Multi-photon, multi-mode polarization entanglement in parametric down-conversion

A. Gatti1, R. Zambrini2, M. San Miguel1 and L. A. Lugiato1

1 INFN, Dipartimento di Scienze CC FF MM, Università dell’Insubria, Via Valleggio 11, 22100 Como, Italy. and 2 IMEDEA, Campus Universitat Illes Balears, E-07071 Palma de Mallorca, Spain.

Parametric down-conversion in a type II crystal is a well-known source of polarization entangled photons. When phase matching conditions are properly chosen, the ordinary and extraordinary emission cones intersect in two regions, as shown by Fig. 1a. When considering photodetection from these regions in the regime of single photon pair production, the two-photon state can be described as the ideal polarization-entangled state [1]. Photons produced by this process has become an essential ingredient in many implementations of quantum information schemes.

The question that we have addressed is whether this microscopic photon polarization entanglement leaves any trace in the regime of high parametric down-conversion efficiency, where the number of down-converted photons can be rather large, and in which form.

To this end parametric down-conversion is described in the framework of a multi-mode model, valid for any gain regime, which includes realistic propagation effects, as diffraction and spatio-temporal walk-off. Quantum-optical polarization properties of the down-converted light are described within the formalism of Stokes operators. These operators obey to angular momentum-like commutation rules, and the associated observables are in general non compatible. We define a local version of Stokes operators and study the quantum correlation between operators measured from symmetric portions of the far field beam cross-section. Two of the Stokes operators, namely \( \hat{S}_0 \) and \( \hat{S}_4 \), represent the sum and the difference between the number of ordinary (vertical) and extraordinary (horizontal) photons. Analytical calculations, performed in the limit of a plane-wave pump, show a perfect correlation, both between \( \hat{S}_0(x) \), \( \hat{S}_0(-x) \), and between \( \hat{S}_4(x) \), \( \hat{S}_4(-x) \) for any choice of the position \( x \) in the far field. This result is a direct consequence of pairwise emission of photons with ordinary and extraordinary polarization propagating in symmetric directions, as required by transverse light momentum conservation.

Quite different is the situation for the other two Stokes operators (namely \( \hat{S}_2 \) and \( \hat{S}_3 \)) that involve measuring the photon numbers in the oblique and circular polarization basis. Figure 1b shows a typical result for the noise in the difference between Stokes operators measured from small symmetric portions of the far-field. Precisely, the figure shows \( \langle |\hat{S}_2(x) - \hat{S}_2(-x)|^2 \rangle = \langle |\hat{S}_4(x) - \hat{S}_4(-x)|^2 \rangle \), scaled to the shot noise level, represented by \( \langle |\hat{S}_0(x) + \hat{S}_0(-x)|^2 \rangle \), as a function of the transverse coordinate \( x \). Parameters are those of a 2 mm long BBO crystal, cut for degenerate type II phase matching at 702nm. For comparison, part (a) of the figure shows the mean intensity distribution in the far field. In plot (b) are clearly visible two large blue zones, in correspondence of the intersections of the emission rings, where the Stokes operator correlation is almost perfect. The figure was obtained by filtering the emitted frequencies in a 5 nm band; if a wider frequency filter is used, our calculations show that the regions where Stokes operators are quantum correlated stretch to form a ring around the system axis.

In conclusion, the polarization entanglement of photon pairs emitted in parametric down-conversion survives in the high gain regime, in the form of non-classical correlations of all light Stokes operators associated to polarization degrees of freedom. Although Stokes parameters are extremely noisy (the state is unpolarized), measurement of a Stokes parameter in any polarization basis in one far-field region determines the Stokes parameter collected from the symmetric region, within an uncertainty much below the standard quantum limit. We believe that this form of entanglement, with its increased complexity in terms of degrees of freedom (photon number, polarization, frequency and spatial degrees of freedom) can be promising for new quantum information schemes.