Numerical simulations of submesoscale balanced vertical velocity forcing unsteady nutrient-phytoplankton-zooplankton distributions

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[1] The effect of submesoscale balanced (void of waves) vertical velocity on initially steady nutrient-phytoplankton-zooplankton (NPZ) distributions is investigated numerically. First, steady vertical NPZ profiles, continuous but not continuously differentiable at several depths, are found as analytical solutions to the NPZ equations in the absence of flow. These solutions admit numerical equivalents that are continuously differentiable in the numerical sense, here meaning convergence of the vertical derivative with respect to increasing vertical resolution. These NPZ solutions require, however, large vertical resolutions, with a grid size of few centimeters, to be properly discretized.

The ecological model is next coupled to a nonhydrostatic Boussinesq $f$ plane physical model that explicitly conserves potential vorticity on isopycnals. Then the NPZ solutions are used as steady initial ecosystem conditions to investigate the role of submesoscale balanced vertical velocity in forcing NPZ anomalies in an idealized case of a baroclinic unstable jet. The results show that the baroclinic flow rapidly develops balanced vertical velocity that in turn favors NPZ anomalies. A large cancelation between the local change and the horizontal advection occurs in the ecosystem variables. This particularly occurs for zooplankton anomalies that therefore behave as better passive tracers of the horizontal flow than phytoplankton anomalies. However, once phytoplankton and zooplankton anomalies develop locally, forced by vertical velocity, they are horizontally advected away from the upwelling or downwelling regions so that spatial distributions of vertical velocity and ecological fields become eventually uncorrelated.


1. Introduction

[2] The dynamics of oceanic planktonic ecosystems is often investigated using nutrient-phytoplankton-zooplankton (NPZ)-type numerical models [Wroblewski, 1977; Franks et al., 1986; Franks, 2002; Newberger et al., 2003]. These models are discrete versions of the continuous partial differential equations for the field variables NPZ. In the absence of flow and horizontal gradients this system of equations has a number of one-dimensional (1D) steady analytical solutions which are very useful both to characterize different planktonic regimes and to serve as initial conditions in coupled physical-ecosystem numerical modeling. It may happen, however, that some of these 1D steady solutions be only continuous, but not continuously differentiable functions of the vertical coordinate $z$ along the water column. This potential lack of differentiability makes these NPZ solutions inappropriate as initial conditions in three-dimensional (3D) coupled physical-ecosystem models. These 3D coupled models, if formulated as it is usual in the spatial description, require existence of vertical derivatives for the vertical advective terms, present in the material rate of change of the ecosystem quantities, make sense.

[3] In this paper we present a particular example of a motionless, 1D steady NPZ solutions which, being continuous but not continuously differentiable at several depths, do however admit numerical equivalents that are continuously differentiable in the numerical sense (here meaning convergence of the vertical derivative with respect to increasing vertical resolution). In the next section the NPZ system of equations is briefly introduced. Though more sophisticated ecological models exist, a simple one is used here because we seek to keep the number of free parameters as small as possible while retaining the essential behavior of the ecological fields, particularly the development of NPZ anomalies due to the vertical advection of nutrients into the euphotic zone. The NPZ-type models are ecological bulk algorithms subjected to significant errors in the mathematical parametrization of the different ecological processes [see Anderson, 2005; Flynn, 2005; Mitra et al., 2007].

[4] The motionless 1D continuous steady ecological solutions and their numerical continuously differentiable equivalents are obtained in section 3. Convergence of this differentiable solution is reached however at vertical reso-
lutions of a few centimeters. As an application example, in section 4 the differentiable steady NPZ solutions are used as initial conditions in a 3D coupled physical-ecosystem model (with poor vertical resolution) to address the role of the vertical velocity in a case of oceanic baroclinic instability. Concluding remarks are given in section 5.

2. NPZ Model

The dependent variables of the NPZ model are the dissolved inorganic nitrogen (N), the phytoplankton (P), and the zooplankton (Z) biomass. These variables are expressed in units of concentration of nitrogen (here always in mmol N m$^{-3}$), and satisfy the system of equations [Wroblewski, 1977; Newberner et al., 2003]

\[
\frac{dP}{dt} = \frac{N}{K_0 + N} \left( GP - R_0 (1 - e^{-\lambda_0 P}) Z - \frac{\Xi_0 P}{P_m} \right),
\]

\[
\frac{dZ}{dt} = (1 - \gamma_0) R - \frac{\Gamma_0 Z}{Z_0},
\]

\[
\frac{dN}{dt} = -U + \gamma_0 R + Z_{st} + P_{sf}.
\]

Above, the material rate of change $d\chi/dt \equiv \partial \chi/\partial t + \mathbf{u} \cdot \nabla \chi$ is the sum of the local and advective rates of change of $\chi$, $\mathbf{u} = (u, v, w)$ is the 3D velocity, and $\nabla$ is the 3D gradient operator. The $P$ increases because of the uptake rate ($U$) of dissolved inorganic nitrogen by phytoplankton and decreases because of the herbivore grazing ($R$) and the phytoplankton mortality rate ($P_{sf}$). Constant $K_0$ is the half-saturation concentration for phytoplankton uptake of nutrients. The $Z$ increases because of the ingestion of phytoplankton (g) and decreases because of the zooplankton specific excretion and mortality rate ($Z_{st}$).

