


These three books by the young Italian logician Nicola Grana (born in 1949) disclose an energetic, restless searching mind, who tirelessly explores new and new fields in mathematical logic and the philosophy of logic, trying out sundry approaches, never stopping by already attained results or consecrated views, evincing a great extent of self-criticism and awareness to any shortcomings that might turn out to flaw previously cherished positions. The author's philosophical outlook takes as its starting point a broadly intuitionistic view of logic, but its thrust is towards a critical complementarism or perspectivism, according to which no system can by itself exhaust the set of discoverable logical truths, and hence no system can become an all-encompassing subsystem, but yet every logical system is somehow or other right on its own, its truth being always limited and grounded on some side of reality (i.e., of how things in themselves are). The human mind cannot straightforwardly get into reality as such and as a whole: it can only get in touch with reality in an indirect and mediated manner, reflecting some side or other of reality and even that only insomuch and in such a way as that side appears to a certain human viewpoint fraught with some theoretic interests and aims. Such a complementarism is a critical one because, far from being content with a soothing recognition of all systems, each of them keeping its own ground, Grana's approach regards every system as unsuccessful even as regards what might seem the modest goal of reflecting but some side of reality relatively to some subjective all happens as if, according to Grana, such an objective both was unattainable and yet betrayed or surpassed itself, leading to more ambitious goals, such as broadening the side of reality we aim at reflecting (or, as Grana puts it, reconstituting), or stripping out views of reality of subjective slants and biases, thus banning all arbitrariness or fancy. (I'm not sure I've portrayed Grana's epistemological views quite correctly. I'm afraid my account thereof is too realistic—being contaminated with my own realistic learning. Perhaps what Grana means when speaking about the need to ban arbitrariness or fancy is just that there are ideal patterns of rational thought which our systems ought to tend to comply with, should they never actually reach that goal. As for the question what such pattern's existence is and whether it is constructed by us, too, Grana is likely to answer that such issues fall outside the scope of a constructive philosophizing about logic and only arise on the ground of an "ontologistics" approach hankering after an ultimate transcendent explanation).

Grana is doubtless aware that such a view of logic (what I'd call "logical criticism"?!) is liable to turn out (over)inconsistent. Still, he apparently thinks that it would be idle and worthless to try to set up a coherent metalogical system apt to live up to its own requirements and thus to become unassailable and definitive. Whatever the purely formal interest of such a construction, it would prove barren and unfruitful for a further development of logic (and philosophy of logic). Accordingly, we had better stick to a fertile if nonformalizable view of logic.
Sentieri della logica is a collection of essays written between 1977 and 1980 on topics such as modality and intuitionism, information logic, Quantum logic, many-valued logic, logic of causality statements. Through the book the author's intuitionistic trends show up: Proofs and derivations are regarded as construction processes, behind or below which no transcendent truth lies. Mathematical existence is thus viewed as mind-dependent, all Russellian "ontology" being waived. As understood by Grana, Heyting's intuitionism's gist consists in putting forward a calculus of problems as against a calculus of Independently existent states of affairs. Such a view can fruitfully interact with Lewis's modal insights, which, according to Grana, are directed toward coherently sorting out thinkable states of affairs from the rest, so that what thus emerges can be viewed as a set of patterns for constructibility. A modalized intuitionistic system is proposed, by adding to Heyting's axioms and primitive inference-rules two modal axioms (\(\neg\neg p \rightarrow p\) and \(p \rightarrow p\neg\neg p\)) and two inference-rules (from \(\neg p \rightarrow q\) to infer \(\neg q\), necq; and from \(\neg p\) to infer \(\neg p\rightarrow q\); even if the author doesn't say so, I take it that in the former rule "\(\neg p\)" is a thoroughly modalized sentence and so is "\(q\)" in the latter rule). The two modal symbols (\(p\neg\neg\) and \(\neg\neg\)) are taken as primitive. Such a system is, according to Grana, an intentional transparent system in the sense of Pavel Tichý.

