

Procurement design with corruption

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Abstract

I investigate the design of optimal procurement mechanisms in the presence of corruption. After contracting with the sponsor, the contractor may bribe the inspector to misrepresent quality. The mechanism affects whether bribery occurs. I discuss the cases of both fixed and variable (with the size of misrepresentation) bribes, and also uncertainty about the bribe amount. In all cases, the optimal contract curtails quality for low efficiency contractors but also for the most efficient contractors. I also present models of bribe negotiations whose reduced form coincide with the model analyzed in the paper, and discuss implementation and the effect of competition.

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1 Introduction

In recent years, there has been growing interest in both the economic effects of corruption and also potential remedies for dealing with it. Corruption ranks high

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among the difficulties when organizing procurement, in general and public procurement in particular.¹ Indeed, officials and contractors are often tempted by the gains from trading in illegal, or at least illegitimate, mutual favors. Bribes are often exchanged for manipulation of either the rules that select the price (and/or identity of the contractor) or the quality of the goods delivered. In this paper, I study the design of optimal procurement mechanisms for the principal (sponsor) in the face of this latter form of manipulation.²

I begin by showing that standard tools in mechanism design may be adapted to characterize and analyze this problem. As we discuss below, the effect of this sort of corruption is to transform an adverse selection problem into an adverse-selection/moral-hazard problem.

Consider the standard procurement design problem, where the principal cares about quality and price, and the cost of delivering quality is the contractor's private information. Quality cannot be directly observed, and the contract supervisor or inspector in charge of assessing delivered quality can be "bought." First, let us take a reduced-form approach and assume that buying the inspector simply entails paying a bribe. In the simplest reduced-form bribe model, this bribe is fixed, and the contractor knows its value. The key, immediate observation is that a (direct) mechanism may then be defined as a triple of functions that specifies not only the quality and the price for each 'type' of contractor, but also whether the contractor should bribe or not—equivalently, deliver the contracted quality or not—. This is the moral hazard dimension that bribery introduces. As with collusion-proofness (see Tirole, 1986), dealing with corruption then adds a new incentive compatibility constraint to the sponsor's choice. In this case, the constraint guarantees that the contractor does not have incentives to disregard the instruction with respect to bribing.

I characterize optimal mechanisms in this scenario and show that bribing imposes a bound on the levels and range of quality that are implementable. In re-

¹As an example, according to the Minister for State-Owned Enterprises, 80% of corruption and fraud in state-owned Indonesian companies occur in public procurement. (Transparency International, 2006.)

²Transparency International lists this as one of the four areas of increased risk in the procurement cycle, together with limited access to information, abuse of exceptions and the budget phase. See Transparency International (2006).

sponse, the principal gives up inducing any (non-trivial) quality for an interval at the low-efficiency tail of types. This is probably not surprising. What is more subtle is that bribery is also problematic at the other end of the type domain. Indeed, due to the possibility of bribing, optimally the sponsor curtails the quality that it contracts with the most efficient contractors. Obtaining higher quality from these types of contractors obviously requires promising them a higher price. That, in turn, makes it more attractive for low-efficiency types to bribe the inspector in order to (claim high efficiency and so) contract for high quality, while delivering less of it. Therefore, the higher the quality contracted with high-efficiency types, the tighter the incentive constraint for lower-efficiency types.

Note that this incentive constraint is not local, but global. Indeed, bribing allows a contractor to target the highest price while avoiding the cost of delivering the corresponding highest quality. Thus, conditional on bribing, a contractor would claim to be the highest efficiency type.

When the cost of manipulation, i.e., the size of the bribe, is fixed and common knowledge, the possibility of bribery binds, but bribery does not occur when the sponsor designs the optimal mechanism.³

Then I look at more general, still reduced-form models and show that, if the required bribe is not fixed but increasing and sufficiently concave in the size of manipulation, the optimal mechanism is still qualitatively the same. Bribery is prevented, and quality distorted for both the low range and the high range of efficiency types. However, things are different when the required bribe is convex in the size of manipulation, or when there is uncertainty about the required bribe, which is only resolved after contracting but before the project is completed.⁴

Totally preventing bribery may not be in the sponsor's best interest in that case. I characterize the optimal mechanism under these alternative scenarios. In that mechanism, bribery may occur with a positive probability for all types. Indeed, after quality is contracted—and the size of the required bribe learnt by the contractor—the contractor will always choose whatever is cheaper: bribing or delivering. Thus,

³The same would be true if there were uncertainty about the size of the required bribe, but the uncertainty were resolved only after the project had been completed.

⁴This is the case if, for instance, the identity (and inclination toward gift exchanges) of the contract supervisor is revealed only after the contract is signed.

bribery can be seen as a smooth modification of the contractor's cost function. Other than that, the problem is technically very similar to the design problem in the absence of corruption. The effect of bribery is that, again, the sponsor optimally gives up inducing delivery of quality by the low tail of efficiency types, and contracts lower quality with all types of contractor, even the most efficient one. In that sense, the effect of bribery is similar whether bribes are paid or not in equilibrium.

The shape of the bribe (i.e., of the cost that bribing represents for the contractor) may be determined by social traits. Indeed, different societies are characterized by different degrees of the incidence and tolerance of corruption.⁵

When it is very costly to lure an official into illegal transactions, bribes are naturally concave in the amount of manipulation. A moderate bribe may be ineffective to induce the official to cross the line, but once crossed it makes sense to make the effort worthwhile. On the contrary, when graft is widely practiced and viewed as acceptable to some degree, bribes are better represented as convex in the size of manipulation. Small manipulation is probably safe, but even if the rule is to break the rule, there are norms as to what it means to *really break* rules.⁶

I illustrate these points by opening the black box and presenting some models of the negotiations between inspector and contractor that fit the reduced-forms analyzed before. After the sponsor designs the mechanism and the contractor signs the contract, the inspector and the contractor bargain over a possible bribe and misreporting. A false report by the inspector is detected with some exogenous probability, perhaps increasing in the distance to the truth. In that case, the contractor pays a fine, possibly increasing in the underprovision of quality, and the inspector loses wages, which may be decided by the sponsor. The inspector may also suffer from shame or other losses related to future prospects. Negotiations may take place either under asymmetric information regarding the "type"—say, cost of shame—of

⁵Several theories have been proposed to explain differences in the prevalence of corruption in societies, both based on rational behavior (e.g., based on reputation, cf. Tirole, 1996; or multiplicity of equilibria, c.f., Acemoglu, 1995) and on social norms. In their study of the empirical relationship between inequality and corruption, You and Khagram (2005) argue that social norms and perceptions are channels through which inequality affects corruption.

⁶Truex (2011) reports the results of a survey of Kathmandu residents that shows that respondents generally agree that large-scale bribery was unacceptable but also that petty corruption, gift giving, and favoritism are much more acceptable.

the inspector, or once this type is learnt by the contractor.

I also study the implementation of the optimal mechanism, and show that a simple menu of contracts, much in the spirit of Che (1993), does the job. Finally, for economy of notation, all the results discussed before are obtained for the case of only one potential contractor. Usually, having more than one potential—symmetric—contractor does not change matters significantly. In particular, competition does not affect the choice of quality as a function of the contractor's type, although it does reduce the expected price for the sponsor conditioned on quality. This is still the case when corruption introduces only locally binding incentive constraints. Indeed, as we have noted, both in the uncertain and fixed bribe and in the convex bribe cases corruption results in a standard design problem, although with a different cost function. This is unaffected by the number of potential contractors, and so bribery does not affect this optimal relationship between quality and type. However, when the bribe is fixed and known or concave, and so optimal procurement calls for avoiding bribes, things are more subtle. Indeed, I show that in these cases the fact that competition reduces the price that is necessary for obtaining high quality from high efficiency types implies that competition reduces the incentives for low efficiency types to imitate high efficiency ones. The result is that competition relaxes the global constraint imposed by corruption. That is, competition not only reduces the price of quality for the sponsor but also allows the sponsor to obtain higher levels of quality from the same type of contractor.

One technical contribution of this paper is to solve one problem where incentive constraints bind not only locally. As we have noted before, in the fixed and concave bribe cases, incentive compatibility related to moral hazard (no bribing) means that no type should have incentives to imitate the most efficient. This constraint is most stringent for the least efficient type. The strategy that I follow for characterizing the optimal mechanism is to fix an arbitrary quality for the most efficient type. Given this arbitrary value, I then characterize the sponsor's problem as a free-time, fixed-endpoint control problem. The optimal mechanism is the solution to this problem when the quality selected for the most efficient type is also optimal. That is, when the sponsor's surplus in the solution to the optimal control problem is maximized with respect to that quality.

