Primordial perturbations in Hybrid (Loop) Quantum Cosmology

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MOTIVATION

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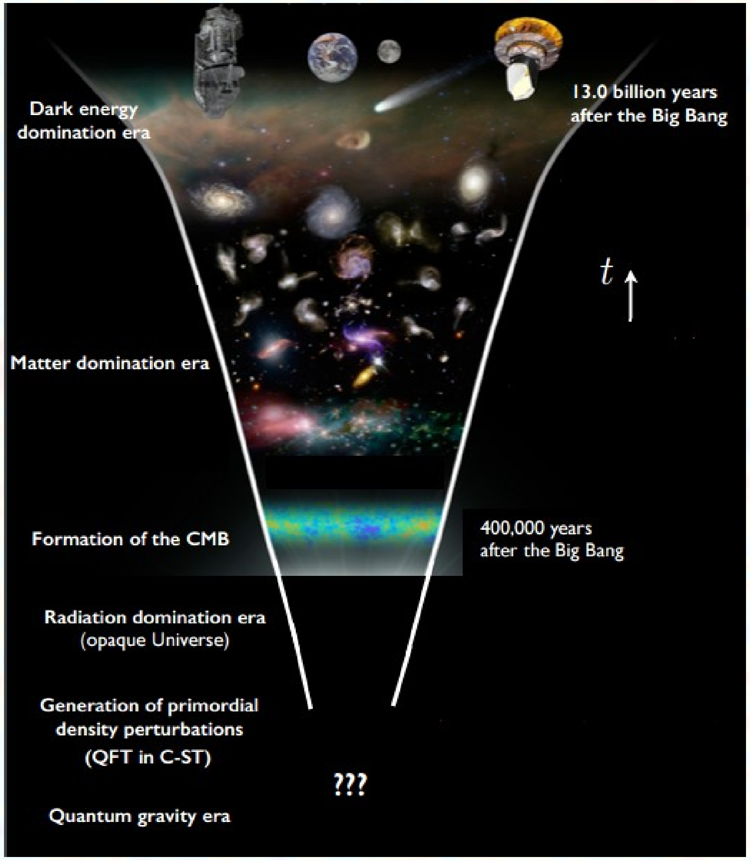
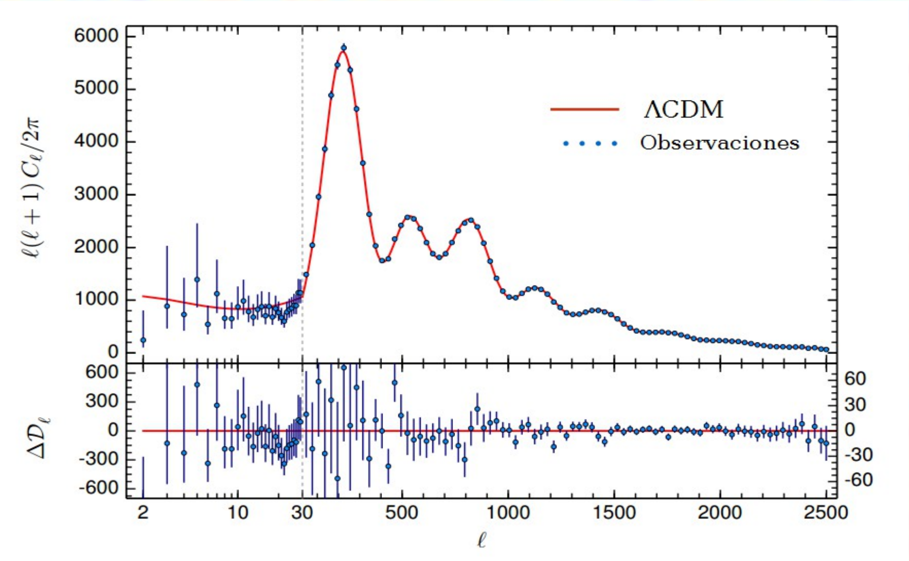


Standard Cosmology
(Lambda-CDM model)



The latest Planck 2015 analysis of the power spectrum of the CMB reveals some **anomalies** at large angular scales.

GOALS

Provide a consistent framework for the quantum description of the evolution of cosmological scalar perturbations in the Early Universe.

- Elaborate a quantization program for a **realistic inhomogeneous cosmological model**, which includes both the background geometry and the perturbations.
- Explore the **quantum nature of spacetime**, beyond treating perturbations as test fields in a generalized QFT.
- Develop theoretical tools to improve our control on the quantitative predictions.
- Investigate whether it is possible to find **observational signatures** in the primordial quantum fluctuations.

