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Abstract

Ultracold atom experiments allow the study of topological insulators, such as the non-interacting Haldane model. In this work we study a generalization of the Haldane model with spin–spin on-site interactions that can be implemented on such experiments. We focus on measuring the winding number, a topological invariant, of the ground state, which we compute using a mean-field calculation that effectively captures long-range correlations and a matrix product state computation in a lattice with 64 sites. Our main result is that we show how the topological phases present in the non-interacting model survive until the interactions are comparable to the kinetic energy. We also demonstrate the accuracy of our mean-field approach in efficiently capturing long-range correlations. Based on state-of-the-art ultracold atom experiments, we propose an implementation of our model that can give information about the topological phases.

1. Introduction

Ultracold atoms trapped in optical lattices allow the simulation of rich physical models that cannot be easily recreated in solid state materials [1, 2]. Great experimental breakthroughs include the simulation of the Bose–Hubbard [3] and Fermi–Hubbard models [4, 5], the implementation of fermionic and bosonic lattice models with artificial gauge fields, such as the Hofstadter model [6–8] or topological Floquet band models [9] and of particular relevance for this work, the realization of the Haldane model [10]. The Haldane model [11] is an example of a 2D topological insulator (TI), a quadratic Hamiltonian with a non-trivial topological invariant which, while being insulating in their bulk, supports protected currents along its edges [12]. An interesting problem is the study of how this, or other TI models, are modified by the presence of interactions, either by destroying the topologically protected phase, or by replacing it with other non-trivial phases. Some work has already been done in the field of interacting TIs, the spinless Haldane model with nearest neighbor interactions [13], the Haldane–Hubbard model with spin and on-site interactions [14–23], the Kane–Mele model [24, 25], pyrochlores [26], the synthetic Creutz–Hubbard model [27] or a general TI in the context of the fractional quantum Hall effect [28].

In this work we study the topological phases of the Haldane model with spin 1/2 fermions for a broad range of on-site interactions using two methods: (i) a mean-field approach in momentum space that effectively takes into account long-range correlations and (ii) a matrix product state (MPS) ansatz in a 2D lattice [29, 30] with 2 × 32 sites. We have found that the topological phases which appear in the non-interacting model extend to values of the interaction which match the kinetic energy of the fermions. In this broad range, the momentum space mean-field theory compares favorably with the more costly and difficult MPS simulations, and the winding number provides an accurate characterization of the TI phase. For stronger interactions, the ground state becomes a Mott insulator, where, in accordance to earlier work, a non-trivial spin order may appear [14–22]. On account of our numerical results, we discuss a possible ultracold atom experiment that can directly simulate our model, based on previous experiments with TIs [6, 7, 10] and the measurement of topological properties [31].
The article is structured as follows. In section 2 we describe a generalization of the Haldane model with spin–spin on-site interactions, how topology arises in the non-interacting limit and how it can be detected by measuring the winding number. In section 3 we begin our study of interactions, introducing a mean-field variational wavefunction that exactly reproduces the non-interacting ground state. This function is a product state in momentum space and may capture the long-range correlations needed to describe a topological phase with interactions. Using a global optimization procedure, we estimate the ground state energy and wavefunction within this ansatz. We find evidence of a topological phase for nonzero interactions, as well as a cross-over into a Mott insulator and double occupancy regions for strongly repulsive and strongly attractive interactions, respectively. In section 4 we study the same problem, using the MPS ansatz spanned over a 2D honeycomb lattice. Computationally intensive simulations with up to 64 lattice sites and bond dimensions up to 180 confirm the predictions of the mean-field ansatz, including the topological phase transition and cross-overs, and show the accuracy of this simple wavefunction when estimating the ground state energy. Section 5 discusses the modifications needed to implement our model using state-of-the-art experiments [10], including state preparation and detection of topological and trivial phases. We close this work with a brief summary and discussion in section 6.

