d\tau} = -\frac{1}{3}\Theta^2 - \sigma_{\mu\nu}\sigma^{\mu\nu} + \omega_{\mu\nu}\omega^{\mu\nu} - R_{\mu\nu}n^\mu n^\nu. \quad (19)$$

The last term of the equation is the only one sensitive to the theory of gravity. For general relativity and geodesic observers it becomes $R_{\mu\nu}n^\mu n^\nu = 8\pi G(T_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T)n^\mu n^\nu = 4\pi G(\rho + 3p)$. It is this term which allows for accelerated expansion in a Λ CDM-FRW model (for which both shear and vorticity vanish).

We can now compute all the terms in this equation in our LTB models. The global rate of expansion becomes $\Theta = H_L + 2H_T$; the spatial shear is non-vanishing, $\sigma_{ij} = (H_T - H_L) M_{ij}$, where $M_j^i = \text{diag}(-2/3, 1/3, 1/3)$ is a traceless symmetric matrix; while the vorticity tensor vanishes, $\omega_{ij} = 0$. Note that the shear component of the congruence vanishes asymptotically as we approach the Einstein-de Sitter universe, as well as locally, close to the center of the LTB void. However at intermediate redshifts it has its maximal deviation from zero, where the longitudinal and transverse rates of expansion differ maximally, corresponding to the edge of the void. This shear component is a distinctive feature of inhomogeneous cosmological models, in fact it is one that could be used to distinguish FRW from LTB models in the near future with the next generation of astronomical surveys like DES, PAU, BOSS and LSST.

In order to be quantitative, we should find a directly measurable quantity that can be computed and compared with observations. From a theoretical point of view, the natural variable to consider is the ratio of shear to expansion,

$$\varepsilon \equiv \sqrt{\frac{3}{2}} \frac{\sigma}{\Theta} = \frac{H_T - H_L}{H_L + 2H_T}, \quad (20)$$

where $\sigma^2 \equiv \sigma_{ij}\sigma^{ij} = \frac{2}{3}(H_T - H_L)^2$. However, these rates are difficult to measure directly (in particular the transverse rate of expansion H_T can only be measured indirectly). More appropriate would be to use physical quantities like angular diameter distances and radial BAO scales, as a function of redshift or its derivatives.

One can indeed find a relation between the variable ε and those quantities, by noting that in LTB models, the angular diameter distance $d_A(z)$ is nothing but $A(r, t)$ in the metric (1). Its derivatives give the various rates of expansion, $H_T = \dot{A}/A$ and $H_L = \dot{A}'/A'$, but note that in general, the time derivatives in an inhomogeneous universe are not directly related to the redshift derivatives since there is also a contribution from spatial gradients,

$$\frac{d}{dN} = \frac{\sqrt{1-k(r)}}{A'H_L} \frac{d}{dr} - \frac{1}{H_L} \frac{d}{dt},$$

where we have used Eqs. (12-13), and therefore H_L and H_T are not directly measurable quantities, contrary to the case in FRW models. Computing explicitly the derivatives

of $A(r, t)$ with respect to redshift, we find

$$\frac{H_T}{H_L} = \frac{\sqrt{1-k(r)}}{AH_L} - \frac{d \ln A}{dN} = \frac{\sqrt{1-k(r)}}{d_A(z)H_L(z)} - \frac{d \ln d_A(z)}{d \ln(1+z)}, \quad (21)$$

which gives for the new variable, the normalized shear ε , as a function of the line of sight rate of expansion and the angular diameter distance,

$$\varepsilon = \frac{\sqrt{1-k(r)} - H_L(z) \partial_z[(1+z) d_A(z)]}{3H_L(z)d_A(z) + 2\sqrt{1-k(r)} - 2H_L(z) \partial_z[(1+z) d_A(z)]}. \quad (22)$$

This quantity still depends on an unknown, the spatial curvature $k(r)$. However, we can integrate Eq. (13) to give

$$\sqrt{1-k(r)} = \cosh \left[H_0(r) \sqrt{\Omega_K(r)} \int_0^z \frac{dz'}{H_L(z')} \right] \approx 1 + \frac{1}{2} H_0^2(r) \Omega_K(r) \left(\int_0^z \frac{dz'}{H_L(z')} \right)^2,$$

which is not far from unity in a wide range of LTB models that agree with observations, see Ref. [23, 24]. In this case, the normalized shear becomes

$$\varepsilon(z) \approx \frac{1 - H_L(z) \partial_z[(1+z) d_A(z)]}{3H_L(z)d_A(z) + 2 - 2H_L(z) \partial_z[(1+z) d_A(z)]}, \quad (23)$$

which can in principle be measured by any of the future surveys. In particular, the rate of expansion along the line of sight has recently been measured by Gaztañaga et al. at two different redshifts using the radial BAO scale [36, 37], and together with measurements of angular diameter distances could give a value for $\varepsilon(z)$. Note that in FRW universes the normalized shear (22) vanishes identically since $H_L = H_T = H$ and

$$(1+z)d_A(z) = \frac{1}{H_0\sqrt{\Omega_K}} \sinh \left[H_0\sqrt{\Omega_K} \int_0^z \frac{dz'}{H(z')} \right].$$

Note also that the function $H_L(z)d_A(z)$ in the denominator is nothing but the Alcock-Paczynski factor $f(z)$, which is used as a geometric test for the existence of vacuum energy in Λ CDM FRW models. On the other hand, it is worth making emphasis on the fact that this normalized shear (23) is independent of the value of H_0 , still a mayor uncertainty in cosmology.

