Recovery of North-East Atlantic temperature fields from profiling floats: determination of the optimal float number from sampling and instrumental error analysis

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Abstract

Argo is an international project that is deploying an array of temperature and salinity profiling floats over the global ocean. Here we use the error formulation derived from Optimal Statistical Interpolation to estimate statistical errors associated with the recovery of the temperature field in the North-East Atlantic ocean. Results indicate that with the present distribution of floats (119 in the considered domain), scales of wavelength larger than 500 km can be recovered with a relative uncertainty (rms error relative to the standard deviation of the field) of about 7\% at 50 m, 8\% at 200 m and 10\% at 1000 m. This corresponds to mean absolute errors of 0.111^\circ C at 50 m, 0.104^\circ C at 200 m and 0.073^\circ C at 1000 m.

The splitting of total errors into instrumental and sampling contributions reveals that, in the present scenario, errors are more due to the small number of floats than to instrumental errors, especially at upper levels. For scales larger than 500 km this will hold true until 200-250 floats are deployed (less than 200 for deep levels). In such a simulated scenario, the number of observations and the technology become approximately equally limiting factors for the accuracy of the temperature field mapping, with total relative errors of less than 2\% at upper levels and about 3\% at 1000 m.

Key words: Profiling floats, objective analysis, error analysis, North Atlantic Ocean

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1 Introduction

Argo is an international project that is deploying an array of temperature and salinity profiling floats over the global ocean. It constitutes a pilot project of the Global Ocean Observing System (GOOS) and a major contributor to the Global Climate Observing System (GCOS) (Gould and Belbeoch, 2003). Applications of the Argo project to climate studies and operational oceanography are being evaluated by the Climate Variability and Predictability Experiment (CLIVAR) and the Global Ocean Data Assimilation Experiment (GODAE) (Le-Traon et al., 1999). One of the capabilities is the recovery of the decadal variability of the Subtropical North Atlantic by comparing float data with previous WOCE hydrographic transects (e.g., Parrilla et al., 1994; Joyce and Robbins, 1996; Vargas-Yáñez et al., 2004). In a more global context, the Argo project is expected to produce an accurate global climatology, with error bars and variability statistics obtained from monthly mean data (Argo Science Team, 1998).

The design of the array of profiling floats has not a unique solution and needs of a continuous revision of over/sub-sampled regions with respect to the expected Argo achievements. Hence, the evaluation of the information contents of the float array is a key issue for the planning of future deployments. This study intends to provide some guidance on the optimal number of floats required for the recovery of the temperature field in the North-East Atlantic ocean. The optimization of the array is here defined as the number of floats required for the recovery of a prescribed spectral range of the temperature field with a given accuracy. The work has been undertaken in the framework of the Gyroscope project (Desaubies, 2003), which is itself a major European contribution to the Argo project.

In a recent paper, Guinehut et al. (2002) evaluated the accuracy of the recovery of North Atlantic temperature fields by sampling model output of the region at different numbers of regularly distributed points. Those synthetic observations were then interpolated (via Optimal Statistical Interpolation, hereafter OI) onto the model grid and compared with the original model values. The result was a table with temperature errors as a function of the station spacing at several levels and for two different low-pass filtering processes (cut-off wavelengths of 1000 and 500 km).

Here we face the problem using a different methodology. First, the error values provided by Guinehut et al. (2002) correspond to a particular set of realizations (assumed to be representative) of the temperature field. Instead, we use the error formulation derived from OI theory to directly compute statistical errors, i.e., those that would be obtained by averaging a very large number of different temperature fields having a common set of prescribed statistical
properties. Second, we use actual float distributions instead of synthetic regular distributions as observation points. Moreover, all the parameters used on input by the OI formulation have been derived from actual float data, instead of from model data.

Finally, and perhaps most important, we not only produce total errors, but also split them into the contribution attributed to instrumental noise (hereafter ‘observational errors’) and that derived from the station distribution (hereafter ‘sampling errors’). This will be a first key issue of this work, given the different impact that the number of floats has on each error contribution. It will be useful, for instance, to decide whether it is more convenient putting the efforts in increasing the number of floats or in improving the accuracy of their sensors (provided both initiatives were equally possible).

A second key issue of this work is the concept of spatial scale. This is crucial when talking about errors, since these depend critically on the ratio between the station separation and the scales intended to be resolved. When detailing the formulation, we will argue that the deviations of the interpolation field should not be measured with respect to the true field, but with respect to the recovered spectral range of the truth. Here we will focus on the recovery of scales with wavelength larger than 500 km, according to the present low resolution deployment strategy (250 km) and also with the purpose of comparing our results with those obtained by Guinehut et al. (2002).