2. The model

The Haldane model is a Hamiltonian that describes the motion of particles in a honeycomb lattice with nearest- and next-to-nearest neighbor hopping amplitudes. The topological nature of the model arises from the existence of complex hopping amplitudes, which in our work we place in the nearest neighbor hopping. The honeycomb lattice is composed of two triangular lattices, which we denote \( A \) and \( B \). Each site will be able to host up to two fermionic particles in state \( |\uparrow, \downarrow\rangle \), which we denote with creation operators \( a_i^\dagger \) and \( b_i^\dagger \) for the two sites of the \( i \)th unit cell. The full model describing this system reads

\[
H = H_0 + U \sum_i (a_i^\dagger a_i^\dagger a_i a_i) + b_i^\dagger b_i^\dagger b_i b_i),
\]

where \( H_0 \) is the kinetic energy of each fermionic species, and \( U \) is the on-site interaction energy between fermions.

The kinetic part has the usual form

\[
H_0 = - \sum_{\langle i,j \rangle} (t e^{i p_{ij}} a_i^\dagger b_j + h.c.) - \sum_{\langle i,j \rangle} (t_a a_i^\dagger a_j + t_b a_i^\dagger b_j) - \sum_i \epsilon (a_i^\dagger a_i - b_i^\dagger b_i).
\]

This non-interacting model contains a first-neighbor hopping, \( t \), which is affected by a phase that grows linearly along the lattice direction \( p \), \( \phi_{ij} = p(x_i - x_j) \). There are also two next-to-nearest neighbor hopping amplitudes \( t_a \) and \( t_b \), on the respective sublattices (see figure 1); and an on-site energy imbalance between lattices \( \epsilon \).

Our model is a variation of Haldane’s original model in that the complex phase is located at the first-neighbor hoppings. When \( |t_a| = |t_b| \) both models are topologically equivalent in the sense that there always exists a continuous path in parameter space that connects one model with the other without breaking the
topological phase. However, when \( |t_a| = |t_b| \) and with the introduction of \( \epsilon \), it is possible to reach other exotic phases, such as a topological semimetal\cite{32}. In this work we set \( t = 1 \) as unit of energy, fixing \( t_a = -t_b = 0.1 \) to small values that support a topological order but do not significantly affect the total bandwidth of the problem. We also set the sublattice imbalance to zero, \( \epsilon = 0 \), to focus on the role of interactions and how it may destroy the topologically protected phases.

The hopping Hamiltonian for each spin population (2) is a two band model that can be written as an effective magnetic field acting in momentum space. Introducing the Fourier transformed operators \( a_{k,s} \) and \( b_{k,s} \), and the pseudospin operators \( \hat{\sigma}^x = a^\dagger b + b^\dagger a, \hat{\sigma}^z = a^\dagger a - b^\dagger b \) and \( \hat{\sigma}^y = -i[\sigma^z, \sigma^x] \), we find

\[
H_0 = \sum_k B(k) \cdot \hat{\sigma}_{k,s} \quad s \in \{\uparrow, \downarrow\}.
\]

(3)

The field \( B(k) \) determines a preferred orientation of the expectation value for the single-particle operators \( \sigma^i \). In absence of chemical potential, the ground state \( \Psi_0 \) of \( H_0 \), is a half-filled state, for which the pseudospin field, \( \hat{S}_s(k) \), which is proportional to the expected value of the pseudospin operators, \( \sigma_{k,s} \), satisfies

\[
\hat{S}_s(k) = \frac{1}{2} \langle \Psi_0 | \sigma_{k,s} | \Psi_0 \rangle \propto -B(k).
\]

(4)

The direction and amplitude of the field \( B(k) \) and of the pseudospin \( \hat{S}_s \) are given by

\[
B(k) = (\text{Re} \{ f(k) \}, \text{Im} \{ f(k) \}, \epsilon + (t_a - t_b) g(k)),
\]

(5)

\[
f(k) = \sum_{i=1,2,3} e^{-i p_i k \cdot \mathbf{v}_i},
\]

(6)

\[
g(k) = \sum_{i=1,2,3} \cos(k \cdot \mathbf{w}_i),
\]

(7)

where \( \mathbf{v}_i \) and \( \mathbf{w}_i \) are the vectors that connect a site with its nearest- and next-to-nearest-neighbors, respectively. The half-filled ground state without interactions is therefore completely determined by the direction of the pseudospin fields \( \hat{S}_s(k) \) of both spin populations over the Brillouin zone.