In Fig. 1 we have plotted the normalized shear ε for a LTB model with cosmological parameters corresponding to the best fit model that accommodates current CMB, SNIa and RBAO observations (see section 5). A survey that can measure ε in Eq. (23) significantly above zero will be able to distinguish homogeneous FRW models from inhomogeneous universes. The maximum value of the shear is attained when the density gradient is maximal, and thus the redshift of the maximum gives information about the size of the void. Note that the typical value of the normalized shear can be small, of order a few percent for LTB models, so this may impose severe requirements on future proposed surveys. A way to characterize the ability of a survey to distinguish between homogenous and inhomogeneous models of the Universe would be to ask them to detect a nonvanishing shear (e.g. $\max(\varepsilon) \simeq 0.03$) at 95% confidence level, see inset of Fig. 1. Note also that the sign of the (normalized) shear parameter can be used to characterize and distinguish a void from an overdensity.

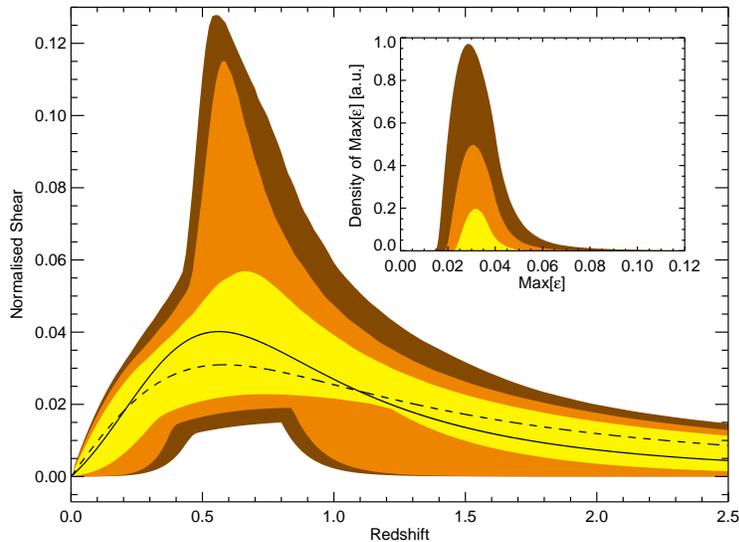


Figure 1. The redshift dependence of the normalized shear (22) for an LTB universe with cosmological parameters corresponding to the best-fit model (full line) and to the model with minimum χ^2 (dashed line) with 1, 2, and 3- σ envelopes, that accommodates current CMB, SNIa and RBAO observations. We also plot in the inset the distribution function of maximum shear in 1, 1+2 and 1+2+3- σ bins.

4. Standard Rulers in LTB models

Many bounds from observational cosmology, such as the sound horizon (a “standard ruler”), the CMB, and applications of the Alcock-Paczinsky test, are derived by considering and calculating physical scales and processes in the early universe, and are based on the implicit assumption of an underlying FRW metric. To test LTB models against these observational data we have to connect distance scales, redshifts, and expansion rates in the early universe to those observed today.

Starting from an approximately uniform universe at a high redshift z_e in the LTB model, the expansion rate and matter density becomes gradually inhomogeneous, and a uniform comoving physical length scale l in the early universe at z_e , for example the sound horizon, is not uniform at some later redshift z , because of the space dependent expansion rate. In particular the comoving length at the current time t_0 depends on how much relative expansion there has been at different positions since the formation of the uniform length scale

$$l(r(z)) = l(r_\infty) \frac{A(r(z), t_0)}{A(r(z), t(z_e))} \frac{A(r_\infty, t(z_e))}{A(r_\infty, t_0)}, \quad (24)$$

where r_∞ is the radial coordinate of an observer far away from the void. While it is clear that the *physical* length becomes scale dependent due to the inhomogeneous expansion of the Universe, the scale-dependence of the *comoving* scale is a consequence of it being measured at t_0 . If instead we *defined* the comoving length scales to be measured in the early universe at $t(z_e)$, then indeed $l(r(z))$ would be independent of the observer

position. The convenience of the above formula is, that the LTB models we consider in this paper are asymptotically Einstein de Sitter, and we can easily compute comoving scales at infinity.

Not only do we have to take into account that scales are different, but we would also like to use the standard framework to compute early universe quantities. Hence, we need to compute the equivalent Hubble constant at the current redshift, that an observer in a Einstein de Sitter universe would have now at t_0 to have the same redshift history far away from the void, that the central observer has in the LTB model. Using the normal FRW equation for an Einstein de Sitter universe we can write:

$$H^2(z_e) = H_{\text{eff}}^2(1 + z_e)^3 \quad (25)$$

where z_e is a high enough redshift where the lightcone is far away from the void, but where the radiation density has not become significant yet, typically $z \simeq 100$. The l.h.s. has to be evaluated in the LTB model, while the r.h.s. gives the equivalent Einstein de Sitter Hubble constant H_{eff} , that can be used as input for calculating e.g. the sound horizon using standard methods[§].

