Abstract—We present a detailed numerical study of the dynamics of two unidirectionally coupled semiconductor lasers subject to filtered optical feedback. We show that this chaos-based communication scheme allows for an improvement in the decoding of encrypted messages in comparison with the conventional feedback scheme. We found that the performance of the system is optimal when the closed-loop configuration and similar filters are used in the emitter and receiver systems.

Index Terms—Chaos, dynamics, optical feedback, semiconductor lasers, synchronization.

I. INTRODUCTION

SYNCHRONIZATION in nonlinear, chaotic systems is a growing field since the appearance of the pioneering work by Pecora and Carroll in 1990 [1]. They showed that two autonomous similar chaotic systems can synchronize to each other if they are properly coupled. However, synchronization of chaotic systems is somehow counterintuitive since the trajectories of two identical chaotic systems started at nearly the same initial points quickly become uncorrelated. For this reason, practical implementations of synchronized nonlinear chaotic systems have drawn much attention since the appearance of the concept of chaos synchronization.

Chaos synchronization was first demonstrated in electronic circuits [1] and later in coupled solid-state lasers [2]. Two semiconductor lasers subject to optical feedback are yet another example of chaos synchronization that can be found when they are unidirectionally coupled in a master–slave configuration [3], [4] or when they are mutually coupled [5]–[7]. Message encoding/decoding via a chaos-pass filtering process was demonstrated in 2000 [8], yielding a very fast growth of the field in the last few years [9]–[12]. Nowadays, mechanically stable integrated sources are being proposed and tested [13]–[15].

To transmit information, the message has to be encoded in the chaotic output of the emitter system. At the receiver side, a system similar to the emitter needs to synchronize mainly to the chaotic output of the emitter, allowing the extraction of the message [8]. The receiver system filters out the message encoded in the transmitted signal, and the information can be extracted by a simple subtraction. Besides the conventional optical feedback scheme, other configurations including optical injection [4], amplified injection [16], amplified optical feedback [13], incoherent optical feedback and injection [17], [18], optoelectronic feedback [19], and multimode lasers [20] [21] have also been investigated.

In this work, we study numerically the possibility of encoding a message using as a carrier the chaotic output of a semiconductor laser subject to filtered optical feedback. The presence of a filter in the feedback loop can strongly modify the dynamical response of the laser [22]–[25]. A major advantage of filtered feedback systems is that they can produce chaotic emission of narrow bandwidth, which could reduce the synchronization degradation due to bandwidth limitations in the communications channel [26] or crosstalk effects. Furthermore, there are additional advantages that motivate the use of semiconductor lasers subject to filtered optical feedback in the context of chaos-based communications. For instance, the fact that the high frequencies are cut favors the synchronization between emitter and receiver [27]. Also, more importantly, the main parameters of the filter in the feedback loop, such as the detuning of the central frequency of the filter with respect to the solitary laser frequency, and the filter bandwidth, can be used as additional keys to improve the security in the communications, as will be shown later.

We focus on a master–slave configuration, i.e., unidirectional coupling, with two possible schemes: 1) open-loop and 2) closed-loop [28]–[30]. In the open-loop scheme, the emitter, i.e., master laser (ML), is subject to filtered optical feedback while the receiver, i.e., slave laser (SL), is not subject to feedback but to the injection of the emitter’s output. In the closed-loop scheme, both lasers are subject to filtered optical feedback, and the SL is also subject to the injection of the emitter’s output.

II. RATE-EQUATION MODEL

The system under consideration is schematically represented in Fig. 1. The ML is always subject to optical feedback, while
the SL can operate in the open loop [Fig. 1(a)] or in the closed-loop configuration [Fig. 1(b)]. In the open-loop configuration, an additional filter can be placed in front of the SL.

