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This version: March 2018

This version: March 2018
(June 2017)

Barcelona GSE Working Paper Series

Working Paper nº 971

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March, 2018

Abstract

We study a model of advertising targeting based on information about the consumer's likely ranking of products. With horizontally differentiated goods and costly search, ads then convey to consumers a noisy, positive signal of their unknown willingness to pay for the firms' products. That implies a higher expected willingness to pay for a yet not sampled firm, which increases the incentives to search, but also a lower expected differentiation of the products that the consumers learn about, which reduces the incentives to search. The first effect is more important for lower search costs, and the second for larger number of products. Also, the equilibrium intensity of advertising will affect the precision of consumers' information. Larger marginal cost of advertising results in larger endogenous segmentation and larger prices.

Keywords: random advertising, targeted advertising, horizontal differentiation, sequential search

JEL classification: L13, D83

*We are grateful to Simon Anderson, Dan Bernhard, José Luis Moraga-González, Régis Renault, Chris Wilson, the participants of the IX$^{\text{th}}$ Workshop on the Economics of Advertising and Marketing, the 15th IIOS Conference, EARIE-2017 and the paricipants of seminars at IAE (CSIC) for their invaluable comments and suggestions. The authors acknowledge the financial support of the Spanish Ministry of Economy and Competitiveness through the Severo Ochoa Programme for Centres of Excellence in R&D (SEV-2015-0563), grants No. ECO2014-59959-P and No. ECO2015-74328-JIN (AEI/FEDER/UE), and 2016 FBBVA grant "Innovación e Información en la Economía Digital".

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1 Introduction

For decades now, search-theoretic models have been a standard tool to study, among other things, the incentives and consequences of advertising (see, for instance, Baye, Morgan, and Scholten, 2006; Stahl II, 1994). More recently, and to a large extent as a response to the availability of enhanced information technologies, search-models have also been standard tools to analyze the consequences for consumers and market outcomes of firms' ability to target their advertising. Targeting has been modeled as the ability to make the (costly) decision of informing one consumer a function of (a perhaps imperfect[1] signal of) her willingness to pay for the firm's product (e.g., Renault (2016), Bergemann and Bonatti (2015)).

The starting point of this paper is the observation that much of the information that firms use to target their ads is not directly related to the consumer's willingness to pay for their product. Instead, in a world of horizontal product differentiation, that information refers rather to the relative fit of a firm's product characteristics to the consumer's tastes. Online search habits, lists of favorite YouTube videos, or likes on Facebook may tell more about the fit of a product's particular characteristics to the consumer's tastes (sport versus formal design shoes, sci-fi versus romantic book, etc.) than about the consumer's (average) willingness to pay for the product. That means that, when targeting, firms may have at their disposal information about the consumer's relative –to rivals' products– willingness to pay for its product rather than information about her absolute willingness to pay for it.

In this paper, we propose a model to analyze the consequences for market outcomes of firms' ability to target their advertising based on this type of information in markets with costly, sequential consumer search and horizontally differentiated products, as in Wolinsky (1986). Similarly to Butters (1977) and Grossman and Shapiro (1984), consumers may visit a firm only after receiving an ad from this firm.[2] Ads do not carry any information about product characteristics or prices.[3]

We analyze two advertising regimes: random and targeted advertising. In the first regime, firms cannot identify any relevant characteristic of a consumer, and therefore they send their ads randomly. In the second regime, firms can identify and target their ads to consumers who rank their products high. Targeting is not perfect, though, and so there is still some residual uncertainty about which product best fits a consumer's preferences.[4] Thus, in both advertising regimes, there is room for consumer search.

The possibility of targeting advertising based on this type of information has several effects

[1]During the experiment of Mediasmith with the data of different consumer data vendors, an increase in advertising accuracy through targeting varied from 5% to 183% (Marshall, 2015). Forbes (2015) notes that 54% of North American companies cite the identification of a target audience as a primary challenge. According to Loechner (2014), "*92% of Americans ignore at least one type of ad seen every day across six different types of media*", and the survey of Reuters shows that 39% and 47% of consumers in the UK and the US respectively use ad-blocking software (Austin and Newman, 2015).

[2]This standard assumption captures the fact that a consumer, due to impatience and the lack of time, often searches within a subset of products instead of all existing goods even if the total number of sellers is known.

[3]In online retail, consumers often receive promotional e-mails containing nothing but invitations to visit stores and check their new products. The ads contain little more than small advertising banners with firms' logos. Thus, there is scant product information (Honka et al. (2017)). Likewise, advertisement in sports events, on sponsored teams uniforms, or on public transport fall comfortably into this category.

[4]In contrast, Athey and Ellison (2011), Chen and He (2011) and Anderson and Renault (2015),Chen and Zhang (2017) assume that consumers know the ranking of products that they search.

on firms' incentives. Keeping the intensity of advertising constant, targeting increases the information content of ads. Consumers learn about the –probabilistically– most relevant competitors, given their idiosyncratic preferences, which raises the value of search. More search increases the competitive pressure on firms, and so acts as a downward pressure on prices. We call this effect a *competition effect*. But in addition, the products that consumers learn about are more valuable and also closer substitutes on average. When this selection effect of advertising is strong –i.e., when there are sufficiently many varieties in the market– the value of search is lower, in expected terms, and demand less elastic. This encourages sellers to raise their prices. We call this the *demand composition effect*. This second effect is closely related to the Diamond (1971) paradox, and its strength increases with the number of firms. Indeed, as this number gets larger, the expected differentiation of the products that best fit the consumer´s preferences gets smaller. Thus, in expectation, the gains from search also shrink and, as the number of firms grows, this demand composition effect eventually dominates the competition effect. As a consequence, the equilibrium price is higher under targeted advertising. If instead search costs and the number of firms are small, then the competition effect dominates, and the equilibrium price is higher under random advertising.

Targeting also affects the incentives of firms to advertise. Moreover, the intensity of advertising affects the information that consumers may infer from receiving ads, and so their incentives to search. Indeed, firms prefer to target those consumers for whom their product is likely to be among the best matches to the consumer's preferences. So, other things equal, firms advertise first to those consumers for whom their product is more likely to be their best match. If they send more ads, then they will target those for whom their product is more likely to be the second best match, etc. In the limit –say, if advertising is very cheap– firms may send ads to all consumers. In the latter case, ads carry the same information as in the case of non targeted (random) advertising. Contrary, if firms base their decisions on highly correlated information and they only send ads to consumers for whom their product is most likely to be the best match, receiving an ad (probably the only one) from a firm tells the consumer that the firm carries the product that most likely fits her preferences. That is, the intensity of advertising endogenously determines the implicit information conveyed by ads, and so –inversely– the consumers' incentives to search. We show that, when the marginal cost of advertising is high, so that firms are selective with their ads, the effect of this endogenous increase of the precision of consumers' information that we term *endogenous monopolization* effect, results in prices that are higher than under random advertising.[5]

As we have already mentioned, our contribution is to study targeting based on information concerning relative ranking of products in the consumer's preferences, rather than the absolute willingness to pay for the firm's product. This takes us away from stationary models of search, where consumers' beliefs do not change as they proceed with search. Indeed, targeting introduces endogenous correlation in the consumer's willingness to pay for the different products she learns about. Spatial models are alternative ways of capturing this correlation, by assuming it exoge-

[5] Galeotti and Moraga-González (2008) and Manduchi (2004) study interesting models with homogeneous products and utility-irrelevant consumers' observable characteristics. Firms may target their ads to different-looking –identical, in all relevant characteristics– consumers, so as to segment the market and reduce competition.

nously, at the outset. That is the modeling choice of Ben Elhadj-Ben Brahim et al. (2011), in a Hotelling model and two firms.[6] de Cornière (2016) uses a Salop model with a continuum of firms, which once more eliminates the ex-post (and endogenous) correlation, as the consumers' beliefs are again unchanged as search proceeds. Contrary, our modeling choice allows to consider a finite number of firms and study how this number of firms, and so underlying competition, affects market outcomes.