5. Constraints from observations

To constrain the parameters of our model we use three data sets: The first acoustic peak in the CMB as measured by WMAP [42], the Union Supernovae data set [5] consisting of 307 Type Ia supernovae, and the radial baryon acoustic scale measured at $z = 0.24$ and $z = 0.43$ using large red galaxies in the SDSS DR6 [36, 37].

The likelihood for the acoustic CMB scale is calculated as in [23]. For the UNION supernovae we have used the full covariance matrix published by the Supernova Cosmology Project [5, 43] including systematic errors, and we have adapted their likelihood code to our `easyLTB` program. We find the radial baryon acoustic differential redshift Δz in our model as

$$\Delta z_{LTB} = \frac{H_L(z)r_s(z)}{c} \quad (26)$$

and compare it to the measurements, including systematic errors, published in [37]. The size of the sound horizon is redshift dependent and computed as detailed in section 4.

Several changes can be observed (see Fig. 4) compared to our previous analysis in [23]: Clearly the UNION supernovae prefer a higher overall density contrast of the void, compared to the Davis et al. compilation [44]. This is curious, given that most of the supernovae are overlapping between the two data sets, and it is not a priori clear where the difference is. A possible cause, apart from the increased number of supernovae in the UNION set, could be that all supernovae in the UNION compilation has been reanalysed with the SALT light curve fitter, presumably giving a more homogeneous data set

[§] See [23] for the general case when the space time is not asymptotically Einstein de Sitter, and Ref. [25] for a treatment, that also fixes the angular diameter distance to the correct value in the effective Einstein de Sitter model, so the full angular powerspectrum can be computed by standard CMB packages

	H_0	$H_{r=0}$	$H_{r=\infty}$	Ω_{in}	r_0	Δr
units	100 km s ⁻¹ Mpc ⁻¹				Gpc	r_0
Priors	0.50–0.95	0.4–0.89	0.33–0.63	0.05–0.35	0.5–4.5	0.1–0.9
Best Fit $\pm 2\text{-}\sigma$	0.67 ± 0.03	0.58	0.45	0.16 ± 0.09	2.7 ± 0.8	$0.44 (> 0.12)$
Minimum χ^2	0.68	0.59	0.45	0.145	2.35	0.85

Table 1. Priors used when scanning the parameters of the model, the marginalised best fit values with $2\text{-}\sigma$ confidence levels (see Fig. 5), and the values at the minimum χ^2 . H_0 is only a pre factor for $H_0(r)$ and the priors on $H_{r=0}$ and $H_{r=\infty}$ are derived from the priors on Ω_{in} and H_0 . Because they have a complicated prior, we give them without confidence limits.

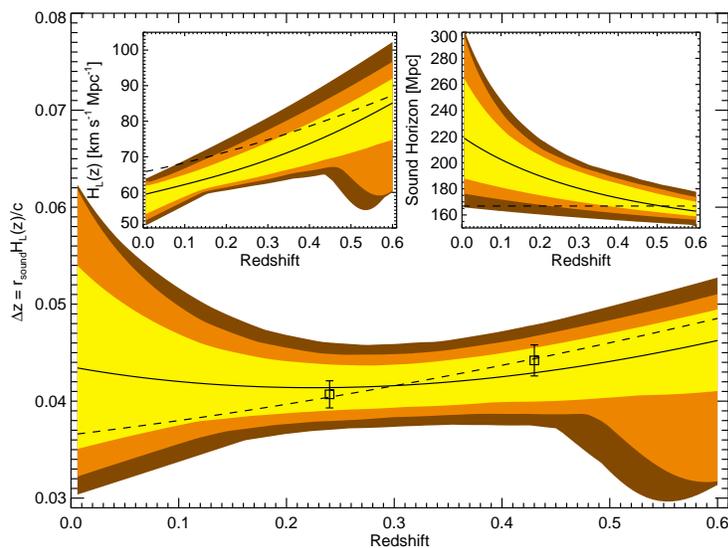


Figure 2. The 1, 2, and 3- σ limits for the sound horizon differential redshift Δz along the line of sight, the longitudinal Hubble rate H_L , and the sound horizon together with the best fit model (full line). The observed Δz with errors are given as boxes, and the equivalent best fit Λ CDM model from [36, 37] is indicated by a dashed line.

without spurious offsets between different groups of supernovae. The magnitude-redshift relation is shown in Fig. 3, and even though we only imposed as good a fit as possible to the data, the curves of the LTB models with the best fit, and the minimum χ^2 are both almost identical to the best fit Λ CDM model, illuminating the very small freedom in the magnitude-redshift relation allowed by current data. Notice also that at $z > 2$ the slopes of the three curves are approximately the same, because at those redshifts all models are effectively Einstein de Sitter. Hence, a ~ 5 Gpc sized void would be able to mimic the Λ CDM magnitude-redshift relation up to any redshift.

