Democracy and the curse of natural resources∗

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Abstract

We propose a theoretical model to explain empirical regularities related to the curse of natural resources. This is an explicitly political model which emphasizes the behavior and incentives of politicians. We extend the standard voting model to give voters political control beyond the elections. This gives rise to a new restriction into our political economy model: policies should not give rise to a revolution. Our model clarifies when resource discoveries might lead to revolutions, namely, in countries with weak institutions. Natural resources may be bad for democracy by harming political turnover. Our model also suggests a non-linear dependence of human capital on natural resources. For low levels of democracy human capital depends negatively on natural resources, while for high levels of democracy the dependence is reversed. This theoretical finding is corroborated in both cross section and panel data regressions.

JEL Codes: D72, H52, O13.

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1 Introduction

Until world war II the economic profession tended to believe that natural resources were an unqualified blessing for the nation owning them. However, in the post world-war II period the evidence against this belief started accumulating: many resource rich countries grew very slowly and economists started to talk about the curse of natural resources. There is a large number of empirical papers which find evidence of this curse (e.g. Sachs and Warner (1995, 1997, 1999, 2001), Mehlum et al. (2006), Gylafson (2004), Strauss (2000)). Some authors (Sala-i-Martin (1997) and Doppelhofer et al. (2000)) have even classified natural resources as one of the ten most robust variables with a significantly negative effect on growth in empirical studies.

To summarize, there seems to be an empirical consensus on the following:

Fact 1 The curse of natural resources: countries rich in natural resources grow slower on average than natural resource poor countries.

However, there are many important outliers. Some resource rich countries have grown very fast (e.g. Botswana, Canada, Australia Norway) while others have grown very slowly (e.g. Nigeria, Zambia, Sierra Leone, Angola, Saudi Arabia, Venezuela). It seems fair to claim that:

Fact 2 The cross-country evidence is inconsistent with a monotonic effect of resources on development/growth: (Robinson et al. (2005))

We therefore need to understand when are natural resources a blessing and when are they a curse. The empirical literature has taken a step in this direction and it defines policy failure as the prime cause of the underperformance of resource rich countries. It also points to a reason why these policy failures occur. Namely:

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1Acemoglu et al. (2003) show that Botswana has the highest per capita growth of any country in the world in the last 35 years. The natural resource of Botswana are diamonds. This country had very bad starting conditions for growth (extremely low education levels, bad infrastructure, etc) but “good” institutions.

2Some countries which have been fairly rich in resources in 1970 that grew rapidly in the next 20 years are Malaysia, Mauritius and Iceland (see Sachs and Warner (2001)). Gylafson (2001a) additionally lists Indonesia and Thailand as countries attaining both long-term investment exceeding 25% GDP and per capita GNP growth exceeding 4% per year on average from 1970 to 1998. Also the so-called Scandinavian catch-up in the late nineteenth century was based on natural resources.
Fact 3 The quality of institutions is decisive in determining whether natural resources are a blessing or a curse.\textsuperscript{3}

Institutions are linked to the behavior of politicians, as they limit their discretion and define the policy space. The quality of institutions is also indicative of the level of democracy of a country. More democratic countries tend to have better institutions and are therefore less likely to be cursed by natural resources. But empirical findings also suggest a reverse causality:

Fact 4 Natural resources have antidemocratic properties: oil and mineral wealth tends to make states less democratic (Ross, 2001, Lam and Wantchekon, 2002)\textsuperscript{4}

Moreover, in countries with weak institutions natural resources are one of the main sources of civil war and revolution.

Fact 5 Many revolutions are linked to rents derived from natural resources (Collier and Hoeffler, 1998).

The contribution of the present paper is threefold:

1. We propose the first theoretical model that incorporates and explains the five empirical facts outlined above.

2. We present an explicitly political model which emphasizes the behavior and incentives of politicians. This is key, since there is a clear understanding that the behavior of government/politicians is fundamental to explain the economic performance in resource abundant countries (Newberry (1986, p.334)).

3. We extend the standard voting model to give voters political control beyond the elections. Democratic institutions are often imperfect, and electoral competition could be weak. But in our model, as in reality, citizens have instruments in addition to elections that allow them to

\textsuperscript{3}Mehlum, Moene and Torvik (2006) show that the effect of resources on growth is positive (negative) when institutions are good (bad) using Sachs and Warner’s (1995) data. The same paper as well as Boschini, Petterson and Roine (2003) show that the direct negative effect is stronger for minerals than other resources and institutions are more decisive for the effect of minerals than other resources.

\textsuperscript{4}This is also known as the political Dutch disease.
avoid policies which could cause them big welfare losses. We introduce these considerations in the model by assuming that citizens can initiate a revolution.\(^5\) This gives rise to a new restriction into our political economy model: policies should not give rise to a revolution. We will refer to this new constraint as the *no-revolution constraint*.

The existing theoretical literature concentrates mainly on explaining the “curse” (Fact 1)\(^6\)\(^7\) and suggests ways to avoid the curse.\(^8\) By ignoring the role of the government, this line of research cannot explain why governments do not choose the good policies in the first place.\(^9\) We need an explicitly political models to understand when natural resources are a blessing and when they are a curse.

To our knowledge the only existing explicitly political model in this area is Robinson et al. (2005), which explains empirical facts 2 and 3. In their paper there are two periods, with elections at the end of the first period. In the first period, natural resources are discovered. The incumbent government has to decide which proportion of the resources to extract then, and how much to leave for the following period. The government can consume the resource income, or use it to influence election outcomes by offering employment in the public sector, which is relatively inefficient. The main result of the paper is that politicians tend to overextract resources in the first period because they only care about the future resources if they remain in power. Moreover,

\(^5\)It need not be violent, although we will assume it causes some economic disruption.
\(^6\)For a list of explanations for the natural resource trap and their empirical support see Strauss (2000).
\(^7\)There is no generally accepted explanation for the curse so far. The one with maybe most empirical support is the “Dutch disease” explanation which goes as follows: the discovery and exploitations of natural resources like oil typically leads to large profits. These profits encourage entry into the industry at the expense of other sectors, expand national income and increase demand with a resulting inflationary pressure. At the same time more foreign currency enters the country which appreciates the real exchange rate. Export profits in the non-boom sector fall sharply which attracts even more capacity into the boom sector. The long-run results once the boom is over are stagflation and an over-valued real exchange rate.

The Dutch disease is preventable by good policies; e.g. Indonesia avoided the disease after its oil discovery by consistently devaluing its currency.