2 Data set and methodology

In the framework of the Gyroscope project, 84 profiling floats were deployed in the North-East Atlantic ocean from summer 2001 to the end of summer 2002. By March 2003 (considered here as the ‘present time’), 75 floats remained active and a total of 2500 profiles had been recorded (Desaubies, 2003). For the purpose of this work, we also used data from other active temperature and salinity profiling floats deployed in the North-East Atlantic in the framework of other projects. A total of 119 floats (including Gyroscope floats) distributed over a domain D2 (see Fig. 1) were obtained from the Coriolis Data Center public ftp server (ftp://www.coriolis.eu.org).

Argo floats work as follows: their mean density is accurately set as to have a neutral buoyancy at a prescribed depth (the so-called ‘parking depth’), set to 1500 m in the Atlantic ocean. Therefore, they drift following approximately the isobaric currents at that depth. Every ten days the density of the floats is first increased and then decreased through volume compression/expansion, so as the float descends to about 2000 m and then goes up to the surface. All sensors are waken up just before the density modifications, in order to measure
the properties of the water column in its way up to the surface. Once there, the float sends the measured profile to data centers by satellite transmission. Obtaining the profile and sending the data takes about 16 h, after which the float descends again to the parking depth. All temperature profiles reported by all active floats from January 2002 to March 2003 constituted the data base for the estimation of the statistical properties of the temperature field required for the application of the OI scheme. In the following, we first describe the OI formulation that constitutes the basis of this work and then compute the parameters required for its application.

2.1 Error formulation derived from Optimal Statistical Interpolation theory

Following OI formulation (see for instance Bretherton et al., 1976), the \( m \times m \) analysis error covariance matrix \( \hat{\Sigma}_{gg} \) associated with the recovery of a 2D field at ‘m’ grid points from a set of ‘n’ scattered observations is given by:

\[
\hat{\Sigma}_{gg} = \begin{bmatrix}
\langle \hat{\phi}_g - \tilde{\phi}_g \rangle \\
\langle \hat{\phi}_g \hat{\phi}_g^T \rangle + \langle \tilde{\phi}_g \tilde{\phi}_g^T \rangle - \langle \hat{\phi}_g \tilde{\phi}_g^T \rangle - \langle \tilde{\phi}_g \hat{\phi}_g^T \rangle
\end{bmatrix} = \langle \hat{\Sigma}_V \rangle - \hat{\Sigma}_{go} \hat{\Sigma}_o^{-1} \hat{\phi}_o \hat{\phi}_o^T \hat{\Sigma}_V + \hat{\Sigma}_{gg} - \hat{\Sigma}_{go} \hat{\Sigma}_o^{-1} \hat{\phi}_o \hat{\phi}_o^T \hat{\Sigma}_V
\]

where \( < . > \) denotes a statistical mean, \( \hat{\phi}_g \) is the m-vector of interpolated values at grid points and \( \tilde{\phi}_g \) is the m-vector of (unknown) true values at grid points, so that the analysis errors are given by \( \langle \hat{\phi}_g - \tilde{\phi}_g \rangle \). According to OI, the interpolated values are given by \( \hat{\phi}_g = \hat{V}_{go} (\hat{V}_{oo} + \hat{E}_{oo})^{-1} \hat{\phi}_o \), where \( \hat{\phi}_o \) is the n-vector of observations. Matrices \( \hat{V}_{gg}, \hat{V}_{oo} \) and \( \hat{V}_{go} \) report the spatial covariance of the true anomaly field. In particular, the symmetric mxm matrix \( \hat{V}_{gg} \) contains the covariance between grid points, the symmetric nxn matrix \( \hat{V}_{oo} \) contains the covariance between observation locations and the mxn matrix \( \hat{V}_{go} \) contains the covariance between grid points and observation points. \( \hat{E}_{oo} \) is a symmetric nxn matrix reporting the spatial covariance of observation errors.
From (1) it follows that, at a given grid point, the statistical deviation between the recovered field and the true field depends on the station distribution through matrices $\vec{V}_{oo}$, $\vec{V}_{go}$ and on the accuracy of observations through matrix $\vec{E}_{oo}$. However, the interpolation process is usually aimed to recover only a part of the spectral contents of the truth, since the discrete sampling of the field imposes a lower limit to the size of the structures that can be properly resolved. Hence, a more fair error measure is given by the statistical difference between the recovered field and the low-pass filtered truth (i.e., the scales intended to be recovered). When the smoothed interpolated field is obtained by applying a linear filter operator $F$ onto the OI output, the corresponding error covariance matrix $\hat{\vec{E}}_{gg}$ is given by (Bretherton et al., 1976; Gomis and Pedder, 2005):

