Nonperturbative effects may lead to an explosive decay of flat direction condensates in supersymmetric theories. We confirm the efficiency of this process with lattice simulations: After only one to five rotations of the condensates in their complex plane, most of their energy is converted into inhomogeneous fluctuations. This generates a gravitational wave background, which depends on the inflaton sector and falls in the hertz–kilohertz frequency range today. These gravity waves can be observable by upcoming experiments such as Advanced LIGO and depend crucially on (i) the initial vacuum expectation value of flat directions when they start to oscillate, (ii) their soft supersymmetry-breaking mass, and (iii) the reheat temperature of the Universe. This signal could open a new observational window on inflation and low-energy supersymmetry.

Gravity-wave (GW) experiments could provide unique information about high-energy phenomena in the early Universe, such as inflation, preheating after inflation, cosmic strings, and first-order phase transitions [1]. In particular, cosmological GW backgrounds will be a target for several high-sensitivity interferometric experiments, planned or proposed, in the frequency range between $10^{-5}$ and $10^{10}$ Hz. In this Letter, we want to consider a new cosmological source of GWs, which emerges naturally in the framework of high-energy physics.

Supersymmetric theories typically involve many flat directions [see 2] for the minimal supersymmetric standard model (MSSM), i.e., directions in field space where the renormalizable part of the scalar potential is exactly flat in the limit of unbroken supersymmetry (SUSY). The flatness is lifted by soft SUSY-breaking terms, nonrenormalizable terms, and SUSY-breaking terms from the finite energy density in the early Universe [3]. Scalar field condensates may develop large vacuum expectation values (VEVs) along these directions during inflation and have several interesting consequences in cosmology [4]. A major example is the Affleck-Dine mechanism for baryogenesis [5]. Condensates with large VEVs start to oscillate when the Hubble rate $H$ becomes of the order of their soft mass $m \sim \text{TeV}$ [6], with an initial amplitude that is model-dependent and can be as high as the Planck scale [3,8]. It has recently been shown [9–11] that nonperturbative resonant effects may lead to an explosive decay of these coherent oscillations. We will show that this generates GWs that can be observable by upcoming experiments [12].

The resonant decay of flat directions shares many similarities with preheating after inflation, when the inflaton decays into large, nonthermal fluctuations of itself and other bosonic fields [13]. GWs from preheating have been intensively studied recently [14–17]. Reference [16] developed theoretical and numerical methods to calculate GW production from dynamical scalar fields in an expanding universe. We will apply and extend these methods to the decay of flat directions.

The resulting GWs, however, will have different properties. GWs from preheating have been studied so far in two main classes of models: parametric resonance after chaotic inflation and tachyonic preheating after hybrid inflation. In the first case, the typical GW frequency today is of the order of $f \sim 10^7$ Hz, which is too high to be observable. Hybrid inflation models may occur at lower energy scales, so it was initially hoped that GWs from tachyonic preheating may fall into an observable range. However, this generically requires extremely small coupling constants [17]. By contrast, we will see that GWs from the nonperturbative decay of flat directions fall naturally in the hertz–kilohertz frequency range today. Another difference—which will be crucial for the GW amplitude—is that, whereas the inflaton dominates the energy density before it decays, flat directions will typically be subdominant. When they decay, the Universe can then be matter-dominated if the inflaton is still oscillating around its minimum or radiation-dominated if it decays earlier and its decay products thermalize when $H > m$. Both cases lead to different GW spectra, which thus also carry information on the inflaton sector.