Corruption has received much attention in economic literature at least since Rose-Ackerman's (1975) pioneering paper. (See Aidt, 2003, for a survey.) Corruption has been characterized in agency models as a form of collusion between two agents in their relationship with the principal. (See, among others, Laffont and Tirole, 1991a, 1991b; Faure-Grimaud et al. 2003; or Che and Kim, 2006.) In this literature, an inspector is nothing but an agent who possesses information relevant for the relationship (e.g., regarding the contractor's costs). That is, the inspector's information refers to the adverse selection parameter in the principal-contractor relationship, and can be used to decide on the terms of this relationship. The question is how to design a mechanism that elicits this information when agents can sign side contracts.⁷

Also on the effects of corruption on procurement, auction theorists have sought to understand how manipulation and bribe-taking affect allocation and surplus-sharing when particular mechanisms are used to allocate contracts or goods. The literature has studied specific ways (typically, bid readjustment) in which an agent acting on behalf of the principal may, in exchange for a bribe, favor a contractor in some particular auction mechanism. (See, among others, Compte et al., 2005; Menezes and Monteiro, 2006; Burguet and Perry, 2007; Koc and Neilson, 2008; Arozamena and Weinschelbaum, 2009; or Lengwiler and Wolfstetter, 2010.) The literature has also considered the effects of manipulation when quality, and not only price, is an argument in the principal's objective function. (See Celentani and Ganuza, 2002; and Burguet and Che, 2004.) With the exception of Celentani and Ganuza (2002), in all these papers, the agent's ability to manipulate is defined in reference to the particular mechanism itself. Thus, the analysis of remedies is limited in scope, and for good reasons: the mere question of what is an optimal mechanism from the principal's point of view may be ill-defined in these settings. For example, assume that an agent in charge of running a sealed-bid, first-price procurement auction is able to learn the bids before they are publicly opened and is also able to adjust downward one contractor's bid, if that bid is not already the lowest. This is

⁷Kessler (2000), discussing a typical two-type, adverse-selection, moral-hazard problem, is an exception. She assumes that the inspector can monitor the contractor's choice of effort—the moral hazard aspect—rather than the cost structure—the adverse selection parameter—. She shows that collusion between agent and firm does not impose any cost on the principal.

the model studied in most of the papers previously mentioned. If instead, the procurement mechanism were an oral auction, this ability would be irrelevant. Even in a second-price auction, altering one contractor's bid would be of no value to that contractor. Perhaps in either of these other auction formats the agent may manipulate the outcome by other means. The question would then become what is the equivalent to bid readjustment in those, or any, other auction formats? That is, the question of what constitutes an optimal procurement format, in the presence of corruption, takes us to a more complex problem: defining what a corrupt procurement agent may do under each of the possible mechanisms the principal may design.⁸

To my knowledge, this is the first paper that investigates the *design* of procurement mechanisms in the presence of bribery when the mechanism affects the incidence of bribery. Note that, contrary to many of the papers cited in the previous paragraph, I consider corruption in the execution of the contract, not in the negotiation of its terms. This means that the design of the procurement mechanism may affect the incidence of corruption, but what manipulation means can be defined independent of the mechanism itself. In their insightful paper, Celentani and Ganuza (2002) do study the optimal mechanism with a corrupt agent who may (adjust bids and) misrepresent quality assessment. However, (potential) corrupt deals pre-date the principal's choice of mechanism, and so the mechanism cannot affect quality, if corrupt deals indeed took place. Moreover, the authors assume that information rents are determined as in the absence of bribery, justifying this assumption on the need for 'hiding' corrupt deals. Under these assumptions, the sponsor knows that she will pay for quality as if no corruption existed, but will obtain that quality with a fixed, exogenous probability lower than one. Therefore, the sponsor optimally induces lower quality from all types: the optimal mechanism coincides with the optimal mechanism in the absence of bribery when quality is multiplied by a constant equal to that exogenous probability of actually getting the contracted quality.

⁸Note that, if the corrupt agent's ability to bend the rules depends on the rules themselves, the problem for the principal is not one where standard tools in mechanism design may be used. Even the revelation principle is problematic, as equivalence of mechanisms cannot be predicated without reference to equivalence of manipulation. That is, given the lack of a simple way of establishing equivalence classes in the set of mechanisms, restricting attention to revelation games may simply be of no use.

In the next section, I present the basic model with a fixed bribe and obtain the optimal mechanism for this case. Then I analyze uncertainty concerning the size of the bribe and also the case where the size of the bribe is a function of the amount of manipulation. Section 3 discusses models of inspector-contractor negotiations that fit the reduced-forms on the previous section. In Section 4, I briefly discuss implementation of the optimal procurement rules, and the effects of an increase in competition. Some concluding remarks close the paper. An appendix contains all proofs.

2 The model

A sponsor procures a contract of fixed size and variable quality, $q \in [0, \infty)$. Quality q measures the sponsor's willingness to pay for the project. That is, the payoff for the sponsor when she makes a total payment of P and receives a project with realized quality q is simply $q - P$. The sponsor's goal is to maximize the expected value of that payoff.

N ex-ante symmetric contractors are able to undertake the project. Contractor i 's cost of delivering quality q is $C(q; \theta_i)$, where $\theta_i \in [\underline{\theta}, \bar{\theta}]$ is the contractor's type and her private information. Each θ_i is the independent realization of some random variable with cdf F and density f in $[\underline{\theta}, \bar{\theta}]$, and this is common knowledge. We postulate a differentiable C with $C_\theta, C_q > 0$, and $C_{q\theta}, C_{qq} > 0$. This guarantees that, in the absence of bribery, monotonicity of q and monotonicity of the allocation with respect to θ is virtually all that is needed for implementation. Also, we assume that $C(0; \theta) = 0$, for all θ , and that

$$(1) \quad C_q(q; \theta) + C_{q\theta}(q; \theta) \frac{F(\theta)}{f(\theta)}$$

is increasing both in q and in θ . This guarantees interior (to monotonicity constraints) solution for the optimal mechanism in the absence of bribery. (See Che, 1993.)

Zero quality should be interpreted as a standard minimum quality, and a positive quality as quality obtained at an additional cost to the contractor. Likewise, a price

of zero corresponds to a payment that equals the (common) cost of honoring the contract at a standard, minimum quality, $q = 0$.

As is usually the case in procurement design, most of our results and insights do not depend on N and so, for simplicity, we will first analyze the case where $N = 1$. However, in Section 4 we discuss a novel effect that an increase in N , i.e., increased contractor competition, brings into the picture.

With $N = 1$, and under observability of quality, and so without any role for an inspector, a direct mechanism is a pair (p, q) , where $p : [\underline{\theta}, \bar{\theta}] \rightarrow \mathbb{R}$ and $q : [\underline{\theta}, \bar{\theta}] \rightarrow \mathbb{R}_+$. $p(\theta)$ represents the payment that the contractor obtains and $q(\theta)$ represents the quality that the contractor provides. Both are functions of the contractor's type. The sponsor needs to consider only incentive compatible, individually rational, direct mechanisms. Among them, the optimal one is characterized by quality $q^{NB}(\theta)$ that satisfies

$$(2) \quad 1 - C_q(q^{NB}(\theta); \theta) - C_{q\theta}(q^{NB}(\theta); \theta) \frac{F(\theta)}{f(\theta)} = 0,$$

as long as

$$(3) \quad q^{NB}(\theta) - C(q^{NB}(\theta); \theta) \geq C_\theta(q^{NB}(\theta); \theta) \frac{F(\theta)}{f(\theta)},$$

and $q^{NB}(\theta) = 0$ otherwise. (See Che, 1993.) In order to simplify the analysis and also concentrate on the interesting results below, we will assume that (3) holds with equality when evaluated at $\bar{\theta}$ and $q^{NB}(\bar{\theta}) = 0$, so that $q^{NB}(\theta) > 0$ for all $\theta < \bar{\theta}$.⁹

2.1 Bribery in reduced form

In this paper, we are interested in the case when quality q is not directly observable. Instead, the sponsor requires an agent, the inspector, who assesses (or certifies) the quality of the completed project. The inspector can observe q without cost. The

⁹If (3) is negative at $\bar{\theta}$ for $q = 0$, everything that follows applies with only redefining the domain of types to the interval where (3) is non negative. Also, if (3) is strictly positive at $\bar{\theta}$ and for $q = 0$, then the value of θ^c defined later may be equal to, not strictly smaller than, $\bar{\theta}$. That "corner" solution would not affect any other insight.

inspector can also accept bribes to be untruthful when reporting quality.

We should model the negotiations between contractor and inspector over bribe and manipulation, including what information each has about the other. Likewise, we need to model penalties and the remuneration of the inspector by the sponsor. We will explicitly do that in Section 3. In this section, we treat these negotiations and payments as a black box. In particular, in this subsection we assume that the agent is willing to make a false report in favor of the contractor in exchange for a fixed bribe B . Moreover, we assume that B is common knowledge at the time the contractor and the sponsor sign any contract. In the next subsection, we will consider the case that the size of B is only learnt by the contractor after signing the contract with the sponsor. The final subsection of this section will discuss the extension to allow for the bribe to depend on the size of manipulation. As we have mentioned, Section 3 will discuss how these specifications map to models of negotiations between contractor and inspector.¹⁰ In any case, we should note that the bribe B will always include any (expected) payment by the contractor associated with bribing the agent.

Thus, assume that B is fixed and common knowledge. Under the threat of bribery, we may define a direct mechanism as a triple (p, q, b) , where p and q are defined and interpreted as above, and $b : [\underline{\theta}, \bar{\theta}] \rightarrow \{0, 1\}$. We interpret $b(\theta) = 1$ as an instruction to bribe and $b(\theta) = 0$ as an instruction not to bribe. Also, we should interpret $q(\theta)$ as the *contracted* quality, not necessarily the delivered one.¹¹

Let

$$\Pi(\theta) = b(\theta) (p(\theta) - B) + (1 - b(\theta)) (p(\theta) - C(q(\theta); \theta)).$$

The mechanism is implementable if it satisfies incentive compatibility (IC) and individual rationality (IR), as in any standard problem:

IR) For all $\theta \in [\underline{\theta}, \bar{\theta}]$, $\Pi(\theta) \geq 0$.

IC) For all $\theta, z \in [\underline{\theta}, \bar{\theta}]$, $\Pi(\theta) \geq p(z) - \min \{C(q(z); \theta), B\}$.