$$\hat{\vec{E}}_{gg} = \vec{F}_{gg} \hat{\vec{E}}_{gg} \vec{F}_{gg}^T = \vec{F}_{gg} \vec{V}_{gg} \vec{F}_{gg}^T - \vec{F}_{gg} \vec{V}_{go} (\vec{V}_{oo} + \hat{\vec{E}}_{oo})^{-1} \vec{V}_{go} \vec{F}_{gg}^T$$  

(2)

A first simplification to expressions (1) and (2) comes from assuming that when observations are obtained with independent instruments, errors can be considered as spatially uncorrelated. In that case the observation error covariance matrix $\vec{E}_{oo}$ is a diagonal matrix. If, moreover, the accuracy of all instruments $\epsilon$ is similar, matrix $\vec{E}_{oo}$ can be expressed simply as $\epsilon^2 \vec{I}$, where $\vec{I}$ is the nxn identity matrix. A second simplification comes from assuming a horizontally constant (depth dependent) value $\sigma^2$ for the anomaly field variance. In this case, all covariance matrices $\vec{V}$ can be expressed as $\vec{V} = \sigma^2 \vec{C}$, where $\vec{C}$ contain correlation elements, and the so-called noise-to-signal ratio $\gamma = \epsilon^2 / \sigma^2$ is also constant.

The most critical limitation to the application of OI formulation in its whole potential is the lack of in situ oceanographic data, which prevents from deriving statistically significant correlations between specific location pairs. Hence, a further, common simplification is to assume that correlation obeys a simple (usually homogenous and isotropic) analytical model. This is usually some kind of function decaying with distance and can be fitted to the correlations computed by averaging all observation anomaly pairs within given distance lags. A positive consequence of assuming a simple correlation model is that the application of the linear filter $\vec{F}_{gg}$ onto the correlation matrix $\vec{C}_{go}$ can often be analytically summarized in a single operator $\vec{C}_{go}$, which eliminates any dependence of the filter on the discrete sampling (Pedder, 1993). The elements of matrix $\vec{C}_{go}$ are the analytical convolution between the filtering
function and the correlation function evaluated at location pairs.

Under all the above assumptions, expression (2) can be written as:

$$\hat{\vec{E}}_{fg} = \sigma^2 [\hat{\vec{F}}_{gg}\hat{\vec{C}}_{gg}\hat{\vec{F}}^T_{gg} - \hat{\vec{C}}_{go}(\hat{\vec{C}}_{oo} + \gamma\hat{\vec{I}})^{-1}\hat{\vec{C}}^T_{go}]$$  \hspace{1cm} (3)

Reminding that $$\hat{\vec{C}}_{gg} = \sigma^{-2} < \vec{\phi}_g\vec{\phi}_g^T >$$, then $$\hat{\vec{F}}_{gg}\hat{\vec{C}}_{gg}\hat{\vec{F}}^T_{gg} = \sigma^{-2} < \vec{\phi}_g\vec{\phi}_g\vec{\phi}_g^T >$$

$$= \sigma^{-2} < \vec{\phi}_g\vec{\phi}_g^T >$$, where $$\vec{\phi}_g$$ is a realization of the filtered truth. It then follows that $$\sigma^2 \hat{\vec{F}}_{gg}\hat{\vec{C}}_{gg}\hat{\vec{F}}^T_{gg}$$ is the covariance matrix of the filtered true anomaly field, its diagonal terms then containing the variance $$(\sigma^f)^2$$ of the resolved spectral range of the truth.

Of the whole error covariance matrix $$\hat{\vec{E}}_{fg}$$, our interest focuses on the diagonal terms, which report the error variance at each grid point $$\hat{\epsilon}_f^2$$. These are given by:

$$\hat{\epsilon}_f^2 = (\sigma^f)^2 - \sigma^2 \hat{\vec{C}}_{go}(\hat{\vec{C}}_{oo} + \gamma\hat{\vec{I}})^{-1}\hat{\vec{C}}^T_{go}$$  \hspace{1cm} (4)

where vector $$\hat{\vec{C}}^T_{go}$$ is the 'g' row of matrix $$\hat{\vec{C}}_{go}$$ (and $$\hat{\vec{C}}_{go}$$ is the 'g' column of matrix $$\hat{\vec{C}}^T_{go}$$). Hence, computing the standard deviation of errors relative to the standard deviation of the (filtered) anomaly field $$\hat{\epsilon}_f^2/\sigma^f$$ is straightforward from (4).