Semiconductor lasers subject to optical feedback can be modeled via rate equations. We use a general rate-equations model for single-longitudinal-mode emission. The optical feedback is introduced using the Lang–Kobayashi approach, which takes into account a single reflection in the external cavity. In addition, the external optical feedback is filtered by an external grating, which has a Lorentzian transmission response \( R(\omega) \) characterized by a filter half width at half the maximum \( \Lambda \) and a central frequency \( \omega_c \) [22], [23], i.e.,

\[
R(\omega) = \frac{\Lambda}{\Lambda + i(\omega_c - \omega)}. \tag{1}
\]

The equations for the carrier number in the ML (denoted by \( m \)) and SL (denoted by \( s \)) then read

\[
\frac{dN_{m,s}(t)}{dt} = \frac{I}{\epsilon} - \frac{N_{m,s}(t)}{\tau_N} - G_{m,s}(t)P_{m,s}(t) \tag{2}
\]

and the equations for the evolution of the electric field in the ML read

\[
\frac{dE_m(t)}{dt} = \frac{1 + i\alpha}{2} \left( G_m(t) - \frac{1}{\tau_p} \right) E_m(t) + \gamma_m F_m(t) \tag{3}
\]

\[
\frac{dE_s(t)}{dt} = \Lambda_m E_m(t - \tau)e^{-i\phi_m} + (i\Delta_m - \Lambda_m) F_m(t). \tag{4}
\]

The evolution of the electric field in the SL in the open-loop scheme read

\[
\frac{dE_s(t)}{dt} = \frac{1 + i\alpha}{2} \left( G_s(t) - \frac{1}{\tau_p} \right) E_s(t) + \kappa_p F_p(t) \tag{5}
\]

\[
\frac{dF_p(t)}{dt} = \Lambda_p E_m(t)e^{i\phi_p} + (i\Delta_p - \Lambda_p) F_p(t) \tag{6}
\]

and those for the evolution of the electric field in the SL in the closed-loop scheme read

\[
\frac{dE_s(t)}{dt} = \frac{1 + i\alpha}{2} \left( G_s(t) - \frac{1}{\tau_p} \right) E_s(t) + \gamma_s F_s(t)
\]

\[
+ \kappa_p E_m(t) \tag{7}
\]

\[
\frac{dF_p(t)}{dt} = \Lambda_s E_s(t - \tau)e^{-i\phi_s} + (i\Delta_s - \Lambda_s) F_s(t). \tag{8}
\]

The electric field \( F_{m,s}(t) \) is the external optical feedback filtered by a grating, which has a Lorentzian transmission response. \( F_p(t) \) is the field emitted by the ML that can also be filtered by a grating prior to the injection into the SL. For filters of infinite width, \( F_{m,s}(t) = E_{m,s}(t - \tau) \) and \( F_p(t) = E_m(t) \). The terms \( \kappa_p F_p(t) \) and \( \kappa_p E_m(t) \) account for the light injected from the ML into the SL. \( \kappa_p \) is the coupling strength, and the detuning between the ML and SL emission frequencies is initially neglected. For simplicity, the laser parameters are considered to be identical for both devices. \( \alpha \) is the linewidth enhancement factor. \( \tau_p \) is the photon lifetime. \( I \) denotes the injected current. \( c \) is the electron charge, and \( \tau_N \) is the carrier lifetime. In the definition of the gain \( G_{m,s}(t) = g(N_{m,s}(t) - N_0)/(1 + sP_{m,s}(t)) \), \( g \) is the differential gain, \( N_0 \) is the carrier number at transparency, \( s \) is the gain compression factor, and \( P_{m,s} = |E_{m,s}|^2 \) is the optical intensity in terms of the number of photons. \( \gamma_{m,s} \) are the feedback strengths. The filters half widths at half the maximum are \( \Lambda_{m,s,p} \) and their central frequency detunings, with respect to the free-running emission frequency of the lasers, are \( \Delta_{m,s,p} = 0 \). \( \tau \) is the external cavity delay and \( \Phi_{m,s} = \frac{\pi c \omega_c}{2\Lambda_{m,s}(\omega)\tau} \) are the phases accumulated by the electric fields in the external cavity round trips. In the long cavity limit, as the one we are considering here, the phases do not play a significant role [11]. Without loss of generality, the flying time between ML and SL is taken as 0. Table I gives the parameter values that are kept constant through this paper. The relaxation oscillation frequency of the solitary laser is \( f_{RO} = 5.9 \) GHz at \( I = 30 \) mA.