\(^8\)See e.g. Birdsall et al. (2000).
\(^9\)Rent-seeking and corruption are explanations that have been put forward. In these models the state is an aggregator of pressure from interest groups (Becker-Olson approach) which as Robinson et al. (2005) pointed out ignores incentives of politicians who often have a large amount of autonomy from interest groups.
the public sector will be inefficiently large. Institutions are decisive for the overall impact of resource booms because they determine the extent to which political incentives can really influence policy outcomes.

While the size of the public sector and the extraction path of natural resources are clearly relevant issues, there are other important channels from natural resources to growth that are unexplored by Robinson et al. (2005). In particular, human capital accumulation or education. One danger of natural resources (Gylafson (2001a)) is the neglect of education, since the country can live well over an extended period even with a weak commitment to education. But since we know that increased education is conducive to higher growth levels (Barro (2001), Barro and Lee (2001), Gylfason and Zoega (2004)), this reduced commitment to education will surely cost those countries in terms of long-run growth. For this reason, it is difficult to explain the higher persistence of growth in resource-rich Scandinavia than in Latin America (especially resource-rich countries such as Argentina and Chile) without remarking on the educational gap that emerged between the two groups of countries over the period 1870-1910 and which remained large throughout the twentieth century (see Bravo-Ortega et al. (2002)).

In this paper we build an explicitly political model to explain when will the finding of natural resources lead to higher or lower education levels. We are only interested in publicly owned resources, such as oil, gas and minerals. Politicians are purely self-interested and would like to consume the resource wealth themselves, but political pressure obliges them to redistribute at least a part of it to voters. This redistribution can take the form of: (i) a direct transfer or (ii) a subsidy for the investment in human capital, which has a positive spillover on the entire population. The incumbent government faces political pressure from two sources: an election and the possibility of a revolution. We model political opposition by the existence of a competitive fringe. The fringe parties compete among themselves and are less efficient than the government at managing natural resources. The underlying idea is that the party in power has access to better information or technologies which are denied to the opposition. The bigger this difference, the less transparent are the country’s institutions and the less democratic is the country. Hence

\[^{10}\text{In all petrostates the government maintains explicit legal ownership of below-ground reserves irrespective of surface property rights (see footnote 12 in Lam and Wantchekon (2002)). Most OPEC governments put the resources under national control in particular in the 60s and 70s.}

\[^{11}\text{E.g. control of the media.}\]
the size of the disadvantage of the fringe gives us a measure for the level of
democracy. While there will be always some gap, human capital can reduce
the comparative disadvantage of the fringe.

Besides the political competition there is always a possibility of a rev-
olution. If the revolution is successful, natural resources fall into the hand
of the voters who divide the gains equally among themselves. These gains
now depend on the management skills of voters. We assume that voters are
better at managing natural resources the higher their level of education.

We establish the following main results:

1. If the fringe wins the election, human capital increases with the size of
   the stock of natural resources.

2. If the government wins the election, human capital is a non-increasing
   function of natural resources.

3. If the government does not have to worry about revolution, human
capital is constant.

4. If revolution is a binding constraint, human capital decreases in natural
   resources.

5. Revolution is less likely to be a threat, the more democratic are a
country’s institution.

6. The probability that the incumbent is reelected may increase with nat-
   ural resources and this is more likely for countries with little democracy.

These results confirm that our explicitly political model captures the
five empirical facts mentioned above. Our model clarifies when resource
discoveries might lead to revolutions (Fact 5), namely, in countries with weak
institutions. In our model, natural resources may be bad for democracy
because they can harm political turnover (Fact 4). Our model suggests a non-
linear dependence of human capital on natural resources (Fact 2). For low
levels of democracy human capital depends negatively on natural resources,
while for high levels of democracy the dependence is reversed (Fact 3). Since
natural resources are bad when the government wins the election, and this
probability may increase with natural resources, especially in countries with
bad institutions, natural resources are a curse on average (Fact 1).
Empirical facts 1 to 3 were stated in terms of growth. We do not model growth directly, but use human capital/education instead, which is an established engine of growth (Barro 2001). Hence, our model yields direct predictions for the effect of natural resources on education (as well as indirect predictions on growth). These direct predictions can be tested empirically. Existing empirical studies report conflicting results of the effect of natural resources on education. The most complete study is the one by Stijns (2004), who discusses the different indicators used for resource abundance and human capital accumulation and shows that the conclusion on the link between these two is sensitive to the indicators chosen. Simple correlation coefficients and regressions switch from positive to negative depending on which resource abundance and which human capital indicator is used. This evidence might be consistent with the non-linear dependence of human capital on natural resources predicted in our model. This, however, has to be tested by including a variable that measures the level of democracy or the quality of institutions of a country in the regressions. There does not exist such an empirical study. We therefore run our own regressions (cross section and panel data) which confirm that the non-linear relationship is indeed driven by the democracy variable.

The remainder of the paper is organized as follows. Section 2 presents the model and solves it. Section 3 test the model empirically. Section 4 concludes.

2 The Model

Assume a country owns a stock of natural resources whose rents generate a discounted present value $W$. These resources are publicly owned and will therefore be managed by politicians. Politicians are motivated solely by self-interest, hence the government would like to keep the gains from the resources for itself, but it will only be able to benefit from the resource discovery if it remains in power. There are two potential threats for the government’s power: an upcoming election and the possibility of a revolution. First the election takes place. Once the election outcome is known, voters still have

\footnote{Gylfason (2001a, 2004) establishes an inverse relationship of human capital measured as public expenditure on education, expected years of schooling for girls and gross secondary-school enrollment with the share of natural capital in national wealth. However, the results seem to be driven by very few countries.}
the possibility to make a revolution. We first describe the electoral process. In the elections, the government $G$ faces the opposition of a competitive fringe. In other words, the opposition consists of several parties that compete among themselves. The unique policy issue is how to distribute the rents generated by the natural resource. We assume that the value of resources depends on the winner of the elections: its value will be $W$ if managed by the incumbent government and $\delta(e)W$ if managed by one of the fringe parties, where $0 < \delta(e) < 1$ for all $e$, indicates that the fringe is at a comparative disadvantage in managing natural resources. We assume that $\delta'(e) > 0$, i.e. the competence of the fringe increases with human capital. The function $\delta(e)$ is a proxy for the strength of democracy. The more democratic the state, the better is the fringe at managing natural resources.