An interesting property of the hopping Hamiltonian (2) is that different configurations of the vector field \( \hat{S}_s \) can be regarded as topologically distinct. More precisely, \( \hat{S}_s(k) \) may be regarded as a mapping from the Brillouin zone onto the sphere, \( |\hat{S}_s| = 1 \), with \( \hat{S}_s \) the normalized orientation of \( S_s \). These mappings may be classified by the number of times the pseudospin \( \hat{S}_s \) wraps around the sphere, what is called the winding number\cite{33}

\[
\nu_s = \frac{1}{4\pi} \int \hat{S}_s(k) \cdot [\partial_{k_x} \hat{S}_s(k) \times \partial_{k_y} \hat{S}_s(k)] d^2k.
\]

(8)

Whenever the band wraps one or more times around the sphere, the model is said to be topologically non-trivial. This results in a nonzero Chern number and the existence of topologically protected edge currents when the model is embedded in a finite domain with boundaries or holes. Figure 1(b) shows the sum of the winding numbers of both spin populations in the non-interacting regime. The winding number is antisymmetric with respect to the flux \( \phi \) and the topological phases survive until the sublattice imbalance \( \epsilon \) becomes of the order of \( |t_a - t_b| \). Contrary to the topological phases of the original Haldane model, there are topologically trivial phases at \( \epsilon = 0 \).

A very important open question is what happens to the TI model (1) when there are active interactions between particles, \( U \neq 0 \). In such situations, the single-particle picture is no longer valid, but the system may still exhibit topological properties, such as a non-trivial pseudospin pattern \( S_s(k) \), which gives rise to different phases, or the existence of edge states, or, in the limit of infinitely strong interactions, perhaps the appearance of true topological order.

In this work we will analyze (1) using different variational methods—mean-field theory and MPSs—with the goal of classifying the approximate ground states of the model. Our main tool in doing this will be the use of the winding number, \( \nu_s \), because it is a topological property that can be experimentally and numerically determined, without access to the wavefunctions or parallel transport.

### 3. Mean-field

We have seen above that the ground state at half-filling and without interactions is uniquely determined by the pseudospin field \( \hat{S}_s(k) \) over the Brillouin zone, with one particle per unit cell in momentum space. If we study the non-interacting case but confine ourselves to half-filling, it makes sense to approximate the ground state at nonzero interactions by a product state in momentum space

\[
|\Psi_0[S_s, S_s]| = \prod_{k \in BZ} c_{k,s}^\dagger [S_s] c_{k,s}^\dagger [S_s] |0\rangle.
\]

(9)
This ground state is constructed by placing one particle per spin in each of the momentum states, $c^\dagger_k |S\rangle = \alpha_k |S\rangle a^\dagger_k + \beta_k |S\rangle b^\dagger_k$. These modes are characterized by the fact that $\frac{1}{\sqrt{2}} \langle 0 | \sigma_k \cdot \epsilon_k^\dagger | 0 \rangle = S_z(k)$, where $S_z(k)$ is a vector of norm $1/2$ and our variational parameter. All the information about the ground state is thus given by two independent pseudospin fields $S_\uparrow$ and $S_\downarrow$, with uniform norm. The variational energy of this wavefunction then reads as the functional

$$E[U, S, \epsilon] = -2 \sum_k B_k \cdot S_k \uparrow - 2 \sum_k B_k \cdot S_k \downarrow + 2 \frac{U}{N} \sum_{kk'} S^z_{k\uparrow} S^z_{k'\downarrow} + \frac{U}{2},$$

where $N$ is the total number of unit cells in our lattice. We remark that, in the absence of interactions $U = 0$ this mean-field approach is exact.