To illustrate why it may be difficult for GWs from preheating to be observable, consider a simple model $V = m^2 \phi^2 + g^2 \phi^2 \chi^2$, with two real scalar fields $\phi$ and $\chi$. In this model, $\chi$ particles are produced due to the nonadiabatic evolution of their frequency as the condensate $\phi$ oscillates around the minimum [13]. For $m \sim \text{TeV}$, this could a priori describe the decay of a flat direction $\phi$. We will denote by an index “$i$” the time $i$, when the flat direction starts to oscillate ($H_i = m$) and by $\Phi_i$, its initial amplitude at that time, and we normalize the scale factor to $a_i = 1$. The spectra of the quanta amplified by preheating are usually strongly peaked around some typical (comoving) momentum $k_*$, which is inherited by the GW spectrum.
[15–17]. For the model above, $k_s \sim \sqrt{g_m \Phi_i} a^{1/4}$ [13]. Assuming for simplicity that the Universe is already radiation-dominated during the decay, this leads to the estimate $f_s \sim \sqrt{g_0 \Phi_0 / M_{Pl}} 10^{14}$ Hz for peak frequency of the resulting GWs today. Taking, for instance, $\Phi_0 = 10^{-2} M_{Pl}$, $f_s < 10^{15}$ Hz requires an extremely small coupling constant $g^2 < 10^{-28}$. If $\Phi_i$ is smaller, the GW amplitude is too low to be observable. Indeed, since the energy density in the flat direction $\rho_{\text{flat}}$ is subdominant, only a small fraction $\rho_{\text{flat}} / \rho_{\text{tot}}$ of the total energy density is available for the production of GWs. We will see that the fraction of energy density in GWs varies as $(\rho_{\text{flat}} / \rho_{\text{tot}})^2$, which can be very small.

However, flat directions are complex fields, and non-renormalizable terms usually generate a velocity for their phase when $H \sim m$ [3,5]. The subsequent dynamics is dominated by the $m^2 |\phi|^2$ term, resulting in a constant, non-zero phase velocity and an elliptical motion for $a^{3/2} \phi$ in its complex plane. This leads to time-dependent mixings between different excitations around flat directions, which may result in a very efficient mechanism of particle production [9,10], significantly different from the model considered above. In particular, these particles have a typical momentum $k_s \sim m$. Formally, this amounts to replacing $\sqrt{g_0 \Phi_0}$ by $\sqrt{m}$ in the previous model. This gives a peak frequency of GWs that is independent of $\Phi_i$ and of order $10^{17}$ Hz today for $m \sim \text{TeV}$.

This encouraging estimate should be complemented by detailed computations. The resonant decay of flat directions has been studied so far only in the linear regime, neglecting any backreaction of the quanta produced. Backreaction is eventually responsible for most of the decay, and we have to use lattice simulations to fully take it into account. We will simulate the model [9,18]

$$V = m^2 |\phi|^2 + m^2 |\chi|^2 + \frac{g^2}{2} (\phi^* \chi + \phi \chi^*),$$  

(1)

where * denotes the complex conjugate, $\phi$ the flat direction field, and $\chi$ its decay products. This potential includes the relevant terms for excitations around MSSM flat directions (where the interaction comes from the $D$ term), but at least two flat directions have to be considered when the symmetry broken by their VEV is gauged [9]. This case was studied in Refs. [10,11] and found to be largely similar to the model (1), so here we focus on (1) for simplicity. We perform lattice simulations with the parallel code CLUSTEREASY [19], and we compute the resulting GW spectrum with the method of Ref. [17] (extended here to an expanding Universe).