CLASSICAL SYSTEM

PERTURBED FLRW

Scalar perturbations in:

FLRW universe + minimally coupled scalar field

Approximation:

Truncation at quadratic perturbative order in the action

- Compact flat spatial topology, T^3 .
- Scalar field is subject to a potential (e.g. a mass term).

PERTURBED FLRW

- Mode expansion of the inhomogeneities: metric and field.

- Adopt the real modes of the Laplace-Beltrami operator compatible with the metric.

- PHASE SPACE
  - Zero-modes $\{\alpha, \varphi\}$
  - Inhomogeneities $\{a_{\vec{n},\pm}, b_{\vec{n},\pm}, f_{\vec{n},\pm}\}$

- We call $g_{\vec{n},\pm}$ and $k_{\vec{n},\pm}$ the (properly scaled) Fourier coefficients of the lapse function and shift vector.

- Truncated action at quadratic order:

$$H = N_0 \left[H_{|0} + \sum H_{|2}^{\vec{n},\pm} \right] + \sum g_{\vec{n},\pm} \tilde{H}_{|1}^{\vec{n},\pm} + \sum k_{\vec{n},\pm} \tilde{H}_{\_1}^{\vec{n},\pm}$$

Inhomogeneous sector

- Consider (for a moment) the homogeneous sector as a fixed background.
- Canonical transformation for perturbations:

$$\{X_l^{\vec{n},\pm}\} \equiv \{a_{\vec{n},\pm}, b_{\vec{n},\pm}, f_{\vec{n},\pm};\ \pi_{a_{\vec{n},\pm}}, \pi_{b_{\vec{n},\pm}}, \pi_{f_{\vec{n},\pm}}\} \longrightarrow \{V_l^{\vec{n},\pm}\} \equiv \{\boldsymbol{v}_{\vec{n},\pm}, C_{|1}^{\vec{n},\pm}, C_{\_1}^{\vec{n},\pm};\ \pi_{v_{\vec{n},\pm}}, \breve{\boldsymbol{H}}_{|1}^{\vec{n},\pm}, \tilde{\boldsymbol{H}}_{\_1}^{\vec{n},\pm}\}.$$

- **Abelianize the algebra of constraints**

$$\breve{H}_{|1}^{\vec{n},\pm} = \tilde{H}_{|1}^{\vec{n},\pm} - 3e^{3\alpha} H_{|0}\, a_{\vec{n},\pm}.$$

Redefinition of the **lapse** function.

- **Mukhanov-Sasaki variables** (gauge invariant)

$$v_{\vec{n},\pm} = e^{\alpha}\left[f_{\vec{n},\pm} + \frac{\pi_{\varphi}}{\pi_{\alpha}}\left(a_{\vec{n},\pm} + b_{\vec{n},\pm}\right)\right].$$

- Complete the transformation and make a specific **choice** of **momentum** for the MS variable.

Homogeneous sector

- Complete the canonical transformation in the entire system.

- Find a transformation for the zero modes,

$$\{w_q^a;\ w_p^a\} := \{\alpha, \varphi;\ \pi_\alpha, \pi_\varphi\} \longrightarrow \{\tilde{w}_q^a;\ \tilde{w}_p^a\} := \{\tilde{\alpha}, \tilde{\varphi};\ \tilde{\pi}_\alpha, \tilde{\pi}_\varphi\},$$

such that the **Legendre term** retains its canonical form (at the considered perturbative order) when expressed in terms of the gauge invariants for the perturbations.

⇓

The difference is precisely QUADRATIC in the perturbations.

Symplectic structure of the total system is preserved.

HAMILTONIAN

- The Hamiltonian constraint in the new formulation: $H_{|0}\left(\tilde{w}^a\right) + \sum \tilde{H}^{\vec{n},\pm}_{|2}\left(\tilde{w}^a, V^{\vec{n},\pm}_l\right),$

  where

$$\tilde{H}^{\vec{n},\pm}_{|2} = H^{\vec{n},\pm}_{|2}\left(\tilde{w}^a, V^{\vec{n},\pm}_l\right) + \sum\left[\left(w^a_q - \tilde{w}^a_q\right)\frac{\partial H_{|0}}{\partial \tilde{w}^a_q} + \left(w^a_p - \tilde{w}^a_p\right)\frac{\partial H_{|0}}{\partial \tilde{w}^a_p}\right]$$

$$= \breve{H}^{\vec{n},\pm}_{|2} + F^{\vec{n},\pm}_{|2} H_{|0} + F^{\vec{n},\pm}_{|1} V^{\vec{n},\pm}_{p2} + \left(F^{\vec{n},\pm}_{-1} - 3\frac{e^{-3\tilde{\alpha}}}{\tilde{\pi}_\alpha} V^{\vec{n},\pm}_{p2} + \frac{9}{2} e^{-3\tilde{\alpha}} V^{\vec{n},\pm}_{p3}\right) V^{\vec{n},\pm}_{p3}.$$