¹⁰In all cases, we do not consider the possibility of "extortion", that is, the possibility that the inspector asks for bribes in order to report the truth, threatening to otherwise report lower quality than what is actually delivered. Note that in this type of corrupt behavior the parties involved, inspector and contractor, have conflicting interest. This is contrary to the cases studied in this paper.

¹¹For the moment, we do not need to specify delivered quality: if $b(\theta) = 1$, then delivered quality will be 0, and otherwise it will be $q(\theta)$.

Incentive compatibility incorporates adverse selection and moral hazard constraints. In particular, for $b(\theta) = 0$, $\Pi(\theta) \geq p(\theta) - B$. That is, the contractor's profits should be at least as large as the profit she could get by simply bribing. On the other hand, if $b(\theta) = 1$, then IC requires that delivered quality is 0: the contractor would not obey any other "instruction". Thus, we do not need to specify delivered quality when $b(\theta) = 1$.

The problem for the sponsor is to design the direct, implementable mechanism (p, q, b) that maximizes her expected payoff, $E[q(\theta)(1 - b(\theta)) - p(\theta)]$.

When the bribe B is known and fixed, that optimal mechanism cannot include $b(\theta) = 1$, i.e., an instruction to bribe, for any θ . The contractor would still have incentives to truthfully report her type if, instead, the mechanism asked her to deliver 0 quality and offered her a price of $p(\theta) - B$. Thus, the sponsor can get the same quality for a lower price. Indeed,

Lemma 1 *For any IC, IR direct mechanism, (p, q, b) , there exists an IC, IR mechanism with $b(\theta) = 0 \forall \theta \in [\underline{\theta}, \bar{\theta}]$, so that $E[q(\theta)(1 - b(\theta)) - p(\theta)]$ is higher for the latter.*

Proof. See the appendix. ■

Thus, in this basic setting, bribery does not occur in the optimal mechanism, and so we can restrict attention to mechanisms with $b(\cdot) = 0$. Bribery does impose restrictions on mechanisms for IC and IR to hold, as we will see.

Standard necessary conditions for IC and IR (under no bribery) are still necessary. Indeed, even without intending to bribe the inspector, the contractor should have incentives to report her type truthfully. Three consequences follow from these necessary conditions. First, $\Pi(\theta)$ must be monotone decreasing. In fact,

$$\pi(z; \theta) \equiv p(z) - C(q(z); \theta)$$

satisfies the conditions of Theorem 2 in Milgrom and Segal (2002), so that

$$(4) \quad \Pi(\theta) = p(\theta) - C(q(\theta); \theta) = \Pi(\bar{\theta}) + \int_{\theta}^{\bar{\theta}} C_{\theta}(q(z); z) dz,$$

and is absolutely continuous. That virtually fixes $p(\theta)$ as a function of $q(\theta)$. Second, and as a result, a necessary but also sufficient condition for IR is $\Pi(\bar{\theta}) \geq 0$. Third, monotonicity can also be extended to q and p in a standard way.

Lemma 2 *In any IC mechanism, $p(\theta)$ and $q(\theta)$ are monotone decreasing.*

Proof. See the appendix. ■

These features, which are shared by any feasible mechanism in the absence of corruption, have important consequences that make our problem tractable. Indeed, since p is monotone, a bribing contractor of any type would maximize profits by claiming type $\underline{\theta}$. Also, since Π is monotone, type $\bar{\theta}$ is the type with the strongest incentives to bribe. Thus, the only binding constraint that bribery imposes is

$$\Pi(\bar{\theta}) \geq p(\underline{\theta}) - B.$$

That is, substituting for $p(\underline{\theta})$ from (4),

$$(5) \quad B \geq C(q(\underline{\theta}); \underline{\theta}) + \int_{\underline{\theta}}^{\bar{\theta}} C_{\theta}(q(z); z) dz.$$

The main insight in this subsection is contained in (5). Bribery constrains the variation allowed to $q(\theta)$. When corruption is absent, optimally the sponsor asks for higher quality when it is less costly to obtain—when θ is lower—. However, increases in quality require faster increases in prices, and high differences in prices cannot be sustained when bribing is inexpensive.

It is a standard result that these conditions are not only necessary but also sufficient for implementation, provided that $C_{q\theta}(q; \theta) > 0$. Thus, the problem that the sponsor solves is

$$(6) \quad \max_{p,q} \int_{\underline{\theta}}^{\bar{\theta}} \{q(\theta) - p(\theta)\} f(\theta) d\theta,$$

s.t. $\Pi(\bar{\theta}) \geq 0$; (4); (5); and subject to $q(\theta)$ being monotone decreasing.

IR binds at the solution to this problem.¹² Thus, $p(\bar{\theta}) = C(q(\bar{\theta}), \bar{\theta})$. The constraint (5) may not bind. Indeed, if (5) holds for $q^{NB}(\theta)$, then this is also the optimal mechanism under bribery. However, the case of interest is when B is sufficiently small, so that (5) does bind. The following proposition characterizes the optimal mechanism also for this case

Proposition 3 *If (5) holds at the optimal quality without bribery, $q^{NB}(\theta)$, then (q^{NB}, p^{NB}, b) with $b(\theta) = 0$ for all θ , is the optimal mechanism under bribery. Otherwise, there exist θ^a and θ^c , with $\underline{\theta} < \theta^a \leq \theta^c < \bar{\theta}$ such that at the optimal mechanism; (i) $q(\theta) = 0$ if $\theta > \theta^c$; (ii) $q(\theta) = q^{NB}(\theta)$ if $\theta \in (\theta^a, \theta^c)$; and (iii) $q(\theta) = q^{NB}(\theta^a)$ if $\theta < \theta^a$.*

Proof. See the appendix. ■

Thus, when the size of the bribe is fixed and known and bribery is a relevant constraint, the sponsor gives up the possibility of obtaining—an increase in—quality from contractors with very high costs. This result is not surprising. Indeed, recall that bribery imposes a limit on the variability of quality levels that can be commanded. If the sponsor is going to shave the range of quality, she optimally does so for types with low efficiency. The loss from such distortion is lowest when the contractor has one of these types. But, perhaps less obviously, optimally quality is also distorted at the top, i.e., when the contractor has high efficiency.

The intuition behind this result is simple. (See Figure 1.) Assume $B > C(q^{NB}(\underline{\theta}); \underline{\theta})$, but also assume that (5) binds for $q^{NB}(\theta)$.¹³ That is,

$$B < C(q^{NB}(\underline{\theta}); \underline{\theta}) + \int_{\underline{\theta}}^{\bar{\theta}} C_{\theta}(q^{NB}(z); z) dz.$$

It is then feasible to distort quality only at the bottom. That is, to select θ^c that solves $B = C(q(\underline{\theta}); \underline{\theta}) + \int_{\underline{\theta}}^{\theta^c} C_{\theta}(q(z); z) dz$, and let $q(\theta) = q^{NB}(\theta)$ for all $\theta \leq \theta^c$,

¹²Indeed, for any mechanism (q, p) so that $\Pi(\bar{\theta}) = p(\bar{\theta}) - C(q(\bar{\theta}), \bar{\theta}) = \epsilon > 0$, we can define a mechanism (q', p') with $q' = q$ and $p' = p - \epsilon$ for all θ . The new mechanism satisfies all the above constraints and results in a higher payoff for the sponsor.

¹³Of course, if $B < C(q^{NB}(\underline{\theta}); \underline{\theta})$ then distortion at the top is imposed by constraint (5) on any implementable mechanism.

and $q(\theta) = 0$ for all $\theta > \theta^c$. This is represented in Figure 1. The thick line represents $q^{NB}(\theta)$ and the thin line is $q(\theta)$. Now consider the effects of reducing $q(\theta)$ for all types in $[\underline{\theta}, \underline{\theta} + \varepsilon)$ to the level $q(\underline{\theta} + \varepsilon)$. Firstly, it implies obtaining a lower quality from types in $[\underline{\theta}, \underline{\theta} + \varepsilon)$ and consequently reducing the payment to those types of contractor, just as in the absence of bribery. This effect is of second order when $q(\theta) = q^{NB}(\theta)$. Secondly, it also implies a more relaxed no-bribing constraint, as it is now less profitable to claim a type $\underline{\theta}$ when the type is in fact higher. That is, the reduction in $q(\underline{\theta})$ allows raising θ^c . This is a first-order effect, as types that fall between the old and the new values of θ^c can now be asked to supply quality $q^{NB}(\theta^c)$ at a price $C(q(\theta^c); \theta^c)$ instead of quality 0 at price 0. In other words, a reduction in $q(\underline{\theta})$ allows the contractor to expect positive levels of quality from the previously marginal type θ^c . This effect is first-order, and therefore distorting at the top is surplus-improving for the sponsor.