Information about rankings is central in models that study online search mediated by search engines, like Athey and Ellison (2011). Our model focuses on firms' pricing incentives and the intensity of advertising, as opposed to the relationship between the engine and firms accessed through the engine (which determines the display of rankings). Also, we assume that ads themselves do not inform the consumer directly of the ranking of valuations, as opposed to Chen and He (2011) (the order of sponsored links in the engine *may reveal* this ranking), who also study a clever model where consumers' beliefs do not change as they proceed with search.[7] Chen and He (2011), also discuss a second equilibrium of their model where sellers appear randomly in the list of sponsored links. However, the particular distribution of willingness to pay in that paper –with some probability, a common knowledge number v, otherwise 0 – means that the price is virtually exogenous (equal to v) in both equilibria. Welfare comparisons are driven by search costs.

We specify our model in Section 2. In the targeting case, firms receive a signal for each consumer. The signal informs the firm, but not the consumer, about the probability that its product is the best match –alternatively, the second best match– for that consumer, if that probability is positive. Otherwise, the firm learns that its product is not among the best two for that consumer. This modeling choice renders our model perhaps the simplest possible with horizontally differentiated producers learning imperfectly about the relative ranking of products for each consumer.

We first characterize consumer search with random and targeted advertising in Section 3. We take a first look at equilibrium prices in Section 4. We do so by fixing the advertising intensity at the level that is simplest to analyze, where all firms send ads to all consumers for whom their product may be among the best two matches. This allows us to clearly identify and discuss, in Section 5, the competition and demand composition effects we mentioned above. Moreover, extending the conditions for equilibrium to the general advertising intensity case is then a simple exercise. We do so in Section 6, where we also characterize optimal advertising strategies. Section 7 solves a parametric example that illustrates the effects of targeting not only on prices, but also on consumer surplus and profits. Section 8 concludes.

2 Model

The basis of our model is Wolinsky's (1986) model. Each of $N > 2$ symmetric firms offers one –horizontally differentiated– product to a continuum of consumers of measure one. Firms' marginal production costs are constant, identical, and normalized to zero, so they only incur advertising costs. Firms face no capacity constraints.

[6]Iyer et al. (2005) can be viewed as a extreme version of this, with consumers only at the extremes of the line or at the middle point.

[7]See also Chen and Zhang (2017), Anderson and Renault (2015), and the references therein.

There is a continuum of consumers of mass one, modeled as the unit interval. Consumer $j \in [0,1]$ has type $(\boldsymbol{z}^j, \beta^j)$, where $\boldsymbol{z}^j = \left(z_1^j, z_2^j, ..., z_N^j\right)$; z_i^j is the realization of a (independent across i and j) random variable with a log-concave distribution function $F(z)$ and a log-concave, increasing density function $f(z)$ in the interval $[0,1]$; and β^j is the realization of a (independent across j) uniform random variable distributed in $\left[\frac{1}{2}, \alpha\right]$ with $\alpha \in \left[\frac{1}{2}, 1\right]$. The value of z_i^j represents the willingness to pay (valuation) of consumer j for the product sold by firm i. Thus, if consumer j buys that product i at a cost—price paid and search costs— of m, her (von-Neumann Morgenstern) utility is

$$u^j = z_i^j - m.$$

We will assume throughout that z_i^j can be observed only by consumer i, and only after visiting firm i. Also, the utility of not buying any product is 0 minus any search cost incurred.

Firms choose the number (measure) of ads they send, $\mu \in [0,1]$, at a cost $C(\mu)$, where C is a differentiable, increasing, strictly-convex function with $C(0) = 0$. Consumers know the number of firms N but do not know either the "location" or the price of any firm.[8] Receiving an ad from firm i allows consumer j to (locate and) visit firm i at a cost s, whereupon she learns both the price P_i and the value of z_i^j.[9] Upon this visit, the consumer may either buy or visit, at cost s, another firm from which she has received an ad. Returning to a previous visited firm is costless, and we assume that firms cannot price discriminate. Therefore, if consumer j buys from firm i after visiting n firms, her total cost of purchase is $m = P_i + n \times s$.

The timing of the game that firms and consumers play is as follows. First, firms simultaneously and independently choose how many ads to send, to whom, and what prices to charge. After receiving ads, consumers decide which firm to visit first, if any. Upon visiting firm i and learning z_i^j and P_i, a consumer decides whether to buy, to search further, or to return to buy from a previously visited firm.

Our modeling innovation is captured by the component β^j in the consumer's type. Given $\boldsymbol{z}^j$, β^j measures the parameter of a (conditional) distribution, $X^j(\boldsymbol{z}^j) = (i_1(\boldsymbol{z}^j), i_2(\boldsymbol{z}^j))$ on $N \times N$, that takes the value $i_1(\boldsymbol{z}^j) = \arg\max_i z_i^j$ and $i_2(\boldsymbol{z}^j) = \arg\max_{i \neq i_1(j)} z_i^j$ with probability β^j and the value $i_2(\boldsymbol{z}^j) = \arg\max_i z_i^j$ and $i_1(\boldsymbol{z}^j) = \arg\max_{i \neq i_1(j)} z_i^j$ with probability $1 - \beta^j$. That is, β^j is the probability that the (partial) observed ranking $X^j(\boldsymbol{z}^j)$ is correct, so that $i_1(\boldsymbol{z}^j)$ is indeed the best match for the consumer and $i_2(\boldsymbol{z}^j)$ is the second best match, and $1 - \beta^j$ is the probability that the ranking $X^j(\boldsymbol{z}^j)$ is reversing the two positions.[10] We will examine—and compare—two versions of the model. In the first, nobody can observe (either β^j or) the realization of $X^j(\boldsymbol{z}^j)$. Thus, to all effects this part of the type is irrelevant, and we recover Wolinsky's (1986) model with the addition of ads. In particular, when firms advertise, they cannot target their ads to consumers.