There are two ways to transfer resource rents to voters, (i) via a direct (per capita) transfer $w$ and (ii) via a per unit subsidy $\pi$ for the investment in human capital. The individual’s level of human capital $e$ together with the average level of human capital $\bar{e}$ determines each individual’s marginal productivity (salary) $\omega$ in the following way (where we assume $\alpha + \beta < 1$):

$$\omega = ke^{\alpha}\bar{e}^{\beta}$$

Hence there is a positive externality (spillover) for society as a whole if an individual invests in human capital. We assume that the monetary cost of acquiring a unit of human capital is $\lambda$. Given the promised transfers, the voter decides on his own level of human capital by maximizing his utility. Hence, the program of the consumer is

$$\max_{e}U(w + ke^{\alpha}\bar{e}^{\beta} - (\lambda - \pi)e)$$

The FOC of this (concave) problem give $\lambda - \pi = \alpha ke^{\alpha-1}\bar{e}^{\beta}$. Since all consumers are identical we can assume that in equilibrium $e = \bar{e}$. Therefore the consumer’s incentive compatibility constraint can be written as

$$\lambda - \pi = \alpha ke^{\alpha+\beta-1}$$

and, using this constraint, we can talk directly about the level of human capital $e$ resulting from the transfers instead of discussing the size of the subsidy $\pi$.

There is a continuum of voters with total mass $n$. Voters care about the promised utility by the competing parties but also have some ideological concerns. The fringe parties are perceived by voters as ideologically equivalent,
hence we can assume that the equilibrium behavior of fringe players will be identical (we focus on a symmetric equilibrium). From now on, all endogenous variables will be indexed by the political actor offering them. Thus, we have $\omega_i, w_i, e_i, \bar{e}_i, \pi_i$ with $i \in \{G, F\}$ where $G$ stands for “Government” and $F$ for “Fringe”.

The electoral process is a version of the probabilistic voting model and works in the following way:

Voters are located in the interval $[0, 1]$. The utility of a voter $v \in [0, 1]$ when offered a policy that delivers “material” utility $U_G$ from the government is denoted

$$u(v, U_G) = U_G - \theta v$$

The utility of a voter $v \in [0, 1]$ when offered a policy that delivers “material” utility $U_F$ from the competitive fringe is denoted

$$u(v, U_F) = U_F - \theta(1 - v)$$

where $\theta$ denotes the strength of purely ideological concerns.

In addition, in every election there is an unexpected “aggregate shock” $\varepsilon \sim U[-A, A]$ to the utility that shifts preferences of all the voters in favor or against the incumbent. We add this shock to the preferences toward the incumbent.

$$u(v, U_G) + \varepsilon$$

The proportion of voters preferring $G$ over $F$ is then:

$$\min\left\{ \max\left\{ 0, \frac{1}{2} + \frac{U_G - U_F}{2\theta} + \frac{\varepsilon}{2\theta} \right\}, 1 \right\}$$

Thus, the ex ante probability that the incumbent wins the election, given promises $U_F, U_G$ is:

$$\Pr\left[ \min\left\{ \max\left\{ 0, \frac{1}{2} + \frac{U_G - U_F}{2\theta} + \frac{\varepsilon}{2\theta} \right\}, 1 \right\} \geq \frac{1}{2} \right].$$

Hence, the incumbent wins for all $\varepsilon > \varepsilon_1$ where $\varepsilon_1$ makes $\frac{1}{2} + \frac{U_G - U_F}{2\theta} + \frac{\varepsilon_1}{2\theta} = \frac{1}{2}$. Thus, $\varepsilon_1 = -(U_G - U_F)$. The probability of winning for the incumbent is equal to
Pr \[\varepsilon > \varepsilon_1 = -(U_G - U_F) \] = \min \left\{ \max \left\{ 0, \frac{A - \varepsilon_1}{2A} \right\}, 1 \right\} (2)
= \min \left\{ \max \left\{ 0, \frac{1}{2} + \frac{U_G - U_F}{2A} \right\}, 1 \right\}.

The incumbent cannot win if \(\frac{1}{2} + \frac{U_G - U_F}{2A} < 0\) which implies that \(A < -(U_G - U_F)\). On the other hand, the incumbent wins with probability 1 for \(A < (U_G - U_F)\).

After the election results, the citizens decide whether or not to make a revolution. Hence, if a revolution takes place, this happens after the acquisition of human capital. We assume that a revolution is costly (its marginal cost is \(c\)) and it is successful with probability \(q\). In case of a successful revolution, the citizens manage directly the natural resources and obtain an equal split of these resources. Just like with the fringe politicians, the natural resources that go to the citizens after the revolution increases with human capital. We model this by assuming that the natural resources that go to citizens after a successful revolution is \(\gamma(e) \frac{W}{n}\) with \(\gamma'(e) > 0\). If the revolution fails, the original contract proposed by the winner of the elections is imposed.

We now make a first observation on this game, that will useful for the discussion.

**Lemma 6** Revolution is a potential threat only to the incumbent government

**Proof.** First notice that competition among the fringe players drives their profits down to zero. The equilibrium offer by the fringe can thus be obtained by maximizing the consumers’ utility subject to the resource constraint (what we call the fringe program). To see why the fringe never takes the no-revolution constraint into account, suppose that the solution to the above described fringe program (call it program 1) does not satisfy the no-revolution constraint (which would be the only problematic case). Then one could obtain an alternative solution by imposing the constraint (call this

\[^{13}\text{We think of a revolution as a threshold public good problem. At least } x \text{ people have to go, or the revolution will not take place. This modeling choice leaves unanswered the question of who does the revolution. In our model there is a natural candidate: the group of voters ideologically most distinct from the winning party.}\]
the solution to program 2). But the solution to program 2 can only decrease
the utility of agents (with respect to the solution of program 1), which can
only worsen the constraint, hence we would entail a contradiction.

The government, on the other hand, does keep some of the resource rents
for itself. Therefore revolution might be a threat for the government. The
no-revolution constraint requires the contract offered by the government
to be at least as good as the outcome of the revolution, i.e.\(^{14}\)

\[
U(w + ke^{\alpha+\beta} - ake^{\alpha+\beta}) \geq qU(\gamma(e)W/n - \lambda e) + (1-q)U(w + ke^{\alpha+\beta} - ake^{\alpha+\beta}) - c
\]

which simplifies to:

\[
U(w + ke^{\alpha+\beta} - ake^{\alpha+\beta}) \geq U(\gamma(e)W/n - \lambda e) - \frac{c}{q} \quad (3)
\]

We are now in the position to state the maximization problems of the
government and of the fringe players.