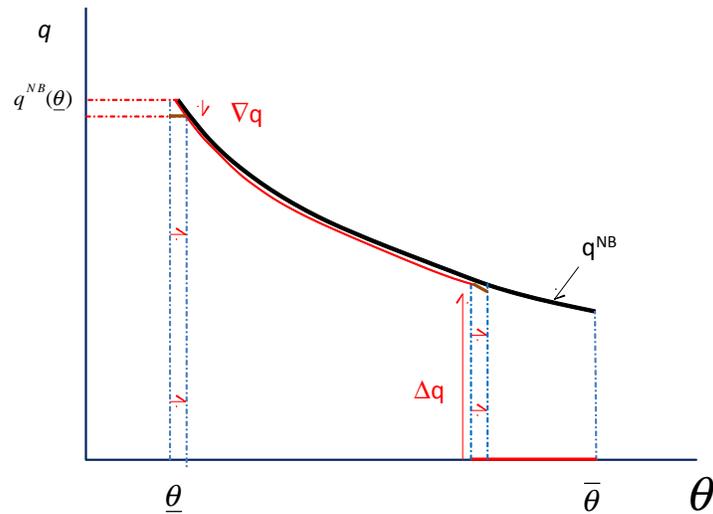


Figure 1

2.2 Bribing in equilibrium

Perhaps the most interesting feature of the optimal mechanism discussed in the previous subsection is the distortion at the high end of the distribution of contractor types. That is, bribery imposes a quality ceiling. However, optimally the sponsor

virtually buys the possibility of bribing. In this subsection we show that this is not the case if we assume that, at the time of contracting, B is uncertain. Thus, assume that B is only learnt by the contractor *after* the terms of the contract have been set and *before* quality is delivered.¹⁴ B takes values in some interval, $[\underline{B}, \overline{B}]$ according to some c.d.f. G with density g such that $\frac{1-G(B)}{g(B)}$ is decreasing in B , and $C_\theta(q; \theta) [1 - G(C(q; \theta))]$ is increasing in q . This latter assumption guarantees that $\Pi(\theta)$ will be monotone.

To simplify the analysis, assume for now that $\underline{B} = 0$. (See below the case $\underline{B} > 0$.) Under these conditions, preventing all types of contractor from bribing in all probability—when B is low—may be too expensive for the sponsor.

Thus, define a contract as a triple (p, q, b) , where $p : [\underline{\theta}, \overline{\theta}] \rightarrow \mathbb{R}$ and $q : [\underline{\theta}, \overline{\theta}] \rightarrow \mathbb{R}_+$, as before, and $b : [\underline{\theta}, \overline{\theta}] \rightarrow [\underline{B}, \overline{B}]$. $b(\theta)$ is now interpreted as the cut-off value for B so that type θ is instructed to bribe if and only if $B < b(\theta)$. It is sufficient to consider mechanisms of this form. Indeed, if a contractor prefers not to bribe when the size of the bribe is B , the contractor will also prefer not to bribe when the size of the bribe is larger than B . On the contrary, if the contractor prefers to bribe when the size of the bribe is B , then the contractor will also prefer to bribe if the size of the bribe is smaller than B . Hence, IC will require that the instruction to bribe takes a cut-off form. Also, as in the previous section, we do not need to specify what quality is to be delivered if bribing: IC requires that this quality is 0.¹⁵

Assuming that the size of B is uncertain at the time of contracting changes the nature of the moral-hazard incentives for the contractor. Indeed, whatever quality the contractor has committed to, the contractor will prefer to bribe if, and only if, the realized value of the bribe is less than the cost of providing that quality. Thus, if the contractor reports her type truthfully, IC requires

$$(7) \quad b(\theta) = C(q(\theta), \theta).$$

¹⁴The case where uncertainty about B is resolved after the quality is delivered is equivalent to the fixed and known B , with only substituting EB for B . Indeed, in that case the decision whether to bribe or not would have to be taken before providing quality. Not providing quality but claiming to do so would require the contractor to pay the bribe B whatever its realization, whereas if the contractor provides quality there is no point in also paying the bribe.

¹⁵Using a similar argument as the one made in the proof of Lemma 1, it is a simple exercise to show that we may restrict attention to mechanisms with $b(\theta) < \overline{B}$.

This moral-hazard, IC constraint, defines $b(\theta)$ once $q(\theta)$ is determined. Moreover, this constraint is equivalent to a *smooth* modification of the contractor's (expected) cost function. Indeed, whatever quality q a contractor of type θ commits to, her—lowest—expected cost of honoring this commitment is now:

$$(8) \quad \Phi(q, \theta) = E_B \min \{C(q, \theta), B\} = C(q; \theta)(1 - G(C(q, \theta))) + B \cdot G(C(q, \theta)).$$

The important observation is that $\Phi(q, \theta)$ is still a (a.e.) differentiable, monotone increasing function of q and θ , which under mild assumptions satisfies the sorting conditions that guarantee that only local *IC* constraints bind. Thus, (sufficient) uncertainty about the size of the bribe renders the problem of designing an optimal mechanism for procurement a standard one with a modified cost function. Consequently, the problem of designing procurement mechanisms under bribery parallels the design problem without bribery.¹⁶ As in the latter, IC requires that:

$$(9) \quad \Pi'(\theta) = \Phi_\theta(q, \theta) = C_\theta(q; \theta)(1 - G(C(q, \theta)))$$

evaluated at $q(\theta)$. The expected surplus for the sponsor can be written as

$$(10) \quad \int_{\underline{\theta}}^{\bar{\theta}} \{[1 - G(C(q(\theta), \theta))] [q(\theta) - C(q(\theta), \theta)] - \Pi(\theta)\} f(\theta) d\theta.$$

The next proposition, the proof of which we omit, simply reports the standard optimal procurement mechanism for the modified cost function Φ .

Proposition 4 *There exists θ^d , with $\underline{\theta} < \theta^d \leq \bar{\theta}$ such that at the optimal mecha-*

¹⁶As in the previous subsection, monotonicity of q and $\Pi(\theta)$ are not only necessary but also sufficient for implementation. This condition requires $C_\theta(q; \theta) [1 - G(C(q; \theta))]$, and not only $C_\theta(q; \theta)$, to be increasing in q . It is satisfied, for instance, if B is disperse, so that g is small for any value of B . (See details in the working paper version.)

nism; (i) $q(\theta) = 0$ if $\theta > \theta^d$; and (ii) if $\theta < \theta^d$, then $q(\theta)$ solves

$$(11) \quad 0 = \frac{1 - G(C(q, \theta))}{g(C(q, \theta))} \left\{ 1 - C_q(q, \theta) - C_{\theta q}(q, \theta) \frac{F(\theta)}{f(\theta)} \right\} - C_q(q, \theta) \left\{ q - C(q, \theta) - C_\theta(q, \theta) \frac{F(\theta)}{f(\theta)} \right\}.$$

The intuition behind this result is relatively straightforward when considering the Hamiltonian for the maximization of (10). Suppressing, for compactness, the variables in the functions and taking into account (9) this Hamiltonian is

$$H = (1 - G(C)) [(q - C) f - FC_\theta].$$

In the absence of bribery, the corresponding Hamiltonian would be $H^{NB} = (q - C) f - FC_\theta$. Thus,

$$H = (1 - G(C)) H^{NB}.$$

The interpretation is simple. Quality $q(\theta)$ will be delivered with probability $1 - G(C)$, and then with that probability, the decision regarding q will have the same consequences on contractor's rents, cost, and on sponsor's surplus, as in the absence of bribery. This is the phenomenon captured in Celentani and Ganuza (2002). In the present setting, this would have no effect on q , but only on the prices, p .¹⁷ However, $q(\theta)$ will also affect the probability of bribery through its effect on the threshold $b(\theta) = C(q(\theta), \theta)$. This represents an additional effect of size $-g(C)C_q H^{NB} < 0$ of a small increase in $q(\theta)$. Indeed, a—small—increase in q will increase by $g(C)C_q$ the probability that the sponsor gets 0 quality, instead of q , in exchange for the price. Thus, for values of q that solve $\partial H^{NB} / \partial q = 0$, the presence of this negative effect of an increase in q would imply that $\partial H / \partial q < 0$.¹⁸ Therefore, the solution under bribery implies lower values of q for any θ .

For high enough types, θ , so that that $q^{NB}(\theta)$ is positive but small, the optimal

¹⁷In Celentani and Ganuza (2002), the effect on q appears from the fact that the price is determined by non-corrupt contractors, who are assumed to act as in the absence of corruption. Therefore, the only way of controlling bribery rents is by affecting quality.

¹⁸Of course, if $G(C(q(\theta), \theta)) = 0$ when evaluated at the $q(\theta)$ that solves $\partial H^{NB} / \partial q = 0$, then bribery does not change $q(\theta)$.

answer is to relinquish the possibility of obtaining any quality above the minimum. Just as in the fixed-bribe case, attempting to do so would be too expensive in terms of rents. Also, as in the fixed bribe case, distortions at the top are part of the optimal mechanism. Indeed, for the most efficient type, $q(\underline{\theta})$ solves

$$\frac{1 - G(C)}{g(C)} [1 - C_q] = C_q [q - C].$$

The left hand side is zero at $q^{NB}(\underline{\theta})$, as under no bribery there is no distortion at the top, but the right hand side is positive evaluated at that same value. Thus, $q(\underline{\theta}) < q^{NB}(\underline{\theta})$.

We have assumed that the lower support of the distribution of B was sufficiently low so as to make it too expensive for the sponsor to avoid bribery completely, even for the most efficient type. If \underline{B} were larger, we could use the same techniques as in Proposition 4 to obtain the optimal mechanism, but this time the constraint would be $\Pi'(\theta) = -C_\theta(q(\theta), \theta) [1 - G(\min\{C(q(\theta), \theta), \underline{B}\})]$. Thus, in the solution to that problem, we will have $q(\theta) = q^{NB}(\theta)$ whenever $C(q^{NB}(\theta), \theta) < \underline{B}$. Note that as long as (2) defines a continuous and monotone $q^{NB}(\theta)$, that solution $q(\theta)$ is also continuous and monotone, so indeed it defines the optimal mechanism.

In summary, an uncertain bribe is equivalent to a smooth modification to the contractor's cost function: local IC constraints are correspondingly modified, which implies reductions of quality, even on *contracted* levels of quality, for all types of contractors, as well as bribe payments with a positive probability.