In the second version, firms, but not consumers, can observe both β^j and the realization of $X^j(\boldsymbol{z}^j)$, so that they can use this information to target their ads. That is, the firms can

[8] Differently, Haan and Moraga-González (2011) assume that consumers give priority to the firms whose ads they have received and may search other firms too.

[9] Firms issue ads that do not contain any information about products, e.g. online banners with firm logos, leaflets inviting to the opening of a store, promotional e-mails inviting to visit an online store to see new arrivals, etc.

[10] Thus, β^j is also the probability that i is the best match for consumer j conditional on the ranking $X^j(\boldsymbol{z}^j)$ with $i_1(\boldsymbol{z}^j) = i$.

(imperfectly, and partially) observe the ranking of their products in the preferences of a consumer. A consumer with a value of β^j close to 1 (only possible when α is close to one) for whom the realization of $X^j(\boldsymbol{z}^j)$ is (i, i') most likely has product i as the most desirable for her, and i' as the runner up. On the contrary, if β^j is close to 1/2 for that consumer, both rankings (i, i') and (i', i) are (almost) equally likely. Firms observe this information (in this version) and so may decide whether to send an ad to consumer j based on the value of β^j and the realization of $X^j(\boldsymbol{z}^j)$.[11] Note that this information does not refer (directly) to the value of any component of $\boldsymbol{z}^j$. (Obviously, conditional on this information the distribution of the random vector $\boldsymbol{z}^j$ changes.)

In the next three sections, we will study the limit case where $\alpha = 1/2$, and so $\beta^j = 1/2$ for all consumers, for a given level of advertising μ. That is, all that $X^j(\boldsymbol{z}^j)$ identifies is the two best matches for consumer j's preferences. That will allow us to most clearly illustrate two of the effects of targeting that we have referred to: the competition and demand composition effects. Then, in Section 6 we will consider the general model, with $\alpha \neq 1/2$ as well as the choice of μ.

3 Search behavior

Let us then assume that $\alpha = 1/2$. That is, in the targeting treatment, when firms observe $X^j(\boldsymbol{z}^j)$ and (trivially in this case) β^j for all consumers, each firm can identify the set of consumers for whom the firm's product ranks among the consumer's best two matches. We call these consumers the *targetable consumers* for the firm. Also, we assume that firms send a measure of ads μ in the non-targeting treatment and also that firms send ads to all their targetable consumers in the targeting treatment.

From a consumers' point of view, all sellers from whom she receives ads are symmetric in both treatments. Thus, a consumer visits firms of which she knows about in a random order. At each step, the customer compares the expected gains from searching one more product with the search cost, s. If the gains are higher than the search cost, the consumer continues searching. Otherwise, the customer stops searching and buys the product providing the highest (positive) observed utility (if any). Thus, the consumer will terminate search if (either she knows of no more firms or if) the highest observed utility is higher than a certain threshold. The value of this threshold depends on whether advertising is targeted or not.

We first analyze consumers' optimal search behavior when conjecturing some symmetric price P^*.

Random advertising. When ads are random, the observed match value for the product of a firm does not give any information about not-yet-searched products. Here, as in Weitzman (1979), a consumer whose best option among the already sampled firms gives her a utility u ($= z_i^j - P \geq 0$ for some visited seller i) expects a gain from an additional visit to a new firm—where she expects price P^*—equal to

$$\int_{u+P^*}^{1} (z - P^* - u)\, dF(z) - s.$$

[11] Note that the ad does not contain information about β^j or $X^j(z^j)$. Unless this was based on "hard signals", which is not the case usually, any such information would not be credible, as any firm i would have incentives to claim $\beta^j = 1$ and $X^j(z^j) = (i, .)$.

Thus, when all firms charge the same price ($P = P^*$), consumers will keep searching a long as (they have a new firm to search and) the highest match value they have found is below the threshold w that solves

$$\int_w^1 (z - w)\, dF(z) = s. \tag{1}$$

The left-hand-side of (1) are the gains from visiting a new firm that is expected to charge P^* when the highest match value sampled so far is w. The right-hand-side of the equation is the search cost paid for the visit.

Targeted advertising. After observing the match value of one firm's product, a consumer makes an inference about the match value of the product sold by the other firm she knows about. Without loss of generality, let us suppose that the consumer has received ads from firms 1 and 2 and she visits firm 1 first.

The match value z_1 may be the highest existing for the consumer. The conditional probability of this event equals

$$\frac{Nf(z_1)\, F(z_1)^{N-1}}{Nf(z_1)\, F(z_1)^{N-1} + N(N-1)\, f(z_1) F(z_1)^{N-2}(1 - F(z_1))}.$$

Likewise, z_1 may be the second highest match value for the consumer, an event with (conditional) probability

$$\frac{N(N-1)\, f(z_1)\, F(z_1)^{N-2}(1 - F(z_1))}{NF(z_1)^{N-2} f(z_1)\,(F(z_1) + N(N-1)(1 - F(z_1)))}.$$

Thus, the density function of z_2 conditional on z_1 is

$$g(z_2|\, z_1) = \begin{cases} \frac{(N-1) f(z_2) F(z_2)^{N-2}}{F(z_1)^{N-1}} \frac{F(z_1)}{F(z_1)+(N-1)(1-F(z_1))} & \text{if } z_2 \le z_1, \\ \frac{f(z_2)}{1-F(z_1)} \frac{(1-F(z_1))(N-1)}{F(z_1)+(N-1)(1-F(z_1))} & \text{if } z_2 > z_1. \end{cases} \tag{2}$$

The expected gains from searching product 2, having observed match value z_1 and price P_1, with $z_1 \geq P_1$, equal

$$\int_{z_1 - P_1 + P^*}^1 (z_2 - P^* - z_1 + P_1)\, g(z_2|z_1)\, dz_2. \tag{3}$$

Similarly to the random advertising case, the consumer will continue searching after the first visit if z_1 is below the threshold that equates (3) to s. The expression (3) is decreasing in z_1. Thus, the threshold is well defined and is decreasing in s. In an equilibrium, where $P_1 = P^*$, that value solves an equation similar to (1) with only substituting $g(z_2|z_1 = w)$ for $f(z)$.

Maximum search costs. To ensure that some consumers search in equilibrium, we need to impose an upper bound on the search cost. In the non-targeting treatment, this upper bound may be $\bar{s} = \int_{P^M}^1 \left(z - P^M\right) dF(z)$, where P^M is an optimal price of a single-product monopolist: $P^M = \arg\max_P P\,(1 - F(P))$.

In the targeting advertising treatment, and since (3) is decreasing in z_1, search (beyond the first visit) may be part of an equilibrium with price P^* if

$$s \le \int_{P^*}^{1} (z_2 - P^*)\, g\,(z_2 | z_1 = 0)\, dz_2 = \int_{P^*}^{1} (z_2 - P^*)\, f\,(z_2)\, dz_2, \tag{4}$$

Thus, much as in the non-targeted advertising case, a sufficient condition for search (beyond the first visit) to be part of any symmetric equilibrium is that $s \le \bar{s}_t = \int_{P_t^M}^{1} \left(z - P_t^M\right) dF(z)$, where P_t^M is the monopoly price when demand comes from targeted consumers. That is, $P_t^M=$ $\arg\max_P P\,(1 - F_t(P))$ where

$$F_t(z) = \frac{1}{2}\left[F\,(z)^N + F(z)^{N-1}\,(N - (N-1)\,F(z))\right]$$

is the distribution of a random variable that equals the first or the second order statistic of N draws from F, each with probability one half. We will then assume that $s \le \min\{\bar{s}, \bar{s}_t\}$.