In addition to the variance of total errors, the OI formulation can also evaluate the impact of instrumental errors as a separate contribution (i.e., how random errors inherent to observations propagate through the interpolation process). To obtain this contribution, one must go back to the derivation of the analysis error covariance matrix (1). If the vector of observations $$\vec{\phi}_o$$ is split into a vector of true value $$\vec{\phi}_o$$ and a vector of instrumental noise $$\vec{\varepsilon}_o$$, one can follow the impact of the latter throughout the analysis (Gomis and Pedder, 2005). Using correlation matrices instead of the covariance matrices initially used in (1), we obtain:

$$\hat{\vec{E}}_{gg} = \hat{\vec{C}}_{go}(\hat{\vec{C}}_{oo} + \gamma\hat{\vec{I}})^{-1} < (\vec{\phi}_o + \vec{\varepsilon}_o)(\vec{\phi}_o + \vec{\varepsilon}_o)^T > (\hat{\vec{C}}_{oo} + \gamma\hat{\vec{I}})^{-1}\hat{\vec{C}}_{go}$$

$$+ \sigma^2 \hat{\vec{C}}_{gg} - \hat{\vec{C}}_{go}(\hat{\vec{C}}_{oo} + \gamma\hat{\vec{I}})^{-1} < (\vec{\phi}_o + \vec{\varepsilon}_o)\vec{\phi}_o^T >$$

$$- < \vec{\phi}_g(\vec{\phi}_o + \vec{\varepsilon}_o)^T > (\hat{\vec{C}}_{oo} + \gamma\hat{\vec{I}})^{-1}\hat{\vec{C}}^T_{go} =$$

$$\hat{\vec{C}}_{go}(\hat{\vec{C}}_{oo} + \gamma\hat{\vec{I}})^{-1}\epsilon^2 I(\hat{\vec{C}}_{oo} + \gamma\hat{\vec{I}})^{-1}\hat{\vec{C}}_{go}$$

6
\[
\begin{align*}
+\tilde{C}_{go}(\tilde{C}_{oo} + \gamma \tilde{I})^{-1}\sigma^2 \tilde{C}_{oo}(\tilde{C}_{oo} + \gamma \tilde{I})^{-1}C_{go}^T \\
+\sigma^2 \tilde{C}_{gg} - 2\sigma^2 \tilde{C}_{go}(\tilde{C}_{oo} + \gamma \tilde{I})^{-1}C_{go}^T
\end{align*}
\] (5)

Therefore the effect of random errors restricts to the first term on the rhs. The diagonal entries of that term (for the case in which correlation matrices are convoluted with a linear filter) are given by:

\[
(\hat{\epsilon}_{f g,obs})^2 = \epsilon^2 C_{T go}^T (\tilde{C}_{oo} + \gamma \tilde{I})^{-2}C_{go}
\] (6)

It follows that the instrumental noise response (referred to as the ‘observational contribution’) is proportional to the noise variance \(\epsilon^2\), but it also depends on the station distribution through the correlation matrices. It may seem that (6) depends also on the anomaly field variance \(\sigma^2\), which is implicit in \(\gamma\) and in the correlation matrices. This would be unreasonable, since (6) intends to give the response of the interpolation scheme to the use of random noise on input. The explanation is that \(\sigma^2\) is in fact dividing all the terms of all matrices, and since \((\tilde{C}_{oo} + \gamma \tilde{I})\) has a \(-2\) power, the effect of sigma cancels out with the other two matrices. In other words, (6) does not actually depends on sigma.

The other terms on the rhs of (5) can be considered as the contribution that would be obtained if the interpolation scheme was applied to perfect observations, but with the same linear estimator derived from noise observations, since they contain the parameter \(\gamma\). This contribution can be obtained in a more easy way as the difference between total errors and the observational contribution, so that the diagonal entries would be given by:

\[
(\hat{\epsilon}_{f g,sam})^2 = (\hat{\epsilon}_{f g})^2 - (\hat{\epsilon}_{f g,obs})^2
\] (7)