2.1 The fringe problem

Due to competition among fringe players, the fringe maximizes the con-
sumers’ utility subject to the resource constraint\(^{15}\):

\[
\max_{e_F, w_F} U(w_F + ke_F^{\alpha+\beta} - ake_F^{\alpha+\beta})
\]

subject to \(\delta(e_F)W/n - w_F - \lambda e_F + ake_F^{\alpha+\beta} \geq 0\)

or equivalently

\[
\max_{e_F, w_F} U \left( \delta(e_F)W/n - w_F - \lambda e_F + ake_F^{\alpha+\beta} + ke_F^{\alpha+\beta} - ake_F^{\alpha+\beta} \right)
\]

\[
\max_{e_F} U \left( \delta(e_F)W/n - \lambda e_F + ke_F^{\alpha+\beta} \right)
\]

\(^{14}\)Here we already introduce the incentive compatibility constraint of consumers, equation 1.

\(^{15}\)Profits are really \(\delta(e_F)W/n - w_F - \lambda e_F + ake_F^{\alpha+\beta}\) min \(\max \left\{ 0, \frac{1}{2} + \frac{U_F - U_G}{A} \right\} , 1 \right\},\)
but notice that min \(\max \left\{ 0, \frac{1}{2} + \frac{U_F - U_G}{A} \right\} , 1 \right\} being a probability is always bigger than
zero, thus it never affects whether the constraint is binding or not.
So the first order condition is

\[ \delta'(e_F) W/n + (\alpha + \beta)k e_F^{\alpha+\beta-1} - \lambda = 0 \]  

(4)

In this way, we have that

\[ \frac{\partial e_F}{\partial W/n} = \frac{-\delta'(e_F)}{(\alpha + \beta)(\alpha + \beta - 1)k e_F^{\alpha+\beta-2} + \delta''(e_F)W/n} \]

Since we know that if the decision is optimal \((\alpha + \beta)(\alpha + \beta - 1)k e_F^{\alpha+\beta-2} + \delta''(e_F)W/n \leq 0\) (to guarantee the satisfaction of second order conditions), then if \(\delta'(e_F) \geq 0\), the effect of increasing \(W\) in \(e_F\) is positive. We summarize this observation in:

**Proposition 7** When the fringe wins the election, human capital is positively related to the amount of natural resources.

Things look very different if the incumbent government wins the election.

### 2.2 The government problem

The government maximizes its own utility subject to the no-revolution constraint:

\[
\max_{e_G,w_G} \left( W/n - w_G - \lambda e_G + \alpha k e_G^{\alpha+\beta} \right) \\
\times \min \left\{ \max \left\{ 0, \frac{1}{2} + \frac{U(w_G + ke_G^{\alpha+\beta} - \alpha ke_G^{\alpha+\beta}) - U_F}{2A} \right\}, 1 \right\}
\]

subject to \(U(w_G + ke_G^{\alpha+\beta} - \alpha ke_G^{\alpha+\beta}) \geq U(\gamma(e_G)W/n - \lambda e_G) - \frac{c}{q}\)

In order to derive some analytical solutions, we further assume that \(U(x) = \ln(x)\). Then

\[
\max_{e_G,w_G} \left( W/n - w_G - \lambda e_G + \alpha k e_G^{\alpha+\beta} \right) \\
\times \min \left\{ \max \left\{ 0, \frac{1}{2} + \frac{\ln(w_G + ke_G^{\alpha+\beta} - \alpha ke_G^{\alpha+\beta}) - U_F}{2A} \right\}, 1 \right\}
\]
subject to $w_G + k e_G^{\alpha+\beta} - \alpha k e_G^{\alpha+\beta} \geq (\gamma(e_G)W/n - \lambda e_G) \exp\left(-\frac{c}{q}\right)$

We have to distinguish two cases: (i) the no-revolution constraint binds at the optimum and (ii) the no-revolution constraint does not bind at the optimum.

Case (i) We have:

$$\max_{e_G, w_G} \left[ \frac{W}{n} \left( 1 - \gamma(e_G) \exp\left(-\frac{c}{q}\right) \right) + k e_G^{\alpha+\beta} - \lambda e_G \left( 1 - \exp\left(-\frac{c}{q}\right) \right) \right]$$

$$\times \min \left\{ \max \left\{ 0, \frac{1}{2} + \frac{\ln \left( \frac{W}{n} \left( 1 - \gamma(e_G) \exp\left(-\frac{c}{q}\right) \right) - U_F \right) }{2A} \right\} , 1 \right\}$$

The FOC are

$$G'(e_G) \equiv -\frac{W}{n} \left[ \gamma'(e_G) \exp\left(-\frac{c}{q}\right) + k(\alpha + \beta) e_G^{\alpha+\beta-1} - \lambda \left( 1 - \exp\left(-\frac{c}{q}\right) \right) \right]$$

$$\left( \frac{1}{2} + \frac{U}{2A} \frac{W/n \left( 1 - \gamma(e_G) \exp\left(-\frac{c}{q}\right) \right) - U_F}{2A} \right)$$

$$+ \left( \frac{-\gamma'(e_G) \exp\left(-\frac{c}{q}\right)}{2A \left( 1 - \gamma(e_G) \exp\left(-\frac{c}{q}\right) \right)} \right) \frac{W}{n}$$

$$= 0$$

which implies that
\[ \frac{\partial e_G}{\partial W/n} = \frac{G''(e_G) \gamma'(e_G) e_G - c q \left( \frac{1}{2} + \frac{U (W/n (1 - \gamma(e_G) e_G)) - U_F}{2 A} \right)}{G''(e_G)} + \frac{\left[ \left( 1 - \gamma(e_G) e_G \right) + ke_G^{\alpha+\beta} - \lambda e_G \left( 1 - \exp \left( -\frac{c q}{q} \right) \right) \right] \left( \frac{\gamma'(e_G) e_G - c q}{2 A (1 - \gamma(e_G) e_G)} \right)}{G''(e_G)} \]

We know that:

1. \( G''(e_G) \leq 0 \) to guarantee the satisfaction of second order conditions.