The uptake $U$ follows the Michaelis-Menten kinetics and depends on $G$, the maximum uptake rate of nutrients by $P$ [Newberner et al., 2003],

\[
G(x,t) \equiv \frac{V_0 \alpha_0 I(x,t)}{\sqrt{\frac{V_0}{2} + \alpha_0^2 I^2(x,t)}},
\]

which depends on the photosynthetically available radiation

\[
I(x,t) \equiv I_0 \exp\{k_w z + k_p \kappa(x,t)\},
\]

\[
\kappa(x,t) \equiv -\int_z^0 P(x,y,z,t) d^2y,
\]

where $z \leq 0$.

Above, $V_0$ is the phytoplankton maximum uptake rate, $\alpha_0$ is the initial slope of the $P - I$ curve, $k_w$ is the extinction coefficient of seawater in the absence of phytoplankton (following the Lambert-Beer law and light variation with day time is not considered), $I_0 \exp\{k_p \kappa(x,t)\}$ is the light attenuation by phytoplankton self-shading, where constant $I_0$ is the surface photosynthetically available radiation, and $k_p$ is the extinction coefficient per unit concentration of phytoplankton.

Adding (1) + (2) + (3) the total nitrogen $N_0 \equiv N + P + Z$ is materially conserved,

\[
\frac{dN_0}{dt} = 0.
\]

It is convenient therefore to define the system of independent equations as (1)−(2)−(6), instead of the original set (1)−(2)−(3). In the next section two particular solutions of these equations are obtained in the case of steady and horizontally homogeneous ecosystem distributions in the absence of flow.

3. Steady Solutions

3.1. Analytical Steady Solutions

In order to obtain analytical steady solutions of horizontally homogeneous distributions we set $dP/dt = dZ/dt = 0$ in (1) and (2), so that spatial functions depend only on $z$. The water column is divided into three layers. In the upper layer $P \neq 0$ and $Z \neq 0$, in the mid layer $P \neq 0$ and $Z = 0$, and in the lower layer $P = Z = 0$.

In the upper layer ($z \in [z_1, 0]$) the steady $P$, obtained directly from (2) [e.g., Busenberg et al., 1990, equation 8; Newberner et al., 2003, equation 18], is independent of $z$

\[
P(z) = P_1 = -\frac{1}{\Lambda_0} \ln \left[ \frac{\Xi_0}{R_0 (1 - \gamma_0)} \right], \quad z \in [z_1, 0].
\]

With the commonly used parameters for upwelling conditions given in Table 1, $P_1 = 8.468$ mmol N m$^{-3}$. The values and a sensitivity analysis of these parameters are given by Newberner et al. [2003]. The maximum depth $z_1$ at which this solution is feasible is determined below. From (1) and (7) the zooplankton $Z(z) = Z_1(z)$ in the upper layer $z \in [z_1, 0]$ is obtained solving the quadratic equation

\[
A_0 Z_1^2 + BZ_1 + C = 0,
\]

where the coefficients

\[
A_0 \equiv \frac{\Gamma_0}{(1 - \gamma_0) P_1},
\]

\[
B(z) \equiv -G(z) - A_0 (K_0 + N_0 - P_1) + \Xi_0,
\]

\[
C(z) \equiv [G(z) - \Xi_0] (N_0 - P_1) - \Xi_0 K_0.
\]

Thus,

\[
Z_1(z) = \frac{-B(z) - \sqrt{B^2(z) - 4A_0 C(z)}}{2A_0}.
\]

The negative root solution above ensures that $Z(z) < N_0$. To further simplify the problem we consider $N_0$ as constant,
independent of \( z \). \( N_0 \) must be such that the discriminant \( B^2(z) - 4A_0C(z) \geq 0 \).