- Hamiltonian:

$$H = \bar{N}_0\left[H_{|0} + \sum \breve{H}^{\vec{n},\pm}_{|2}\right] + \sum G_{\vec{n},\pm} V^{\vec{n},\pm}_{p2} + \sum K_{\vec{n},\pm} V^{\vec{n},\pm}_{p3}.$$

  - Redefiniton of the zero-mode of the lapse function and the Lagrange multipliers.
  - $\breve{H}^{\vec{n},\pm}_{|2}$ is the **Mukhanov-Sasaki Hamiltonian** and it has no linear contributions of the MS momentum, just quadratic ones. In addition, it is linear in the momentum of the homogeneous scalar field.

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- Hamiltonian:

$$H = \overline{N}_0\left[H_{|0} + \sum \breve{H}^{\vec{n},\pm}_{|2}\right] + \sum G_{\vec{n},\pm} V^{\vec{n},\pm}_{p2} + \sum K_{\vec{n},\pm} V^{\vec{n},\pm}_{p3}.$$

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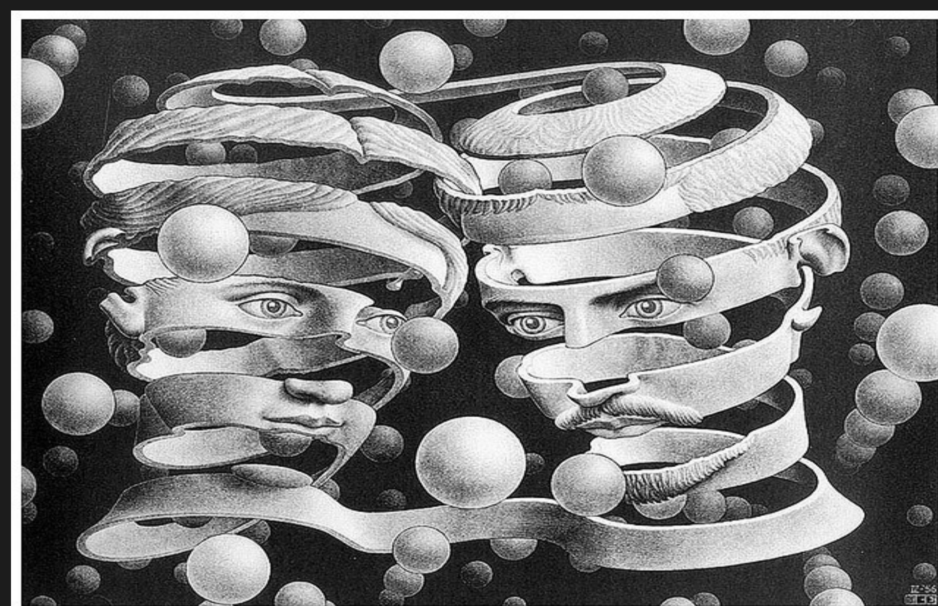
HYBRID QUANTIZATION



HYBRID QUANTIZATION

Assumption:

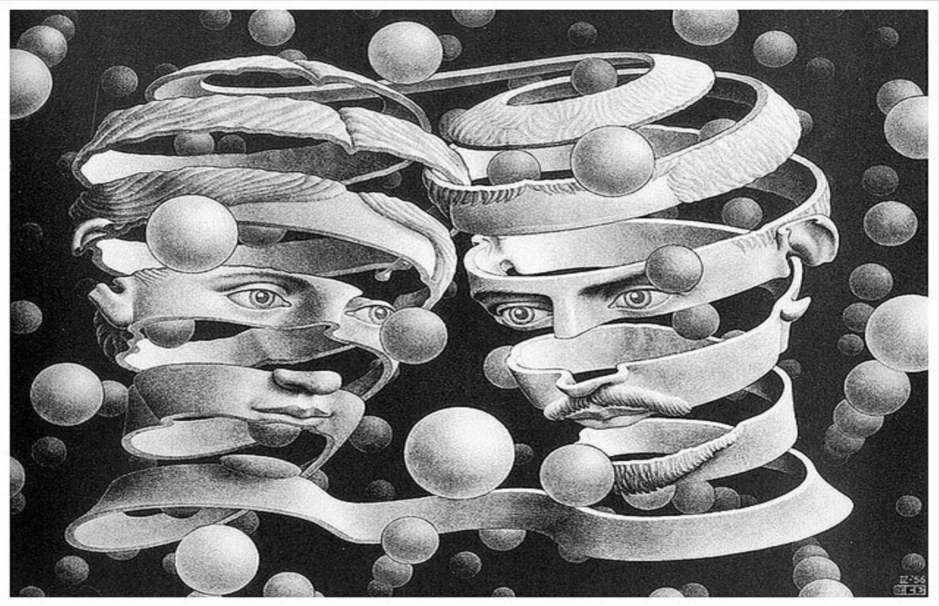
Quantum geometry effects are especially relevant in the background.