Consider first the nonperturbative decay itself. The background evolution of the condensate is given by $\dot{\phi} = (\cos \epsilon t + i \epsilon \sin \epsilon t) \Phi_i / \sqrt{2a^3}$. The “ellipticity” $\epsilon \ll 1$ is model-dependent but typically of order $\epsilon \sim 0.1$ [3]. An analytical study of particle production in this background will appear elsewhere. Here we just present the results. It is convenient to introduce the parameter $q = q_i / a^3$, with $q_i = g^2 \Phi_i^2 / m^2$. Note that $q_i$ can be as high as $10^{30}$ for $\Phi_i$ close to the Planck scale. In the realistic case $g e^\epsilon \gg 1$ [20], particle production occurs because the physical eigenstates vary with time [9]. Figure 1 shows the spectra of the $\chi$ modes in a Minkowski background when backreaction is still negligible. The resonance is characterized by large instability bands separated by narrow stability bands, contrary to parametric resonance but similar to the tachyonic resonance studied in the second reference of [21]. When the trajectory of the condensate in its complex plane is circular $\epsilon = 1$, there is a tachyonic amplification of the modes with $k^2 + m^2 < m^2$ [9]. When $\epsilon < 1$, the amplification is more efficient, and extra instability bands appear in the UV, roughly up to $k < m / \epsilon$. The IR band has the highest amplitude, except if $m_\chi / m < 1$. The peak of the spectrum is located at $\kappa_s = k_s / m = 0.1$. In an expanding Universe, the physical momenta $k / a$ “move” in the resonance pattern, and the peak is shifted to a comoving momentum $k_{\text{tot}} \sim 1$. When the $\chi$ modes have been sufficiently amplified, they backreact on the condensate and convert its energy into inhomogeneous fluctuations of both fields. This will be responsible for most of the GW production. We show the evolution of the mean of $a^{3/2} \phi$ in its complex plane in Fig. 2. The field first follows its elliptical trajectory, but the coherent motion is then quickly destroyed by backreaction. This typically happens after only 1–5 rotations of the condensate (the higher $g^2$, the sooner it occurs).

We now turn to the actual computation of GWs. Their evolution from the time of production (that we will denote with an index “p”) up to now depends on the background equation of state. Their present-day frequency and energy density per logarithmic frequency interval [1] then read

$$f = \frac{k}{\rho_i^{1/4}} \left( \frac{a_i}{d_{\text{RD}}} \right)^{1/4} \times 10^{10} \text{Hz},$$

$$h^2 \Omega_{\text{GW}} = 9.3 \times 10^{-6} \frac{1}{\rho_i} \left( \frac{d_i}{d_{\text{RD}}} \right)^3 \frac{1}{d_{\text{ink}}} \left( \frac{\rho_i}{d_{\text{RD}}} \right),$$  

(2)

where $k$ is the comoving wave number, $\rho_i$ is the total
energy density at $t_i$ and we used $h^2 \Omega_{\text{GW}}(g_i/g_*)^{1/3} \approx 9.3 \times 10^{-6}$. The factors in $a_i/a_{RD}$ occur if the Universe is matter-dominated (MD) during the decay and becomes radiation-dominated (RD) later, when $a = a_{RD}$. They are absent if it is already RD at $t_i$ [22].

We can estimate how the GW spectrum depends on the parameters from the characteristic physical length $R_\gamma = a_i/k_\gamma$ of the scalar field inhomogeneities [15–17], but we have to take into account that the flat direction energy density $\rho_{\text{flat}}$ is usually subdominant. We then estimate the fraction of energy density in GWs at the time of production as $[23] \rho_{\text{GW}}/\rho_{\text{tot}} \sim 0.1(R_iH_i^2)(\rho_{\text{flat}}/\rho_{\text{tot}})^2$. By using $\rho_{\text{flat}} = m^2 \Phi_i^2$, $k_\gamma = k_\gamma m$, and $H_i = m$, Eqs. (2) give

$$f_* \approx \kappa_a \left( \frac{a_i}{a_{RD}} \right)^{1/4} 5 \times 10^2 \text{ Hz},$$

$$h^2 \Omega_{\text{GW}} \sim \frac{10^{-4}}{\kappa^2} \left( \frac{\Phi_i}{M_{Pl}} \right)^{4} \left( \frac{a_i}{a_{RD}} \right)$$

for the frequency and amplitude of the GW spectrum’s peak. Note that the amplitude is very sensitive to $\Phi_i$. Note also that Eq. (4) cannot be extrapolated to arbitrary large $\Phi_i$. If a flat direction acquires an initial VEV of order $M_{Pl}/3$ or larger, it dominates the total energy density before decaying and its oscillations are delayed until $\Phi_i \sim M_{Pl}/3$. Thus Eq. (4) with $\Phi_i = M_{Pl}/3$ can be considered as an upper limit on the GW amplitude, i.e., $h^2 \Omega_{\text{GW}} \lesssim 10^{-6}$. The same bound follows from Ref. [23] with $R_i \lesssim 1/H$ and $\rho_{\text{flat}} \leq \rho_{\text{tot}}$ [17].