The radial baryon acoustic (RBAO) scale is pushing the void size upwards compared to the isotropised BAO data of Percival et al [45]. This can be understood in terms of the higher redshift of the second data point ($z=0.45$), compared to the second data point

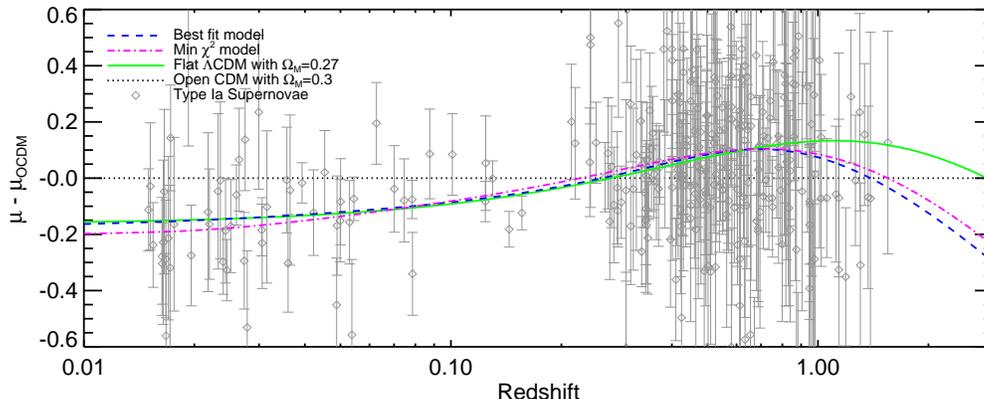


Figure 3. The UNION supernova data together with the magnitude-redshift relations for the best fit and minimum χ^2 LTB models compared to the standard Λ CDM model.

of Percival et al ($z=0.35$). Given that the RBAO measurements are in perfect agreement with a Λ CDM model, the void has to be bigger, to have the same RBAO differential redshifts, Δz , at the two different redshifts. In Fig. 2 is shown the envelope curves of Δz , $r_s(z)$, and $H_L(z)$ as a function of redshift derived from models which are inside the 1, 2, and 3- σ likelihood contours. While the best-fit LTB model is perfectly inside the error bars of the two data points, it is very different from the Λ CDM model at higher and lower redshifts, and just another redshift bin, outside current measurements, would help considerably in discriminating between the models. While Δz is monotonically increasing in Λ CDM, it is almost constant at $z < 0.5$ in the best-fit LTB model. This can be understood by looking at the inserts in Fig. 2: The sound horizon is much larger at lower redshift, because of the higher expansion rate near the centre of the void, and this is almost compensated by the longitudinal Hubble rate, giving a practically flat RBAO differential redshift. Unfortunately it is difficult due to measure the RBAO feature at lower redshifts, because of cosmic variance, but in the near future dedicated RBAO surveys will detect it at higher redshifts.

The CMB acoustic scale fixes the overall Hubble parameter H_0 , because our models are asymptotically flat, and because we have fixed $\omega_{\text{baryon}} = 0.0223$ to the best fit WMAP3 value. Overall the model is capable of yielding a very good fit to data with a minimum $\chi^2 = 307.7$ for 310-4 d.o.f. In figure 5 we show the one dimensional likelihoods, and it can be seen that the different data sets yield consistent and overlapping marginalised likelihoods, further supporting that the model is a very good and consistent fit to the observations.

6. Discussion and conclusions

As an alternative explanation of the dimming of distant supernovae it has recently been advocated that we live in a special place in the Universe near the centre of a large void.