- Strategy:

PHASE SPACE
- Homogeneous sector → Quantum Cosmology.
- Inhomogeneous sector → Fock quantization.

Fock quantization → UNIQUENESS criteria

QUANTUM GRAVITY



QUANTUM FIELD THEORY
on curved spacetime

Backreaction is included at considered truncation order.

- The system is a constrained symplectic manifold.
- The Hamiltonian constraint **couples** both sectors, encoding the backreaction of the perturbations on the homogeneous background.
- We assume:

  i. The zero-modes commute with the perturbations under quantization.

  ii. Operators that represent functions of $\tilde{\varphi}$ act by multiplication.

Fock representation for the inhomogeneities

- **Ambiguity** in selecting a **Fock representation** in QFT can be removed by:

  i. **INVARIANCE** under spatial symmetries of the field equations.

  ii. **UNITARITY** of the quantum evolution in a finite time interval.

- **Annhilation** and **creation operators** for our (rescaled) MS modes, naturally associated with the massless scalar field.

  - The introduced **scaling** is essential for unitarity.

- **Fock space** F : basis of occupancy number sates.

$$\left\{ |N\rangle = |N_{(1,0,0),+}, N_{(1,0,0),-}, \dots\rangle ; \quad N_{\vec{n},\pm} \in \mathbb{N}, \ \sum N_{\vec{n},\pm} < \infty \right\}.$$

Quantum representation of the constraints

- **Linear perturbative constraints** are represented by momenta operators that act as derivatives.

  → Physical states only depend on the homogeneous variables and the MS gauge invariants.

- **Hilbert space**: $H_{kin}^{FLRW} \otimes H_{kin}^{matt} \otimes F$.

- Physical states still must satisfy the scalar constraint: $\hat{H} = e^{3\alpha}\left(\hat{H}_{|0} + \hat{\breve{H}}_{|2}\right),$

with

$$e^{3\alpha}\hat{H}_{|0} \propto \left[\hat{\pi}_{\tilde{\varphi}}^2 - \hat{H}_0^{(2)}\right],$$

$$e^{3\alpha}\hat{\breve{H}}_{|2} \propto -\left[\hat{\Theta}_e + \left(\hat{\Theta}_o \hat{\pi}_{\tilde{\varphi}}\right)_S\right].$$

☆ Quadratic in the MS modes and independent of the momentum of the homogeneous field.

BORN-OPPENHEIMER ANSATZ

- Consider states whose evolution in the inhomogeneities and the FLRW geometry presents different rates of variation:

$$\Psi=\chi(\tilde{\alpha},\tilde{\varphi})\psi(N,\tilde{\varphi}),\qquad \chi(\tilde{\alpha},\tilde{\varphi})=\hat{U}(\tilde{\alpha},\tilde{\varphi})\chi_0(\tilde{\alpha}).$$

  - ☆ The FLRW state is **normalized**, **peaked** and evolves **unitarily**.

- Disregard **nondiagonal** elements for the FLRW geometry sector in the constraint.

↓

$$\hat{\pi}_{\tilde{\varphi}}^2\,\psi+\left(2\left\langle\hat{H}_0\right\rangle_\chi-\left\langle\hat{\Theta}_o\right\rangle_\chi\right)\hat{\pi}_{\tilde{\varphi}}\,\psi=\left[\left\langle\hat{\Theta}_e+\left(\hat{\Theta}_o\hat{H}_0\right)_S\right\rangle_\chi+i\left\langle d_{\tilde{\varphi}}\hat{H}_0-\frac{1}{2}d_{\tilde{\varphi}}\hat{\Theta}_o\right\rangle_\chi\right]\psi,$$

where $-i\,d_{\tilde{\varphi}}\hat{O}:=\left[\hat{\pi}_{\tilde{\varphi}}-\hat{H}_{0,}\hat{O}\right].$