### III. Synchronization Properties

Under the influence of a filtered optical feedback, the dynamics of the laser can be modified, as can be seen from the optical spectra calculated for different filter widths in Fig. 2. Fig. 2(a) shows the spectrum for conventional optical feedback, and Fig. 2(b) shows the spectrum for the filtered optical feedback case with \( \Lambda_m/2\pi = 15 \) GHz. From Fig. 2(b), it can be seen that the spectrum is narrower due to the effect of the filter in the

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
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<tbody>
<tr>
<td>( \alpha )</td>
<td>linewidth enhancement factor</td>
<td>5</td>
</tr>
<tr>
<td>( \tau_p )</td>
<td>photon lifetime</td>
<td>2 ps</td>
</tr>
<tr>
<td>( \tau_N )</td>
<td>carrier lifetime</td>
<td>2 ns</td>
</tr>
<tr>
<td>( g )</td>
<td>differential gain coefficient</td>
<td>( 1.5 \times 10^{-8} ) ps(^{-1} )</td>
</tr>
<tr>
<td>( N_0 )</td>
<td>carrier transparency</td>
<td>( 1.5 \times 10^8 )</td>
</tr>
<tr>
<td>( s )</td>
<td>gain compression coefficient</td>
<td>( 5 \times 10^{-7} )</td>
</tr>
<tr>
<td>( I_{th} )</td>
<td>threshold current</td>
<td>14.7 mA</td>
</tr>
<tr>
<td>( I )</td>
<td>bias current</td>
<td>30 mA</td>
</tr>
<tr>
<td>( \tau )</td>
<td>feedback delay time</td>
<td>1 ns</td>
</tr>
</tbody>
</table>

![Fig. 1. ML and SL configurations used for all-optical chaos communication systems in a back-to-back configuration: (a) open-loop and (b) closed-loop.](image)

![Fig. 2. Optical spectra of a semiconductor laser subject to (a) conventional optical feedback and (b) filtered optical feedback. Parameters: (a) \( \gamma_m = 25 \) ns\(^{-1} \), (b) \( \gamma_m = 25 \) ns\(^{-1} \) and \( \Lambda_m/2\pi = 15 \) GHz.](image)
external cavity. In this paper, we have chosen filter widths of 15 and 30 GHz. The bandwidth of 15 GHz, which is around twice the relaxation oscillation frequency, is the minimum filter width for which the dynamics of the laser is still highly chaotic. The bandwidth of 30 GHz is an intermediate value between the minimum bandwidth of 15 GHz and a filter bandwidth of around 60 GHz, whose corresponding dynamics is almost indistinguishable from the one of a laser subject to conventional optical feedback for the parameters considered here.

By narrowing the bandwidth of the dynamics, larger correlations between the ML and SL are expected. In other words, the lasers will synchronize more easily. The cross correlation used to measure the synchronization is defined as

$$C(t) = \langle P_m(t') P_n(t' - t) \rangle_t / \langle \sigma_{P_m} \sigma_{P_n} \rangle$$

(9)

where $\langle \ldots \rangle_t$ stands for time average, $P_{m,n} (\sigma_{P_{m,n}})$ are the mean (standard deviation) of the ML and SL output powers, respectively.

We first focus on the open-loop scheme. In this configuration, two solutions are typically found: 1) the isochronous synchronization, where $E_m(t)$ is compared with $E_n(t)$, and 2) the achronal synchronization, where $E_m(t)$ is compared with $E_n(t - \tau)$. In filtered optical feedback systems, the achronal synchronization improves when a filter is placed in front of the SL [27]. Note that in this study the current is set to $I = 2,041 I_{th} = 30$ mA, while in [27] the current is set to $I = 1,53 I_{th}$. Fig. 3 shows the correlation between the ML and SL in the cases of conventional feedback and filtered feedback with filters of 15- and 30-GHz widths. As can be seen in Fig. 3, the relation $\gamma_m = \kappa_m$ [31], [32] must be satisfied in order to obtain the largest possible correlation in the achronal solution. In general, we see that the narrower the filter, the larger the correlations that are obtained.

In the case of the isochronous synchronization and the open-loop configuration, the correlation values are not sufficiently large, when using reasonable values of $\kappa_m$, for message encoding and decoding purposes [27], [28]. Only for very large values of the coupling strength is it possible to obtain correlations that can be used for message decoding [27]. Even in this case, large amplitude messages that can compromise the security of the system have to be used [33].