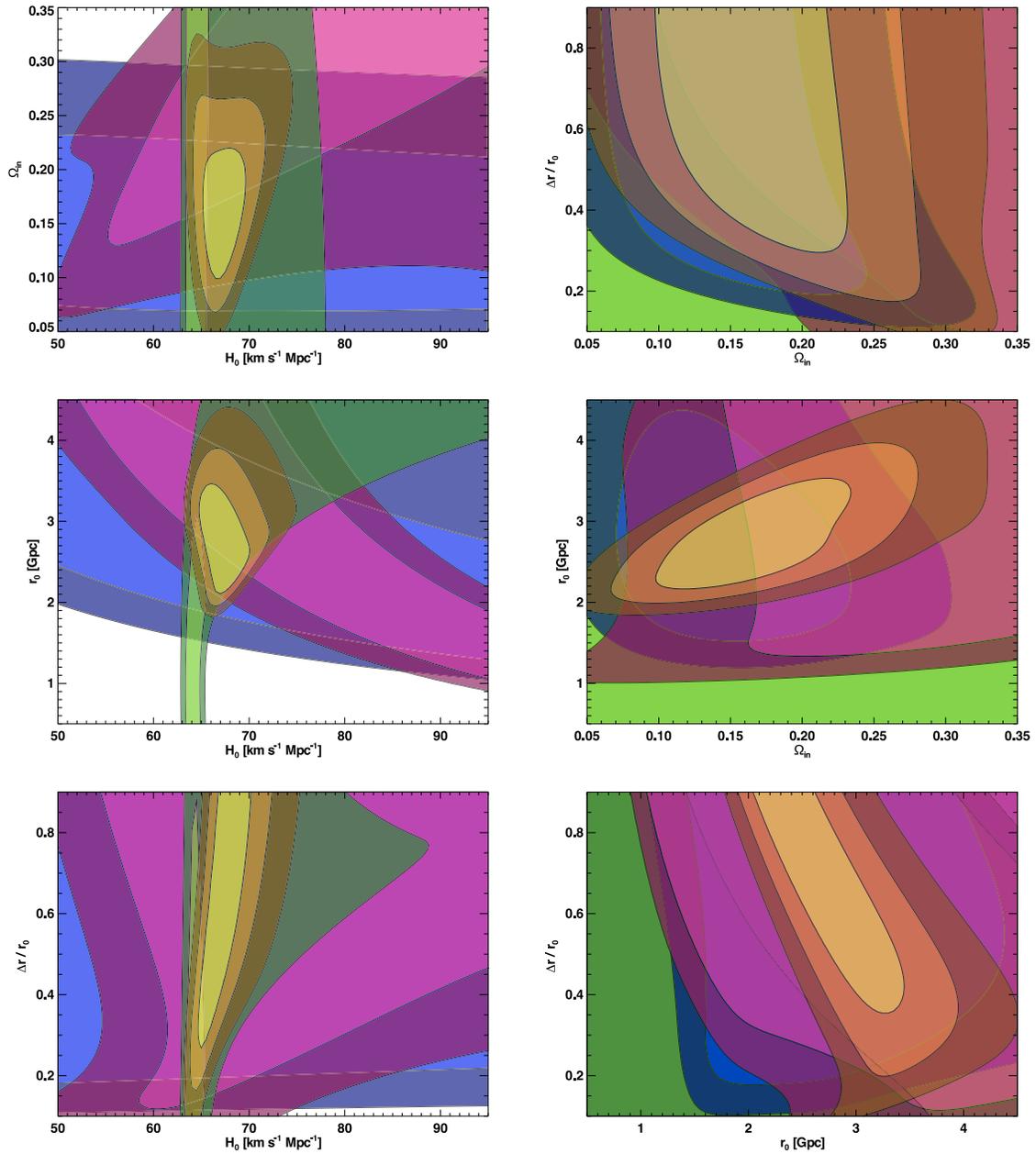


Figure 4. Likelihood contours for the combined data set is shown in yellow with 1-, 2-, and 3- σ contours, while the individual likelihoods for the SNIa, BAO, and CMB data sets are shown in blue, purple, and green respectively with 1- and 2- σ contours.