2.3 Variable bribe

Let us return to the case where the cost of manipulation is common knowledge at the time of contracting. Suppose now that this cost of manipulation is increasing with the size of that misrepresentation. That is, a quality overstatement of size m requires a bribe $B(m)$, where $B' > 0$. Generalizing the model to include this case is straightforward for both sufficiently concave and sufficiently convex $B(m)$. Note that the fixed, known-bribe case in Subsection 2.1 is an extreme case of a concave, increasing $B(m)$. Indeed, there $B(0) = 0$, and $B(m) = B$ for all $m > 0$.

Also, although more subtle, the fixed, unknown bribe case in Subsection 2.2 can be thought of as a convex, increasing $B(m)$. Indeed, a contractor (with type θ) who contracts delivery of quality q will buy expected manipulation $G(C(q; \theta))q$. The ratio of expected bribe to expected manipulation,

$$\int_0^{C(q; \theta)} \frac{Bg(B)}{G(C(q; \theta))q} dB$$

is increasing in q , and the expected value of manipulation is also increasing in q .

We now show that, more generally, the curvature of B will be key to whether bribery will take place in the optimal mechanism or not. As argued in the Introduction, concave $B(m)$ occurs when, for instance, it is quite costly to engage the inspector in illegal activities, but increasing degrees of manipulation can be obtained at decreasing additional cost.¹⁹

On the other hand, a convex $B(m)$ better represents realities where 'petty corruption' is a kind of accepted, or tolerated, social norm, but large misrepresentations may risk crossing the line. Thus, although corruption is a threat in all societies, the curvature of B may be linked to the extent to which honesty is a standard in society.²⁰ We will discuss this issue in the next section.

Assume that $B(m)$ is concave. A direct mechanism specifies both p and q , but also $m : [\underline{\theta}, \bar{\theta}] \rightarrow \mathbb{R}_+$ (i.e., how much manipulation a contractor is expected to buy). Lemma 1 extends to the increasing but concave $B(m)$ case. Indeed, concavity is sufficient to guarantee that optimally $m(\theta) = 0$, for all θ . (See the online appendix for a proof.) Sufficient concavity also guarantees that, if bribing, the contractor will do so to claim the most efficient type. That is, the contractor will offer the highest quality by claiming the lowest type $\underline{\theta}$, and in fact deliver quality 0. Thus,

¹⁹Transparency International (2006) notes: "Experience in industrial countries shows clearly that—apart from facilitation payments—the majority of corrupt people (both on the private and the government side) are not junior or subordinate staff, but people in the higher echelons, including many senior managers." This form of 'grand corruption' is probably best described by a concave bribe function.

²⁰Obviously, the type of project may also affect the 'shape'—and size—of the bribe function. For example, the probability of catastrophic failure if quality is not as specified, may be concave—large risk for even small amounts of manipulation—or convex—lower but increasing sensitivity to manipulation—in manipulation, and this may result in 'bribe' functions with corresponding shapes.

once again, corruption will introduce only one additional, global constraint similar to (5). A sufficient set of conditions for this to be the case is:

$$\text{A1) } -B''(m) > C_{qq}(x; \theta) \text{ for all } x \leq q^{NB}(\underline{\theta}), \forall \theta;$$

$$\text{A2) } B'(q^{NB}(\theta)) < C_q(q^{NB}(\theta); \theta) \text{ for all } \theta; \text{ and}$$

$$\text{A3) } B(q^{NB}(\theta)) > C(q^{NB}(\theta); \theta).$$

Under these conditions, (5) simply becomes:

$$(12) \quad B(q(\underline{\theta})) \geq C(q(\underline{\theta}); \underline{\theta}) + \int_{\underline{\theta}}^{\bar{\theta}} C_{\theta}(q(z); z) dz.$$

Thus, to all effects, the bribe is still fixed and equal to $B(q(\underline{\theta}))$, *once $q(\underline{\theta})$ is determined*. As a consequence, if $q^{NB}(\underline{\theta})$ violates (12), and as in Proposition 3, there exist θ^a and θ^c , with $\underline{\theta} < \theta^a \leq \theta^c < \bar{\theta}$ such that at the optimal mechanism; (i) $q(\theta) = 0$ if $\theta > \theta^c$; (ii) $q(\theta) = q^{NB}(\theta)$ if $\theta \in (\theta^a, \theta^c)$; and (iii) $q(\theta) = q^{NB}(\theta^a)$ if $\theta < \theta^a$. (See the online appendix for a proof.)

The only difference with the case of fixed B is that now $q(\underline{\theta})$ affects also the left hand side of the no-bribe constraint (12). Yet, A2 still guarantees that $d\theta^c/dq(\underline{\theta}) < 0$ when evaluated at $q(\underline{\theta}) = q^{NB}(\underline{\theta})$, and so indeed $\theta^a > \underline{\theta}$.

If A1, A2, or A3 fail, the optimal contract may be a little more involved. A violation of A1 means that the best deviation for some type θ may involve a mixture of bribing and positive quality delivery. This, in turn, implies that (12), plus local IC and monotonicity, do not guarantee IC for all types, and further upper constraints on q may apply. The consequence would be the need for 'ironing' q . Similarly, if A3 were violated for some type θ , then we would have to consider an additional constraint on the quality that could be obtained from that type. Finally, if A2 does not hold, then the best bribing deviation for any type of contractor may be to claim a type θ below the type θ^a obtained above. The optimal mechanism could still be constructed from $\bar{\theta}$ down at a cost of complexity. (See the working paper version for details.)

Let us now turn to the convex $B(m)$ case, and also assume that $B'(0) = B(0) = 0$. A contractor of type θ can choose to honor a commitment to quality q by buying manipulation and/or delivering quality. Therefore, the contractor's actual cost of

contracting any level of quality is

$$(13) \quad \Phi(q, \theta) = \min_{m \in [0, q]} C(q - m; \theta) + \bar{B}(m).$$

Note that the right hand side is convex, and so the solution is interior since $C_q(0; \theta) = B'(0) = 0$. Thus, much as in Subsection 2.2, bribery is akin to a smooth modification of the contractor's cost function: (13) is the equivalent to (8). Again, solving for the optimal contract is a standard problem with this modified cost function.

The solution to (13) implicitly defines the optimal $m(q; \theta)$ as an increasing function of quality q , given type θ . IC requires that $\Pi'(\theta) = \Phi_\theta(q, \theta) = -C_\theta(q - m(q; \theta); \theta)$, and the expected surplus for the sponsor is

$$\int_{\underline{\theta}}^{\bar{\theta}} \{[q - m - C(q - m; \theta) - B(m)] - \Pi(\theta)\} f(\theta) d\theta,$$

where $m = m(q; \theta)$ and $q = q(\theta)$. Therefore, the first order condition for the maximization of this function, taking into account that $C_q(m(q; \theta); \theta) = B'(m(q; \theta))$, is

$$1 - m_q(q; \theta) - C_q(q, \theta) - C_{\theta q}(q, \theta) \frac{F(\theta)}{f(\theta)} = 0.$$

An increase in one unit of q increases the manipulation that the contractor buys, and so it only increases the delivered quality by $1 - m_q(q; \theta)$. This is true for any type θ and so, as in Subsection 2.2, optimally, the sponsor reduces the *contracted* quality even at the top.

3 Bribery models

In this section, we will open the black box represented by B . What I offer here is a family of models of the sponsor-inspector relationship, of the contractor-inspector negotiations over bribes, and of policing and fines that fit the models of B discussed in the previous section. This family will not cover all possible models. Some alternative models cannot fit our analysis, as we discuss below. Likewise, this family

does not cover all models that do fit. The purpose of the exercise is to show that our insights on the relationship between sponsor and contractor are an exact description of sensible models that consider all the links between the three agents involved. Also, the exercise offers some raw examples of how the insights on the contract design may be taken into account when discussing more detailed policy questions, like inspector remuneration or fines.

The following time-line describes the game that the three agents play. After the sponsor designs the mechanism and the contractor signs the contract, the inspector and the contractor bargain over a possible bribe, β , that the latter would pay the former. If an agreement is reached, the bribe is paid and the inspector reports that quality is as contracted. After the report is sent, and if the report is not truthful, misreporting is detected with a probability μ , in which case the inspector is fired and the contractor pays a fine λ . If a misreport is not detected, then the inspector obtains a wage payment, w . Both μ and λ may depend on m . For simplicity, we will assume that the contractor pays the same fine, λ , if the inspector truthfully reports a lower than contracted quality q . Moreover, for compactness, we will omit the variable m in most of the expressions below.

The inspector's payoff—apart from bribe rents—if not fired is w , which for the time being we consider as given, and otherwise $-\widehat{s}(m; \xi)$, a cost associated to shame or diminished future employment prospects, which may depend on m . The cost may also depend on ξ , the inspector's type. We will assume ξ to be the realization of a uniform random variable—with almost no loss of generality—and also that $\widehat{s}(m; \xi) = s(m)\xi$ for some—weakly—increasing function s . That is, higher ξ represents—a weakly—higher cost.