Going into the relationship between many-valued and modal logics, Grana sets forth one of his main philosophical theses (p. 56):

Consié che anche nelle analisi più astratte della logica si annidano motivi extra logici ed esigenze diverse, ... non abbiamo postato di nuova logica né di logica, ma piú propriamente di sistemi e di dimensioni della logica che hanno la loro unità ideale, ... nell'intenzione costruttiva e nell'esigenza di strutture del pensiero umano.

Ad later on (on p. 66) he says: La pluralità dei sistemi non significa opposizione di essi e di conseguenza un dover scegliere l'uno o l'altro, ma diversità, che diversità non deve risultare in correlazione, far form it.

When reviewing systems of quantum logic, Grana favours Reichenbach's, disliking nondistributive approaches, such as Birkhoff and von Neumann's. That seems to the present writer an unfortunate and gratuitous choice, since nondistributive approaches, even if wrong (as I'm sure they are), can be backed up with cogent arguments (cogent, that is, within the framework of presuppositions not easily disposed of). Moreover, such a cut rejection misbecomes someone of Grana's perspectivistic leanings, since nondistributive approaches can be (and have been) framed to capture a complementaristic perspectivism (as was clearly shown by Apeastel in Matière et forme). What is the status of quantum logic, for Grana? Both Reichenbach's system and classical logic have their own field of applicability, doesn't prevent each system from being partially translated into the other. (Since the possibility of a mutual partial translation is thus laid down, I wish Grana had outlined what he means by 'partial translation' and how such translations are to be devised in the case under consideration).

In Filosofia della logica Grana develops his ideas, dispelling some of the doubts or difficulties they naturally give rise to.

Even if it's more systematic than the previous publication, this book claims to be, not a treatise, but a set of remarks pivoting upon some basic ideas. Human understanding is creative, its products
being structures far from being tied to some immutable unique fabric, as Kant would have it, human thought is manifold in its structures as well as in its fields of application. It starts from intuition and returns thereto. Formalization itself being a product of intuition. But what is intuition? Grana says what it is not; it is neither a mystical look at the world, nor an intellectual feeling as Bergson supposed, nor a fundamental a priori element of human thinking activity residing in time-flowing (Brouwer’s view); it’s nothing else but a sheer beginning of thinking when constructing many dynamic structures, which are then embodied in sundry formal, rigorous discourses. A lot of Grana’s remarks, which make up most of the book, are attempts to make such ideas clearer as well as more cogent and precise, providing many examples and showing a number of applications. Thus, e.g., the issue of the existence of sets is settled by pointing out two different senses of existence: once the mind-dependence of mathematical objects is acknowledged, the problem vanishes. What to the present reviewer’s mind, makes such a solution most unsatisfactory is that any problem whatever may thus be solved or rather shunned. Furthermore, those purportedly different senses of existence ought to be explained and elucidated.

Most of the book is put forward but tentatively, as is hinted at in the chapters’ titles (‘A path still to be travelled along’, ‘A working hypothesis’, and so on). A number of different approaches and systems defile in turn, their respective merits being assessed and correlated. Grana favours such approaches as, being nonstandard, are most apt to disclose some hidden sides of human reasoning (e.g., Keams’ semantics for modal logic without possible-worlds, von Wright’s view of change as contradictory in ‘Time, Change, and Contradiction’, Bonomi’s three-valued temporal logic etc.). What to myself seems most interesting is that already in this book Grana duly gauges von Wright’s arguments for the contradictoryness of change, thus evincing a starting inclination towards paraconsistency, which is strengthened by a sympathetic account of a paper by da Costa and Wolff on the dialectical unity of opposites. Such insights are put in correlation with Croce’s dialectical ideas, with Spisard’s productive logic and with Angel’s and McColl’s connexivist systems. I confess that sometimes I feel dizzy on reading those pages—too much sand—yearning for some firm ground or achievement, for some effective synthesis. Grana displays a great ingenuity in canvassing so many different systems, but he hasn’t managed to find any clear integration or even convergence. What alone emerges is a multifarious swarming of approaches loosely connected among themselves.