The form of the variational energy suggests that, when the interactions are weak enough compared to the kinetic energy terms, $|U| \ll |B|$, the topological phase will remain unaffected, and the ground state will adopt a spin order determined by the effective magnetic field $S_{k\uparrow} = S_{k\downarrow} \propto -B_k$. For strong repulsive interactions $U \gg |B|$, the mean-field model suggests that the vector field develops an antiferromagnetic order, where all spin components point along the $Z$ direction, $S^z_{k\uparrow} = -S^z_{k\downarrow}$. If we inspect the expression for the pseudospin

$$S^z_{k,i} = \frac{1}{2} \langle (a^\dagger_k a^\dagger_i) - (b^\dagger_k b^\dagger_i) \rangle \sim \sum_i \langle a^\dagger_{i\uparrow} a_{i\downarrow} \rangle - \langle b^\dagger_{i\uparrow} b_{i\downarrow} \rangle, \quad |U| \gg t,$$

we realize that this implies a phase separation between spin components, whereby different spin orientations sit on different sublattices, i.e. particles $\uparrow$ and $\downarrow$ are placed on sublattices $A$ and $B$, respectively, or vice versa. Moreover, because the transverse components are zero $S^{x,y} = 0$, the winding number vanishes in this phase. Similarly, for strong attractive interactions, $U \ll -|B|$, we expect a ferromagnetic order to develop, which amounts to having bunching of particles on the same lattice sites, i.e. all particles tend to sit on the same sublattice.

We have optimized numerically the variational energy for lattices with up to 400 sites, using a linear search algorithm that starts with a random distribution of vector fields. For each of the variational ground states, we have computed both a discrete approximation to the winding number (8), as well as an order parameter that detects phase separation and bunching. In the figure 2(a) we plot the winding number of the mean-field ground state of a lattice with $20 \times 20$ unit cells. We see that the topological phases survive until the strength of the interactions, $|U|$, is around three times bigger than the parameter $t$. In figure 2(b) we plot the double occupancy
and changes at the points at which the topological order disappears, becomes dominant and nonlocal expectation values. However, for practical applications one usually sets a maximum size, called bond dimension, around which the dimension of the tensors corresponding to the same site, whose dimensions depend on the bipartite entanglement of the state around. This ansatz is an exact representation of every quantum state for sufficiently large tensors. Discarding configurations that contribute the least to the ground state.

As hinted in our mean-field approach we have used an MPS ansatz to compute the ground state of quantum lattice models that relies on the low scaling of the entanglement entropy in ground states in 1D and 2D. It describes a quantum state as a product of tensors on every lattice site,

$$\Psi = \sum_{i_0,i_1} A^{i_0}[0]A^{i_1}[1] \cdots |i_0,i_1\cdots\rangle, \quad (13)$$

where the dimension of the indices are the physical dimensions of the system at the $j$th lattice site, and $A^{i_j}[j]$ are the tensors corresponding to the same site, whose dimensions depend on the bipartite entanglement of the state around $j$. This ansatz is an exact representation of every quantum state for sufficiently large tensors. However, for practical applications one usually sets a maximum size, called bond dimension, $\chi$, for every $A^{i_j}[j]$, discarding configurations that contribute the least to the ground state.

In our calculations we have constructed an MPS by mapping a lattice with $N \times N$ unit cells to two consecutive 1D systems in a ‘snake’ configuration, with a total of $2 \times 2 \times N^2$ sites, one for each spin component (see figure 3). We have focused our efforts in diagonalizing a $4 \times 4$ lattice using a maximal bond dimension of $\chi = 180$. Lattices smaller than $4 \times 4$ unit cells did not output correct results for the winding number, already in the non-interacting analytical solution. Bigger lattices demanded a too large bond dimension that exceeded our computational resources. As in the mean-field simulations, we have restricted ourselves to situations with half-filling, which are of particular interest for large repulsive interactions.