For a given correlation model, this contribution mainly depends on the spatial distribution of observation points. In this sense it can be regarded as measuring the error associated with inadequate spatial sampling of all those spatial scales which influence the observations, and therefore it will be referred to as the ‘sampling contribution’. Since the observational contribution (6) depends on the station distribution (provided \(\epsilon\) is not equal to 0), and (7) depends on the noise-to-signal ratio \(\gamma\), the partition cannot be strictly considered as fully independent. Some more comments on the proposed partition are given in Gomis and Pedder (2005).
A first parameter to be determined is the field variance. This must be measured with respect to some kind of statistical mean field, so that the resulting anomalies at observation points have null statistical mean as prescribed by OI. Therefore, we computed the departures of observed profiles with respect to the Levitus 94 climatology. This implies that the anomalies cannot be ensured to have a null mean for our particular data set, but only for a longer, statistically significant data set, and this is precisely the requirement of OI. Moreover, we compared the results obtained from computing the anomalies in two different ways: from the Levitus climatology and from a low-order polynomial fitted to observations (as described in Thiébaux and Pedder, 1987). The comparison showed that the choice of the background fields is not crucial for the main results of this work.

The spatial distribution (averaged over the quoted 15 month period) of the anomaly field variance at 50 m is shown in Fig. 1. As expected, largest variances are observed in the Gulf Stream region, with maximum values of about $4.5(°C)^2$. However, within the domain D1, the variance is more homogenous, ranging between 2.0 and 2.5$(°C)^2$ over most of the domain (maximum values are between 3.0 and 3.5$(°C)^2$). When averaged over the domain but not in time, the anomaly field variance exhibits some seasonal variability (not shown).

As stated when presenting the formulation and despite that in principle the OI scheme allows to consider location-dependent variances, we averaged all profile departures in (the horizontal) space and time, in order to obtain a single mean value $\sigma^2$ at each level depth. Three levels were examined: at 50 m, the mean anomaly field variance $\sigma^2$ was estimated to be 2.62$(°C)^2$, decreasing to 1.75$(°C)^2$ at 200 m and to 0.55$(°C)^2$ at 1000 m (Table 1).

The correlation between temperature anomalies was assumed to be homogeneous and isotropic (i.e., depending only on distance). It is worth recalling that this does not imply that observed fields are homogeneous and isotropic; the assumption refers to their departures with respect to the statistical mean field (the Levitus climatology in our case). We assumed a gaussian model $C(r) = exp[-r^2/2L^2]$, the characteristic scale $L$ being set equal to 350 km. This was the value providing the best fit to observed lag correlations, which were computed by averaging all temperature anomaly pairs within 20 km distance lags.

A first reason to use a Gaussian function is that it fulfils all the assumptions required for the correlation between observations (e.g., a continuous derivative at zero distance). It is of course a simple model compared to other functions
also fulfilling the assumptions (see for instance Thiébaux and Pedder, 1987), but it usually provides a reasonable approximation for the scale range studied in this work (scales larger than 500 km). A second reason to use a Gaussian is that it can be analytically convoluted with a normal error filter (see Pedder, 1993).

In order to eliminate non resolvable scales, the lag correlation function was convoluted with a normal error filter $F$ with cut-off wavelength equal to 500 km. We fixed this cut-off as twice the mean separation distance between observations, which determines the smallest resolvable wavelength (the Nyquist wavelength). In this way, structures with typical radius of less than 125 km would be filtered out from the interpolated field. The fraction of variance retained by the interpolation can be obtained from the diagonal terms of the filtered correlation matrix ($\mathbf{\tilde{C}}_{gg}$ in the the previous section). In our case these are equal to 0.956, which leads to variances $(\sigma_f)^2$ of 2.50($^\circ C)^2$, 1.68($^\circ C)^2$ and 0.54($^\circ C)^2$ at 50, 200 and 1000 m, respectively. Therefore, the applied scale selection will in principle retain most of the variability of the field. However, these values are obtained under the assumption that actual fields really obey the prescribed correlation model. In practice, the variance retained after the interpolation could be lower/higher, since part of the observed variance might not actually be associated with the scale range implicit in the assumed correlation model, but with smaller/larger scales.

Finally, we assumed an initial value for the instrumental noise variance of $(0.01^\circ C)^2$. Although the temperature accuracy of APEX floats is claimed to be $0.002^\circ C$ for laboratory controlled conditions, errors are likely to be significantly larger at open sea, due to sensor ageing and bio-fouling. Hence, by assuming a value of $(0.01^\circ C)^2$ we are conservative in estimating the errors. The noise-to-signal parameter $\gamma$ then varies from about $4 \cdot 10^{-5}$ at upper levels (50 m) to about $2 \cdot 10^{-4}$ at deep levels (1000 m), due to the decrease of the anomaly field variance with depth.