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$$\Psi = \chi(\tilde{\alpha}, \tilde{\varphi})\psi(N, \tilde{\varphi}), \qquad \chi(\tilde{\alpha}, \tilde{\varphi}) = \boldsymbol{P}\left[\exp\left(i\int_{\tilde{\varphi}_0}^{\tilde{\varphi}} d\bar{\varphi}\;\hat{H}_0(\tilde{\alpha}, \bar{\varphi})\right)\right]\chi_0(\tilde{\alpha}).$$

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$$\hat{\pi}_{\tilde{\varphi}}^2\psi + \left(2\langle\hat{H}_0\rangle_\chi - \langle\hat{\Theta}_o\rangle_\chi\right)\hat{\pi}_{\tilde{\varphi}}\psi = \left[\left\langle\hat{\Theta}_e + \left(\hat{\Theta}_o\hat{H}_0\right)_S\right\rangle_\chi + i\left\langle d_{\tilde{\varphi}}\hat{H}_0 - \frac{1}{2}d_{\tilde{\varphi}}\hat{\Theta}_o\right\rangle_\chi\right]\psi,$$

where $-i\,d_{\tilde{\varphi}}\hat{O} := \left[\hat{\pi}_{\tilde{\varphi}} - \hat{H}_{0,}\hat{O}\right].$

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$$\hat{\pi}_{\tilde{\varphi}}^2\, \psi + \left(2\left\langle \hat{H}_0 \right\rangle_\chi - \left\langle \hat{\Theta}_o \right\rangle_\chi\right) \hat{\pi}_{\tilde{\varphi}}\, \psi = \left[\left\langle \hat{\Theta}_e + \left(\hat{\Theta}_o \hat{H}_0\right)_S \right\rangle_\chi + i \left\langle d_{\tilde{\varphi}} \hat{H}_0 - \frac{1}{2} d_{\tilde{\varphi}} \hat{\Theta}_o \right\rangle_\chi\right] \psi,$$

- Besides, if we can **neglect**: a) $\hat{\pi}_{\tilde{\varphi}}^2\, \psi$
  b) The total $\tilde{\varphi}$-derivative of $2\hat{H}_0 - \hat{\Theta}_o$,

$$\hat{\pi}_\varphi \psi = \frac{\langle \hat{\Theta}_e + (\hat{\Theta}_o \hat{H}_0)_S \rangle_\chi}{2\langle \hat{H}_0 \rangle_\chi}\, \psi$$

Schrödinger-like equation

EFFECTIVE MUKHANOV-SASAKI EQUATIONS

- Assuming a direct **effective** counterpart for the **inhomogeneities**:

$$d^2_{\eta_\chi} v_{\vec{n},\epsilon} = -v_{\vec{n},\epsilon} 4\pi \left[\omega_n^2 + \frac{\left\langle \hat{\theta}_e^q + (\hat{\theta}_o \hat{H}_0)_S + \frac{1}{2}\left[\hat{\pi}_\phi - \hat{H}_{0,} \hat{\theta}_o\right] \right\rangle_\chi}{\langle \hat{\theta}_e \rangle_\chi} \right],$$

with $2\pi d\eta_\chi = \left\langle \hat{\theta}_e \right\rangle_\chi dT,$

and where $\theta_o = -12 e^{4\tilde{\alpha}} \frac{1}{\pi_{\tilde{\alpha}}} W'(\tilde{\varphi}),$

