Use of disdrometer data to evaluate the relationship of rainfall kinetic energy and intensity (KE-I)

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Abstract

Determination of rainfall kinetic energy (KE) is required to calculate erosivity, the ability of rainfall to detach soil particles and initiate erosion. Disdrometers can measure rainfall KE by measuring raindrop size and velocity. In the absence of such devices, KE is usually estimated with empirical equations that derive KE from measured rainfall intensity (I). We evaluated the performance of 14 different KE–I equations to estimate the 1 min KE and event total KE, and compared these results with 821 observed rainfall events recorded by an optical disdrometer in the inner Ebro Basin, NE Spain. We also evaluated two sources of bias when using such relationships: bias from use of theoretical raindrop terminal velocities instead of measured values; and bias from time aggregation (recording rainfall intensity every 5, 10, 15, 30, and 60
min). Empirical relationships performed well when complete events were considered ($R^2 > 0.90$), but performed poorly for within-event variation (1 min resolution). Also, several of the KE-I equations had large systematic biases. When raindrop size is known, estimation of terminal velocities by empirical laws led to overestimates of raindrop velocity and KE. Time aggregation led to large under-estimates of KE, although linear scaling successfully corrected for this bias.

**Keywords:** Rainfall kinetic energy; rainfall erosivity; drop size distribution (DSD); raindrops fall velocity; kinetic energy-intensity (KE-I) relationship

1. **Introduction**

Determination of rainfall kinetic energy (KE) and other rainfall parameters, such as amount (P) and intensity (I), is essential for estimation of soil erosion. Rainfall KE depends on raindrop size and velocity. The KE of raindrops causes splashing of soil particles and initiates soil erosion due to disaggregation and mobilization of the soil (Wang et al., 2014). Measurement of rainfall KE and I allows estimation of rainfall erosivity -- the ability of rainfall to erode soil by splash and runoff (Cevasco et al., 2015; Martín-Fernández and Martínez-Núñez, 2011). Rainfall erosivity is a key parameter used to estimate soil loss by water in spatially-distributed water erosion models such as the USLE (Wischmeier and Smith, 1958), RUSLE (Renard et al., 1996), LISEM (De Roo et al., 1996), EUROSEM (Morgan et al., 1998a and b), and WaTEM/SEDEM (Van Oost et al., 2000; Van Rompaey et al., 2001, Verstraeten et al., 2002). Once all parameters are properly estimated, these models allow estimation of erosion rates at relevant spatial scales, which is crucial for developing
sustainable conservation practices and land-use planning (Pimentel et al., 1999; Bilotta et al., 2012).

The KE \((ke, \text{measured in } J)\) of a raindrop is defined as:

\[
ke = \frac{1}{2}mv^2 = \frac{1}{12}10^{-3}\pi \rho v^2 D^3 ,
\]

where \(m\) is mass (g), \(\rho\) is the density of water (1 g cm\(^{-3}\)), \(v\) is velocity (m s\(^{-1}\)), and \(D\) is diameter (mm).

Many methods can be used to measure raindrop size and velocity, such as the flour pellet or stain paper methods (Wischmeier and Smith, 1958), electronic devices such as high-speed video cameras (Kinnell, 1980; McIsaac, 1990), acoustic disdrometers (Rosewell, 1986), and optical disdrometers (Cerro et al., 1998; Petan et al., 2010; Angulo-Martínez et al., 2012). These methods have certain limitations, including: \((i)\) the sample interval at which measurements are taken, which can be configured if the instrument is automatic (cameras or disdrometers), but is unknown in other cases (Salles et al., 2002); \((ii)\) difficulties in measuring raindrop velocity (Randue et al., 2002); and \((iii)\) uncertainties of instrument accuracy (Angulo-Martínez and Barros, 2015). All of these methods and instruments provide accurate measurements of raindrop size, but only optical disdrometers and video recorders can accurately measure raindrop velocity. If measurements of raindrop size are only available, then terminal velocity can be estimated from empirical laws based on raindrop diameter (e.g. Atlas et al., 1973; Beard, 1976; Atlas and Ulbrich, 1977; Uplinger, 1981).