4 Pricing

Next, we turn to analyzing firms' pricing decisions given the levels of advertising.

4.1 Pricing under random advertising

The derivations for this case are very similar to those in Wolinsky (1986). We provide the details here for convenience. Because ads land on consumers randomly, there are consumers who receive one, more than one, or no ads.

Suppose that a common price P^* is set by all other firms and conjectured by all consumers, and consider firm i's best decision on its price P_i. The probability that a consumer who has received an ad from firm i and $k \in [0, N-1]$ other firms buys from firm i is

$$\Phi(P_i; k, P^*) = \int_{P}^{w-P^*+P} f(z)F(z + P^* - P_i)^k dz + (1 - F(w - P^* + P_i)) \sum_{l=0}^{k} \frac{1}{k+1} F(w)^l. \tag{5}$$

Indeed, firm i will be "selected to be visited" after $l = 0, 1, ..., k$ other firms with probability $1/\,(k+1)$, a visit that will then happen only if the consumer has not found a match value above w in one of its l previous visits. If this visit takes place and the match value at firm i is above $w - P^* + P_i$, then the consumer will immediately buy from firm i. Otherwise, she will continue searching but may return to buy from firm i if all other k firms of which she knows about happen to offer her a utility below what she would get at firm i.

The firm's expected demand from one consumer who gets its ad is then

$$D(P; P^*, \mu) = \sum_{k=0}^{N-1} \binom{N-1}{k} \mu^k (1-\mu)^{N-k-1} \Phi(P; k, P^*). \tag{6}$$

Thus, a condition for (symmetric) equilibrium is that

$$D(P^*;P^*,\mu)+P^*\frac{\partial D(P;P^*)}{\partial P}\bigg|_{P=P^*}=0. \tag{7}$$

Proposition 1. *For a fixed value of $\mu \in (0,1)$, a symmetric equilibrium price with random advertising, P^*, is characterized by*

$$\sum_{k=0}^{N-1}\binom{N-1}{k}\mu^{k+1}(1-\mu)^{N-k-1}\left\{\frac{1-F(P^*)^{k+1}}{k+1}+P^*\int_0^{w-P^*}F(z+P^*)^k f'(z+P^*)\,dz\right\}=0.$$

As we assume an increasing $f(z)$, firm i's pay-off function is quasi-concave and there is a unique solution to (7).[12]

4.2 Pricing under targeted advertising

We will denote equilibrium variables in the targeting treatment with a subscript t. Since we are assuming that firms send ads to all their targetable consumers, all consumers obtain two ads. These are from the firms carrying the two products for which they have the highest willingness to pay (match value).

Again, let us consider symmetric equilibria where all N firms charge the same price P_t^*. Consider the decision by firm i to set a price $P > P_t^*$.[13] A targetable consumer of firm i who visits the firm first, an event with probability 1/2, does not visit any other firm and buys from firm i if $z_i \geq w_t(P;P_t^*)$, where $w_t(P;P_t^*)$ is the value of z_1 that makes (3) equal to s. For $1/N$ consumers, z_i is the highest, and for $1/N$ consumers z_i is the second-highest match value. Then, the number of consumers who visit firm i first and buy there without visiting other firms is

$$D_{1t}^i(P;P_t^*)=\frac{1}{2N}\left(\int_{w_t(\cdot)}^1 NF(z)^{N-1}f(z)\,dz+\int_{w_t(\cdot)}^1 N(N-1)F(z)^{N-2}f(z)(1-F(z))\,dz\right). \tag{8}$$

If $z_i < w_t(P;P_t^*)$, then the consumer visits the other firm, say firm l, from which she received an ad. In that case, the consumer buys from firm i only if the utility $z_i - P$ exceeds $\max\{z_l - P_t^*, 0\}$. (Note that since $P > P_t^*$ only consumers for whom i is the best match may return.) The number of consumers who visit firm i first, then visit the other firm, and finally return to buy from firm i is

$$D_{Rt}^i(P;P_t^*)=\frac{1}{2N}\int_P^{w_t(\cdot)}\left(\int_0^{z-P+P_t^*}\frac{(N-1)F(z_l)^{N-2}f(z_j)}{F(z)^{N-1}}dz_l\right)NF(z)^{N-1}f(z)\,dz. \tag{9}$$

Some consumers receive an ad from firm i but first visit the other firm, l, from which they have received ads as well. Firm i is visited by one of these consumers if $z_l < w_t \equiv w_t(P_t^*;P_t^*)$. Note that the value w_t does not depend on P_t^*. It does not depend on P either, since the consumer, before visiting i, conjectures that firm i sets a price P_t^*. After the consumer observes z_i, she is fully

[12] More discussion on the subject can be found in Wolinsky (1986) and Anderson and Renault (1999).

[13] A downward deviation leads to slightly different demand expression. However, the limit of the demand and its slope when $P \to P_t^*$ is the same from both directions.

informed about both products. Thus, firm i sells to that consumer only if $z_i - P > \max\{z_l - P_t^*, 0\}$ and $z_l < w_t$. The number of these consumers is

$$D_{2t}^i(P; P_t^*) = \frac{1}{2N}\int_{w_t - P_t^* + P}^{1} NF(z)^{N-1} f(z) \int_0^{w_t} \frac{(N-1)F(z_l)^{N-2} f(z_l)}{F(z)^{N-1}} dz_l dz + \frac{1}{2N}\int_{P}^{w_t - P_t^* + P} NF(z - P + P_t^*)^{N-1} f(z)\, dz. \tag{10}$$

The total demand for product i is $D_t^i(P; P_t^*) = D_{1t}^i(P; P_t^*) + D_{Rt}^i(P; P_t^*) + D_{2t}^i(P; P_t^*)$. The symmetric equilibrium price P_t^* is obtained by setting $P = P_t^*$ in

$$D_t^i(P_t^*; P_t^*) + P_t^* \left.\frac{\partial D_t^i(P; P_t^*)}{\partial P}\right|_{P=P_t^*} = 0.$$

The following proposition characterizes the solution to that equation.[14]

Proposition 2. *If firms advertise to all their targetable consumers, then a symmetric equilibrium price P_t^* satisfies*

$$\frac{1}{N}\left(1 - F(P_t^*)^N\right) - P_t^* F(P_t^*)^{N-1} f(P_t^*) - P_t^*\left[\int_{P_t^*}^{w_t} (N-1)F(z)^{N-2} f(z)^2\, dz + \frac{N-1}{2} F(w_t)^{N-2} f(w_t)(1 - F(w_t))\, dw_t\right] = 0, \tag{11}$$

where dw_t is obtained by implicitly differentiating $w_t(\cdot)$ with respect to P:

$$dw_t = \left.\frac{dw_t(P; P_t^*)}{dP}\right|_{P=P_t^*} = \frac{1}{1 - s\frac{N-2}{N-1}\frac{f(w_t)}{1-F(w_t)}}.$$