[13] The maximum depth \( z_1 \) of the upper layer is defined as the shallower depth at which \( Z(z_1) = 0 \). Using (1) or (8) this condition implies \( C(z_1) = 0 \), and therefore

\[
G_1 \equiv G(z_1) = \Xi_0 \left( 1 + \frac{K_0}{N_0 - P_1} \right).
\]

Inverting \( G(z_1) \) using (4) the depth \( z_1 \) defining the lower boundary of the upper layer is

\[
z_1 = \frac{1}{2(k_wP_1 + k_w)} \ln \left( \frac{V_0^2G^2_1}{(P_1^2 - G_1)^2C_0^2P_1^2} \right).
\]

In the mid layer, \( z \in (z_2, z_1) \), \( Z(z) = Z_2(z) = 0 \) and \( P(z) = P_2(z) \neq 0 \). In this layer, because of (3) or (1), \( P_2 \) satisfies the relation

\[
P_2(z) = N_0 - \frac{\Xi_0K_0}{G(z) - \Xi_0},
\]

where \( G(z) \) is given by (4) and (5). The solution \( P_2(z) \) is found here solving (15) numerically. This solution is feasible as long as \( P_2(z) \leq N_0 \) which, from (15), implies that \( G(z) > \Xi_0 \). Consequently, the maximum depth \( z_2 \) of this mid layer is defined as that at which \( G(z_2) = \Xi_0 \). Inversion of this equation implies that \( z_2 \) satisfies the relation

\[
z_2 = \frac{k_w}{k_{w}} \int_{z_2}^{z_1} P_2(z') \, dz' = \frac{1}{k_w} \ln \left( \frac{V_0 \Xi_0}{\alpha_0 I_0 \sqrt{V_0^2 - \Xi_0^2}} \right) - \frac{k_w}{k_{w}} P_1 z_1.
\]

For its later use it is convenient to define the maximum depth \( z_3 \) of this mid layer as that obtained by neglecting the phytoplankton self-shading only in this mid layer,

\[
z_3 = \frac{1}{k_w} \ln \left( \frac{V_0 \Xi_0}{\alpha_0 I_0 \sqrt{V_0^2 - \Xi_0^2}} \right) + \frac{k_w}{k_{w}} P_1 z_1.
\]

Clearly, \( z_3 < z_2 \). Finally, in the lower layer, \( z \in [z_{min}, z_2] \), there is neither \( P \) nor \( Z (P_3 = Z_3 = 0) \), so that the dissolved inorganic nitrogen \( N(z) = N_0 \). The choice of \( N_0 \) is particularly important in this NPZ solution since it must be such that the resulting distance between \( z_1 \) and \( z_3 \) be large enough to be properly discretized using a finite grid size.

On the basis of the behavior of \( z_1 \) and \( z_3 \) as functions of \( N_0 \) (Figure 1), \( N_0 \) must be close to \( P_1 = 8.468 \, \text{mmolN m}^{-3} \). We select \( N_0 = 8.8 \, \text{mmolN m}^{-3} \). With this choice the values of the transition depths are \( z_1 = -15.26 \, \text{m}, z_2 = -25.13 \, \text{m}, \) and \( z_3 = -36.52 \, \text{m} \). The maximum \( Z \), obtained from (12) at \( z = 0 \), becomes \( Z_0 = Z(0) = 0.25 \, \text{mmolN m}^{-3} \). The solutions NPZ at the three layers are shown in Figure 2. In the upper layer the amount of \( Z \) decreases with depth (from \( Z_0 \) to 0) which is compensated (since both \( P \) and \( N_0 \) are constant) by an equally small increase of \( N \). In the mid layer, \( P \) decreases from \( P_1 \) to 0, and is compensated (since \( Z = 0 \)) by an equally large increase of \( N \).

[14] In this steady solution, and in the upper layer, the large \( z \)-dependent \( U(z) \) is mainly balanced by the large constant \( P_M \) (Figure 3). The remanent, smaller part of \( U(z) \) is balanced by the small \( z \)-dependent grazing \( R(z) \). In the \( Z \) balance, the small growth \( g(z) \) of zooplankton is balanced by \( Z_M(z) \). In the mid layer \( R = 0 \), so that \( U(z) \) and \( P_M(z) \) exactly balance.

[15] These steady NPZ solutions are continuous functions of \( z \), but they have been obtained without any requirement on differentiability conditions. It is clear, at least visually from Figure 2, that these functions are not vertically differentiable at \( z = z_1 \) or \( z = z_2 \). As a simple proof consider differentiability of \( P(z) \) at \( z_2 \). The vertical derivative of (15) is

\[
\frac{\partial P_2}{\partial z} = \frac{\Xi_0K_0}{G(z) - \Xi_0} \frac{\partial G}{\partial z} = \frac{\Xi_0K_0}{G(z) - \Xi_0} \frac{V_0^2 \alpha_0}{(V_0^2 + \alpha_0^2 P^2)^2} \frac{\partial I}{\partial z} = \frac{\Xi_0K_0}{G(z) - \Xi_0} \frac{V_0^2 \alpha_0 [k_w + k_w P_2(z)]}{(V_0^2 + \alpha_0^2 P^2)} I(z).
\]

As \( z \to z_2 \), we have \( P_2(z) \to 0 \) and \( G(z) \to \Xi_0 \), and therefore \( \partial P_2/\partial z \to \infty \) in the mid layer, as observed in Figure 2a.
This limit does not match with the vertical derivative of $P$ as $z \to z_2$ in the lower layer, where $\partial P_3/\partial z = 0$. This lack of differentiability (the functions are continuous but not continuously differentiable) implies that these steady solutions are questionable as initial conditions in many coupled physical-ecosystem models. Most of these models are formulated in the spatial (Eulerian) description and require that solutions NPZ be continuously differentiable for the vertical gradients present in the advective derivative on the left hand side of (1)–(2) make sense. In the next section NPZ solutions, similar to the ones described above but continuously differentiable, are numerically obtained.