We need to specify three more elements: 1) whether bargaining over a bribe takes place before or after quality is produced (and so the cost incurred); 2) the protocol that these negotiations follow; and 3) the information that each of the agents has at the times they make decisions. On the first issue, we assume that bargaining takes place after production. We later discuss the consequences of the alternative assumption. With respect to the bargaining protocol, we will simply assume that the contractor makes a take it or leave it offer to the inspector with probability α , and with probability $(1 - \alpha)$, it is the inspector who makes a take it or leave it

offer. Finally, regarding information, we assume that the inspector does not observe the contract signed by the contractor (and, if bribed, is instructed as to what quality to report). Also, we will assume that ξ is revealed to the contractor after signing the contract with the sponsor, but this may occur at different moments of time: before production, after production but before negotiating with the inspector, or after these negotiations —i.e., effectively, never—. We will also comment on the consequences of modifying these assumptions.

Case 1: The contractor learns ξ after production, but before negotiations.

Suppose the contractor has produced a level of quality that is m short of what was contracted, and also suppose that $\mu(w + s\xi + \lambda) < \lambda$. The contractor would offer a bribe $\mu(w + s\xi)$ that leaves the inspector indifferent between accepting and rejecting the offer. The inspector, on her part, would ask for a bribe $\lambda(1 - \mu)$. Also, if $\mu(w + s\xi + \lambda) > \lambda$, then there is no room for mutually profitable negotiations. We assume that $\lambda(1 - \mu) \geq w\mu$ for any value of m , so that at least there is some possibility that there are gains from bribing—i.e., when $\xi = 0$. Thus, the probability that there are gains from bribing is

$$\bar{\xi} = \min \left\{ \frac{\lambda(1 - \mu) - w\mu}{\mu s}, 1 \right\},$$

a (weakly) monotone, decreasing function of m . (Recall that we are assuming ξ to be distributed as a uniform random variable.) Then, $E_{\xi \leq \bar{\xi}} \xi = \bar{\xi}/2$ and, for any $m > 0$, the expected "bribe" (cost of manipulation) for the contractor at the time of signing the contract with the sponsor is

$$(14) \quad B(m) = \bar{\xi} \left[\alpha \left(\mu(w + s \frac{\bar{\xi}}{2}) + \mu\lambda \right) + (1 - \alpha)\lambda \right] + [1 - \bar{\xi}] \lambda.$$

For interior values of $\bar{\xi}$,

$$B(m) = \lambda - \alpha \frac{(\lambda(1 - \mu) - \mu w)^2}{2\mu s}.$$

Depending on the behavior of μ , λ , and s as functions of m , the model will behave as in the models of Section 2. For instance, if all three variables are in-

dependent of m , we have the constant and (to all effects) known B . On the other hand, if μ and λ are independent of m but s is concave in m , then $B(m)$ is also concave.^{21,22}

Case 2: The contractor learns ξ before production, which is followed by negotiations.

Given m , the offers by both contractor and inspector would be as in case 1. Assume that λ is sufficiently large that it pays to deliver rather than paying the fine for sure: e.g., $C(q^{NB}(\theta); \theta) < \lambda$ for all θ . Thus, given ξ , it never pays to produce with quality m lower than contracted unless $\mu(w + s\xi + \lambda) < \lambda$. For m such that this holds, the expected cost of manipulation is

$$B(m; \xi) = \lambda - \alpha [\lambda - \mu(w + s\xi + \lambda)],$$

so that, having contracted quality q and then observed inspector's type ξ , the contractor with type θ would choose m as to

$$(15) \quad \min_m C(q - m; \theta) + B(m; \xi).$$

When λ , μ , and s are all independent of m , this case coincides with the fixed, uncertain bribe case of Section 2, with $\underline{B} = \lambda(1 - \alpha) + \alpha\mu(w + \lambda)$. In general, if m affects the parameters, the behavior is still analogous. Indeed, define

$$(16) \quad \Phi(q; \theta) = E_\xi[\min_m C(q - m; \theta) + B(m; \xi)].$$

As in Subsection 2.2, the problem for the sponsor is a standard problem with a modified cost function, where (16) is the analogue to (8).

We could also consider the case where the contractor learns ξ only after bargaining. In that case, when the contractor makes the offer having produced quality

²¹The same is true if μ and s are independent of m but λ is concave. In this case, $B(m)$ is concave both for interior values of $\bar{\xi}$ and for $\bar{\xi} = 1$.

²² $B(0) = B'(0) = 0$ and $B(m)$ is convex if, for instance, w is negligible, s is independent of m , and λ is a convex, increasing function of m with $\lim_{m \rightarrow 0} \lambda(m) = 0$, with μ constant or convex in m . That is, a case where the inspector, once fired, incurs no significant cost—from stigma or from loss of wages—of a new job. Again, this is consistent with our interpretation of what a convex $B(m)$ means.

m short of contracted, she solves

$$\max_{\beta} \Pr [\beta \geq \mu(w + s\xi)] (\lambda(1 - \mu) - \beta),$$

Substituting the solution to this problem, we have that the contractor's expected cost of delivering quality m short of contracted is

$$B(m) = \lambda - \alpha \frac{(\lambda(1 - \mu) - \mu w)^2}{4\mu s}.$$

Therefore, in this case, $B(m)$ is larger than in (14) but it has the same qualitative properties.

Also, consider the alternative assumption that bargaining over bribes takes place after the contract with the principal is signed but before the contractor commits resources to production. Arguably, this alternative assumption is less realistic. However, it can still be accommodated by our model in Section 2 as long as we set $\alpha = 1$. That is, as long as it is the contractor who always makes a take it or leave it offer.

On the contrary, our model cannot accommodate a bribe model where the inspector makes a bribe demand after *observing* the terms of the contract signed by the contractor and before the latter incurs the cost of production. This model introduces complications that require a conceptually different treatment. Indeed, under that assumption, bargaining not only would take place under asymmetric information, but also the contract between the sponsor and the contractor would signal the type of the contractor. That, in turn, implies that the procurement mechanism designed by the sponsor would affect the information conditions under which the negotiations between contractor and inspector take place, and so would also affect the cost and probability of bribery. The mechanism and the contractor's—observable—choices would affect the inspector's beliefs, and so the outcome of negotiations with the contractor. These beliefs, in turn, would affect the contractor's choices in the procurement mechanism, so that choices and beliefs need to be consistent with each other. This is a phenomenon that is absent in our model. We discuss this point in

Section 5.²³

Returning to the bribe models that we do consider, and as we mentioned in the Introduction, we may link the curvature of $B(m)$ to certain traits in society. We have seen that concavity of $B(m)$ may be a consequence of concavity of s —and/or λ —. In a society where the rule of law is a strong social norm, we would expect the cost of shame/stigma to be large even for small deviations from the rule, and that is best modeled as concave s . On the other hand, if the rule of law is not a widely and strongly held social norm, we should expect that violations of the rule are easily overlooked and carry little shame even if detected, as long as they are held by boundaries. Only large violations risk reporting and high penalties.²⁴

Note that our results point toward a reinforcing effect of social norms and equilibrium behavior. Indeed, —the possibility of—corruption imposes costs whether $B(m)$ is concave or convex, but in fact, optimally, corruption is prevented when $B(m)$ is—sufficiently—concave but accepted to some extent when $B(m)$ is convex.

Once we consider the linkage between mechanism design and the bribery game, we may also consider the possibility that the sponsor can affect the parameters of this bribery game. As Mookherjee and Png (1995) argue, fines and the standard of proof, typically set by the legislative and judiciary respectively, may be instruments beyond the control of the sponsor, even when the sponsor is a public agency. Thus, it may be reasonable to assume that λ and μ are exogenous for all the agents. However, the inspector's wage w may be a policy instrument for the sponsor. In that

²³Our model of bargaining does capture more elaborate bargaining procedures. For instance, suppose the inspector, not having additional information on the contractor's type—i.e., before observing the quality delivered—, can design and commit to a screening mechanism, which in general allows for a probability of "selling manipulation". This is equivalent to the problem of a seller facing a privately informed buyer. As auction theory shows, the optimal mechanism for the inspector is still a take it or leave it offer in exchange for probability one manipulation. Things would be more complex if bribe payments could be set contingent on delivered quality. In this case, the bribe function $B(m)$ could be designed by the inspector in order to maximize her expected payoff and, once more, the sponsor would have to consider how her contract would affect these choices and the corresponding choices by the contractor. Also, the inspector's beliefs with respect to the contractor's willingness would have to be consistent with the sponsor's—equilibrium— contract.

²⁴For an interesting discussion and overview of the relationship between "culture" and the informal "norms of governance," including corruption, and an empirical analysis of this relationship, see Litch et al. (2007).

case, our model may be used as a building block to analyze optimal procurement mechanisms together with compensation policies for the inspector.