The contrast between the assumed noise-to-signal values and those used by Guinehut et al. (2002) (between 0.1 and 10) may be surprising. The reason is that in many works the desired smoothing of the output field is not achieved by explicitly including a spatial filter $F$ in the interpolation process. Instead, the noise-to-signal parameter $\gamma$ is increased until the interpolation output shows a convenient degree of smoothing. Although this way also yields a spatially smoothed field, we have important reasons to prefer the smoothing strategy described in the previous section (some of them were already reported in Gomis et al., 2001, and Gomis and Pedder, 2005).

A first reason is that the desired scale selection is not strictly ensured by increasing the noise-to-signal parameter, since this is only suitable for suppressing purely white (spatially uncorrelated) noise. It must be admitted, however,
that treating small scales as spatially uncorrelated noise is not a particularly strong assumption compared with others made in this work. A second, more practical reason is that both the instrumental noise variance and the cut-off wavelength of the spatial filter can be set in an objective way, as the first depends only on the instrument accuracy and the second is constrained by the station separation. Instead, determining the variance associated with small scales (and use this measure to increase the parameter $\gamma$) is more cumbersome; it is not surprising, therefore, that $\gamma$ is very often increased in a rather subjective way. Last and perhaps most important regarding this work: if non resolvable scales were treated as spatially uncorrelated noise (an implicit assumption when increasing the value of $\gamma$), the distinction between instrumental and sampling errors would no longer be possible. The reason is that the two contributions would be mixed up in the noise-to-signal ratio $\gamma$.

In the results section, the primary computations will be obtained for Gyroscope floats only (75) and for all active floats (119). Next, the dependence of errors on the number of floats will be examined. To simulate future scenarios, additional floats will be randomly located within the domain under the constraint of being separated by at least 1.5° (in both longitude and latitude) from existing floats. It is worth noting that the errors provided by the OI formulation are statistical estimates; hence, they do not depend on the observed values themselves, but only on the location of the observations and on the instrumental noise. Therefore, we did not need to assign any T/S value to the simulated buoys, only a location and the same instrumental noise (and the same anomaly field variance) assumed for the actual buoys. Also scenarios with subsets of the presently active floats will be considered in the next section.

Regarding the error values reported in the following, two points must be noted. The first one is that since ocean boundaries are badly sampled, spatial mean values will not be obtained averaging the error field over the whole domain $D_2$, but over an inner domain ($D_1$) (see Fig. 1). The second one is that results will be mainly expressed as relative errors, i.e., as rms analysis errors divided by the standard deviation of the (filtered) anomaly field.

3 Results

Figure 2 shows the relative error field at 50 m corresponding to all active floats, split into observational and sampling contributions. The values are smaller for the first than for the second, but the two patterns are rather similar within the sampled domain. Maximum observational errors are about 11% (i.e., temperature errors of $0.175^{\circ}C$) in an isolated poorly sampled region in the middle of the domain. However, the mean observational error (averaged
over domain D1, see Table 1) is only 3% (or 0.048°C). Sampling errors are more
dependent on the station distribution, and hence they are larger in data voids
(e.g., 16% or 0.254°C in the middle of the domain) and near the boundaries.
The mean sampling error (see Table 1) is about 6% (or 0.095°C).

It is worth noting that far away from data points, the influence of instrumen-
tal error decreases, indicating that random errors inherent to observations can
not propagate much further than the correlation scale length. Instead, sam-
pling relative errors (and therefore total errors) approach 100% outside the
sampled domain, due to the absence of observations. Averaging total (observ-
utional plus sampling) errors over the inner domain D1 gives a value of about
7%. [Note that the partition of total errors into observational and sampling
contributions is formulated in terms of error variances (formula (7) in section
2.1), i.e., in terms of the squares of the rms values quoted in Table 1. ] This
is, the spatial mean of statistical errors associated with the recovery of the
temperature field from the March 2003 float distribution is about 0.111°C
at 50 m. This value is about 11 times larger than the accuracy assumed for
observations (0.010°C).

At deeper levels (200 and 1000 m) the error patterns obtained for the present
distribution of all active floats (not shown) are similar to that obtained at 50
m (Fig. 2). The magnitudes of relative errors are slightly larger (see Table 1),
although absolute temperature errors are smaller: 8% or 0.104°C at 200 m and
10% or 0.073°C at 1000 m. There are two reasons explaining why relative errors
are larger. First, some floats do not report data at those levels (especially at
1000 m), so that the effective number of observations is slightly smaller and,
consequently, sampling errors become slightly larger. The second reason is
that the smaller field variance at deep levels translates into a larger value
for the noise-to-signal parameter \( \gamma \), which mainly increases the observational
contribution.