For closed-loop systems, we first analyze the scenario where the ML and SL are identical. Fig. 4 shows the correlation coefficient for different coupling values. As the bandwidth of the filter in the feedback arm is reduced, larger correlations are obtained for smaller coupling strengths, as it can be seen in Fig. 4. This is due to the combination of the small reduction in the complexity of the dynamics and the filtering of the high frequencies [27].

Let us now consider a parameter mismatch in the bandwidth of the filters used in the ML and SL. While the filter of the SL is varied, the bandwidth of the ML is kept constant. Fig. 5 shows the correlations obtained when the bandwidths of the ML are 15 and 30 GHz. We can see that the maximum correlation is always obtained when the feedback loops of both lasers have identical characteristics. We have observed that for larger bandwidths, the correlation coefficient is less sensitive to the filter mismatch. When the bandwidth of the SL filter is smaller than that of the ML, the correlation and synchronization between the two lasers drastically decreases. On the contrary, when the bandwidth of the filter in the SL is larger than that of the ML, large correlations can still be obtained, although they slowly decrease as the filter of the SL is increased. We attribute that lack of symmetry to the asymmetries induced by the linewidth enhancement factor.

It is worth mentioning that, by using a filtered optical feedback loop, more variables are involved in the generation of the chaotic carrier. This would allow for a more secure transmission of messages embedded in chaotic carriers. Particularly, if a narrow filter is used in the ML, both lasers need to have almost no mismatch in their filter parameters to have enough correlation to recover a message. This would add an extra difficulty to an eavesdropper trying to recover the message being transmitted, since he would also need to know all the operating characteristics of the filter in the feedback loop of the ML. A more detailed study of the synchronization properties of two unidirectionally coupled lasers subject to filtered optical feedback can be found in [27].

IV. MESSAGE ENCODING AND DECODING

The scheme we have chosen to encode the information is chaos modulation (CM), since it is known to perform better
than other codification schemes [34]. In this scheme, the message is added by modulating the emitter’s chaotic carrier, according to the expression \( H(t) = (1 - \epsilon M(t)) P_m(t) \) [35], where \( \epsilon \) is the amplitude of the message encoding, \( M(t) \) is the message being transmitted, and \( P_m(t) \) is the power of the ML. In all of the simulations presented here, \( \epsilon = 0.04 \). The extracted message \( M'(t) \) is recovered at the receiver side as follows: \( M'(t) = (1 - \langle H(t)/P_s(t) \rangle) / \epsilon \), where \( P_s(t) \) is the power of the SL. The extraction of the message relies on the chaos synchronization, where the SL mainly reproduces the chaotic part of the injected signal \( H(t) \), i.e., \( P_s(t) = P_m(t) \) for perfect synchronization. Finally, the extracted message, \( M'(t) \), is filtered with a fifth-order Butterworth low-pass filter. The cutoff frequency of this filter is set to 1.5 times the message-encoding bit rate. From now on, we denote the decoded message after filtering as \( M''(t) \). We first study the open-loop scheme and, afterwards, the closed-loop scheme. In the open-loop scheme, we concentrate on the achronal solution, since the isochronous solution yields poor synchronization [27], [28].

### A. Open-Loop Configuration

As we said before, in this study, we focus on the achronal solution when a filter is placed in front of the SL. Fig. 6 shows the decoding of a message when a filter of 15-GHz width is used in the external cavity, for a 5-Gb/s encoding bit rate, and for the optimal condition \( \kappa_r = \gamma_m \) (see Fig. 3).

In Fig. 6, we can see by comparing the original and decoded messages that the information is not completely recovered. In particular, the ML undergoes a chaotic trajectory with higher frequency at the middle of the time series that prevents the SL to properly synchronize to it. Therefore, the decoding of the message is seriously affected. To quantify the performance of the decoding we calculate the \( Q \)-factor [35], which is used as a conventional parameter in the evaluation of the performance of the communication systems, given by the expression

\[
Q = \frac{S_1 - S_0}{(\sigma_1 + \sigma_0)}
\]

where \( S_1 \) and \( S_0 \) are the average optical power of bits “1” and “0,” and \( \sigma_1 \) and \( \sigma_0 \) are the corresponding standard deviations.