5 A first take: competition effect and demand composition effect of targeting

Before turning to the general model, with $\alpha \in [1/2, 1]$, we may use the analysis in the previous sections to discuss two of the effects that targeting based on (imperfect, partial) ranking information has on firms' incentives when setting their prices. Suppose that, as we have been assuming, firms send ads to all their targetable consumers when they are able to target them, that is, $\mu_t = \frac{2}{N}$, and send $\mu \in [0, 1]$ when targeting is not possible. Note that μ may well be larger than μ_t. Now suppose that the cost of search is very small, say $s = 0$. In this case, consumers will visit all firms whose locations they know before deciding whether and where to buy.[15] Even though the consumers can only locate two firms in the targeting case, that is all that the consumers need to know (in any symmetric equilibrium) in order to find the best bargain! On the contrary, unless

[14] The assumption $f'(z) > 0$ does not immediately guarantee existence of equilibrium. Numerically, we have tested existence and uniqueness for a wide array of log-concave distributions of z, finding that indeed the symmetric equilibrium characterized in Proposition 2 does exist and is unique in all those cases.

[15] Note that in this limit case, firms will never have an incentive to send an ad to non targetable consumers, in the targeting treatment.

$\mu = 1$, in the random advertising case a consumer may not receive an ad from the firm offering the best match for her preferences, and so will never be able to find it.

The consequence is that, although $P_t^* = P^*$ when $\mu = 1$ (and $s = 0$),[16] whenever $\mu < 1$ the demand is less elastic in the non-targeting case and so $P_t^* < P^*$. For values of s positive but sufficiently small that continues to be so.

We call this effect of targeting on prices, associated to the higher informativeness of ads in that treatment, the *competition effect* .

However, the higher informativeness of targeted ads has another (direct) effect on the firms' pricing incentives. Indeed, consider a consumer who obtains two ads in both regimes. Also, assume that the consumer visits one firm in both cases and conjectures equal prices by both firms. The gains from conducting a second visit after learning z_1 is given by (3), setting $P_1 = P_t^*$. Since $g(z_2|z_1)$ is increasing in N (for $z_2 \geq z_1$), these gains are increasing in N for each given value of z_1. However, the probability of observing low values of z_1 which induce the consumer to search, i.e., $z_1 < w_t$, is also decreasing in N. In fact, as N grows, the value of w_t converges to the solution to

$$\int_{w_t}^{1} (z - w_t) \frac{f(z)}{1 - F(w_t)} dz = s,$$

a value strictly lower than 1, whereas the density of the match value in the first visit, z_1, an average of the first and second order statistic of N realizations of match values, approaches zero for all values of z_1 other than 1. That is, for large enough N, consumers will not search unless the price is very close to 1.

Indeed, from (11), multiplying both sides by N and taking into account that $N(N-1)x^{N-2}$ approaches 0 as N grows large, for any $x < 1$, we conclude that the equilibrium price converges to 1 as N converges to ∞.

Thus, although the price under targeted advertising could be decreasing in N, eventually, for large values of N the price will approach 1, the monopoly price (for a very inelastic demand composed of targetable consumers).

This, in fact, is the result of a well understood phenomenon usually referred to as the Diamond paradox. Indeed, two products that have been selected as the ones offering the highest match value for a consumer are more similar (their difference in their match value for the consumer is expected to be smaller) than two products chosen at random. This selection effect is stronger the larger the number of firms (the stronger the selection). We have learnt from Diamond (1971) that, when search is an issue (i.e., when $s > 0$), and differentiation vanishes, we should expect prices to be close to monopoly prices. In our model, as N grows large, we approximate this scenario. Thus, for any s, there is a number of firms N large enough so that in fact $P_t^* > P^*$. We call this effect a *demand composition effect.*

Given the firms' advertising effort, the relative strength of these two effects will determine the relative prices in equilibrium, and so the sign of targeting on equilibrium prices. Summarizing,

Proposition 3. *Suppose $\alpha = 1/2$, $\mu_t^* = \frac{2}{N}$, and a given μ^*. Then for small search costs and a small number of firms, $P_t^* < P^*$, whereas for large search costs and a larger number of firms*

[16] The reader may check that in the previous section.

$P_t^* > P^*$.

The model with $\alpha = 1/2$ (and assuming a given intensity of advertising) allowed us to illustrate, in the simplest possible framework, two of the effects of targeting. Moreover, analyzing the more general model of information precision, $\alpha > 1/2$, and endogenous advertising effort μ is now a simple extension of this analysis, which will allow us to discuss the incentives for advertising in both regimes.

6 Targeted advertising with endogenous intensity (and so precision of) advertising

Thus, let $\alpha \in [1/2, 1]$, and consider the firms' advertising decision in the targeted advertising treatment. We study the more interesting case where the cost of advertising is significant, so that firms do not send ads to non targetable consumers.[17]

If the firm decides to send $\mu_t \leq \frac{1}{N}$, then only those consumers for whom $i_1(\boldsymbol{z}^j) = i$ and β^j is larger than some threshold β_1—i.e., those for whom i's product is most likely the best match—will receive an ad from firm i, where β_1 satisfies

$$\mu_t = \frac{1}{N}\frac{\alpha - \beta_1}{\alpha - \frac{1}{2}}.$$

If the cost of advertising is lower, then the firm will also send ads to some targetable consumers for whom i appears in the second position in the realization of $X^j(\boldsymbol{z}^j)$ but for whom β^j is low (close to $\frac{1}{2}$). That is, if $\mu_t \in (\frac{1}{N}, \frac{2}{N}]$, then firm i sends ads to all consumers for whom $i_1(\boldsymbol{z}^j) = i$ (all $\frac{1}{N}$ of them) and to those consumers for whom $i_2(\boldsymbol{z}^j) = i$ (again, there are $\frac{1}{N}$ of them) *and* $\beta^j \leq \beta_2$, where

$$\mu_t = \frac{1}{N} + \frac{1}{N}\frac{\beta_2 - \frac{1}{2}}{\alpha - \frac{1}{2}}. \tag{12}$$

We will again consider a symmetric equilibrium where all firms choose the same advertising effort μ_t. Note that the interesting case is $\mu_t > \frac{1}{N}$. Indeed, for a high enough cost of advertising, so that $\beta_1 > \frac{1}{2}$, each consumer will receive only one ad in equilibrium, so that monopoly pricing (for a demand given by targetable consumers with $i_1(\boldsymbol{z}^j) = i$ and $\beta^j \geq \beta_1$) would ensue. If the cost of advertising is not as high, then ($\beta_1 = \frac{1}{2}$ and) $\beta_2 > \frac{1}{2}$ in equilibrium, and so there may be consumers who obtain only one ad, and consumers who obtain two ads. Note that the average precision of the information a consumer infers when receiving an ad from a firm depends on the intensity of advertising μ_t, as this determines the probability that the consumer learns what is the best match or only which are the best two matches.