When increasing the number of data points with fictitious floats, sampling er-
rors reduce quite significantly, whereas observational errors reduce more moder-
ately. At upper levels (50 m), both contributions become of the same order
(1% ) for about 233 floats (see Table 1). In such a scenario, total errors would
be less than 2%, namely of the order of 0.026°C. At 200 m and 1000 m both
contributions become equal for a smaller number of floats (about 195) due to
the larger magnitude of observational errors relative to sampling errors.

Figure 3 summarizes the dependence of the two error contributions as well as
of total errors on the number of floats. A first point to be recalled is that some
of the presently active floats (119) do not report data at deep levels, so that
the error values could actually have been plotted as corresponding to a smaller
number of observations. Instead, added simulated floats have been assumed to
report data at all levels. This is the reason for the apparent sudden change in
the slope of the deep level plots around the number of presently active floats.

Figure 3 confirms that observational errors decrease smoothly when increasing the number of floats. On the other hand, sampling errors decrease rapidly at the beginning, e.g. from almost 40% to less than 20% when the number of floats increases from 20 to 40 units (not shown). For the 75 Gyroscope floats, observational errors are about 6% and sampling errors about 10%. From 100 floats onwards, sampling errors continue decreasing but more smoothly, until intersecting with observational errors. This occurs for about 200-250 floats at 50 m and for less than 200 floats at deeper levels. In such a scenario, total errors are of the order of 1.5% at 50 m, 1.75% at 200 m and about 3% at 1000 m. From then on, a large increase in the number of floats is required to obtain smaller errors. In this sense, the reported values can be regarded as the best feasible accuracy for the recovered temperature field. In absolute terms they correspond to about 0.024°C at 50 m, 0.023°C at 200 m and 0.022°C at 1000 m, i.e., which is slightly more than twice the assumed instrumental accuracy.

4 Discussion and conclusions

A first point for the practical significance of this work is the determination of statistical errors associated with the recovery of a given spectral range of true fields. Applied to the temperature field in the North-East Atlantic ocean, we have found that for the present profiling float array, scales larger than 500 km can be recovered with a relative uncertainty (rms error relative to the standard deviation of the anomaly field) of about 7% at 50 m, 8% at 200 m and 10% at 1000 m. This corresponds to mean absolute errors of 0.111°C, 0.104°C and 0.073°C, respectively (all values obtained excluding the boundaries, which are poorly sampled by the floats).

A second, key result has been the splitting of total errors into instrumental and sampling contributions. This has revealed that, in the present scenario, errors are more due to the small number of floats than to instrumental errors, especially at upper levels. Consequently, efforts should be devoted to the deployment of more floats rather than to the improvement of their sensors from the point of view of temperature mapping. For scales larger than 500 km this will hold true until 200-250 floats are deployed (less than 200 for deep levels). In such a simulated scenario, the number of observations and the technology would become approximately equally limiting factors for the accuracy of the temperature field mapping. Total errors have been estimated in less than 2% (at 50 m), which is comparable to the results at 20 m obtained by Guinehut et al. (2002) for a 3° x 3° array of profiling floats (about the mean separation distance of presently active floats).
Regarding the reliability of the presented results, some considerations need to be made. The first relates to the use of a constant variance. The impact of this assumption is that in a particular region where the anomaly field variance is significantly higher than the assumed mean value, the actual noise-to-signal fraction of observations will be lower than the value assumed for $\gamma$. Consequently, the computed relative errors will overestimate actual relative errors, since the effect of $\gamma$ is to increase the errors. The opposite holds for regions where the anomaly field variance is significantly lower than the assumed mean value. However, this assumption is not critical for the nucleus of the work, since it does not significantly affect the mean values given in Table 1 and used to construct Fig. 3.

Another consideration comes from the fact that errors depend not only on the number of floats, but also on their distribution. Therefore, for each number of floats, computations were repeated for different float distributions. Results showed that despite there were obvious differences in the local error distributions, all spatial mean errors were very similar to those reported above.

Most important, when a simple correlation model is assumed, as in our case, the obtained errors usually underestimate actual errors (Daley, 1991). The reason is that the OI formulation provides the ‘best’ linear estimator (i.e., the one producing the smallest statistical errors) provided the field actually fulfils the assumed statistical properties (in particular the covariance model). In practice, that is not strictly the case, and therefore the OI interpolation is expected to yield larger analysis errors than the theoretical estimations provided by the theory. On the other hand, the spatial structure of the error field is usually less sensitive to the actual non-optimality of the OI scheme. And in any case, statistically significant location-dependent correlations can hardly be obtained at present from in situ data alone.