Measurement of raindrop KE requires great experimental efforts, as well as expensive and accurate instruments. Historically, measurement of rainfall KE was confined to research studies, and many of them focused on finding the best way to estimate rainfall KE from more readily available records, such as rainfall intensity \((I)\) (Wischmeier and Smith, 1958; Hudson, 1963; Kinnell, 1980). Marshall and Palmer (1948) found a relationship between raindrop size
and rainfall intensity and proposed a two-parameter exponential equation as the best descriptor. Their research indicated that the number of drops and drop sizes increased with rainfall intensity to a certain threshold, after which the number of drops remains constant or only increases slowly. Since this first study, other researchers have proposed several KE–I equations in which the relationship was: (i) logarithmic (Wischmeier and Smith, 1958; Davison et al., 2005); (ii) power-law (Park et al., 1980); (iii) polynomial (Carter et al., 1974); (iv) linear (Sempere-Torres et al., 1992); or (v) exponential (Kinnell, 1980; Brown and Foster, 1987). The exponential function is currently preferred (Van Dijk et al., 2002). The power-law and logarithmic functions imply an infinite increase of rainfall KE with intensity, but research has shown that rainfall KE reaches an upper limit (Hudson, 1963; Carter et al., 1974; Kinnell, 1980; Rosewell, 1986; Brown and Foster, 1987) at an intensity of about 70 mm h⁻¹ (Hudson, 1963; Wischmeier and Smith, 1978).

Rainfall KE and I are functions of the local climate and precipitation microphysics of the location where they are measured. Thus, empirically-derived KE–I equations are only valid for the regions where the data was measured or for regions with similar geographical and meteorological characteristics. This has motivated researchers to develop individual KE–I formulas for different geographic locations. The most-used KE–I relationship is the exponential equation proposed by Brown and Foster (1987). Renard et al. (1997) used this equation to estimate the rainfall erosivity (R-factor) in the Revised Universal Soil Loss Equation (RUSLE). The Brown and Foster (1987) equation is valid for the continental United States. For parameter determination, they enlarged the data used in Wischmeier and Smith (1958) by adding new measurements taken at Holy Springs, Mississippi (US); however, the intensity range of their data was relatively low, so they used the upper KE limit indicated by Rosewell (1986) from studies in Australia. The Brown and Foster equation has been widely applied worldwide for estimation of rainfall erosivity. Some authors used correction factors
when the rainfall intensity input data had a different time resolution than that needed for this equation (Panagos et al., 2015), but others did not use this correction (Diodato, 2004; Angulo-Martínez and Beguería, 2009).

Our comprehensive review of the literature indicated that researchers have proposed over 14 different KE–I exponential relationships worldwide. The objective of the present research is to critically analyze rainfall KE estimates from these equations and compare them with accurate measurements of rainfall KE recorded during 4 years of monitoring natural rainfall with an optical disdrometer in Zaragoza (NE Spain). Our purpose is not to identify the equation that provides the best fit to the local characteristics of rainfall KE, but to address several more general questions. In particular, (i) we investigated the influence of measured vs. estimated raindrop terminal velocity, (ii) compared and evaluated several rainfall KE estimates, and (iii) investigated the influence of time aggregation on the application of the different KE–I formulas.

2. Materials and methods

2.1. Experimental setup and measurement of rainfall kinetic energy

Rainfall characteristics under natural conditions were monitored at Aula Dei Experimental Station in the central Ebro Valley, NE Spain (41°43’30”N, 0°48’39”W, 230 m. a.s.l.) using a Thies Clima laser precipitation monitor (LPM) (Adolf Thies GmbH & Co. KG, 2011) from 04/03/2010 to 28/05/2014. One-minute rainfall observations were recorded, and rainfall episodes were identified according to the following criteria: a rainfall episode began when rainfall was continuous for at least 10 min; two separate rainfall episodes were separated by a period of at least 1 h without rain.
The LPM is an Optical Spectro Pluviometer (optical disdrometer) that measures the diameter and velocity of every raindrop whose diameter is greater than 0.16 mm at ground level. Donnadieu et al. (1969) initially developed an LPM to derive the velocity and diameter of hydrometeors from measurements of the duration and amplitude of obscurations between an infrared laser beam and a receiver (sample area: 51.4 cm²). The geometry of the beam limits the estimation of fall velocity to the vertical component (Salles and Poesen, 1999), and one input point. Hauser et al. (1984) provides more details on the optical disdrometer. The Thies Clima LPM classifies each measured drop into 1 of 22 diameters (mm) and 1 of 20 velocities (m s⁻¹) (see details in Table 6 of Adolf Thies GmbH & Co. KG, 2011). From these data, several rainfall variables are integrated every minute. In this study we focused on rainfall intensity (I, mm h⁻¹), rainfall amount (P, mm), and rainfall KE expressed in units of Joules per square meter per millimeter of rain (KE, J m⁻² mm⁻¹).