As an example, consider the model in Subsection 2.1, and suppose the sponsor selects the wage w that she will pay the inspector, unless the inspector is—duly—fired. Suppose the choice of mechanism and w are simultaneous. Also, suppose the right model of bribery is our case 1 above with both μ and s independent of m , and normalize the inspector’s reservation wage to 0. Finally, we should modify the sponsor’s payoff in (6) to include the cost of inspection. Given any value of w , it is straightforward to extend Lemma 1 to the case that B is a function of w as defined in (14).²⁵ Thus, we need only add to (6) the term $-w$. Given the optimal choice of w , the model in Subsection 2.1 describes the optimal mechanism and the expected surplus for the sponsor. Thus, applying the envelope theorem and the derivations in the proof of Proposition 3, the optimal w satisfies

$$\frac{C_\theta(q^{NB}(\theta^c); \theta^c)}{f(\theta^c)} \left(\frac{dB}{dw} \right)^{-1} = q^{NB}(\theta^c) - C(q^{NB}(\theta^c); \theta^c) - \frac{F(\theta^c)}{f(\theta^c)} C_\theta(q^{NB}(\theta^c); \theta^c),$$

where dB/dw is the derivative of the right hand side of (14),

$$\frac{dB}{dw} = \alpha \frac{\lambda(1 - \mu) - \mu w}{s},$$

and θ^c is as defined in Proposition 3 for B given by (14). The right hand side measures the effect of relaxing the no bribing constraint. Recall that this (first order effect) was linked to having the opportunity to obtain second best quality from type θ^c , instead of zero quality. The left hand side measures (in terms of the sponsor’s surplus) the cost of relaxing this constraint by increasing w . *Given* θ^c , the higher this cost, the lower the optimal wage, and vice versa. That cost is higher the higher is $(dB/dw)^{-1}$, which measures the increase in w necessary to produce a unit increase

²⁵A sufficient condition for this is that

$$B = \lambda - \alpha \frac{(\lambda(1 - \mu) - \mu w)^2}{2\mu s} > \mu w.$$

It can be shown that this is the case if w is not larger than λ/μ . On the other hand, for $w > \lambda/\mu$, there are never gains from bribing, so that, indeed, bribery never happens in equilibrium.

in the cost of bribing. Larger values of s or μ , or lower values of α or λ will result in larger necessary increases in w . They all result in lower probability of gains from bribing and (high bribes and so) small gains when they are positive. Thus, a one unit increase in the expected cost of bribing requires a larger increase in w , and so a higher cost of relaxing the no bribing constraint, when it is binding.

Of course, θ^c is not *given*, but depends on B , and so on these same variables. However, one can still obtain some immediate comparative statics, only considering this analysis. For instance, consider an increase in α and λ so that B (which is increasing in λ and decreasing in α) remains unaffected (and so does θ^c , for a given value of w). This unambiguously results in a reduction in the cost of relaxing the no bribing constraint through increases in w , and so in higher optimal values of w .

4 Discussion: implementation and competition

We have shown that the optimal mechanism with fixed, known bribe or with sufficiently concave bribe coincides with the optimal mechanism under no corruption in the interior of some interval of types. For types above this interval, quality should be zero and for types below this interval, quality should be a constant. This may be implemented by the menu of contracts—similar to a first-score auction— $(q, p(q))$ for $q \in (q^{NB}(\theta^c), q^{NB}(\theta^a))$, where θ^c and θ^a are defined in Section 2, and for each of these quality levels, $p(q) = q - \Delta(q)$, where

$$\Delta(q) = q^{NB}(\theta^c) - C(q^{NB}(\theta^c), \theta^c) + \int_{q^{NB}(\theta^c)}^q C_{q\theta}(z; q_{NB}^{-1}(z)) \frac{F(q_{NB}^{-1}(z))}{f(q_{NB}^{-1}(z))} dz,$$

and where, q_{NB}^{-1} represents the inverse function of q^{NB} . The menu should be complemented with the contract $(p, q) = (0, 0)$. This is a straightforward corollary of Proposition 4 in Che (1993).

Likewise, the optimal procurement rules when the size of the bribe is uncertain, or the bribe is known but convex, may be implemented with the menu of contracts that implements the optimal mechanism with the modified cost function discussed in Section 2.

Introducing competition, that is, $N > 1$, is also straightforward. A mechanism should now determine not only price, quality, and whether bribes should be paid, but also the probability that each contractor i is selected, $x_i(\theta_i)$. Still, under our assumptions, it is optimal to treat all contractors symmetrically $x_i(\theta) = x(\theta)$. (With $N > 1$, we should understand $p(\theta)$ as the expected payments, i.e., price times probability of winning the competition for the contract, to a contractor of type θ .) It is also optimal to select the most efficient firm, i.e., $x(\theta) = (1 - F(\theta))^{N-1}$, for all θ such that $0 < q(\theta) < q(\underline{\theta})$.²⁶

Before discussing the effects of larger values of N , recall that $q^{NB}(\theta)$, the quality in the optimal contract when corruption is absent, is independent of N . (See Che, 1993.) Only $x(\theta)$, and then $p(\theta)$, are affected: IC imposes a constraint similar to (4):

$$p(\theta) - x(\theta)C(q(\theta); \theta) = \int_{\theta}^{\bar{\theta}} x(z)C_{\theta}(q(z); z)dz,$$

which indeed depends on N , since x depends on N . But the fact that p is affected is key under the threat of corruption: the incentives to bribe are related to the prospect of obtaining the highest price, $p(\theta)/x(\theta)$, so it is this highest price that should be lower than B . Thus, in the case of fixed, known bribe, competition will modify (5) to:²⁷

$$B \geq C(q(\underline{\theta}); \underline{\theta}) + \int_{\underline{\theta}}^{\bar{\theta}} \frac{x(z)}{x(\underline{\theta})} C_{\theta}(q(z); z)dz.$$

Bribery is not binding when this expression is satisfied for $q^{NB}(\theta)$. Thus, the presence of $x(z)/x(\underline{\theta}) < 1$ in the expression above implies that bribery is a less serious constraint when $N > 1$.²⁸ Moreover, as $x(z)$ is decreasing in N , the stronger the competition, i.e., the larger N , the larger the chances that bribery is not an issue.

²⁶If the quality is constant in an interval of types, as we will have with corruption at the high-end of efficiency types, and if there are more than one firm with types in the interval, then the selection should be random among these firms.

²⁷Incentive compatibility requires that $\pi(z; \theta)$ is maximized at $z = \theta$, which implies that, if x is monotone decreasing and q is constant, still $p(\theta)x(\theta)$ is decreasing in θ . Note that, in case of bribing, a contractor obtains an expected profit of $x(\theta)(p(\theta) - B)$, and so again when bribing the contractor will claim type $\underline{\theta}$.

²⁸In this case, $x(\underline{\theta}) = 1$, since there is no "pooling at the top", so that $x(z)/x(\underline{\theta}) = x(z)$.

That is, competition may eliminate the threat of bribery.

Perhaps more interesting, even when bribery binds, so that the expression above is violated, the constraint imposed by bribery is weaker the larger the value of N . Indeed, as when $N = 1$, $q(\theta)$ is distorted down to $q^{NB}(\theta^a)$ in an interval $[\underline{\theta}, \theta^a]$. The larger N , the lower this optimal value of θ^a . That is, the stronger the competition, the smaller the range of types at the top for which quality is distorted.

The intuition for both results is simple. As we have already mentioned, the only reason to distort quality at the top is to reduce the incentives for lower efficiency types to bribe, so that positive quality may be obtained from these types. When there are N contractors, this gain is realized from a contractor only with probability $[1 - F(\theta^c)]^{n-1}$. This probability is obviously smaller the stronger the competition, and so is the return of distortions at the top.²⁹ As a consequence, and contrary to what happens in the absence of bribery, competition increases the optimal value of $q(\theta)$ for a range of types.

The same result can be obtained when B is increasing in m , but concave. However, when B is convex or uncertain at the time of contracting, we have observed that the optimal contract is equivalent to the optimal contract in the absence of bribery, but with a modified cost function. The modification is independent of N , and so, as in the case of no bribery, competition has no effect on the optimal levels of q , but only on prices.

5 Concluding remarks

We have characterized the optimal procurement rules when the contractor may bribe the inspector so that the latter manipulates quality assessments once the contract between contractor and sponsor has been signed. Even when it is optimal to prevent bribery, the optimal rules distort quality downwards, not only for low-efficiency types, but also for the most efficient contractors. This is the case when the bribe is of known and of fixed size or, more generally, under sufficient concavity of the bribe

²⁹The direct effect of N on θ^c is also positive. However, the increase in N induces an increase in $q(\underline{\theta})$ which has an additional, negative effect on θ^c . However, in expected terms, the overall effect on quality is positive.

necessary to secure a given level of manipulation. In these cases, bribery imposes a global (as opposed to local) incentive-compatibility constraint, which compresses the variability of quality that may be obtained. Thus, the sponsor optimally sets quality floors, but also quality ceilings, to what absent corruption would be optimal.

When the bribe is sufficiently convex, or it is uncertain at the time of contracting, totally avoiding bribery may be too costly, and so the optimal mechanism is characterized by some manipulation and bribe payments in equilibrium. In these cases, bribery simply affects local incentive-compatibility constraints. Still, quality is curtailed as a means of reducing the incidence of manipulation.

We have discussed the effects of contractor competition. On the one hand, when bribery imposes a global constraint, this constraint is weakened as the number of potential contractors increases. Correspondingly, quality distortions are reduced. On the other hand, when only local constraints matter, so that bribery is part of the optimal outcome, the number of contractors does not affect the quality contracted with each type of contractor, and so competition does not help reduce quality distortions.

In most of our analysis, we have taken a parsimonious, reduced-form approach to bribery. Then, we have discussed a model of bribery negotiations between contractor and inspector that fits this reduced form. The crucial assumption in this model is that negotiations take place once the project is finished or, at least, resources are committed. In most instances, this is probably the most realistic timing. However, our model would not fit well a case in which negotiations take place before the project is carried out and the inspector observes—at least part of—the contract between sponsor and contractor. In that case, the contractor's choices may signal her type. Moreover, the informativeness of the signal depends on the mechanism that the sponsor designs. The first important consequence of this is the lack of a Revelation Principle that applies to the case. Even if we are satisfied with restricting attention to menus of contracts, so that the choice of p and q by the contractor, not her type, is all that the inspector may observe, the problem would include issues that have been absent in our analysis.³⁰

³⁰At least in public procurement, we should expect these to be required to be made public. Even without this requirement, the inspector may insist on having access to the contract signed by the

Incentive compatibility would have to also contemplate the incentives for the contractor to signal types to the inspector. Also, as in Baliga and Sjoström (1998), the sponsor (principal) would benefit from designing a mechanism that preserves the asymmetry of information between inspector and contractor about the type of the latter. Moreover, as in Celik (2009), the mechanism would determine the contractor's disagreement utility. Both things, information asymmetry and disagreement utility, affect the gains from trading bribes for favorable reports.