### 3.2. Numerical Steady Solutions

In order to obtain continuously differentiable steady $P(z)$ and $Z(z)$ solutions, the prognostic equations (1)–(2) are numerically integrated in time, using as initial conditions smooth profiles $P^{(0)}(z)$ and $Z^{(0)}(z)$, until a steady state is reached. The initial profiles $P^{(0)}(z)$ and $Z^{(0)}(z)$ are identical to the solutions obtained in the previous section (Figures 2a and 2b) except that in the non homogeneous layers, that is, the mid layer in the case of $P$ and the upper layer in the case of $Z$, the $z$-dependent profiles are replaced by transition cosine functions. Since $\partial \cos(z)/\partial z = -\sin(z) = 0$ at $z = \{0, \pi\}$, $P^{(0)}$ and $Z^{(0)}$ have continuous (zero) derivatives at $z_1$ and $z_3$, and at 0 and $z_1$, respectively. Specifically, we define the initial profiles

$$P^{(0)}(z) = \begin{cases} P_1, & z \in [z_1, 0] \\ \frac{1}{2} P_1 \left[ 1 + \cos \left( \frac{z - z_1}{z_3 - z_1} \pi \right) \right], & z \in [z_3, z_1] \\ 0, & z < z_3, \end{cases} \quad (19)$$

and

$$Z^{(0)}(z) = \begin{cases} Z_1, & z \in [z_1, 0] \\ \frac{1}{2} Z_1 \left[ 1 + \cos \left( \frac{z - z_1}{z_3 - z_1} \pi \right) \right], & z \in [z_3, z_1] \\ 0, & z < z_1. \end{cases} \quad (20)$$

These initial profiles are shown in Figure 4. Next, $P^{(0)}(z)$ and $Z^{(0)}(z)$ are vertically discretized and integrated forward using (1)–(2) with $u = 0$. Nine different numerical resolutions, ranging from constant grid size $\Delta z = 4$ m ($i = 1$) to $\Delta z = 1.5625$ cm ($i = 9$), are implemented (Table 2).

Time integration is carried out using an explicit leapfrog scheme, together with a Robert-Asselin time filter to avoid the computational mode [see, e.g., Durran, 1998, p. 62]. The time integration was $10^4$ days, at the end of which the maximum forcing term in the local rate of change...
of $P$ or $Z$ was of the order of $10^{-7}$ mmolN m$^{-3}$d$^{-1}$. During the integration time $P$ and $Z$ monotonically converged to the steady solutions $P^{(i)}$ and $Z^{(i)}$ ($i = 1, \ldots, 9$) shown in Figure 4. After a first look these solutions seem to be very similar to the non continuously differentiable solutions obtained in the previous section. However, a closer view around the layer boundary depths (zooms on previous section. But the initial physical conditions lead to an idealized case of computational limita-
tions is restricted to 65 cm, are addressed. The theoretical basis of the numerical model are explained in detail in the references above and only a brief summary is given next to introduce the indispensable symbol definitions and mathematical expressions.

The Froude number $F \equiv \omega_f/N$ and the Rossby number $R \equiv \zeta/f$, where $\omega$, $\zeta$ are the horizontal and vertical components of the relative vorticity $\omega \equiv \omega_{x} + \zeta k$, and $f$ and $N$ are the Coriolis and total Brunt-Väisälä frequencies, respectively. The isopycnal vertical displacement $\mathcal{D}$ with respect to a reference density configuration $\mathcal{D}(x, t) = z - d(x, t)$ where $d \equiv (\rho - \rho_0)/\rho_0$ is the depth, or vertical location, that an isopycnal located at $x$ at time $t$ has in the reference density configuration defined by $\rho_0 + \zeta k$, where $\rho$ is the mass density, and $\rho_0 > 0$ and $\zeta < 0$ are constant values that do not need to be specified in the Boussinesq approximation. Therefore, static instability occurs when the stratification number $D_2 > 1$, and inertial instability when $R < -1$.