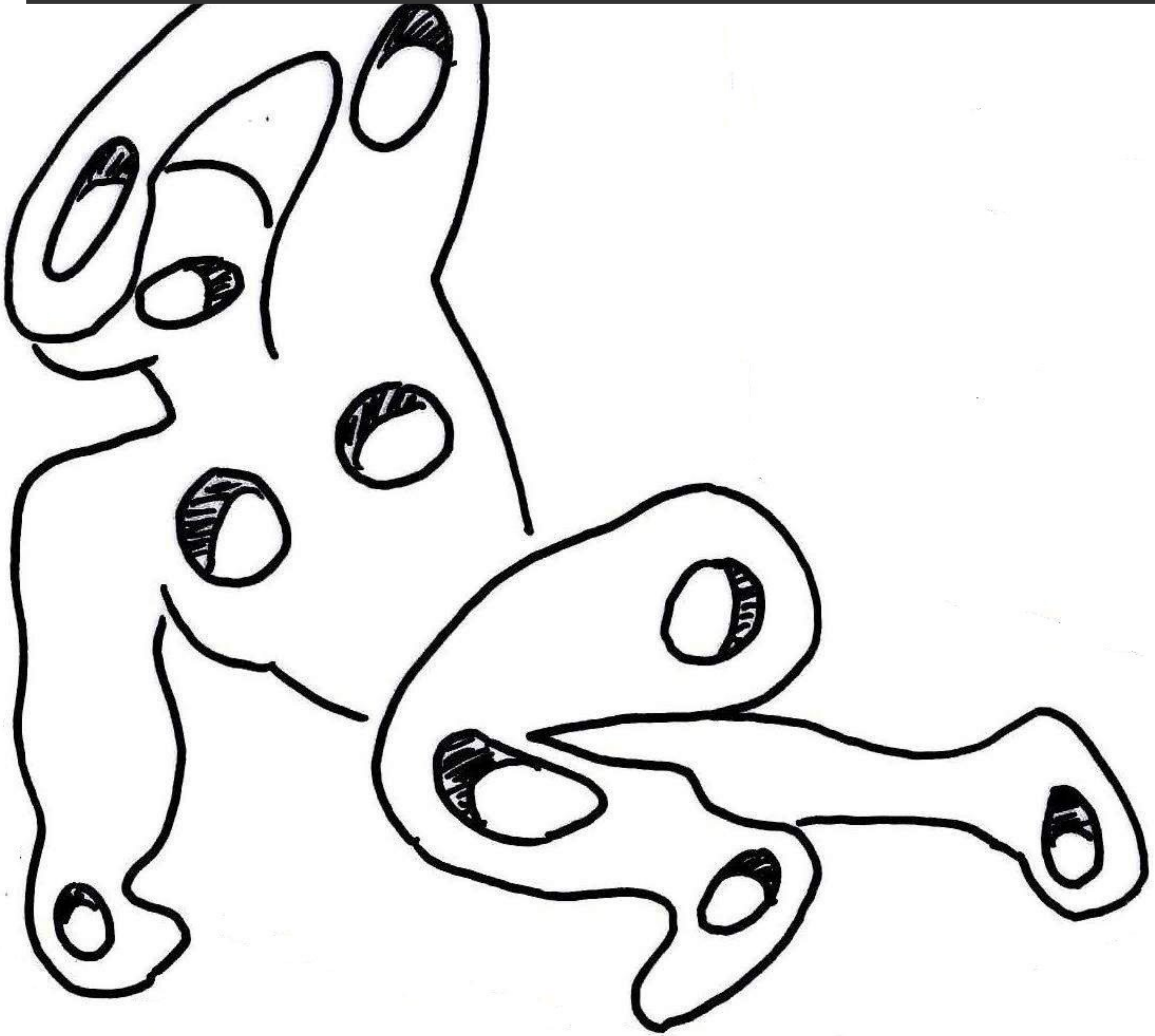
$$\theta_e = e^{2\tilde{\alpha}},$$

$$\theta_e^q = e^{-2\tilde{\alpha}} H_0^{(2)} \left(19 - 18 \frac{H_0^{(2)}}{\pi_{\tilde{\alpha}}^2} \right) + e^{4\tilde{\alpha}} \left[W''(\tilde{\varphi}) - 4 W(\tilde{\varphi}) \right].$$

POTENTIAL

Master equation to calculate the power spectrum of the CMB.

HYBRID LOOP QUANTUM COSMOLOGY



- Homogeneous and isotropic spacetimes (FLRW): flat and spatial curvature ($k = 1$), positive and negative cosmological constant.
- Anisotropies: Bianchi I, Bianchi II, Bianchi IX.
- Inhomogeneities: Gowdy model, cosmological perturbations.

  ⟶ **Hybrid Quantization**

- Inflation.