We confirmed the predictions (3) and (4) with intensive lattice simulations. Two different scales have to be kept under control numerically: the mass and the VEV of the condensate, whose ratio is measured by $q_i$. In our simulations, $m$ defines the natural unit of time, while $1/\sqrt{q_i}$ defines the time step required to accurately follow the dynamics. Clearly, we cannot simulate values of $q_i$ as high as $10^{60}$. However, we saw above that the linear stage of particle production is independent of $q_i$; see also [9]. Indeed, the GW spectra that we obtained for different $q_i$ (by varying $g^2$) were nearly identical, as far as $q e^{\epsilon} \gg 1$ was satisfied. We were able to simulate values of $q_i$ ranging from $10^4$ to $10^8$, which allowed us to take $\epsilon$ ranging from 0.2 to 1. Smaller values of $\epsilon$ lead to faster particle production, which could increase the GW amplitude. Increasing the mass of $\chi$ has the opposite effect and makes the UV modes relatively more important (which makes this case more difficult to simulate). However, this becomes negligible when $m_q$ differs significantly from $m$. This is probably more natural in models with multiple flat directions [10]. We varied $g^2$ from $10^{-2}$ to 0.5, $\Phi_i$, from $10^{-13} M_{Pl}$ to $10^{-1} M_{Pl}$, and $m$ from 100 GeV to 10 TeV, finding excellent agreement with Eqs. (3) and (4).

In Fig. 3, we compare our results with the sensitivity of interferometric experiments. We consider the soft masses $m = 100$ GeV and $m = 10$ TeV for different initial VEVs $\Phi_i$ and reheat temperatures $T_R$. For $T_R \gtrsim 0.2\sqrt{m M_{Pl}}$, the Universe is RD during the decay and the GW amplitude is independent of $m$. If $T_R$ is lower, GWs are further diluted and redshifted by the factors in $a_i/a_{RD}$ in Eqs. (2), which depend on $m$. Assuming a fast transition from MD to RD when the Universe reheats, we have $a_i/a_{RD} \approx (5 T_R^2/\sqrt{m M_{Pl}})^{2}$. We show the results for $T_R = 10^5$ [24] and $10^7$ GeV. These GWs can be observable for a reasonable range of soft masses and reheat temperatures, but this generically requires that some flat directions acquire a very large initial VEV. A detection would be easier for low $m$ or high $T_R$, but we should have at least $\Phi_i > 10^{17}$ GeV.

The initial amplitude of flat direction oscillations depends on the lowest order of the nonrenormalizable terms for a given direction [3]. A superpotential term like $d^n/M_{n^{n-3}}$ (with $M$ the cutoff scale) leads to $\Phi_i \sim (m M_{n^{n-3}})^{1/(n-2)}$ [3] when $H \sim m$. In the MSSM, $n = 9$ is the minimal value allowed by gauge invariance for the flattest direction [2]. This gives $\Phi_i \sim 10^{16}$ GeV for $M$ the reduced Planck mass, which would be too small for the GWs computed here to be observable by currently proposed experiments. However, the fact that this term is allowed by gauge invariance does clearly not imply that it is indeed present; e.g., it can be forbidden by other symmetries. Higher-order terms lead to larger values of $\Phi_i$ and

![FIG. 3](color online). Sensitivity sketch of Advanced LIGO and Einstein Telescope [1] compared to GW spectra produced in the model (1). In each case, the spectrum at the left is for $m = 100$ GeV and the one at the right for $m = 10$ TeV.)
therefore to GWs that can be observable. Indeed, initial VEVs can be as high as the Planck scale [3], e.g., in no-scale supergravity models [8].