2. \( \frac{1}{2} + \frac{U (W/n (1 - \gamma(e_G) e_G)) - U_F}{2 A} \geq 0 \), since it is a probability,

3. \( \left[ \left( 1 - \gamma(e_G) e_G \right) + ke_G^{\alpha+\beta} - \lambda e_G \left( 1 - \exp \left( -\frac{c q}{q} \right) \right) \right] \geq 0 \), as those are the profits of the incumbent if she wins the elections

Thus, if \( \gamma'(e_G) \geq 0 \), the effect of increasing \( W \) in \( e_G \) is negative.\(^{16}\)

**Case (ii)** If the constraint does not bind (true for sufficiently low values of \( \exp \left( -\frac{c q}{q} \right) \)), we have:

\[
\max_{e_G, w_G} \left( W/n - w_G - \lambda e_G + \alpha k e_G^{\alpha+\beta} \right)
\times \min \left\{ \max \left\{ 0, \frac{1}{2} + \frac{\ln(w_G + k e_G^{\alpha+\beta} - \alpha k e_G^{\alpha+\beta}) - U_F}{2 A} \right\}, 1 \right\}
\]

\(^{16}\)When calculating the first order conditions we implicitly assume that \( \frac{1}{2} + \frac{U_G - U_F}{4 A} < 1 \). If the expression becomes bigger than 1, the government wins the elections for sure. In this case it only has to take the no-revolution constraint into account. We show in the Appendix that also in this case increasing natural resources has a negative effect on human capital.
and the FOCs are:

\[
0 = -\left(\frac{1}{2} + \frac{\ln(w_G + ke_G^{\alpha+\beta} - \alpha ke_G^{\alpha+\beta})}{2A} - U_F\right) + \\
\left(W/n - w_G - \lambda e_G + \alpha ke_G^{\alpha+\beta}\right)\left(\frac{1}{2A(w_G + ke_G^{\alpha+\beta} - \alpha ke_G^{\alpha+\beta})}\right)
\]

\[
0 = (-\lambda + \alpha k(\alpha + \beta)e_G^{\alpha+\beta-1})\left(\frac{1}{2} + \frac{\ln(w_G + ke_G^{\alpha+\beta} - \alpha ke_G^{\alpha+\beta}) - U_F}{2A}\right)
\]

\[
+ \left(W/n - w_G - \lambda e_G + \alpha ke_G^{\alpha+\beta}\right)\left(\frac{k(1 - \alpha)(\alpha + \beta)e_G^{\alpha+\beta-1}}{2A(w_G + ke_G^{\alpha+\beta} - \alpha ke_G^{\alpha+\beta})}\right)
\]

Thus

\[
k(\alpha + \beta)e_G^{\alpha+\beta-1} = \lambda \iff e_G = \left(\frac{k(\alpha + \beta)}{\lambda}\right)^{1-\alpha-\beta}
\]

(5)

In this case human capital is independent of the amount of natural resources.

We summarize these remarks in the following proposition.

**Proposition 8** If the government wins the election, human capital is a non-increasing function of natural resources. If revolution is no threat, human capital does not depend on natural resources. If revolution is a threat, human capital decreases with natural resources.

#### 2.2.1 When does the government worry about revolution?

One thing that remains unclear in the previous exposition is the conditions under which the no-revolution constraint is binding. We now explore this issue.

Rewriting the FOCs for case (ii) (the one where the constraint is not binding) allows us to calculate \(w_G\).

\[
0 = -\left(\frac{1}{2} + \frac{\ln(w_G + (1 - \alpha)e_G^{\alpha+\beta} - U_F)}{2A}\right) + \frac{W/n - w_G - \beta e_G^{\alpha+\beta}}{2A(w_G + (1 - \alpha)e_G^{\alpha+\beta})}
\]

The solution is:

\[
\begin{cases}
  w_G = \exp\left(\text{LambertW}\left(\left(W/n + (1 - (\alpha + \beta))ke_G^{\alpha+\beta}\right)\exp^{A-U_F-1}\right) - A + U_F + 1\right)
  \\
  -k(1 - \alpha)e_G^{\alpha+\beta}
\end{cases}
\]

(6)
The salary $w_G$ we obtained assuming the no-revolution constraint is not binding satisfies that constraint when the following inequality is true.

$$w_G \geq (\gamma(e_G)W/n - k(\alpha + \beta)e_G^{\alpha+\beta}) \exp\left(-\frac{c}{q}\right) - k(1 - \alpha)e_G^{\alpha+\beta} \equiv NR(e_G) \quad (7)$$

To get some insight when revolution is a concern for the government, we perform some numerical simulations, using condition (7). In those simulations we will always vary the value of $W/n$ and some other exogenous variable simultaneously. Similarly, the figures we show depict the value of $w_G$ and of $NR(e_G)$, as a function of $W/n$ and some other exogenous variable. We group these other exogenous variables into four categories depending on their economic meaning. For the simulations we use the functions\textsuperscript{17} $\gamma(e) = 10^{-4} + e^2$, and $\delta(e) = \hat{\delta} e^{\alpha+\beta}$ and the basic parameters, which are then varied individually (along with $W/n$) to observe the different comparative statics are: $(\alpha, \beta, \hat{\delta}, \lambda, A, c/q, k) = (0.5, 0.2, 0.15, 1, 1, 10)$.

1. The variables $\frac{c}{q}$ and $\gamma(e_G)$ determine the strength of the threat of revolution. The larger is this threat, the more likely is the no-revolution constraint to bind. In other words, as the citizens become better at managing natural resources (high $\gamma(e_G)$ for all $e_G$), the no-revolution constraint becomes more relevant. Similarly, for low values of $\frac{c}{q}$ (the cost of revolution is low and/or the probability of success is high) the no-revolution constraint will always bind. When $\frac{c}{q}$ increases, low values of $W/n$ give rise to the unconstrained solution while the constraint binds for high values of $W/n$. Given that revolution is already costly, it is only worthwhile if there is a lot to gain (high $W/n$). For sufficiently high $\frac{c}{q}$ revolution is never an issue; it is simply too costly or too unlikely to be successful.

Figure 1 shows the impact of $\frac{c}{q}$ on both $w_G$ and of $NR(e_G)$ and illustrates graphically the previous discussion.

2. The variables $k, \lambda, \alpha$ and $\beta$ determine the returns and costs of investment in human capital.

\textsuperscript{17}We tried other functional forms, in particular $\delta(e) = \hat{\delta}$, and the qualitative results in terms of comparative statics are similar.
(a) The effect of a change in $k$, which increases (linearly) the marginal return to human capital, depends crucially on the function $\gamma(e)$. An increase in $k$, leads to higher $e$ and thus an increase in $\gamma(e)$. Both the unconstrained transfer $w_G$ and $NR(e_G)$ increase with $k$ (and with $W/n$). Whether or not the latter increases more strongly, depends on $\gamma(e)$.

i. For low $\gamma(e)$, the no-revolution constraint never binds. Citizens are simply too bad in managing natural resources.

ii. Suppose $\gamma(e)$ is sufficiently large. Then, if $k$ or $W/n$ are sufficiently low, the no-revolution constraint never binds. However, if both $k$ and $W/n$ are sufficiently high, the constraining binds. The reason for this is that the average slope of $NR(e_G)$ with respect to both $k$ and $W/n$ is higher than that of $w_G$. To understand this, notice that on the right hand side of equation (7) we have the term $\gamma(e_G)W/n$. This means that the human capital of the government $e_G$, and natural resources $W/n$ are complements in the technology for revolutions, so a simultaneous increase of $k$, and thus $e_G$, and $W/n$ are bound to have a higher effect on the possibility of revolutions than on $w_G$.