Under the assumption of a simple correlation model and for a given scale selection, the most critical input parameter is the noise-to-signal fraction $\gamma$. Hence, we also recomputed all errors assuming a higher instrumental noise variance: we took a value of $(0.025^\circ C)^2$, which can be considered as an upper boundary for the accuracy of APEX floats and close to the estimated accuracy of other type of floats (PROVOR). Figure 4 shows that both observational and sampling errors are larger than those of Fig. 3 (e.g., from 6% to 8% for observational errors obtained considering only the Gyroscope floats at 50 m). It is worth noting, however, that despite the asymptotic values of both contributions are larger (about twice the values of Fig. 3 at 50 and 200 m) the intersection between them occurs at about the same number of floats. Hence, the number of floats at which instrumental errors become an equally limiting factor than sampling errors seems quite robust.

As stated along this work, all results refer to the recovery of scales larger than
about 500 km. If smaller scales were to be recovered, total errors would be larger. The increase would be mainly due to the sampling contribution, since the aspect ratio between observation separation and scales to be recovered has a much stronger impact on sampling errors than on observational errors. Instead, errors will be smaller if the targeted scales are larger. Results for any prescribed scale selection can be easily obtained using the formulation detailed in section 2.1. It will be as simple as changing the cut-off wavelength of the filtering operator $F$.

Finally, we have to state that float data are rapidly increasing, to the point that they have already become the most important source of in situ data. In this context, this work has intended to provide some guidance for future deployment strategies. But moreover, the information provided by the float array can be combined with remote sensing data, e.g. with SST data for the temperature field and with satellite altimetry for dynamic height (Guinehut et al., 2004; Willis et al., 2003). In such cases, the instrumental noise could no longer be considered constant (a different value for each type of data should be considered) and neither spatially uncorrelated (for satellite data). On the positive side, satellite data sets could allow to derive more realistic covariance matrices. Despite the relatively large noise variance of satellite data, the significant increment in the number of observations usually yields smaller total error values. Hence, the combination of float and remote sensing data can be envisaged as a promising way of attempting the recovery of surface fields at the mesoscale.

**Acknowledgments**

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References


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Figure captions

Figure 1. Spatial distribution of the temperature field variance at 50 m with respect to the Levitus’ 94 climatology (units are $(^\circ C)^2$). The location of all active floats in the North-East Atlantic Ocean during March 2003 has been overplotted. Solid line and dashed line enclose the inner domain D1 and total domain D2 respectively.

Figure 2. Distribution of observational (a) and sampling (b) relative errors (their standard deviation divided by the standard deviation of the (filtered) anomaly field). They correspond to the recovery of the temperature field at 50 m from all active floats during March 2003. The accuracy of observations has been assumed to be 0.01°C.

Figure 3. Mean observational (♦) and sampling (∗) relative errors against the number of profiling floats at a) 50 m, b) 200 m and c) 1000m. The accuracy of observations has been assumed to be 0.01°C. Solid line corresponds to the actual float set and a few subsets, whereas the dashed line denotes the inclusion of additional, simulated floats.

Figure 4. As in figure 3, but assuming an instrument accuracy of 0.025°C.
Table 1
Observational, sampling and total rms errors involved in the recovery of the temperature field in March 2003. They are expressed relative to the standard deviation of the field associated with scales larger than 500 km (specified for each level).

<table>
<thead>
<tr>
<th></th>
<th>Mean error</th>
<th>Mean error</th>
<th>Mean error</th>
</tr>
</thead>
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<tr>
<td>obs.</td>
<td></td>
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<td></td>
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<tr>
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<td></td>
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<tr>
<td>total</td>
<td></td>
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<td></td>
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<td>0.11</td>
<td>0.13</td>
</tr>
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<td>Active plus simulated floats$^b$</td>
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<td>0.02</td>
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<tr>
<td>Active plus simulated floats</td>
<td>0.02</td>
<td>0.02</td>
<td>0.03</td>
</tr>
</tbody>
</table>

$^a$ All T/S active floats operating in the North-East Atlantic Ocean (119)

$^b$ All T/S active floats operating in the North-East Atlantic Ocean (119) plus 124 simulated floats located randomly but separated by at least 1.5° lat,lon from existing floats.
All active floats in the NE Atlantic Ocean

Fig. 3.
Gyroscope floats

All active floats in the NE Atlantic Ocean

Fig. 4.