The $AB\bar{\varpi}$ model integrates the dimensionless ageostrophic horizontal vorticity $\mathcal{A}_h = (A, B) \equiv \bar{\omega}_h - c^2\nabla \mathcal{U}_h$ using the equation

$$\frac{d\mathcal{A}_h}{dt} = -f k \times \mathcal{A}_h + (1 - c^2) \nabla w + \bar{\omega} \cdot \nabla u_h + c^2 \nabla u \cdot \nabla \mathcal{D},$$

(21)

where the Prandtl ratio $c \equiv N/f$, the relative vorticity $\omega \equiv \nabla \times u = (\xi, \eta, \zeta)$, and $\chi \equiv \chi/f$ for any quantity $\chi$. The third prognostic equation is the explicit conservation of PV.


4.1. Dynamical $AB\bar{\varpi}$ Model

The physical model (hereinafter referred to as the $AB\bar{\varpi}$ model) is nonhydrostatic and simulates the volume-preserving flow of a stratified rotating fluid under the Boussinesq and $f$ plane approximations [Dritschel and Viúdez, 2003]. The initial flow is specified by the potential vorticity (PV) using the PV initialization approach [Viúdez and Dritschel, 2003]. This initialization technique largely avoids the initial generation of inertia–gravity waves which otherwise could contaminate the balanced vertical velocity.
anomaly \( \pi \) through PV contour advection on isopycnals,
\[
d\pi/dt = 0,
\]
where
\[
\pi \equiv \Pi - 1 = \frac{\omega + f k}{f} \cdot \nabla \theta - 1 = \begin{pmatrix} \omega + f k \end{pmatrix} \cdot \begin{pmatrix} k - \nabla D \end{pmatrix} - 1 = \frac{\partial D}{\partial z} - \omega \cdot \nabla D,
\]
and \( \Pi \equiv (\omega + k) \cdot \nabla \theta \) is the total dimensionless PV.

The state variables are the components of the three-dimensional vector potential \( \varphi = (\varphi, \psi, \phi) \) which provide the velocity \( \mathbf{u} = - \nabla \times \varphi \) and the vertical displacement \( c^2 \mathbf{D} = - \nabla \cdot \varphi \). The horizontal components of the vector potential \( \varphi_h = (\varphi, \psi) \) are diagnosed every time step by inversion of \( \varphi_h = \nabla^2 \varphi_h \), while the vertical component \( \phi \) is obtained from the inversion of the \( \pi \) definition (22) as a function of \( (\varphi, \psi, \phi) \). This procedure, based on the explicit conservation and inversion of PV, allows long-term simulations of large PV gradients as those happening during baroclinic instability processes. Also, owing to the fact that the divergenceless condition \( \nabla \cdot \mathbf{u} = 0 \) is implicitly satisfied using the vector potential \( \varphi \), this algorithm is specially precise in obtaining the vertical velocity field which, though crucial for the development of NPZ anomalies, is typically three orders of magnitude smaller than the horizontal velocity in submesoscale balanced flow.

### 4.2. Numerical Parameters

The domain is a triply periodic box of vertical extent \( L_Z = 2\pi \) (which defines the unit of space) and horizontal extents \( L_X = L_Y = cL_Z \), with \( c \equiv N/f = 10 \). The number of grid points is \( (n_X, n_Y, n_Z) = (128, 128, 128) \), and the number of isopycnals \( n_L = 128 \). The background Brunt-Väisälä frequency \( N = 2\pi \), which defines the background buoyancy period as the unit of time, \( T_{bp} = 2\pi/N = 1 \). Thus one inertial period \( T_I = cT_{bp} \). The time step \( \delta t = 0.01 \). In order to relate the nondimensional quantities of the physical model to the dimensional ones of the NPZ model only two dimensional parameters must be specified, namely the maximum depth \( z_{\text{min}} \) and the mean latitude \( \theta_0 \). These two parameters specify the length \( L \) and time \( T \) conversion factors, being \( L/\pi \approx 31.83 \text{ m} \), for \( z_{\text{min}} = -100 \text{ m} \), and \( T \equiv 24 \times 3600/(c \sin \theta_0) \approx 6109.5 \text{ s} \) for \( \theta_0 = 45^\circ \text{ N} \). The dimensional quantities in the previous section, including the constants in Table 1, are made dimensionless using the appropriate combination of \( L \) and \( T \) factors. The dimension of amount of substance (mmolN) is unchanged since it exclusively belongs to the NPZ equations. The relation between the physical and ecosystem models is that the \( AB\pi \) model provides the three-dimensional velocity \( \mathbf{u} \) for the advective terms in the NPZ model. The \( AB\pi \) model is still adiabatic, since heating by solar radiation is ignored, and diffusion processes are neglected so that PV is materially conserved.

### 4.3. Numerical Results

We simulate a shallow current containing both positive and negative \( \pi \). The current is set up by placing two horizontal PV cylinders of opposite sign, slightly perturbed, next to each other (Figure 7). Each PV cylinder has a horizontal and vertical semiaxis of 0.5 and 0.7, respectively.
Figure 7. PV contours at the surface \( z = 0 \). Horizontal extent is \( \Delta x = \Delta y = [-\pi, \pi] \). Distance \( D_x = D_y = [C_0, p] \). Time is shown in \( T_{ip} \).