Figure 2 shows the impact of $k$ on both $w_G$ and of $NR(e_G)$ and illustrates graphically the previous discussion.

(b) The parameter $\lambda$ measures the individual’s marginal cost to acquire human capital. The effects of changing $\lambda$ are, thus, the reverse effects of changing $k$ (which, remember, is a proportionality constant on human capital returns). More precisely:

i. For low $\gamma(e)$ the no-revolution constraint never binds.

ii. If $\gamma(e)$ is sufficiently large, the no-revolution constraint binds when both $\lambda$ and $W/n$ are sufficiently low and does not bind if either $\lambda$ or $W/n$ are sufficiently high.

Figure 3 shows the impact of $\lambda$ on both $w_G$ and of $NR(e_G)$.

(c) The parameters $\alpha$ and $\beta$ determine the returns to scale of human capital. We assume $\alpha + \beta < 1$, hence returns to scale will always be decreasing. Since both parameters have the same qualitative effect, we will describe only the effects of $\alpha$. For low $\alpha$, the no-revolution constraint always binds except for very high values of
When $\alpha$ increases, the fraction of values of $W/n$ in which the unconstrained solution holds increases, until $\alpha$ is so high that only the unconstrained solution holds. The intuition is as follows: returns to human capital are not relevant when revolution is successful since there is no productive activity of workers in case of revolution. Hence, the revolution is more attractive for low values of $\alpha$. When $\alpha$ increases, it is more costly to forgo the returns from productive activity, and revolution will only be attractive if there are sufficient natural resources to be managed. For high enough $\alpha$, it is simply too costly not to engage in the productive activity, hence revolution is never an issue.

Figure 4 shows the impact of $\alpha$ on both $w_G$ and of $NR(e_G)$ and illustrates graphically the previous discussion.

3. The function $\delta(e)$ is a measure of democracy. The closer this function is to 1 for all $e$, the better are the democratic institutions. Better democratic institutions allows the fringe to offer a higher utility $U_F$ to voters. Hence, the government has to react with a higher direct transfer $w_G$ which implies that the no-revolution constraint will bind less often. In other words, with good democratic institutions, revolution will not occur.

4. The aggregate shock $A$ to voters’ preferences measures the extent to which policies matter for winning the elections. The bigger the shock, the less important are the promised utilities to voter. For very low $A$, we always have the unconstrained solution. When $A$ increases, the constraint soon bites and we only get the unconstrained solution for low $W/n$. The higher $A$, the smaller the fraction of value for which the unconstrained solution holds. This happens because $w_G$ decreases with $A$, since promised utilities have a smaller effect on the probability of winning the elections, while $NR(e_G)$ is independent of $A$.

Figure 5 shows the impact of $A$ on both $w_G$ and of $NR(e_G)$ and illustrates graphically the previous discussion.

### 2.3 Determining the winner of the elections

The probability that the government wins the election is directly related to $U_G - U_F$. To gain some insight we will discuss the case when the no-revolution
constraint does not bind. From (6) we can conclude that

\[ U_G - U_F = \text{LambertW} \left( \left( W/n + (1 - (\alpha + \beta))ke_G^{\alpha+\beta} \right) \exp^{A-U_F-1} \right) - A + 1 \]

Since the LambertW function is increasing we only have to look at the derivative of its argument. Thus we have

\[
sign \frac{\partial (U_G - U_F)}{\partial (W/n)} = \text{sign} \left( \exp^{A-U_F-1} \left( 1 - \left( \frac{W/n + (1 - (\alpha + \beta))ke_G^{\alpha+\beta}}{\exp^{A-U_F-1}} \right) \frac{\partial U_F}{\partial W/n} \right) \right) = \text{sign} \left( 1 - \frac{(W/n + (1 - (\alpha + \beta))ke_G^{\alpha+\beta}) \delta(e_F)}{\delta(e_F)W/n -}\delta e_F + ke_F^{\alpha+\beta} \right) \]

(8)

Whether this sign is positive or negative, hence whether the probability that the government wins the election is increasing or decreasing is generally going to depend on the parameters of the model. However, a couple of things can be deduced from this expression. For \( W/n = 0 \) we know from equation (4) that

\[ e_F = \left( \frac{(\alpha+\beta)}{\lambda} \right)^{1/(1-\alpha-\beta)} \]

Clearly, if \( \frac{(\alpha+\beta)}{\lambda} \) is low enough, (8) is positive. On the other hand, for very large \( W/n \) when the variation of \( e_F \) is smaller than that of \( W/n \) then (8) will asymptote to zero. From this argument it is not clear whether it could ever be decreasing. To confirm that in fact it can, we perform a numerical simulation using the same basic parameter values and functional forms as in subsection 2.2.1. The result of this simulation is shown in Figure 6. The figure displays the two features we uncovered analytically and also shows that for sufficiently high \( W/n \) the sign is negative.

The fact that the derivative can be both positive and negative reflects that two economic forces are at work. On the one hand, as resources increase, the government can pay higher direct transfers \( w_G \), thus increasing its chances of winning. On the other hand, the fringe can also offer better terms, especially through the channel of human capital \( e_F \), which also enhances its probability of winning and makes the fringe a better administrator of natural resources. The concavity of the effort function makes it more likely that the first effect dominates in the beginning. The effect of direct transfers hits the margin directly from the beginning, whereas the effect of human capital needs more natural resources to have the same marginal impact.
Clearly, by bounding the $\delta(e)$ function one could ensure that (8) is never negative, which seems to be the relevant case according to the empirical evidence. Recall that we interpreted the $\delta(e)$ function as a proxy for the strength of democracy. $\delta(e)$ low and bounded corresponds to a country where democratic institutions are weak and the Fringe cannot manage natural resources as efficiently as the government even for high levels of education. In this case, any natural resource finding increases the chances of the incumbent government to stay in power. Only strong democracies make it less likely that natural resources will allow the incumbent to become more entrenched.