Finally, we have not considered the possibility that the sponsor designs a—simultaneous—mechanism to deal both with inspector and contractor. A direct mechanism there would ask the contractor to reveal her type and the inspector to declare the delivered quality, and make the compensation of both agents a function of both messages. Although theoretically attractive—and complex—, it seems difficult to envision practical problems where the inspector's compensation may depend on her—and the contractor's—report. However, these may be interesting lines for future research that are beyond the scope of this paper.

6 References

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7 Appendix

7.1 Proof of Lemma 1

Given an IC, IR mechanism (p, q, b) , consider the new mechanism (p', q', b') where $b'(\theta) = 0 \forall \theta \in [\underline{\theta}, \bar{\theta}]$; $q'(\theta) = q(\theta)$, $p'(\theta) = p(\theta)$ if $b(\theta) = 0$; and $q'(\theta) = 0$, $p'(\theta) = p(\theta) - B$ if $b(\theta) = 1$. It is straightforward that $\Pi(\theta)$ is the same under both mechanisms for all θ . Also, $p'(z) - \min_z \{C'(q(z); \theta), B\} = p(z) - \min_z \{C(q(z); \theta), B\}$ if $b(\theta) = 0$, and $p'(z) - \min_z \{C'(q(z); \theta), B\} = p(z) - B \leq p(z) - \min_z \{C(q(z); \theta), B\}$ if $b(\theta) = 1$, and so the mechanism is incentive-compatible and individually rational. Finally, the sponsor's payoff is larger under (p', q', b') if the probability that $b(\theta) = 1$ is positive.

7.2 Proof of Lemma 2

IC applied to types $\theta + \Delta$ and θ , require $\pi(\theta; \theta) \geq \pi(\theta + \Delta; \theta)$ and $\pi(\theta; \theta + \Delta) \leq \pi(\theta + \Delta; \theta + \Delta)$, which simplifies to

$$0 \leq C(q(\theta + \Delta), \theta) - C(q(\theta + \Delta), \theta + \Delta) - (C(q(\theta), \theta) - C(q(\theta), \theta + \Delta)),$$

so that and since $C_{q\theta} > 0$, we conclude that $q(\theta)$ is indeed monotone decreasing. We may bound $q(\theta)$ without loss of generality, and thus, both $\Pi(\theta)$ and $q(\theta)$ are a.e. differentiable, and from (4) so is p . Also using (4), at any differentiability point of $\Pi(\theta)$,

$$(A1) \quad p'(\theta) = C_q(q(\theta), \theta)q'(\theta) \leq 0.$$

At any non-differentiability point, we can rule out a jump upwards from continuity of $\Pi(\theta)$ and $C(q; \theta)$, and monotonicity of $q(\theta)$.

7.3 Proof of Proposition 3

The proof when (5) holds for the optimal mechanism without bribery is trivial. Now, assume that (5) is violated by the optimal mechanism without bribery, and

consider the following free-time, fixed-endpoint control problem

$$\begin{aligned} & \max_{q(\theta) \in [0, q(\underline{\theta})]} \int_{\underline{\theta}}^{\tau} \{q(\theta) - C(q(\theta); \theta) - X(\theta)\} f(\theta) d\theta \quad (\text{A2}) \\ \text{s.t. } & X' = -C_{\theta}(q(\theta); \theta), \end{aligned}$$

with initial condition $X(\underline{\theta}) = B - C(q(\underline{\theta}); \underline{\theta})$ and target point $X(\tau) = 0$, for some given parameter $q(\underline{\theta})$. Note that

$$(\text{A3}) \quad X(\theta) = X(\underline{\theta}) + \int_{\underline{\theta}}^{\theta} X'(z) dz = \int_{\theta}^{\bar{\theta}} C_{\theta}(q(z); z) dz (= \Pi(\theta)).$$

The quality choice q^* in the optimal mechanism ($p^*, q^*, b^* = 0$) with $p^*(\theta)$ defined by (4) is a solution to this optimal control problem for $q(\underline{\theta}) = q^*(\underline{\theta})$ if it is monotone. Also, at that solution $q(\theta) = 0$ for $\theta > \tau$. The Hamiltonian for this optimal control problem is

$$H(\mu, X, q) = \{q(\theta) - C(q(\theta); \theta) - X(\theta)\} f(\theta) + \mu(-C_{\theta}(q(\theta); \theta))$$

where μ is the costate variable. Necessary conditions for interior solution include:

$$\mu' = f(\theta),$$

$$-\mu C_{\theta q}(q(\theta); \theta) + (1 - C_q(q(\theta); \theta)) f(\theta) = 0.$$

Integrating for μ' , we obtain $\mu = F(\theta)$, and substituting in the second equation, we obtain the same marginal condition (2) as without bribery for values of $\theta < \tau$ at points where the solution in q is interior to $[0, q(\underline{\theta})]$. Given our assumptions, this solution to (2) is monotone decreasing. Therefore, if $q^{NB}(\theta) > q(\underline{\theta})$ at some θ , then the solution to the control problem at that θ is at a corner, $q(\underline{\theta})$, and when $q^{NB}(\theta) < q(\underline{\theta})$ but $\theta < \tau$, then q^* coincides with q^{NB} . Thus, defining θ^a as

$$(\text{A4}) \quad q^{NB}(\theta^a) = q(\underline{\theta}),$$

the solution for $\theta < \theta^a$ is $q(\theta) = q(\underline{\theta})$. If $X(\tau) = 0$ binds ($\tau < \bar{\theta}$ at the solution of the control problem), and from (A3) then $\tau \equiv \theta^c$, satisfies

$$(A5) \quad B - C(q^*(\underline{\theta}); \underline{\theta}) - \int_{\underline{\theta}}^{\theta^a} C_{\theta}(q^*(\underline{\theta}); z) dz - \int_{\theta^a}^{\theta^c} C_{\theta}(q^{NB}(z); z) dz = 0.$$

We now characterize the value $q^*(\underline{\theta})$ in the optimal mechanism, and so the values of θ^a and θ^c at the optimal mechanism. The sponsor's expected surplus in the solution to the control problem for a given value of $q(\underline{\theta}) \leq q^{NB}(\underline{\theta})$ is:

$$\begin{aligned} & \int_{\underline{\theta}}^{\theta^a} \left\{ q(\underline{\theta}) - C(q(\underline{\theta}); \theta) - \int_{\theta}^{\theta^a} C_{\theta}(q(\underline{\theta}); z) dz - \int_{\theta^a}^{\theta^c} C_{\theta}(q^{NB}(z); z) dz \right\} f(\theta) d\theta + \\ & \int_{\theta^a}^{\theta^c} \left\{ q^{NB}(\theta) - C(q^{NB}(\theta); \theta) - \int_{\theta}^{\theta^c} C_{\theta}(q^{NB}(z); z) dz \right\} f(\theta) d\theta, \end{aligned} \quad (A6)$$

where θ^a is a function of $q(\underline{\theta})$ defined in (A4), and θ^c is then also a function of $q(\underline{\theta})$ defined in (A5). The derivative of (A6) with respect to $q(\underline{\theta})$ is

$$\begin{aligned} & \frac{d\theta^c}{dq(\underline{\theta})} \{ [q^{NB}(\theta^c) - C(q^{NB}(\theta^c); \theta^c)] f(\theta^c) - C_{\theta}(q^{NB}(\theta^c); \theta^c) F(\theta^c) \} \\ & \int_{\underline{\theta}}^{\theta^a} \left\{ 1 - C_q(q(\underline{\theta}); \theta) - \int_{\theta}^{\theta^a} C_{\theta q}(q(\underline{\theta}); z) dz \right\} f(\theta) d\theta. \end{aligned} \quad (A7)$$

When evaluated at $q(\underline{\theta}) = q^{NB}(\underline{\theta})$, and so at $\theta^a = \underline{\theta}$, the second line vanishes. Also, from (A5), $d\theta^c/dq(\underline{\theta}) < 0$. The curly brackets in the first line is positive if $q^{NB}(\theta^c) > 0$. Note that indeed $q^{NB}(\theta^c) > 0$ for θ^c evaluated at $q(\underline{\theta}) = q^{NB}(\underline{\theta})$ since otherwise (5) would hold. Therefore, $q^*(\underline{\theta}) < q^{NB}(\underline{\theta})$. Also, changing the order of integration, the second line in (A7) is

$$\int_{\underline{\theta}}^{\theta^a} \{ [1 - C_q(q(\underline{\theta}); \theta)] f(\theta) - C_{\theta q}(q(\underline{\theta}); \theta) F(\theta) \} d\theta > 0,$$

where the inequality follows from the fact that $q(\underline{\theta}) < q^{NB}(\theta)$ for all $\theta \in [\underline{\theta}, \theta^a)$ and the integrand is zero evaluated at $q = q^{NB}(\theta)$. Thus, at the optimal mechanism

$$[q^{NB}(\theta^c) - C(q^{NB}(\theta^c); \theta^c)] f(\theta^c) - C_\theta(q^{NB}(\theta^c); \theta^c) F(\theta^c) > 0,$$

and so $\theta^c < \bar{\theta}$, since we are assuming that $q^{NB}(\bar{\theta}) = 0$.