Figure 8. (a) Horizontal distribution of \( u_h = (u, v) \) at \( z = 0 \). Only every 4 vectors is plotted. Contours are speed \( u_h \equiv |u_h| \) \( \max\{u_h\} = 1.7 \). (b) The \( u_h(y, z) \) on vertical section \( x = 0 \) \( \max\{u_h\} = 1.52 \). (c) Horizontal distribution of \( w \) at \( z_b \simeq 23.4 \text{ m} \) \( (w \in [-2.9, 4.5] \times 10^{-3}) \). The PV contours \( \omega = \pm 0.05 \) at \( z = 0 \) are included for reference. (d) The \( w(y, z) \) on vertical section \( x = 0 \) \( (w \in [-3.9, 3.6] \times 10^{-3}) \). Time \( t = 8 T_{ip}, z_{min} = -100 \text{ m}, z_a = -12.5 \text{ m}, \) and \( z_b \simeq -23.4 \text{ m} \).
The minimum and maximum \( \bar{w} \) at the center of the cylinders is \( \{ \bar{w}_{\min}, \bar{w}_{\max} \} = \{ -0.75, 0.75 \} \). This PV distribution induces a northward current which, during the baroclinic instability process, rolls up into a street of cyclonic and anticyclonic vortices that pair forming vortex dipoles (Figures 7, 8a, and 8b). During this process vertical velocity is generated (Figures 8c and 8d). We choose a shallow jet, located in the first 50 m (Figure 8b) in order to increase the numerical resolution in the first 100 m, and a small Prandtl ratio \( c = 10 \) in order to favor vertical velocities.

[25] Vertical velocities reach maxima at \( t = 8 \) and \( t = 9 \) \( T_P \) at the northern side of the northern anticyclone (Figure 8c). At this stage \( F_{\max} = 0.41 \) and \( R_{\min} = -0.67 \). A vertical section along \( x = 0 \) (Figure 8d) shows that \( w \) maxima occur between \( z \simeq -13 \) m and \( z \simeq -25 \) m, and is three orders of magnitude smaller than \( |u_h| \). The vertical velocity is mostly in balance in the sense that inertia–gravity waves, either because of unbalanced initial conditions or to spontaneous generation, have very small amplitude.

[26] As expected, \( P \) anomalies \( P'(x, t) \equiv P(x, t) - P_s(z) \), where \( P_s(z) \) is the stationary \( P \) profile defined in the previous section, reach maxima at \( z \simeq z_2 \) while \( Z \) anomalies \( Z'(x, t) \equiv Z(x, t) - Z_s(z) \) reach maxima at \( z \simeq z_1 \) (Figure 9a). These anomalies have been forced by \( w \) since the horizontal advection of the stationary profiles \( P_s(z) \) and \( Z_s(z) \) is zero. The standard deviation of \( U \) has two maxima (Figure 9b), the largest one is located at \( z \simeq z_2 \) and is related to \( P' \), while the secondary one, located some meters above at \( z \simeq -19 \) m, is related to \( Z' \) through the term \( N(K_0 + N) \) in (1).

[27] Grazing anomalies \( R \) reach maxima at the \( Z \) transition depth \( z \simeq z_1 \) (Figure 9b). \( R \), however, remain small, roughly a 25% of \( U \) at \( z_1 \), so that the phytoplankton balance involves mainly \( dP/dt, U \), and \( P_{\zeta} \). Horizontal advection \( u_h \cdot \nabla_h P \) and local change \( \partial P/\partial t \) (Figure 9c) are the largest contributions to the material rate of change of \( P \). Vertical advection \( w\partial P/\partial z \) is about 50%, while \( dP/dt \) is only 10%, which corresponds to the phytoplankton physiological reaction to light absorption changes due to vertical displacement. There is therefore a large cancelation between \( u_h \cdot \nabla_h P \) and \( \partial P/\partial t \) which, together with the smallness of \( w\partial P/\partial z \) and \( dP/dt \), implies that horizontal distributions of \( P \) resemble a tracer of the horizontal flow (i.e., \( \partial P/\partial t \simeq -u_h \cdot \nabla_h P \)).

[28] A similar fact occurs with the local and advective terms of \( Z \) (Figure 9d). Here the difference is that \( dZ/dt \) is only about a 0.1% of \( \partial Z/\partial t \) or \( u_h \cdot \nabla_h Z \). Thus \( Z \) behaves as a tracer of the horizontal flow better than \( P \).