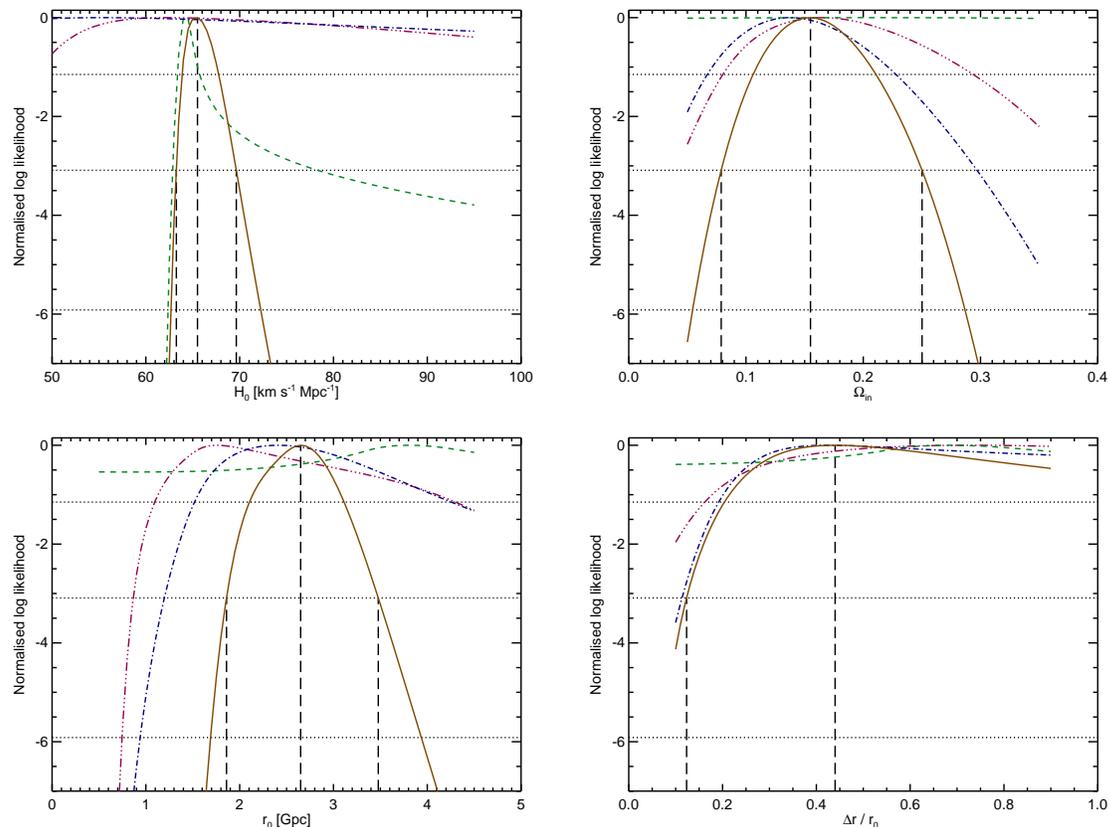


Figure 5. Marginalised one dimensional likelihood curves for the combined data set is shown in yellow (full line), while the individual likelihoods for the SNIa, BAO, and CMB data sets are shown in blue (dot-dashed), purple (dot-dot-dashed), and green (dashed) respectively. The dotted lines indicate the 1-, 2-, and 3- σ levels, while the long dashed lines gives the best fit and the 2- σ values.

In this scenario, the universe is no longer homogeneous and isotropic and the apparent late time acceleration is actually a consequence of spatial gradients in the metric.

If, at the end, local spatial curvature explains away the need for a cosmological constant we would be living through a paradigm shift similar to the one that occurred with the discovery of the expansion of the universe by Hubble and others. At that time, the metric of the universe was assumed to have maximal symmetry (time and space translations plus rotations), suggesting a static, homogeneous and isotropic universe, in agreement with the Perfect Cosmological Principle. The observed expansion of the universe removed the need for time translation invariance and only spatial sections were maximally symmetric, and we are left with a homogeneous and isotropic universe satisfying the usual Cosmological Principle, which states that any observer in the universe is equivalent to any other one. However, if the universe around us is actually curved, within a large void in an otherwise EdS universe, then it is no longer homogeneous, and we are left only with rotational invariance. The metric corresponds to that of a Lemaitre-Tolman-Bondi model. These models can accommodate the observed

dimming of distant supernovae, without the need for a cosmological constant, because spatial curvature also increases the luminosity distance. In fact, LTB models fit well the present SN data, even better than Λ CDM for some models [26].

In this paper we have tested a class of simple LTB models, against different cosmological data, that probes the overall geometry of the universe, as observed in the lightcone, namely: The Union supernovae data set of 307 Type Ia SNe [5], the recent measurement of the radial BAO scale in two different redshift bins [36], and the CMB acoustic scale as measured by WMAP [42]. We have shown that LTB models yield a convincing fit to current observations with only 4 parameters in the model, and an excellent χ^2 for the best-fit model of $\chi^2/\text{d.o.f.} = 307.7/(310 - 4)$. While not ruling out the void model, the observations do require a void of at least $r_0 \geq 2$ Gpc (at 95% C.L.), and a central underdensity that is in accordance with local galaxy survey measurements of $\Omega_{\text{in}} \approx 0.16 - 0.25$. The predicted local Hubble parameter of $59 \text{ km s}^{-1} \text{ Mpc}^{-1}$ is within $2\text{-}\sigma$ of the Hubble key project value of $72 \pm 8 \text{ km s}^{-1} \text{ Mpc}^{-1}$.

Moreover, there are new specific predictions that these inhomogeneous models give for physical observables, the most important one being that the overall rate of expansion is not homogeneous and thus longitudinal and transverse expansion rates along the line of sight are different. In the language of differential geometry, this implies that the congruence of geodesic observers has a non-vanishing shear. If we normalize this shear w.r.t. to the expansion (they both appear on equal footing in the Raychaudhuri equation that governs the acceleration of the universe) then the normalized shear can be used as an observable for future surveys. We have found a very compact form of the normalized shear that can be measured in principle, written in terms of directly observable quantities. In fact, if future surveys measure at high confidence level a non-vanishing value for this new observable, then we will have to accept that we live in an inhomogeneous universe, since any homogeneous model (be it Λ CDM or any other generalized vacuum energy models) has vanishing shear. In a particular class of inhomogeneous models describing large spherical voids, also known as LTB models, the normalized shear has a maximum at the position of the steepest gradient in the void profile. Supernovae observations are compatible with an LTB model with void size of order a few gigaparsecs, well beyond present galaxy catalogs but within reach of the next generation. In those models, the maximum shear is of order a few percent, and may be difficult to measure over the whole redshift range. In the future, the ability of a galaxy survey to measure the normalised shear could be used as a discriminator between different proposals, in a way similar to (and independent of) the figure of merit for the characterization of the nature of dark energy in terms of an equation of state parameter and its derivative, as advocated by the Dark Energy Task Force.

Remote measurements of the CMB anisotropies through precise determinations of the kinematic Sunyaev-Zel'dovich effect in a handful of clusters, as recently proposed by us [24], or via spectral distortions in the local CMB radiation induced by CMB photons passing through the void that are later rescattered back towards us during reionisation, proposed by Caldwell and Stebbins [46] are other new ways of testing the Copernican

Principle and constraining general void models.