[17] Note that, whatever the consumers' –symmetric– conjectures in equilibrium are, an ad sent to a targetable consumer has an expected return that is always higher than an ad sent to a non targetable consumer. Thus, firms will send ads first to targetable consumers. If the cost of advertising is very low, we may also have equilibria where firms send ads to other (random) consumers. This will only blur the signal for consumers, induce then to search more and so reduce, but not affect qualitatively, the effects discussed here. A more general treatment may also include information about lower positions in the ranking of consumers' preferred products, which would be simply more involved.

In any such equlibrium, a consumer receiving two ads will be able to infer exactly the same information as in the case $\alpha = 1/2$. Indeed, all the consumer can infer is that the two firms adertising to her are her two best matches. Moreover, given the symmetric structure of signals, and for any firm i, for each consumer with $\beta \leq \beta_2$ (those receiving two ads) and $i_1(\boldsymbol{z}^j) = i$ firm i also sends an ad to a consumer with the same $\beta \leq \beta_2$ and $i_2(\boldsymbol{z}^j) = i$. Thus, ignoring the information in the realization of $X^j(\boldsymbol{z}^j)$, a consumer with $\beta \leq \beta_2$ arriving at firm i has the same probability of having i's product as the best match as having it as the second-best match. Therefore, the (per consumer) demand of such consumer is simply $D_t^i(P; P_t^*)$ computed in Subsection 4.2.

Consumers who only receive an ad from firm i are such that $i_1(\boldsymbol{z}^j) = i$ and $\beta^j > \beta_2$. These consumers either buy from the firm or take the outside option providing zero utility. If firm i sets price P, then the demand from one of this consumers with type β^j *and* $i_1(\boldsymbol{z}^j) = i$ is

$$D_{tM}^i(P;\beta^j) = \beta^j \int_P^1 NF(z)^{N-1} dF(z) + \left(1-\beta^j\right) \int_P^1 N(N-1) F(z)^{N-2} (1-F(z))\, dF(z).$$

Thus, given the equilibrium value of β_2, a price P_t^* for the rest of firms, and a price P for firm i, firm i's expected demand is

$$\frac{1}{N\left(\alpha - \frac{1}{2}\right)} \int_{\beta_2}^{\alpha} D_{tM}^i\left(P, \beta^j\right) d\beta^j + \frac{\beta_2 - \frac{1}{2}}{\alpha - \frac{1}{2}} D_t^i\left(P; P_t^*\right). \tag{13}$$

and so the first order condition for equilibrium price P_t^* is

$$\begin{aligned} \frac{1}{N\left(\alpha - \frac{1}{2}\right)} \int_{\beta_2}^{\alpha} \left[D_{tM}^i\left(P_t^*, \beta^j\right) + P_t^* \left.\frac{\partial D_{tM}^i\left(P, \beta^j\right)}{\partial P}\right|_{P=P_t^*} \right] d\beta^j + \\ \frac{\beta_2 - \frac{1}{2}}{\alpha - \frac{1}{2}} \left[D_t^i\left(P_t^*; P_t^*\right) + \left.\frac{\partial D_t^i(P,;P_t^*)}{\partial P}\right|_{P=P_t^*} P_t^* \right] = 0. \end{aligned} \tag{14}$$

The first term is (a multiple of) the derivative of $P\int_{\beta_2}^{\alpha} D_{tM}^i\left(P, \beta^j\right) d\beta^j$ with respect to P. The price that maximizes this expression is larger than P^M for $N > 2$, as the demand of a targeted consumer with $\beta^j > \beta_2$ and $i_1(\boldsymbol{z}^j) = i$ is less elastic than the demand of a randomly chosen consumer. As we will discuss below, when the marginal cost of advertising is sufficiently large, the solution β_2 approaches $\frac{1}{2}$: firm i sends ads to—almost—only those consumers j for whom $i_1(\boldsymbol{z}^j) = i$, and so almost all consumers get only one ad. Thus,

Proposition 4. *If $s > 0$, $\alpha > \frac{1}{2}$, and the marginal cost of advertising is sufficiently high, then in any symmetric equilibrium, $P_t^* > P_t$.*

This new effect of targeting, that we can term the *endogenous monopolization effect*, may result in higher prices under targeted advertising whatever the search costs are. (This is simply the effect analyzed in, for instance, Iyer et al. (2005).) As a result of endogenously finer targeting, firms may end up drastically increasing market segmentation, and so monopolization of a less elastic demand ensues.

In order to close the model, we also characterize the equilibrium value of μ_t (i.e., the equilibrium value of β_2) and μ. In the targeting case, and assuming that every other firm sets a threshold β_2, suppose a firm increases its threshold for consumers with $i_2(\boldsymbol{z}^j) = i$ to $\overline{\beta} > \beta_2$. Then, every consumer with $\beta^j \leq \beta_2$ and realizations $i_2(\boldsymbol{z}^j) = i$ or $i_1(\boldsymbol{z}^j) = i$ still receives two ads, and every consumer with $\beta^j > \beta_2$ and $i_1(\boldsymbol{z}^j) = i$ receives one ad only from firm i. However, consumers with $\beta^j \in (\beta_2, \overline{\beta})$ and $i_2(\boldsymbol{z}^j) = i$, but not those with $i_1(\boldsymbol{z}^j) = i$, receive two ads. These consumers will still behave as in the previous section, but the expected demand from each of them will now be a function of β^j. (I.e., in computing their demand, we cannot ignore the realized $X^j(\boldsymbol{z}^j)$).

The expected demand from such consumer is

$$D^1_{tD}(P^*_t, \beta^j) = \beta^j \int_{w_t}^{1} \frac{1}{2} N (N-1) F(z)^{N-2} (1 - F(z)) \, dF(z) \tag{15}$$
$$+ (1 - \beta^j) \left(\frac{1}{2} \int_{P^*_t}^{1} N F(z)^{N-1} dF(z) + \right.$$
$$\left. + \frac{1}{2} \left[\int_{P^*_t}^{w_t} N F(z)^{N-1} dF(z) + \int_{w_t}^{1} N F(w_t)^{N-1} dF(z) \right] \right)$$

The first term represents the probability that this consumer has firm i as her second best match (β^j), in which case she buys from the firm only if she visits the firm first (probability 1/2) and her match value is above w_t.[18] With probability $1 - \beta^j$, firm i is the consumer's best match. Then, if she visits firm i first, she buys from firm i if her match value is above P^*_t, whether on a first visit (if the match alue is above w_t) or after also visiting some other firm. That is the second line. Finally, if she visits the other firm first, then she buys from firm i if her match value at that firm is between P^*_t and w_t (so that the match value with the alternative firm is certainly below w_t) or if it is above w_t but the match value with the alternative firm is below w_t (and so visits firm i as well). That is the third line.[19]

Thus, the total demand after this deviation will include (13) –which contains only terms independent of $\overline{\beta}$– plus an additional term:

$$\frac{1}{N(\alpha - \frac{1}{2})} \int_{\beta_2}^{\overline{\beta}} D^1_{tD}(P^*_t, \beta^j) d\beta^j.$$

Thus, (symmetric) equilibrium μ_t (and so β_2) will be determined by the first order condition

$$D^1_{tD}(P^*_t, \beta_2) P^*_t - C'(\mu) = 0.$$

This first order condition is intuitive enough. Indeed, sending an additional ad to an additional consumer (or to the $\frac{1}{N(\alpha - \frac{1}{2})}$ additional consumers with some particular value of β^j and, say,

[18] Obviously, if the consumer was interested in guessing what the value of βj was for her, the information on z_1 would be relevant. But the consumer is only interested in the probabilities of the realization of z_2, and for this, z_1 is indeed a sufficient statistic, just as in the baseline model.