[29] The horizontal distributions of \( P' \) and \( Z' \) (Figures 10a and 10c) are related to \( w \) (Figure 8c). Positive anomalies occur in downwelling areas and negative anomalies in upwelling areas. The correlation between \( w \), \( P' \), and \( Z' \) along the water column can be noticed comparing the vertical velocities.
Figure 10. (a) $P(x, y)$ at $z_a \approx -23.4$ m ($P' \in [-3.3, 3.1]$ mmolN m$^{-3}$). PV contours $\varpi = \pm 0.05$ at $z = 0$ are included for reference. (b) $P'(y, z)$ on vertical section $x = 0$ ($P' \in [-3.3, 4.8]$ mmolN m$^{-3}$). (c) $Z(x, y)$ at $z_a = -12.5$ m ($Z' \in [-9.0, 8.3] \times 10^{-2}$ mmolN m$^{-3}$). (d) $Z'(y, z)$ on vertical section $x = 0$ ($Z' \in [-8.9, 11.2] \times 10^{-2}$ mmolN m$^{-3}$). Time $t = 8 T_{ip}$. 
sections of $w$ (Figure 8d), $P'$ and $Z$ (Figures 10b and 10d). In the large shear regions $Z'$ has horizontal gradients larger than those of $P'$, which is a consequence of the fact that $Z'$ is materially conserved better than $P'$ as we have commented above.

[30] The existence of two maxima in $\sigma \{U'\}(z)$ (Figure 9b) is explained through the spatial distributions shown in Figure 11. In upwelling regions $U' > 0$ and $U' < 0$ above and below $z_b$, respectively, while the opposite happens in downwelling regions. This behavior is mainly due to the nonlinear relation between $Z$ and $P$ through the term $(N_0 - P - Z)/\{K_0 + N_0 - P - Z\} \times \{P\}$ included in $U$ in (1) since the product $\sigma \{P\} \sigma \{Z\}$ (not shown) reaches a maximum at $z \approx -19$ m. Grazing anomaly $R'$ at the $Z$ transition depth $z \approx z_a = -12.5$ m (not shown) is obviously spatially correlated with $Z'$ (Figures 10c and 10d). Grazing, however, continues having a minor role in the $P$ budget because of the relatively small amount of $Z$ in the upper layer.

[31] The relevant quantities are however $dP/dt$ and $dZ/dt$ (Figure 12) since they represent the total $P$ and $Z$ changes (the right-hand side of (1) and (2), respectively) forced by $w$ and are independent of the effect of mere advection. Ascending fluid particles experience an increase of their $P$ content while descending particles experience a decrease of $P$. Note particularly the large $P$ decrease at the northern side of the domain (Figures 12a and 12b), where $w < 0$ (Figures 8c and 8d). Positive $P$ budgets occur at depths a bit shallower than negative $P$ budgets, which explains why $dP/dt$ is mostly negative at the depth shown in Figure 12a. The material rate of change of $Z$ (Figures 12c and 12d) and $w$ are also clearly correlated. However, $dZ/dt$ usually displays a minimum and a maximum along the water column, which is consistent with the two maxima in $\sigma \{dZ/dt\}$ (Figure 9d).

[32] The analysis of $dP/dt$ and $dZ/dt$ into their local and advective changes (Figure 13) shows that, as inferred from their standard deviations (Figures 9c and 9d), there is a large cancelation between the local change and the horizontal advection of $P$ and $Z$. The vertical advection is smaller. Consistently also with the $P'$ and $Z'$ distributions (Figure 10) the local change and horizontal advection of $Z$ present patterns more elongated than those of $P$. This is a consequence of the better material conservation of $Z$ in comparison with $P$. Large local rates occur in the frontal areas, where both horizontal velocity and horizontal gradients of $P$ and $Z$ are large. The vertical advection of $P$ and $Z$ have however similar patterns. This is so because $\partial P/\partial z \approx \partial P'/\partial z > 0$ at $z = z_b$ and $\partial Z/\partial z \approx \partial Z'/\partial z > 0$ at $z = z_a$, so that the vertical advection patterns (Figures 13c and 13f) resemble the $w$ pattern (Figure 8c).

[33] The time evolution of $\sigma \{dP/dt\}$, $\sigma \{dZ/dt\}$, and $\sigma \{w\}$ (Figure 14) show that the ecosystem time response to $w$ maxima is about 5 $T_w$. The second $w$ maximum at $t \approx 37$ $T_w$ is related to the flow enhancement due to the fusion of two anticyclones.