3 Empirical evidence

In order to test empirically our theory we have conducted some regressions on available data. We first report a cross section regression of primary school enrollment (human capital) on a measure of natural capital share (naturalk), a measure of political rights (polriginv), the cross product of both variables (cross) and additional controls. Natural capital is taken from Gylfason and Zoega (2004) who constructed this measure from World Bank Data. Natural capital is the sum of “subsoil wealth”, timber, non-timber benefits of forests, cropland, pasture land, and the opportunity cost of protected areas. In turn, subsoil wealth is the present value of a constant stream of economic profits on “resource rents” on various fuels and minerals; that is, gross profit on extraction less depreciation of capital and normal return on capital. The political rights variable is the inverse of the Gastil Index of Political Rights constructed by the Freedom House.\footnote{For a detailed explanation of the index and the methodology see http://www.freedomhouse.org/template.cfm?page=35&year=2005} Thus, in our regressions political rights are measured on a one-to-seven scale, with one representing the lowest degree of freedom and seven the highest. Political Rights’ data for a specific year is the previous five years’ average. The source for the data on primary school enrollment is the World Development Indicators from UNESCO. The additional controls are taken from the World Development Indicators (WDI) and are the log of GDP, measures of fertility, mortality and birth rates and the pupil teacher ratio in primary schools (pupil). Natural capital is available for 1994. All other variables are typically available every 5 years from 1970 to 1995. In the regression we use per country averages for this period and only include countries which have at least 3 observations for all variables.
This makes 78 countries.

The results are reported in table 1. Most importantly, natural capital has a negative and significant effect and the cross variable has a positive and significant effect. This means that the higher the level of democracy, the lower the negative impact of natural resources. In fact, when the index for political rights is four or above the net effect is positive. This is consistent with the predictions of our model. Other controls also make sensible predictions: log GDP has a positive effect on human capital, fertility and mortality have negative effects.

Table 1: Cross section regression.

|          | Coeff.  | Std. Err. | t     | P(\(|\beta_i| > 0\)) |
|----------|---------|-----------|-------|-----------------------|
| naturalk| -1.153341 | .4238907  | -2.72 | 0.008                 |
| cross2   | .3395294 | .1244045  | 2.73  | 0.008                 |
| polriginv| -7.585228 | 2.195964  | -3.45 | 0.001                 |
| lgdp     | 3.541507 | 2.756116  | 1.28  | 0.203                 |
| fertility| -20.29832 | 5.814107  | -3.49 | 0.001                 |
| mortality| -.2884667 | .0950492  | -3.03 | 0.003                 |
| pupil    | .2558133 | .1964023  | 1.30  | 0.197                 |
| birth    | 2.493832 | .8983785  | 2.78  | 0.007                 |
| cons     | 116.8061 | 24.83554  | 4.70  | 0.000                 |

One problem with this regression is its cross-section nature. We have also done a fixed effects panel data regression. The main difference is that the variable for natural resources is now a measure of primary export intensity (exports of primary products divided by GDP, source: WTO). The main drawback of this variable is that it also includes agricultural products and not only fuel and mining products. This is the variable used by Sachs and Warner (1995). All variables are typically available from 1970 to 1995 for a total of 346 observations for 84 countries. The results (reported in table 2) are very similar to those of the previous regression. Primary export intensity (sxpwon) has again a negative and significant effect (at a 10% level) and the cross variable has a positive and significant effect. Again, when the index for political rights is four or above the net effect is positive, consistently with
our model. log GDP still has a positive effect on human capital, fertility and mortality still have negative effects. Pupil is now significant and positive.\footnote{A further difference of the panel data regression with the cross section regression is that in the panel data regression (Table 2) we do not include the polriginv variable (our measure for political rights) directly but only the cross effect with primary export intensity. We chose to report the regression of Table 2 since the regression with polriginv reveals that polriginv is not significant (t-value of 0.64) and the cross variable loses significance (t-value of 1.25) when polriginv is included.}

|            | Coeff.  | Std. Err. | t    | \(P(|\beta| > 0)\) |
|------------|---------|-----------|------|---------------------|
| sxpown     | -21.15012 | 12.92005  | -1.64 | 0.103               |
| cross      | 6.758621  | 3.141805  | 2.15  | 0.032               |
| lgdp       | 5.355735  | 3.070228  | 1.74  | 0.082               |
| fertility  | -2.134075 | 2.241165  | -0.95 | 0.342               |
| mortality  | -.4958456 | .0728834  | -6.80 | 0.000               |
| pupil      | .4484188  | .1273336  | 3.52  | 0.001               |
| birth      | .7433418  | .4282405  | 1.74  | 0.084               |
| cons       | 53.89933  | 25.74006  | 2.09  | 0.037               |

Table 2: Panel data regression.

To summarize, the above regressions clearly corroborate our theory: the quality of institutions and the level of democracy is decisive in determining whether natural resources are a blessing or a curse. In democratic countries natural resources enhance education. In non-democratic countries natural resources are detrimental to education. In both the cross section and the panel data regression the cutoff value for our index of political rights is 4. Hence, in all countries clearly classified as free by the Freedom House (which correspond to our index 5.5 to 7) natural resources are a blessing and are a curse in all countries the Freedom House classifies as not free (which correspond to our index 1 to 2.5). The turning point lies in the partly free countries (with an index 3 to 5).
4 Conclusion

In this paper we have presented a formal political-economy analysis of the impact of natural resources on human capital accumulation. In our model, citizens exert control over politicians via an election and can always initiate a revolution if they are dissatisfied with the proposed policies. Since it is a well-documented fact that natural resources have led to civil unrest, it is important to incorporate this possibility into the model. To our knowledge this is the first paper to do so.\textsuperscript{20} We propose to model the possibility of revolution by introducing a new constraint into the model, which we denote the \textit{no-revolution constraint}. Under this constraint politicians select their policies so that there are no sufficiently large sectors of the population who want to block this policy by starting a revolution. In the context of natural resources, this constraint can be taken literally. However, we would like to emphasize that this constraint might be introduced in many other models: the economic literature is full of policy recommendations which no sane politician has dared to implement even if a majority of the population would benefit from them. This sounds contrary to both economic and political theory, but we would argue that there are good practical reasons for the outcome that the models overlook.

These policy recommendations arise in models where the policy resulting from the voting mechanism (e.g. the policy preferred by the median voter) would harm a sizable proportion of the population. Such policies are not implemented because the sector that would be harmed has pressure instruments on top of their votes to block them, and these pressure instruments can be modeled by the no-revolution constraint.\textsuperscript{21} Hence, the importance of

\textsuperscript{20}Introducing revolution in political economy models is not an innovation \textit{“per se.”} Acemoglu and Robinson (2001) explain the “extension of the franchise” in precisely this way. But notice that in their work, revolution is a threat from citizens “excluded” from the vote, who thus have no alternative. In our work, “revolution” is an added tool for all citizens, not an alternative when there is not a chance to vote. Acemoglu and Robinson (2006) do include the possibility of revolting in democracy. However, this possibility does not operate as a constraint for the government. It is simply a binary choice for the poor (already the median voter and thus the tax setters in the democracy).