[19] We could also look at $\overline{\beta} \leq \beta_2$. The conclusions would be the same.

$i_2(\boldsymbol{z}^j) = i$) has an additional cost of $C'(\mu)$ and will result in a sale with probability $D^1_{tD}(P^*_t, \beta_2)$.

Note that the lower C' (i.e., the more profitable advertising is), the higher the solution in μ_t (and β_2), and then the weaker the monopolization effect. Likewise, as the number of firms increases (and so the demand composition effect becomes stronger), the endogenous monopolization effect is also weakened. Indeed, the demand composition effect increases the profit per ad, and so firms' advertising intensity also increases. That raises the expected number of ads that each consumer receives, and so reduces the monopolization of the market that we have just analyzed. As a result, P^*_t may well be non monotonic in N.[20]

We also close the model in the random advertising treatment. In that case, if all other firms are sending μ^* ads, one of firm i's ads will result in a sale with a probability equal to $D(P^*; P^*, \mu^*)$, as defined in (6). Thus, if firm i sends μ' ads, its profits are

$$\mu' D(P^*; P^*, \mu^*) P^* - C(\mu'),$$

and so equilibrium advertising intensity is characterized by

$$D(P^*; P^*, \mu^*) P^* - C'(\mu^*) = 0.$$

7 An example

In this section, we illustrate the effects of targeting on market outcomes by numerically solving a parameterized example.

Let $F(z) = z^r$ for $r \geq 1$ and $z \in [0, 1]$, and $C(\mu) = c\mu^b$, for $c > 0$ and $b > 1$. The example could be computed for values of the parameters (N, α, s, c, b). We present a few figures summarizing the behavior of market outcomes for some of these parameters.

Our first group of figures shows comparative statics in the cost of advertising of prices, profits, consumer surplus, and advertising intensity for both regimes, random and targeted advertising. Figure 1 is computed for $N = 4$, $\alpha = 0.7$, $s = 0.001$, and $b = 2$. As could be expected, larger values of c result in lower levels of advertising in both regimes, although the reduction is more acute in the random advertising case. Prices are lower under targeted advertising for low values of c, where the monopolization effect and the demand composition effects are weaker. However, for larger values of c, and so lower advertising intensity, the two effects are stronger and as a consequence, prices are higher under targeted advertising. This price effect may even compensate for the reduction in search costs and better matches that are associated with targeting, so that consumer surplus may in fact be lower under targeted advertising.

Figure 2 shows a comparative statics exercise in s, the cost of search, for $N = 6$, $c = 0.37$, $\alpha = 0.7$, and again $b = 2$. As search becomes more costly, demand becomes more inelastic in both regimes, so that prices (and also advertising, as it becomes more profitable) increases. The increase in advertising is not sufficient to outweigh the increase in search cost and prices, so that consumer surplus decreases. Prices may again be larger under targeted advertising for larger values of s.

[20]The example that we study next shows this lack of monotonicity.

Interestingly, prices may also be larger in this regime for very low values of s. Indeed, for these values, the marginal profitability of advertising is low, particularly when advertising is targeted, since consumers have a strong incentive to search. Thus, the monopolization effect is particularly strong, more than compensating the (also strong) competition effect. As a result, targeting may result in higher prices even when the cost of advertising is very small and firms advertising is (relatively) intense under random advertising.