It is worth mentioning here the recent independent observations of a systematically large bulk flow on large scales both above and below 100 Mpc [47, 48, 49], based on the kinematic Sunyaev-Zel'dovich effect in X-ray clusters, and various peculiar velocity surveys in the IRAS-PSCz density field respectively. The size of the flow is significantly above that predicted in the Standard Model and constitutes a challenge for Λ CDM. Whether this new observation is due to a large void at distances of order gigaparsecs is still very uncertain, and we will probably have to wait for the next generation of galaxy surveys to rule out this possibility.

The combination of the steadily increasing body of Type Ia Supernovae, constraining the transverse Hubble rate, together with upcoming galaxy surveys that will measure the radial BAO scale in many redshift slices, such as PAU and BOSS, constraining the longitudinal Hubble rate, will put significant pressure on LTB models, or measure for the first time the shear component of cosmic expansion. If not confirmed by observations, then only a small corner of parameter space – with very smooth void profiles and $H_T \approx H_L$ – will be left.

Note added: When we were finalizing this work, we learned of a similar analysis made by Zibin et al. [25], where they also used the UNION supernova and the radial BAO of Ref. [36, 37] to constrain void models. They also tested their void model against the CMB anisotropies, and used a toy model for the ISW effect. They get similar void sizes, albeit they have bigger difficulties in fitting the data, due to their inclusion of the anisotropies in the CMB.

Acknowledgments

We thank James Zibin, Tirthabir Biswas, and Alesio Notari for interesting discussions. We acknowledge the use of the computer resources at the Danish Centre of Scientific Computing. We also acknowledge financial support from the Spanish Research Ministry, under contract FPA2006-05807 and the Consolider Ingenio 2010 programme under contract CSD2007-00060 “Physics of the Accelerating Universe” (PAU).

References

- [1] <http://www.mso.anu.edu.au/2dFGRS/>.
- [2] <http://www.sdss.org/>.
- [3] <http://www.ctio.noao.edu/wproject/>.
- [4] <http://www.cfht.hawaii.edu/SNLS/>.
- [5] M. Kowalski et al. “Improved Cosmological Constraints from New, Old and Combined Supernova Datasets”. 2008, arXiv: 0804.4142 [astro-ph].
- [6] <http://sdssdp47.fnal.gov/sdsssn/sdsssn.html>.
- [7] <http://www.darkenergysurvey.org>.
- [8] <http://www.ice.csic.es/pau>.
- [9] N. Benitez et al. “Measuring BAO along the line of sight with photometric redshifts: The PAU survey”. 2008, arXiv:0807.0535 [astro-ph].

- [10] <http://cosmology.lbl.gov/BOSS/>.
- [11] <http://www.lsst.org/>.
- [12] Nazeem Mustapha, Charles Hellaby, and G. F. R. Ellis. “Large scale inhomogeneity versus source evolution: Can we distinguish them observationally?”. *Mon. Not. Roy. Astron. Soc.*, 292:817–830, 1997, arXiv: gr-qc/9808079.
- [13] Marie-Noelle Celerier. “Do we really see a cosmological constant in the supernovae data?”. *Astron. Astrophys.*, 353:63–71, 2000, arXiv: astro-ph/9907206.
- [14] Kenji Tomita. “A Local Void and the Accelerating Universe”. *Mon. Not. Roy. Astron. Soc.*, 326:287, 2001, arXiv: astro-ph/0011484.
- [15] John W. Moffat. “Cosmic Microwave Background, Accelerating Universe and Inhomogeneous Cosmology”. *JCAP*, 0510:012, 2005, arXiv: astro-ph/0502110.
- [16] Havard Alnes, Morad Amarzguioui, and Oyvind Gron. “An inhomogeneous alternative to dark energy?”. *Phys. Rev.*, D73:083519, 2006, arXiv: astro-ph/0512006.
- [17] David Garfinkle. “Inhomogeneous spacetimes as a dark energy model”. *Class. Quant. Grav.*, 23:4811–4818, 2006, arXiv: gr-qc/0605088.
- [18] Kari Enqvist and Teppo Mattsson. “The effect of inhomogeneous expansion on the supernova observations”. *JCAP*, 0702:019, 2007, arXiv: astro-ph/0609120.
- [19] Kari Enqvist. Lemaitre-Tolman-Bondi model and accelerating expansion. *Gen. Rel. Grav.*, 40:451–466, 2008, 0709.2044.
- [20] Teppo Mattsson. “Dark energy as a mirage”. 2007, arXiv:0711.4264 [astro-ph].
- [21] David L. Wiltshire. “Dark energy without dark energy”. 2007, arXiv:0712.3984 [astro-ph].
- [22] Stephon Alexander, Tirthabir Biswas, Alessio Notari, and Deepak Vaid. “Local Void vs Dark Energy: Confrontation with WMAP and Type Ia Supernovae”. 2007, arXiv: 0712.0370 [astro-ph].
- [23] Juan Garcia-Bellido and Troels Haugboelle. “Confronting Lemaitre-Tolman-Bondi models with Observational Cosmology”. *JCAP*, 0804:003, 2008, arXiv: 0802.1523 [astro-ph].
- [24] Juan Garcia-Bellido and Troels Haugboelle. “Looking the void in the eyes - the kSZ effect in LTB models”. *JCAP*, 0809:016, 2008, arXiv: 0807.1326 [astro-ph].
- [25] J. P. Zibin, A. Moss, and D. Scott. “Can we avoid dark energy?”. 2008, arXiv: 0809.3761 [astro-ph].
- [26] Timothy Clifton, Pedro G. Ferreira, and Kate Land. “Living in a Void: Testing the Copernican Principle with Distant Supernovae”. 2008, arXiv: 0807.1443 [astro-ph].
- [27] M. Cruz, M. Tucci, E. Martinez-Gonzalez, and P. Vielva. “The non-Gaussian Cold Spot in WMAP: significance, morphology and foreground contribution”. *Mon. Not. Roy. Astron. Soc.*, 369:57–67, 2006, arXiv: astro-ph/0601427.
- [28] M. Cruz, L. Cayon, E. Martinez-Gonzalez, P. Vielva, and J. Jin. “The non-Gaussian Cold Spot in the 3-year WMAP data”. *Astrophys. J.*, 655:11–20, 2007, arXiv: astro-ph/0603859.
- [29] M. et al. Cruz. The CMB cold spot: texture, cluster or void? 2008, arXiv: 0804.2904 [astro-ph].
- [30] William J. Frith, G. S. Buswell, R. Fong, N. Metcalfe, and T. Shanks. “The Local Hole in the Galaxy Distribution: Evidence from 2MASS”. *Mon. Not. Roy. Astron. Soc.*, 345:1049, 2003, arXiv: astro-ph/0302331.
- [31] Benjamin R. Granett, Mark C. Neyrinck, and Istvan Szapudi. “An Imprint of Super-Structures on the Microwave Background due to the Integrated Sachs-Wolfe Effect”. 2008, arXiv:0805.3695 [astro-ph].
- [32] R. Brent Tully. “The Local Void is Really Empty”. 2007, arXiv:0708.0864 [astro-ph].
- [33] Havard Alnes and Morad Amarzguioui. “CMB anisotropies seen by an off-center observer in a spherically symmetric inhomogeneous universe”. *Phys. Rev.*, D74:103520, 2006, arXiv: astro-ph/0607334.
- [34] <http://www.physics.princeton.edu/act>.
- [35] <http://spt.uchicago.edu>.
- [36] Enrique Gaztanaga, Anna Cabre, and Lam Hui. “Clustering of Luminous Red Galaxies IV: Baryon