Quantum bounce

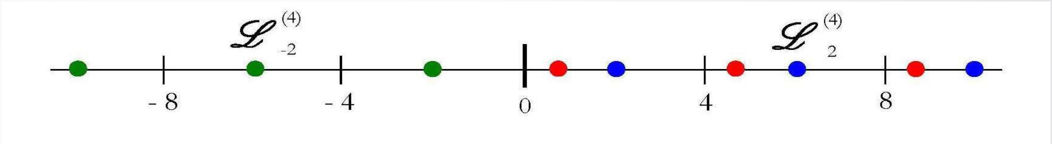
FLRW IN LQC

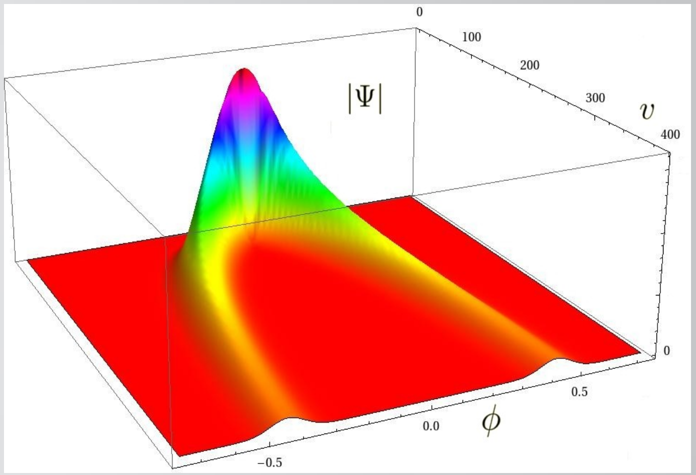
Massive scalar field ϕ minimally coupled to a compact, flat FLRW universe:

- Geometry: $\{c, p\} = 8\pi G\gamma/3, \quad V = |p|^{3/2} = a^3.$

- Contribution to the Hamiltonian constraint: $\hat{H}_0^{(2)} = \frac{3}{4\pi G}\left(\frac{3}{4\pi G\gamma^2}\hat{\Omega}_0^2 - m^2\hat{V}^2\hat{\phi}^2\right).$

- The gravitational part Ω_0 is a difference operator:





- Regularized operator of the inverse of the volumen.

HYBRID LOOP QUANTIZATION

- To quantize the quadratic contribution of the perturbations to the Hamiltonian we adapt the **proposals of the homogeneous sector** and use a symmetric factor ordering:

  - **Symmetrized products** of the type $f(\hat{\phi})\hat{\pi}_{\phi}$.
  - **Symmetric geometric** factor ordering $V^k A \rightarrow \hat{V}^{k/2}\hat{A}\hat{V}^{k/2}$.
  - Adopting the LQC representation $(cp)^{2m} \rightarrow [\hat{\Omega}_0^2]^m$.
  - In order to preserve the FLRW **superselection sectors**, we adopt the prescription $(cp)^{2m+1} \rightarrow [\hat{\Omega}_0^2]^{m/2}\hat{\Lambda}_0[\hat{\Omega}_0^2]^{m/2}$, where $\hat{\Lambda}_0$ is defined like $\hat{\Omega}_0$ but with double steps.

EFFECTIVE MUKHANOV-SASAKI EQUATIONS IN LQC

$$d^2_{\eta_\chi} v_{\vec{n},\epsilon} = -v_{\vec{n},\epsilon} 4\pi \left[\omega_n^2 + \frac{\left\langle \hat{\theta}_e^q + (\hat{\theta}_o \hat{H}_0)_S + \frac{1}{2}\left[\hat{\pi}_\phi - \hat{H}_{0,} \hat{\theta}_o\right] \right\rangle_\chi}{\langle \hat{\theta}_e \rangle_\chi} \right],$$

with $2\pi d\eta_\chi = \langle \hat{\theta}_e \rangle_\chi dT$.

- If we adopt the LQC representation for the homogeneous sector:

$$\hat{H}_0^{(2)} = \frac{3}{4\pi G}\left(\frac{3}{4\pi G \gamma^2} \hat{\Omega}_0^2 - \hat{V}^2 m^2 \hat{\phi}^2 \right),$$