Within the parameter range considered in our simulations, the performance of the open-loop scheme for decoding a message is very poor. \( Q \)-factors are as low as 1.17 for conventional feedback and 1.37 for filtered optical feedback with a filter of 15-GHz width. These values of the \( Q \)-factors yield closed eye diagrams. Hence, the open-loop configuration does not meet the requirements needed to achieve reliable transmission and reception systems [33].

### B. Closed-Loop Configuration

Due to the best performance of the closed-loop scheme [27], we focus on this scheme for the remainder of the paper. Fig. 7 shows the decoding of a 5-Gb/s encoded message when the bandwidth of the filter is 15 GHz in both ML and SL.

In order to estimate the performance of the closed-loop scheme, we present several examples of eye diagrams and \( Q \)-factors computed for various configurations. Fig. 8(a) and (b) shows the eye diagrams when a filter of 15-GHz width and a coupling strength of \( \kappa_r = 70 \) ns\(^{-1} \) are used. For Fig. 8(a), an encoding bit rate of 1 Gb/s is considered and for Fig. 8(b) a 5-Gb/s rate is used. Open and clean eye diagrams, with \( Q \)-factors as large as 16.28 and 13.39, respectively, are found. Fig. 8(c) and (d) shows the eye diagrams when a filter of 30 GHz width is placed in the feedback arms. By looking at

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**Fig. 6.** CM message encoding and decoding in an open-loop scheme where the feedback signal of the ML is filtered with \( \Lambda_m/2\pi = 15 \) GHz, and \( \kappa_r = \gamma_m = 25 \) ns\(^{-1} \). \( P_m(t) \): ML optical intensity, \( M(t) \): transmitted message, \( H(t) \): chaotic carrier with the message, \( P_s(t - \tau) \): \( \tau \)-delayed SL optical intensity, and \( M''(t - \tau) \): \( \tau \)-delayed recovered message.

**Fig. 7.** CM message encoding and decoding in a closed loop scheme where \( \Lambda_m/2\pi = 15 \) GHz, \( \gamma_m = 25 \) ns\(^{-1} \), and \( \kappa_r = 70 \) ns\(^{-1} \). \( P_m(t) \): ML optical intensity, \( M(t) \): transmitted message, \( H(t) \): chaotic carrier with the message, \( P_s(t) \): SL optical intensity, and \( M''(t) \): recovered message.
Fig. 8. Eye diagrams with $\kappa_r = 70$ ns$^{-1}$ and (a) $\Lambda_{m,s}/2\pi = 15$ GHz, 1 Gb/s, (b) $\Lambda_{m,s}/2\pi = 15$ GHz, 5 Gb/s, (c) $\Lambda_{m,s}/2\pi = 30$ GHz, 1 Gb/s, and (d) $\Lambda_{m,s}/2\pi = 30$ GHz, 5 Gb/s.

Fig. 9. Q-factor as the coupling strength $\kappa_r$ is varied for (a) 1-Gb/s and (b) 5-Gb/s message-encoding bit rates.

Fig. 10. Q-factor as a function of the SL filter bandwidth mismatch, with $\kappa_r = 70$ ns$^{-1}$, for (a) 1-Gb/s and (b) 5-Gb/s message-encoding bit rates.

Fig. 11. Q-factor versus bias current in the closed-loop configuration for a constant coupling strength $\kappa_r = 60$ ns$^{-1}$ and an encoding bit rate of 1 Gb/s.

Fig. 8(c) and (d), one can see that the eye diagrams are still very open, with Q-factors as large as 15.35 for the 1-Gb/s encoding bit rate and 10.36 for the 5-Gb/s rate. However, the quality of the decoding is better for the narrow filter. We have found that the larger the bandwidth of the filter, the larger coupling strength is required to obtain a comparable Q-factor. It is worth mentioning that the Q-factors for the conventional feedback case are 13.31 for a 1 Gbit/s encoding bit rate and 9.17 for a 5-Gb/s rate.