As hinted in our mean-field computation (figure 2), the non-trivial topological phases extend until the interactions are comparable to the kinetic energy, $|U| \sim 3t$. For stronger interactions, the term $U \sum_{i,j} n_{i} n_{j}$ becomes dominant and nonlocal expectation values $\langle c_i^c c_j \rangle$ approach zero, where different ferro- or antiferromagnetic orders appear, which correspond to different distributions of the spin components in the two sublattices. We identified the double occupancy as an appropriate order parameter to measure these distribution effects.

In figure 4 we show the main results of our MPS simulations. Due to the big computational effort involved in the MPS, we have studied two representative lines in parameter space, demarked in figure 4(a). The winding numbers along these lines are shown in figures 4(b) and (d). These winding numbers have been computed by extracting the expectation values $\langle c_i^c c_j \rangle$ from the MPS and then Fourier transforming them into bigger lattices.

$$D = \frac{\sum_{i,j} \langle n_i n_j \rangle}{\sum_{i,j} \langle n_i \rangle} - \frac{1}{4} = \frac{1}{N^2} \sum_{k \ell} S_k^x S_{k'}^x,$$  

(12)

This order parameter is strictly zero when there are two particles per unit cell and both are equally distributed over both sites, as it is the case of the non-interacting region $U = 0$. In the topological phase, $D$ is strictly zero and changes at the points at which the topological order disappears, $|U/t| \gg 0$. For repulsive interactions, $U \gg 0$, the quantity $D$ becomes negative with a discontinuous derivative. This behavior shown in figure 2(d) is consistent with a second order phase transition to a Neel phase [14, 18], confirmed by our own simulations with MPS in the following sections. When interactions are attractive, $U < 0$, the expectation value $D$ becomes positive, implying that pairs of particles with opposite spin become correlated and tend to share the same lattice sites. Note that in the non-topological regions, the mean-field does not show the same abrupt behavior, but predicts a cross-over from $D = 0$ to positive or negative values depending on the sign of $U$.  

4. MPS simulations

Figure 3. Scheme of our ‘snake’ MPS spanning over a $4 \times 4$ unit cell honeycomb lattice with spin. The order in which the physical indices are covered is indicated by the thick green line. The MPS goes first through the lattice with spin up and then through the lattice with spin down.
of $8 \times 8$, $12 \times 12$ or $16 \times 16$ unit cells in momentum space. This Fourier interpolation of the pseudospin field allows us to make a more accurate determination of the winding number in momentum space. We only keep simulation results that have converged under this interpolation procedure. The selected values compare favorably with the mean-field predictions for large lattices $20 \times 20$, away from the phase transition.

Near the topological phase transitions, the winding cannot be successfully interpolated from the MPS simulations (see gray shaded areas in figures 4(b), (d)). However, this is not surprising. As a mean-field shows, it is difficult to determine accurately the winding number with just $4 \times 4$ pseudospins (see dashed orange line in figures 4(b), (d)). Right on the topological phase transitions, the gap closes and it is therefore difficult for an algorithm to determine whether the pseudospin field maps into a closed or an open surface (see figures 2(b) and (c)).

In figure 4(c) we plot the double occupancy of the ground state for the MPS computation and the mean-field at the line $\phi = \pi/2$. The expectation value of this observable should approach 0 when the interactions have no effect on the ground state and the two TIs are uncoupled (2). However, D should become $1/4$ in the strongly attractive phases and $-1/4$ in the strongly repulsive phases, where particles with opposite spin are either grouped or separated, respectively. We observe a good agreement between both simulations, specially at the plateau $D = 0$, where our results show a non-trivial topological phase.