\textsuperscript{21}One example of such a policy recommendation is the abolition of capital taxes. Lucas (1990) has shown that the optimal capital tax is zero. It has also been shown that the representative consumer would vote for a capital tax of zero. Even in a model with heterogeneous agents (Garcia-Mila, Marcet and Ventura, 2001) the median voter is likely to vote in favor of abolishing capital taxes. This, however, can harm as much as a third of
our proposed modeling innovation lies far beyond the topic studied.

In terms of the topic we study, our contribution is to incorporate simultaneously the five empirical facts on natural resources presented in the introduction. We can explain when natural resources are a blessing and when they are a curse (Fact 2) and we capture the importance of the quality of institutions and level of democracy (Fact 3). A further result of our model is that natural resources may be bad for political turnover and will be so in countries with little democracy / worse institutions (Fact 4). In those countries natural resources strengthen the position of the incumbent government, who typically chooses policies which do not enhance, or are even detrimental, to human capital accumulation and therefore growth. If the majority of countries with natural resources have bad institutions we can expect that natural resources are bad for growth on average (Fact 1: the curse). Our model also answers the question of when do natural resources lead to a revolution: in countries with little democracy and bad institutions (Fact 5).

Our model links natural resources to education which is an established engine for growth. Nevertheless, we want to be sure that the empirical facts which are stated in terms of growth are also valid if we use education. We therefore tested these facts for education in a series of regressions, and we find that they indeed hold.

Some authors have suggested that the size of a country matters for the effect of natural resources. This is captured in our model, where country size is measured by \( n \). Increasing \( n \) has the same effect as decreasing natural resources \( W \).

In our model, the income of the government stems only from natural resources. In a more complete model the government can also receive income by taxing productive activity. This is one of the extensions we would like to study in the future. The existence of productive activity has an effect on the incentives of politicians to encourage human capital accumulation: better education should enhance productive activity, which in turn enables the government to extract more taxes. But better education also strengthens the opposition and the ability of citizens to engage in a successful revolution. We expect that the incumbent government will prefer not to enhance education, since education weakens its political position and it is easier for them to increase their income from natural resources than by taxing productive the population. This part of the population would probably go to great lengths in order to avoid the zero capital tax.
activity. Natural resources are easily appropriated by corrupt politicians. So are some unnatural resources, like foreign aid. Is there a link between natural resources and foreign aid? Can our model make predictions about the effects foreign aid might have on education or growth?

The answer is yes. Once the foreign aid is granted it is very difficult for international institutions to avoid that politicians steal foreign aid. Empirical evidence suggests that only a small percentage of the aid actually reaches its desired objective. In Uganda only 13% of foreign aid granted for education in 1991-1995 actually reached primary schools (Reinikka and Svensson (2004). The evidence for other African countries is similar. As with natural resources the quality of institutions is crucial in limiting stealing from foreign aid. But similarly to natural resources, foreign aid tends to be detrimental to democracy: studying 108 recipient countries of foreign aid in the period 1960 to 1999 Djankov, Montalvo and Reynal-Querol (2005) find a negative effect of foreign aid on democracy which is much bigger than the negative effect of natural resources. Like natural resources foreign aid can be the cause of civil war and revolution.22

Given these empirical similarities between the effects of natural resources and foreign aid, we can use our model to make predictions about when foreign aid is a blessing and when it is a curse. In democratic countries with good institutions, foreign aid will enhance growth, while the opposite will happen in countries with bad institutions and little democracy. Typically it is the latter group of countries that receives foreign aid. Our model recommends that only poor countries that are democratic and have good institutions should be granted foreign aid.

22Maren (1997) provides evidence that the cause of the civil war in Somalia was the control over foreign aid.
A Appendix

A.1 If the Fringe cannot win the election

If G wins the election with probability 1, it only has to take the no-revolution constraint into account. Therefore, G’s problem is as follows:

\[
\max_{e,w} \frac{W}{n} - w - \lambda e + \alpha ke^{\alpha+\beta} \\
\text{subject to} \ U(w + ke^{\alpha+\beta} - \alpha ke^{\alpha+\beta}) \geq U(\gamma(e)\frac{W}{n} - \lambda e) - \frac{c}{q}
\]

Assume now that \( U(x) = \ln(x) \).

Then

\[
\max_{e,w} \frac{W}{n} - w - \lambda e + \alpha ke^{\alpha+\beta} \\
\text{subject to} \ w + ke^{\alpha+\beta} - \alpha ke^{\alpha+\beta} \geq (\gamma(e)\frac{W}{n} - \lambda e) \exp\left(-\frac{c}{q}\right)
\]

Thus an equivalent way of writing the problem is:

\[
\max_{e,w} \frac{W}{n} \left(1 - \gamma(e) \exp\left(-\frac{c}{q}\right)\right) + ke^{\alpha+\beta} - \lambda e \left(1 - \exp\left(-\frac{c}{q}\right)\right)
\]

The FOC in this case are:

\[
G'(e) \equiv -\frac{W}{n} \left(\gamma'(e) \exp\left(-\frac{c}{q}\right)\right) + k(e^{\alpha+\beta} - \lambda e) \left(1 - \exp\left(-\frac{c}{q}\right)\right) = 0
\]

\[
\frac{\partial e}{\partial \frac{W}{n}} = \frac{\gamma'(e) \exp\left(-\frac{c}{q}\right)}{G''(e)}
\]

\[
G''(e) = -\frac{W}{n} \left(\gamma''(e) \exp\left(-\frac{c}{q}\right)\right) + k(\alpha + \beta - 1)(\alpha + \beta)e^{\alpha+\beta-2}
\]

As before, \( G''(e) \leq 0 \) in the optimum, which implies that \( \frac{\partial e}{\partial \frac{W}{n}} \leq 0 \).

This assumes that we have an interior solution. A sufficient condition for this is, \( \gamma''(\hat{e}) \geq 0 \) for \( \hat{e} \) with \( G'\hat{e}) = 0 \).
Bibliography


Figure 1: Impact of $\xi$ on $w_G$ and $NR(e_G)$
Figure 2: Impact of $k$ on $w_G$ and $NR(e_G)$
Figure 3: Impact of $\lambda$ on $w_G$ and $NR(e_G)$
Figure 4: Impact of $\alpha$ on $w_G$ and $NR(e_G)$
Figure 5: Impact of $A$ on $w_G$ and $NR(e_G)$
Figure 6: Impact of $W/n$ on probability to win elections by incumbent.