We calculated rainfall KE per minute by first determining the total kinetic energy (ke_sum) per minute from multiplication of the kinetic energy of each drop in each diameter and velocity class by the number of drops in each size and velocity class. Then, we obtained the rainfall KE per surface area and volume of rain by dividing by the sample area of the device (a, 0.00514 m²) and rainfall per minute (Pr):

\[
KE_0 = \frac{ke_{sum}}{a Pr} = \frac{\sum N_i D_i^2 \rho v_j}{12 \times 10^{-3} a Pr},
\]

where \(N\) is the number of drops in a size and velocity class, \(D_i\) is the mean diameter for class \(i\) (mm), \(\rho\) is the density of water (1 g cm⁻³), and \(v_j\) is the mean speed for velocity class \(j\) (m s⁻¹).

We use the term \(KE_0\), as in eq. (2), to refer to the KE estimated from direct disdrometric drop size distribution (DSD) and \(V\) every minute.
To determine the effect of raindrop velocity on the parameterization of rainfall erosivity in comparison with raindrop theoretical terminal velocity, we calculated rainfall KE using 3 empirical raindrop terminal velocity equations that calculate the relationship between raindrop diameter and terminal velocity under stable conditions (for which we use the terms $KE_1$, $KE_2$ and $KE_3$).

Based on data from Gunn and Kinzer (1949), Atlas and Ulbrich (1977) proposed a power law equation to estimate raindrop terminal velocity:

$$V_{Atl} = 17.67(D_i/10)^{0.67},$$  \hspace{1cm} (3)

where $D_i$ is the mean raindrop diameter of class i. $D_i$ is expressed in cm in the original formulation, so we divided $D_i$ by 10 for use with our data. This equation was used by Wischmeier and Smith (1958, 1978), Brown and Foster (1987), and others in their calculations of rainfall KE.

Cerro et al. (1998) compared the terminal velocity equation proposed by Uplinger (1981) with data from Beard (1976) and raindrop velocities measured with an optical disdrometer. They found a difference of 17%, and used Uplinger’s equation in their estimation of unit energy. This led to the following estimate of raindrop terminal velocity:

$$V_{Ulp} = 4.874D_i \exp(-0.195D_i),$$  \hspace{1cm} (4)

Van Dijk et al. (2002) used a cubic polynomial function as the best descriptor of raindrop terminal velocity based on the investigations of Beard and Pruppacher (1969) and Beard (1976):

$$V_{VD} = 0.0561D_i^3 - 0.912D_i^2 + 5.03D_i - 0.254,$$  \hspace{1cm} (5)
Thus, rainfall kinetic energy estimates ($KE_1$, $KE_2$, and $KE_3$ in J m$^{-2}$ mm$^{-1}$) were obtained by replacing $v_j^2$ in equation 2 with $V_{Ati}$, $V_{Upl}$, or $V_{VD}$, respectively.

### 2.2 Rainfall kinetic energy–intensity relationships

In the absence of disdrometric data on raindrop size and velocity, $KE$ can be estimated by empirical equations that use rainfall intensity ($I$) at high temporal resolution (less than 30 min between consecutive records) as input data. The high correlation between $KE$ and $I$ (Kinnell, 1973) allows calculation of empirical equations that best describe the relationship of these variables. Thus, there is general agreement on use of an exponential equation of the following form as the best descriptor of the KE–I relationship (Van Dijk, et al., 2002):

$$KE \ (J \ m^{-2} \ mm^{-1}) = e_{max} \ [1 - a \ \exp(-bI)],$$

where $e_{max}$ is the average maximum $KE$ value measured at high intensity rates, and $a$ and $b$ are coefficients modelling the form of the curve. The parameters $e_{max}$ and $a$ determine the minimum energy and parameter $b$ models the general shape of the curve.