We have also studied the staggered magnetization

$$m = \sum_i (-1)^i (n_i^+ - n_i^-),$$

which is shown in figure 5(a). We observe that the ground state remains in a paramagnetic phase until the repulsive interactions destroy the topological phase. The MPS results show, however, that the staggered magnetization begins to grow slightly before the repulsive topological phase transition converts the state in a topologically trivial one, $U/t \sim 3$, which could signal the existence of an intermediate topological phase, as reported by other authors [15, 18, 20, 21] when the sublattice imbalance $\epsilon$ is nonzero. In the parameter space studied in this work, $\epsilon = 0$, this phase reduces to a single point in parameter space, which is absent in the mean-field calculations. This observation is therefore consistent to both the effective disappearance of that phase, or it being observed as a finite-size effect of the simulation. Beyond that point the image shows a strong evidence of a second order topological phase transition at $U/t \sim 3$. In figure 5(b) we observe the quasiparticle gap, defined as the gap that opens when we either add or remove a particle to the ground state

$$\Delta = \min \{E(N_p+1) - E(N_p), E(N_{p-1}) - E(N_p)\}.$$  

The adding or removal of a quasiparticle depends on the sign of the interactions. At $U > 0$ the gap is formed by removing a particle, which effectively decreases the total energy of the system at $U \gg t$ compared to the state with one more particle. Conversely, at $U \ll 0$ the system can only lower its energy by adding a particle. We show in figure 5(b) the behavior of the gap in the mean-field thermodynamic limit to remark that the gap closings are strongly size dependent, which is a consequence of finite-size effects. Note that the quasiparticle gap does not
close at the topological phase transitions because these transitions occur in the subspace of equal particle number, while the quasiparticle gap results from states with different particle number.

We have analyzed the convergence of our MPS simulations up to bond dimensions of \( \chi = 180 \) using two different approaches. The simplest approach involves a finite-size scaling of the ground state energy as estimated by the MPS (see figure 6(c)). We successfully fit the energy as \( E_\chi = a e^{-b\chi} + c \), as a function of the bond dimension. This behavior is a direct consequence of the MPS truncation error, but we find that the results up to \( \chi = 180 \) are almost converged. We can estimate those errors by analyzing the eigenstate fidelity, defined as

\[
\xi_F = 1 - \frac{\langle H^2 \rangle}{\langle H \rangle^2}.
\]

This quantity measures how close our MPS is to a true eigenstate of the Hamiltonian, \( H \). This value is shown in figure 6(a). The fidelity is minimal around the non-interacting phase and increases as we move towards the second order phase transition at \( U \approx 3t \). This decreased fidelity is an evidence of stronger correlations.

We have also analyzed the performance of the mean-field estimates against the extrapolated MPS energy \( E_\infty = \lim_{\chi \to \infty} E_\chi \). Figure 6(b) shows that the mean-field ansatz is marginally better than the MPS close to the non-interacting region. This is an evidence of the long-range correlations that can be captured by this momentum space Gaussian state. As we approach the phase transition, the mean-field becomes less accurate, but the difference is very small.

5. Experimental setup

Ultracold atoms trapped in optical lattices provide an excellent tool to study the topological properties of the Haldane model, in any of its flavors. This has been demonstrated by the recent experimental work in [10]. A crucial step to realize topological order in the lattice is to implement an effective gauge field on the phases of the hopping amplitudes when a particle moves through the lattice. There are two essential ways in which this is done. The first one is through laser assisted hopping [6, 7], in which the lattice is divided into two sublattices which host atoms in different hyperfine states. A laser beam then transfers atoms between sublattices, while imparting a complex phase. A second way to make complex hoppings is to periodically shake the lattice [10, 34]. This creates an effective Hamiltonian such that particles acquire a complex phase as they hop between sites. This second method was the one used to recreate the Haldane model in [10]. In that work the authors also measured...
the Berry phases acquired by the atom while moving through the Brillouin zone, an evidence of the topologically protected phase.