- Acoustic Peak in the Line-of-Sight Direction and a Direct Measurement of $H(z)$ ". 2008, arXiv: 0807.3551 [astro-ph].
- [37] Enrique Gaztanaga, Ramon Miquel, and Eusebio Sanchez. "First Cosmological Constraints on Dark Energy from the Radial Baryon Acoustic Scale". 2008, arXiv: 0808.1921 [astro-ph].
- [38] George Lemaître. "The Expanding Universe". *Gen. Rel. Grav.*, 29:641–680, 1997.
- [39] Richard C. Tolman. "Effect of inhomogeneity on cosmological models". *Proc. Nat. Acad. Sci.*, 20:169–176, 1934.
- [40] H. Bondi. "Spherically symmetrical models in general relativity". *Mon. Not. Roy. Astron. Soc.*, 107:410–425, 1947.
- [41] James P. Zibin. "Scalar Perturbations on Lemaitre-Tolman-Bondi Spacetimes". *Phys. Rev. D* 78:043504, 2008, arXiv: 0804.1787 [astro-ph].
- [42] D. N. Spergel et al. "Wilkinson Microwave Anisotropy Probe (WMAP) three year results: Implications for cosmology". *Astrophys. J. Suppl.*, 170:377, 2007, arXiv: astro-ph/0603449.
- [43] <http://supernova.lbl.gov/Union>
- [44] Tamara M. Davis et al. "Scrutinizing exotic cosmological models using ESSENCE supernova data combined with other cosmological probes". *Astrophys. J.*, 666:716, 2007, arXiv: astro-ph/0701510.
- [45] Will J. Percival et al. "Measuring the Baryon Acoustic Oscillation scale using the SDSS and 2dFGRS". 2007, arXiv:0705.3323 [astro-ph].
- [46] R. R. Caldwell and A. Stebbins. A Test of the Copernican Principle. *Phys. Rev. Lett.*, 100:191302, 2008, arXiv: 0711.3459 [astro-ph].
- [47] A. Kashlinsky, F. Atrio-Barandela, D. Kocevski, and H. Ebeling. "A measurement of large-scale peculiar velocities of clusters of galaxies: results and cosmological implications". *Astrophys. J. Lett.*, 686:L49–L52, 2008, arXiv: 0809.3734 [astro-ph].
- [48] A. Kashlinsky, F. Atrio-Barandela, D. Kocevski, and H. Ebeling. "A measurement of large-scale peculiar velocities of clusters of galaxies: technical details". 2008, arXiv: 0809.3733 [astro-ph].
- [49] Richard Watkins, Hume A. Feldman, and Michael J. Hudson. "Consistently Large Cosmic Flows on Scales of 100 Mpc/h: a Challenge for the Standard LCDM Cosmology". 2008, arXiv: 0809.4041 [astro-ph].