We thus come to Logica paraconsistente, by far the best of the three books. It is the first book to have been published on this topic, which seems the more striking since unlike people like Newton da Costa, or the present reviewer, or Richard Routley, the author is not yet a long-practiced researcher on this field or even one whose scientific activity has been so far devoted to paraconsistent insights or systems. This little book is frankly worth reading. It critically scans several paraconsistent systems, such as Asenjo’s, Baten’s, da Costa’s systems Cω, da Costa & Arruda’s relevant systems P and the present reviewer’s system A of transitive logic.

The study of da Costa and Arruda’s set theory systems NF∗∗, based on first-order calculus Cω, seems too me a little hasty, as it takes no account of the many revisions Arruda was compelled to make them
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undergo without ever reaching any satisfactory solution. I also feel bound to take issue with Grana's assessment of the relationship between set-theory and ontology:

Se presuponemos la ontología, emergeremos de nuestro viaje de investigación. El núcleo de las críticas violentas a sistemas paraconsistentes ha sido en su raíz la presuposición dogmática de la ontología, mientras que debiera ser el punto de arranque y no de partida (p. 39)

And later on:

non si può fare ontologia se non si chiuisce prima la logica dell'universo linguistico.

And he adds that such a position results 'da una deduzione su dati rigoramente incontrovertibili'. The present writer believes in no incontrovertible ascertaining, but, be it as it may, what strikes me as most unwarranted in these remarks is the thesis of a priority of logic (or set-theory) over ontological reflections, while the manifold development of contemporary set-theory (classical and nonclassical) shows that preference of one particular formal system of set-theory is to be determined, or brought about, by certain ontological (metaphysical) considerations - which by no means rules out a further refurbishing of such considerations owing to new insights gained off symbolisation and the formal calculations it gives rise to. What is more, one of the grounds for choosing paracconsistent approaches to set-theory is that they (hopefully) allow us to cleave to "intuitive" ontological views which turned out to be impossible coherently to maintain within classical set-theories (e.g., the existence of a universal class, without which 'exists' is bound to be a nonunivocal predicate). In fact, choosing among competing paraconsistent systems is to be done on some ground or other, and whatever the ground(s) may be, philosophical (and indeed metaphysical) considerations ought to be ingredients thereof, to say the least. Should Grana retort that no choice is necessary, every such system keeping its own application field, I'd reply that, besides other serious difficulties besetting it, that compatibilistic and localistic view is flawed by the very fact that even for some particular field there is contest among differing approaches and systems; adding then a further relativization, such as the one expressed by "for such and such purposes" would trigger an infinite regress.

The most interesting chapter in the book is the one devoted to proposing Grana's own original system IDL of intuitionistic paracconsistent logic. His philosophical motivations for building such a system ought by now to be clear to the reader. Unfortunately, though, system IDL is negationally saturated, i.e., for every \( \neg p \), \( \neg \neg p \) is a theorem thereof. IDL is built like this: add to Johansen's axiom-schemata and inference-rules (i.e., to the result of exscising the axiom-schema \( \neg p \vdash p \neg p \neg p \) from Heyting's sentential calculus) the following axiom-schemata:

\[
\neg (p \lor q), \neg p, \neg \neg p, \neg q \vdash \neg (p \lor q), \neg (p \lor q)
\]

(I use my own notation which should be clear: '1' is conditional, while '1' is disjunction; conventions concerning parentheses and (strengthening) dots are a la Church). I think that on p. 93, when setting forth IDL's primitive inference-rules, Grana has inadvertently skipped adjuction \( p, q \vdash p \neg q \), since it is included in Heyting's system's standard presentation and is doubtless needed for proving theorems, and Grana says can be drawn from his axioms. The proof that IDL is negationally saturated
is like this, using just the six axiom-schemata (1) through (6) (viz., res.