and

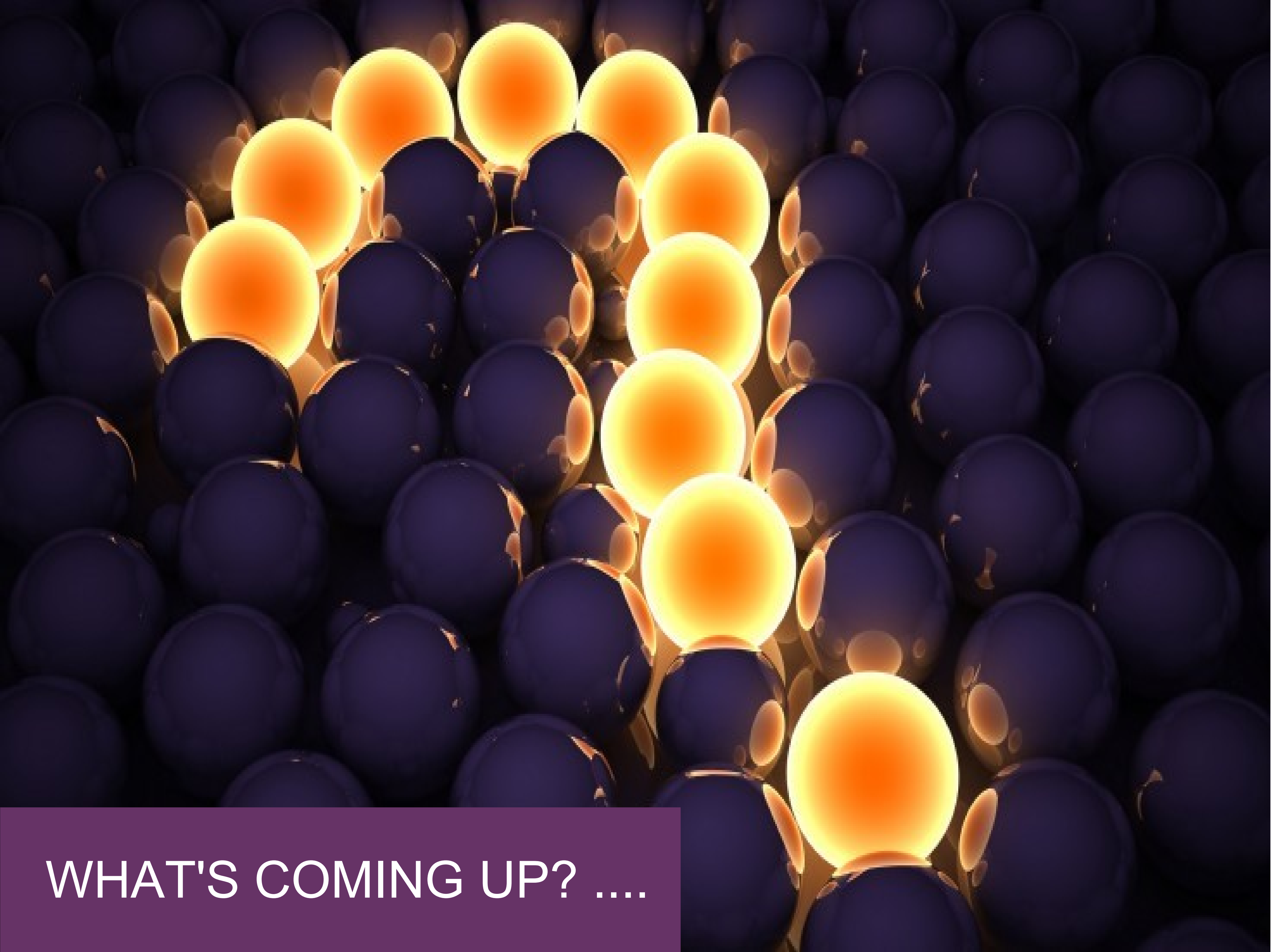
$$\hat{\theta}_o = 4\sqrt{\frac{3G}{\pi}} \gamma m^2 \hat{\phi} \hat{V}^{2/3} \left|\hat{\Omega}_0\right|^{-1} \hat{\Lambda}_0 \left|\hat{\Omega}_0\right|^{-1} \hat{V}^{2/3},$$

$$\hat{\theta}_e = \frac{3}{2G} \hat{V}^{2/3},$$

$$\hat{\theta}_e^q = \frac{1}{2\pi}\left[\frac{\hat{1}}{V}\right]^{1/3} \hat{H}_0^{(2)} \left(19 - 24\pi G \gamma^2 \hat{\Omega}_0^{-2} \hat{H}_0^{(2)}\right)\left[\frac{\hat{1}}{V}\right]^{1/3} + \frac{3m^2}{8\pi^2 G} \hat{V}^{4/3}\left(1 - \frac{8\pi G}{3}\hat{\phi}\right).$$

CONCLUSIONS

- We have considered a FLRW universe minimally coupled to a scalar field perturbed at **quadratic** order in the action.
- We have found a canonical transformation for the **full system** that respects *covariance* at the perturbative level of the truncation.
- Perturbations are describe in terms of MS gauge invariants, linear perturbative constraints, and variables canonically conjugate to them. The zero-modes get quadratic corrections from perturbations.
- We have proposed a **generalized hybrid quantization** of the system. This can be adapted to a variety of quantum FLRW cosmology approaches. Physical states only depend on the MS modes and the homogeneous sector.
- A **Born-Oppenheimer** ansatz leads to **Mukhanov-Sasaki equations** that include quantum corrections, with no dissipative terms. The ultraviolet regime is **hyperbolic**.
- We have proposed methods, based on an **interaction picture**, for the computation in LQC of the expectation values that appear in the effective equations.



WHAT'S COMING UP? ....