Fig. 9 shows how the Q-factor depends on the coupling strength between the ML and SL. Fig. 9(a) is for a 1-Gb/s bit rate, and Fig. 9(b) for a 5-Gb/s bit rate. From these two panels, it can be seen that narrow filters yield better Q-factors; moreover, smaller coupling strengths are needed. For conventional optical feedback, the values of the coupling strength needed to obtain comparable Q-factors are slightly larger than those needed for the filter of 30-GHz width. If we keep on increasing the coupling strength, the Q-factor degrades, i.e., there is an optimum coupling strength where the largest Q-factor is obtained [33]. We have observed that the synchronization error increases as well when the coupling strength is larger than this optimum value, making it more difficult to extract the message.

Now, we check the influence of a mismatch in the filter bandwidths. Fig. 10 shows the Q-factor when a parameter mismatch in the bandwidth of the filters is introduced. In Fig. 10(a) and (b), the bandwidths of the ML are kept constant to 15 and 30 GHz, respectively, while the bandwidth of the SL is scanned. In Fig. 10(a), we use a 1-Gb/s bit rate and, in Fig. 10(b), a 5-Gb/s bit rate is used. From these panels, it can be seen that a mismatch between the filters degrades significantly the Q-factor, and a good message recovery is then almost impossible. The x-axis in Fig. 10 has been renormalized to ease the comparison between the two filters. For a filter of 15-GHz width, the Q-factor drops below 5 when $|A_m - A_s| = 2$ GHz for the 1-Gb/s rate, and $|A_m - A_s| = 0.9$ GHz for the 5-Gb/s rate. For a filter of 30-GHz width, we can see that the Q-factor drops below 5 when $|A_m - A_s| = 4$ GHz for the 1-Gb/s rate, and $|A_m - A_s| = 1.8$ GHz for the 5-Gb/s rate. In general, the use of a narrower filter in the ML makes it more difficult for an eavesdropper to find a matching filter in the receiver.

In Fig. 10, we see that a mismatch in the bandwidths of ML and SL can break the message decoding so that the filter bandwidths can be used as an additional key in chaos-based communications. Besides the filter bandwidth, the detuning between the solitary laser frequency and the filter central frequency ($\Delta_m$) can also be used as an additional secure key. We have checked that values of $\Delta_m$ in a range from $-10$ to 10 GHz do not significantly change the ML dynamics. Hence, any value of $\Delta_m$ in the previous range can be used as a secure key.

We end this section by showing the influence of the bias current on the message-decoding performance. In Fig. 11, we present the results for conventional optical feedback and filtered optical feedback, with filters of 15- and 30-GHz widths. We have chosen an intermediate value of the coupling strength, $\kappa_r = 60$ ns$^{-1}$. Fig. 11 shows that the Q-factor is larger for larger injection currents. This can be explained in terms of the
complexity of the dynamics of the ML. It has been shown that the complexity decreases at large currents as a consequence of the gain compression coefficient [36].

V. CONCLUSION

We have studied two unidirectionally coupled semiconductor lasers that are driven into a chaotic regime by means of a filtered optical feedback loop. First, we have shown that these lasers can synchronize provided that the filter parameters are matched in the ML and SL. Second, we have shown that the encoding and decoding of a message embedded in a chaotic carrier, generated by a filtered optical system, is feasible.

Our results show that the decoding of encrypted messages yields larger $Q$-factors when a closed-loop scheme is used. The open-loop scheme would be preferred for a preliminary practical implementation since it is known to be mechanically more stable. In this scheme, the achronal solution provides identical synchronization when a filter, similar to the one in the ML, is placed in front of the SL. However, a good message decoding with the achronal solution is, in practice, very difficult.

The synchronization properties of filtered optical feedback systems are similar to the ones found in conventional optical feedback schemes. However, the bandwidth reduction, larger $Q$-factors, and the requirement of smaller coupling strengths make this new scheme very attractive for practical applications.

Finally, we have reported on the influence of a parameter mismatch in the filters of the ML and SL. The $Q$-factors decrease rapidly when the filters are different. This decrease is directly related to a degradation of the synchronization with the mismatch. A filter in the feedback loop of the ML adds an extra degree of difficulty for an eavesdropper trying to extract the encoded information. Actually, it is very demanding to extract the bandwidth of the filter from the time series. We are aware that other alternatives to enhance the security of the communication system can be used, but the system we propose is still a very interesting all-optical option that can be even integrated in compact sources if necessary.

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