In summary, we performed the first nonlinear study of the nonperturbative decay of supersymmetric flat directions with lattice simulations. This very efficient process can have important consequences in cosmology, in particular, for baryogenesis and reheating after inflation. We showed that it generates a stochastic background of GWs that depends crucially on the initial VEV of flat directions when they start to oscillate, on their soft SUSY-breaking mass, and on the reheat temperature of the Universe. The specific spectral properties that we computed can be used to distinguish these GWs from other possible cosmological backgrounds. Of particular interest is that they fall in the LIGO frequency range, where stochastic backgrounds are not expected to be significantly screened by astrophysical sources. For these GWs to be observable by upcoming experiments, it is necessary that some flat directions acquire a large enough initial VEV; see Fig. 3 and Eqs. (3) and (4). This typically requires the flatness of the potential to be protected up to high order by symmetries, which is model-dependent. Our results strongly motivate the study of more complex models, with multiple flat directions and local gauge invariance [9,10]. In particular, vector fields are likely to increase the range of parameters leading to an observable signal [16]. Any possibility of direct detection has to be studied in depth, because it would provide crucial information on both supersymmetry and the very early Universe.

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[6] Condensates with VEVs smaller than the ones we will be interested in here may start to oscillate earlier due to thermal effects [7]. We assume that inflation occurs at \( H_{\text{inf}} > \text{TeV} \) and gravity-mediated SUSY breaking.
[12] Kusenko and co-workers consider GWs from the fragmentation of condensates into \( Q \) balls. [A. Kusenko and A. Mazumdar, Phys. Rev. Lett. 101, 211301 (2008); A. Kusenko, A. Mazumdar, and T. Multamaki, Phys. Rev. D 79, 124034 (2009)]. The nonperturbative decay that we consider occurs on significantly shorter time scales, so the fragmentation into \( Q \) balls is not relevant here.
[18] In a complete model, the potential includes also non-renormalizable and Hubble-dependent terms, which generate an initial phase velocity for the condensate when it starts to oscillate. However, these model-dependent terms become quickly negligible as the Universe expands. We thus neglect these terms in the simulations and treat the initial phase velocity as a free parameter.
[20] The case \( q \epsilon^4 \ll 1 \) is similar to the model with two real scalar fields mentioned earlier.
[22] We found that the equation of state of the flat direction decay products jumps quickly towards \( w = 1/3 \), as in Ref. [21], so usually the Universe remains RD if it is so at \( t_i \).
[23] It is instructive to derive this estimate as follows [see also C. Caprini, R. Durrer, and G. Servant, Phys. Rev. D 77, 124015 (2008)]. In the notations of Ref. [16], the GW equation gives \( \dot{h}_{ij} \sim h_{ij}/R \sim 16\pi\rho\epsilon^2 T_{ij}^{\text{GR}} \) when the source is growing. We estimate \( T_{ij}^{\text{GR}} \sim a^2 \rho_{\text{grad}} \), where \( \rho_{\text{grad}} \) is the gradient energy density of the scalar fields. This is a fraction of order 0.1–1 of the flat direction energy density \( \rho_{\text{ad}} \) (condensate + inhomogeneities). This gives \( \rho_{GW} = (h_{ij}h_{ij})/(32\pi G) \sim GR^2 \rho_{\text{flat}} \). Dividing by \( \rho_{\text{tot}} \sim 0.1H^2/G \) gives the estimate of the main text.
[24] This corresponds to an inflaton with a mass of order \( 10^{13} \text{ GeV} \) and only gravitational couplings. The usual gravitino bound is marginally satisfied in this case.