[34] The vertical resolution used in this simulation ($l_z = 65$ cm) is not good enough to fully resolve the large vertical gradients of $P$ and $Z$ at transition depths. On the basis of Figures 5b and 6b, vertical gradients are underestimated by a 50%. Larger vertical resolutions would correctly resolve the vertical advection of $P$ and $Z$ which would cause an important increase in $P'$ and $Z'$ distributions. However, as another consequence of an increased vertical resolution,
Figure 12. (a) The $dP/dt$ at $z_b \approx -23.4$ m ($i_z = 50$, $dP/dt \in [-0.82, 0.081]$), (b) $dP/dt$ at $x = 0$ ($dP/dt \in [-0.82, 0.42]$), (c) $dZ/dt$ at $z = -12.5$ m ($i_z = 57$, $dZ/dt \in [-19.8, 5.3] \times 10^{-5}$), and (d) $dZ/dt$ at $x = 0$ ($dZ/dt \in [-19.6, 10.5] \times 10^{-5}$). Time $t = 8 T_{ip}$. 
(a) The $\frac{\partial P}{\partial t}$ at $z \simeq z_0 = -23.4 m$ ($\partial P/\partial t \in [-8.3, 7.4]$), (b) $u_h \cdot \nabla_h P$ ($\in [-9.4, 8.3]$), (c) $w \frac{\partial P}{\partial z}$ ($\in [-1.8, 4.2]$), (d) $\frac{\partial Z}{\partial t}$ at $z_a = -12.5 m$ ($\partial Z/\partial t \in [-0.51, 0.30]$), (e) $u_h \cdot \nabla_h Z$ ($\in [-0.32, 0.53]$), and (f) $w \frac{\partial Z}{\partial z}$ ($\in [-0.042, 0.14]$). Time $t = 8 T_{ip}$. 

Figure 13.
these larger anomalies would be restricted to thinner ocean layers, so that only quantitative changes are expected in the ecosystem variables.

[35] These numerical results correspond to a non diffusive NPZ ecosystem model coupled to an adiabatic inviscid physical model. These results will not apply when vertical mixing is added to the NPZ model [see Edwards et al., 2000] since in such a case the large vertical NPZ gradients found here would turn $P\nabla^2 Z$ into unsteady solutions. We note that the mere existence of vertical eddy diffusion in a numerical model already requires vertical differentiability. Vertical diffusion is not included here because the NPZ model is kept as simple as possible in order to analyze the vertical velocity forcing of NPZ anomalies. Including vertical diffusion will add new free parameters (the vertical diffusivity coefficients) to the already large list of NPZ parameters on Table 1. Furthermore, the absence of NPZ diffusion is also consistent with the inviscid nature of the PV-conserving dynamical model (only a very small amount of numerical diffusivity is included to avoid grid size noise).

5. Concluding Remarks

[36] We have first shown that 1D steady and continuously differentiable (in a numerical sense) solutions to the NPZ equations are possible. These solutions are potentially useful as initial steady ecosystem conditions to investigate the role of horizontal and vertical advection in 3D coupled physical-ecosystem numerical models. An example of ecological development due to vertical velocity enhancement during a baroclinic instability process has been presented.

[37] This example shows that once phytoplankton and zooplankton anomalies develop locally forced by balanced vertical velocity they are horizontally advected away from the upwelling or downwelling regions so that spatial distributions of vertical velocity and ecological fields become eventually uncorrelated (for experimental evidence of this process, see, e.g., Ruiz et al. [2001]). Thus the biological distributions are more related to PV gradients than to PV itself. This fact, and the submesoscale vertical origin of NPZ anomalies, is consistent with Lévy et al. [2001], who used a primitive equations model with vertical diffusion. However, the experimental work of Lévy et al. [2005] questions the contribution of submesoscale total advection in the phytoplankton variability over large timescales.

[38] The physical-ecological model used here has several limitations. On the one hand, these NPZ solutions require very good vertical resolutions, with a grid size of few centimeters, to be properly discretized. This imposes a severe handicap to the available random access memory of current computers running 3D coupled physical-ecosystem models. Though from a strict numerical perspective this fact is a serious modeling limitation, from a wider perspective other handicaps, for instance, errors in the mathematical parametrization of the different NPZ processes, are likely to be of larger relevance [see Anderson, 2005; Flynn, 2005; Mitra et al., 2007]. It is nevertheless important to know the degree at which ecosystem modeling solutions faithfully reproduce the underlying ecosystem dynamics and that, even with poor vertical resolution, it is possible to obtain good qualitative results from these models.

[39] On the other hand, our results show that $P$ and $Z$ approximately behave as passive tracers while organisms are in fact active tracers. This is so because the simple initial steady NPZ profiles let little interaction between $P$ and $Z$. In future work we will address these interactions using both more realistic initial NPZ profiles and a more complex biological model.

[40] To conclude, these results are a first approximation toward a better understanding of biological processes forced by vertical velocity at the submesoscale. Many questions still remain to be answered in this context. How much does the submesoscale vertical velocity contribute to primary and secondary productivity in comparison to the mesoscale? Does the vertical advection induce different biological patterns in eutrophic and oligotrophic regimes? Is the submesoscale important in the seasonal biological variance?

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References


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