The model implemented by Jotzu et al is topologically equivalent to the Hamiltonian that we have studied (1) in absence of interactions: both are related by a smooth, unitary deformation of the pseudospin field that does not change the topological invariants. Our goal would be then to add interactions to the experiment, so as to observe the survival of topological invariants and the phase transitions that have been evidenced in previous sections. In [10] the authors used $^{40}$K atoms in two internal hyperfine states, $|F, m_F\rangle = |9/2, -9/2\rangle$ and $|9/2, -7/2\rangle$, but their interactions were suppressed using Feshbach resonances so that the effective model was a non-interacting version of (1). The introduction of interactions would ‘simply’ require adiabatically moving away from the Feshbach resonance. As we have shown in previous sections, the ground state in the interacting regime $|U| < |t|$, is smoothly connected to the $U = 0$ many-body state. We therefore expect that adiabatically increasing the interactions should not introduce significant heating. Regarding the experimentally achievable parameters, the authors report to the first-neighbor hoppings to be $t_{\text{FN}}/\hbar = [-746(81), -527(17), -527(17)]$ Hz and the second neighboring hoppings $t_{\text{SN}}/\hbar = [14, 14, 61]$ Hz, where $\hbar$ is Planck’s constant. The ratio $t_{\text{SN}}/t_{\text{FN}} \sim 0.05$ is comparable to our simulations, and the experimental parameters set a limit of interaction strength of about $U \sim 3t_{\text{FN}} \sim \hbar \times 1.5$ kHz.

As discussed in [10], one of the limiting factors in the extension of this experiment to $U \neq 0$, is the increased heating arising from having two fermionic species in the lattice. Provided this can be solved as in other experiments with Hubbard models, we would have access to study the physics of interacting TIs. The results shown in this work demonstrate that it would be possible to experimentally characterize such systems using the pseudospin field, $S(k)$. TOF images [32, 35] which reproduce the momentum density distributions in both sublattices, $n_{\alpha, b}(k)$, can reveal through the winding number the existence or not of a topological phase. Those experiments would also provide access to the study of non-equilibrium and transport physics in TIs, a topic that reaches beyond the simulation capabilities of mean-field and DMRG methods.

6. Summary and discussion

In this work we have studied the topological phase diagram of a TI with spin–spin on-site interactions. To identify these topologically non-trivial phases we have estimated the winding number. Our main result is the observation that the topological phases from the non-interacting TI model are extremely robust against the introduction of repulsive and attractive interactions, disappearing when interactions and hopping are comparable, $|U|/t \sim 3$. This prediction is supported by mean-field and MPS calculations, both of which show a remarkably good agreement in the detection of the topological phases. We have further characterized the topological phase transitions using the double occupancy (12), whose first order derivative undergoes a discontinuity when the winding number changes, giving us a strong signal of a transition into a topologically

![Figure 6. Convergence and fidelity of the MPS computations in the region $\phi = \pi/2$. (a) Eigenstate fidelity of the MPS states at different interaction energies. In green, the topological phase transitions of the mean-field model. (b) Comparison of the extrapolated MPS energy at infinite bond dimension, $E_{\text{MF}}$, with the mean-field energy. The parameter $\theta$ in the horizontal axis corresponds to the fitting parameter of the energy, $E_{\text{MF}}$, and is dependent on the interaction, $U$. Circles correspond to an MPS in a topological phase while triangles are a topologically trivial MPS. The dashed lines are the extrapolation of the respective energies to the limit of infinite bond dimension.](image-url)
trivial phase. We have also made a proposal to study these phase transitions using an existing, state-of-the-art implementation of the Haldane model [10], with the only requirement that interactions are reintroduced in the experiment. Finally, the remarkable agreement between our mean-field ansatz and the MPS simulations hints at the capacity of the former to capture long-range correlations and entanglement, making this ansatz a simple method to study topological phases with great accuracy. Further work to improve the mean-field ansatz, and study time-dependent variations is in order.

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