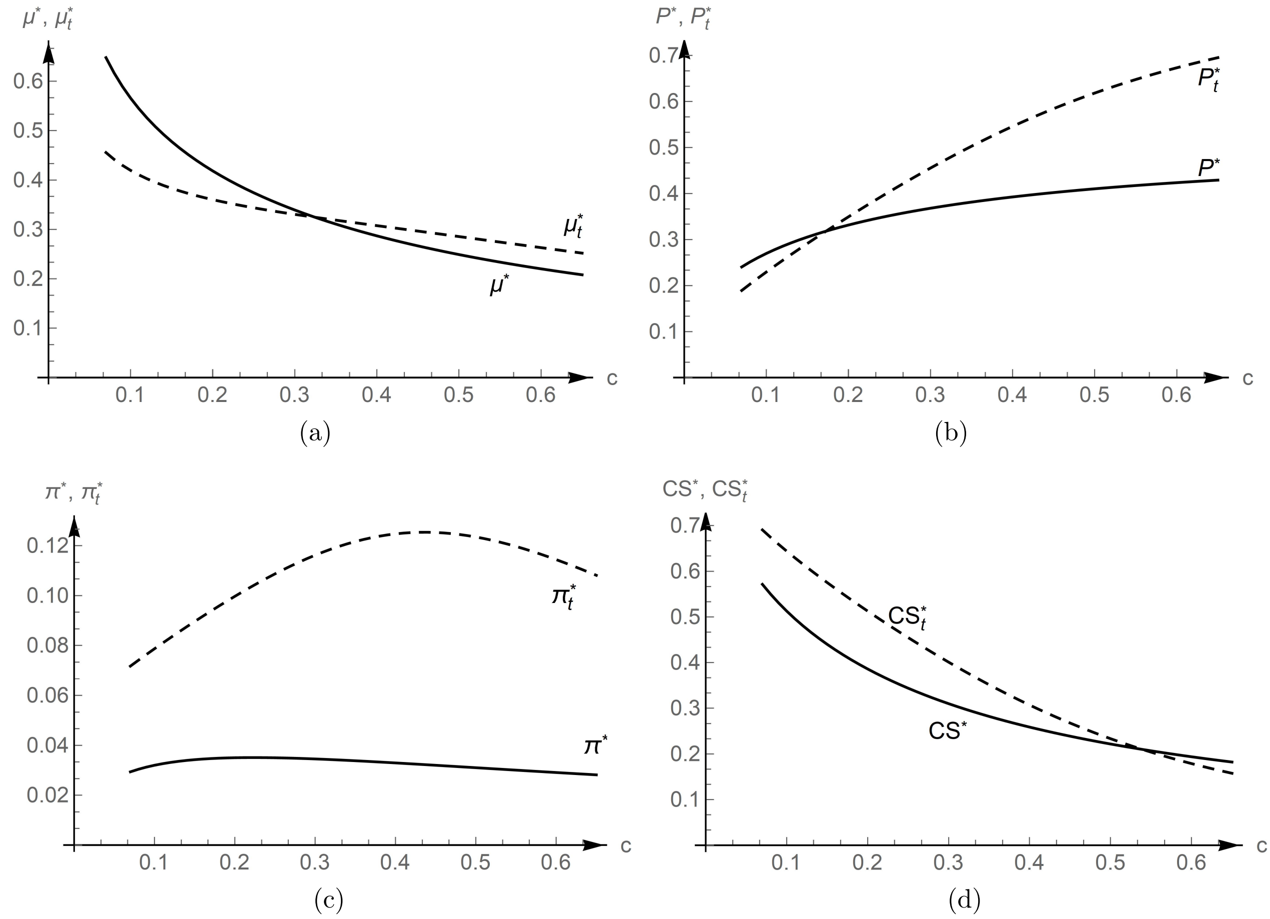

Figure 1: Comparative statics on c, $N = 4$, $s = 0.001$, $b = 2$, $\alpha = 0.7$.

8 Concluding remarks

We have presented and analyzed a model of advertising targeted based on information about the—expected—relative match of the firm's product and the consumer's tastes, but not directly about the consumer's willingness to pay. A consumer who receives an ad has a higher incentive to search firms that so target their ads. More intense search tends to drive prices down. However, the consumer also perceives the products of two firms from whom she receives ads as less differentiated, in expectation, which depresses her incentives for search. That tends to drive prices up. Indeed, as consumers receive ads corresponding to high order statistics of their match values for products in the market, these order statistics are in expected terms closer than random realizations of match values. Thus, targeting has an ambiguous effect on prices. Our study shows that indeed, whether targeted advertising leads to higher or lower prices depends, among other things, on the size of

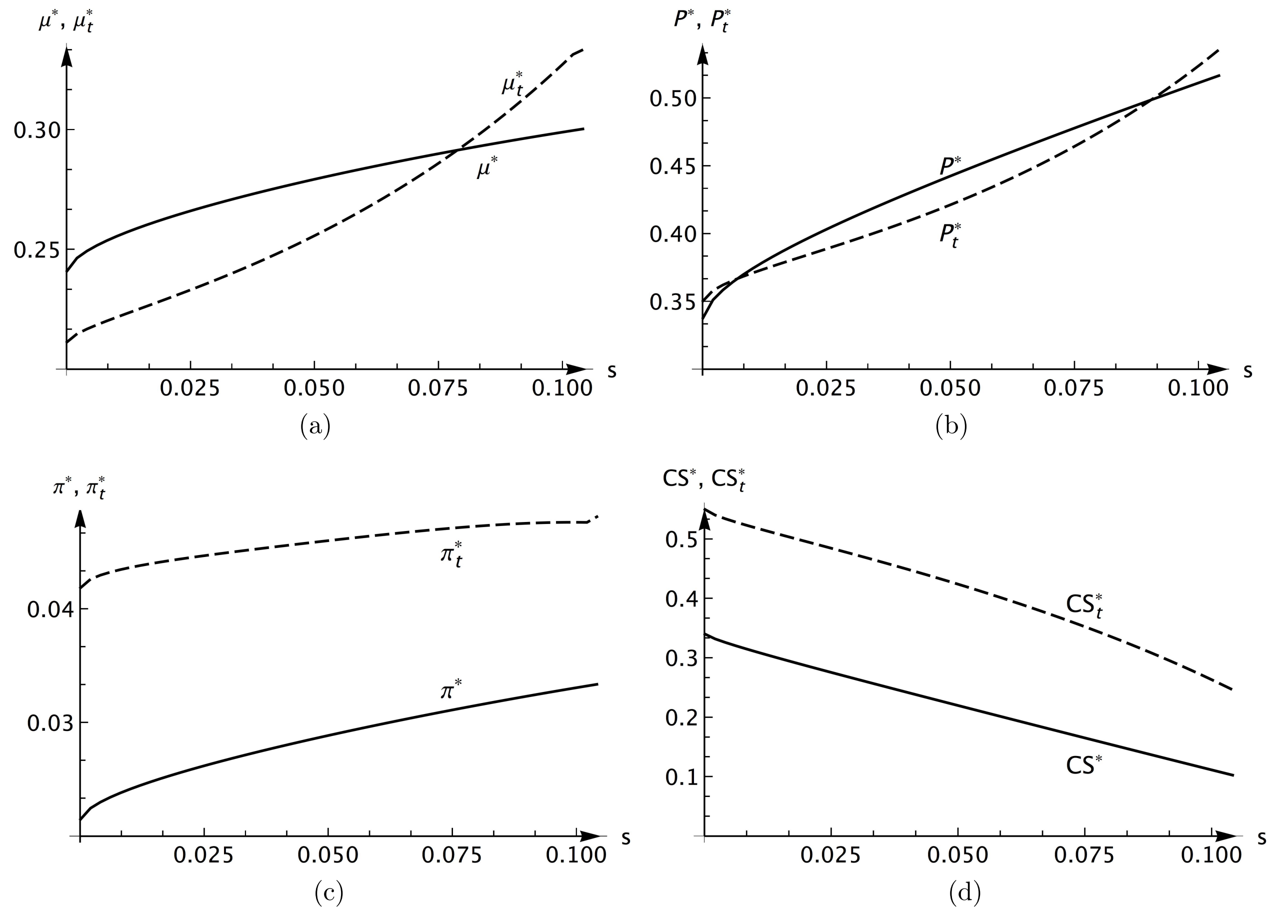

Figure 2: Comparative statics on s, $N = 6$, $c = 0.37$, $b = 2$, $\alpha = 0.7$.

consumer search costs. In particular, for a given advertising intensity, the equilibrium price and the profits of firms are lower under targeted advertising when the search costs are low. On the contrary, if the search costs are high, firms charge higher prices and earn higher profits when ads are targeted.

We have analyzed imperfect targeting, i.e. firms cannot exactly identify the consumers who like their products the most, and consumers cannot infer the ranking order of products whose ads they have obtained. Then the precision of the information conveyed by ads is endogenous and depends on the advertising intensity (how many ads a consumer receives on average). When advertising costs are high, so that "poaching" a rival's consumers is more costly, consumers infer more information from receiving an ad. That strengthens market segmentation through advertising and drives prices up.

We have obtained these results using a novel, simple model that explicitly solves for non stationary search rules. As consumers observe a match value in a firm, their prior on the expected match on not yet searched firms is affected. Despite this lack of stationarity, we have been able to obtain explicit solutions for the pricing and advertising game. Also, we have provided some numerical illustrations for a parameterized example. Our results show that imperfect targeting is welfare improving if search and advertising costs are low, as the informativeness of advertising increases competition among relevant suppliers. However, consumers may be worse off (and welfare may be lower) with targeted advertising if search and advertising are sufficiently costly.

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