\[ p \land (q \land r) \Rightarrow (p \land q) \land r \]

and MP we prove \( p \land (q \land r) \Rightarrow p \land r \), whence applying axiom-
schema (11) (viz., \( p \land r \Rightarrow q \land r \Rightarrow q \land p \)) we prove \( p \land q \Rightarrow q \land p \) (having
previously derived the rule of transitivity of the conditional, RTC
for short, viz., \( p \land r \Rightarrow q \land r \Rightarrow q \land p \), which is through MP derived from axiom-
schema (4), i.e., \( p \land q \Rightarrow q \land p \), applying adjunction-rule). Finally, we
take axiom-schema (10) (viz.,

\[ p \Rightarrow (q \Rightarrow p) \land (q \Rightarrow p) \]

thus proving \( p \Rightarrow q \land p \), whence, owing to the theorem \( p \Rightarrow q \Rightarrow q \land p \), and
MP, we prove the theorem-schema \( q \Rightarrow p \). Notice that adjunction be
dropped, either RTC would still remain derivable otherwise— which I’ve
been unable to find out— or else the whole proof of \( q \Rightarrow p \) would all
the same carry through (without, from axiom-schema (10) and (12)
alone one immediately proves \( q \Rightarrow p \Rightarrow q \Rightarrow p \), as shown above; thus either
IDL lacks simplification as a derived inference-rule or else \( q \Rightarrow p \) is
an axiom-schema after all, even if adjunction and RTC cannot be der-
ived.)

From \( q \Rightarrow p \) we of course get \( q \Rightarrow p \) and then, applying axiom-
schema (7) and (8) \( (p \land q \Rightarrow r \land q) \) and \( (p \land q \Rightarrow r) \), we get \( p \Rightarrow q \land p \), i.e.,

excluded middle. And, as goes without saying (due to axiom-schema
(5)), we also prove \( q \Rightarrow p \land q \Rightarrow p \), owing to transitivity of the
conditional. Noncontradiction comes out as a particular case: \( q \Rightarrow q \land p \Rightarrow q \)

therefore \( \neg (q \land p \Rightarrow q) \), while, as we’ve seen, \( q \Rightarrow p \Rightarrow q \) is a theo-
rem-schema (and would even remain so should adjunction be junked
and should thus RTC become unavailable).

Does such a result spell disaster? No. We’re thus faced with
a Gorgian system, a system, that is, capturing Gorgias’ thesis that all
is false: which thesis doesn’t say or imply that all is true, since double
negation (\( \neg \neg p \Rightarrow p \)) is not available in the system. This result is simi-
lar to the fate befalling any other negationally incoherent extension
of Johansson’s minimal logic. Nevertheless, whoever wants to embrace
such a system should be prepared forcefully to argue for a Gorgian
metaphysics (Gorgias’ own arguments are not so easily disposed of after
all but, if the Gorgian logician is ready to accept them, he needs to
tell us why; if he isn’t, he needs to find other arguments). Should you
reject such an alethic metaphysical nihilism, you cannot hold on to IDL.
To myself, this shows that intuitionism and paraconsistency are very
bad bedfellows.

The final chapter is devoted to my own systems of transitive
logic. It wouldn’t seem decent for me to take here advantage to com-
ment therein, but I cannot forbear saying that Grgas seems to have
grasped the core ideas underlying those systems. (I only regret that
technical details have been missed out, as a result of which the outline
becomes a little sketchy or allusive, even cryptic.)

I warmly recommend Grgas’s three books to anyone interest-
ed in nonclassical logics and philosophy of logic. His is a philosophically
winsome way of looking at logic and bringing problems out, which is
all the fairer for me to emphasize since— as the reader has been able
to make out—the points where I disagree with Grgas are many.

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