A literature review indicated that previous researchers had proposed 14 different exponential KE–I relationships, calibrated at different sites worldwide, including the universal equation from Van Dijk et al. (2002) (Table 1). Most of these studies measured DSD using acoustic or optical disdrometers at resolutions of 1 min. These equations were derived at locations with altitudes of 3 to 1230 m a.s.l. and rain rates of 0 to 300 mm h$^{-1}$.

We compared the KE estimates from each of the 14 equations with observations at our monitoring site ($KE_0$).
Table 1. Exponential terms used to describe the relationship of rainfall kinetic energy (KE, J m^(-2) mm^(-1)) and rainfall intensity (I, mm h^(-1)) (KE-I). Abbreviations: M, method of DSD measurement; FP, Flour Pellet; AD, Acoustic Disdrometer; OD, Optical Disdrometer; C, Camera; raindrop velocity (V); Atl, terminal velocity from Atlas and Ulbrich (1977); Upl, terminal velocity from Uplinger (1981); T, theoretical; m, measured; SI: sampling interval; NA, data not available.

<table>
<thead>
<tr>
<th>Reference</th>
<th>KE — I relationship</th>
<th>Characteristics</th>
<th>Rain range (mm h^(-1))</th>
<th>Altitude m.a.s.l.</th>
<th>Nº obs</th>
</tr>
</thead>
<tbody>
<tr>
<td>E1</td>
<td>29[1 - 0.72 exp(-0.05I)]</td>
<td>M=FP V=Atl SI=NA</td>
<td>0 – 161</td>
<td>180</td>
<td>315</td>
</tr>
<tr>
<td>E2</td>
<td>38.4[1 - 0.54 exp(-0.029I)]</td>
<td>M=OD V=Upl SI=30s.</td>
<td>1 - 150</td>
<td>25</td>
<td>3600</td>
</tr>
<tr>
<td>E3</td>
<td>35.9[1 - 0.56 exp(-0.034I)]</td>
<td>M=AD (Joss) V=T SI=1 min</td>
<td>4 - 103</td>
<td>21</td>
<td>8190</td>
</tr>
<tr>
<td>E4</td>
<td>30.8[1 - 0.55 exp(-0.03I)]</td>
<td>M=AD (Joss) V=Atl SI=1 min</td>
<td>2.8-142</td>
<td>44</td>
<td>8030</td>
</tr>
<tr>
<td>E5</td>
<td>36.8[1 - 0.69 exp(-0.038I)]</td>
<td>M=AD V=T SI=NA</td>
<td>12 - 120</td>
<td>50</td>
<td>18</td>
</tr>
<tr>
<td>E6</td>
<td>29.3[1 - 0.28 exp(-0.018I)]</td>
<td>M=C V=NA SI=NA</td>
<td>2 - 309</td>
<td>3</td>
<td>200</td>
</tr>
<tr>
<td>E7</td>
<td>29.2[1 - 0.89 exp(-0.048I)]</td>
<td>M=FP V=NA SI=NA</td>
<td>19 - 229</td>
<td>1230</td>
<td>50</td>
</tr>
<tr>
<td>E8</td>
<td>25.75[1 - 0.54 exp(-0.05I)]</td>
<td>M=OD V=m SI=5 min.</td>
<td>0.1 - 142</td>
<td>58</td>
<td>4241</td>
</tr>
<tr>
<td>E9</td>
<td>29.8[1 - 0.60 exp(-0.07I)]</td>
<td>M=OD V=Atl SI=1 min.</td>
<td>0.1 - 288</td>
<td>405</td>
<td>15821</td>
</tr>
<tr>
<td>E10</td>
<td>31.9[1 - 0.60 exp(-0.055I)]</td>
<td>M=OD V=Atl SI= 1 min.</td>
<td>0.1 - 220</td>
<td>595</td>
<td>15757</td>
</tr>
<tr>
<td>Reference</td>
<td>KE — I relationship</td>
<td>Characteristics</td>
<td>Rain range (mm h(^{-1}))</td>
<td>Altitude m.a.s.l.</td>
<td>Nº obs</td>
</tr>
<tr>
<td>--------------------</td>
<td>---------------------------------------------------------</td>
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<td>-----------------------------</td>
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<td>-------</td>
</tr>
<tr>
<td>E11 Rosewell, 1986a. Queensland, Australia</td>
<td>(26.4[1 - 0.67 \exp(-0.035I)])</td>
<td>M=AD V=NA SI= 1 min.</td>
<td>1 - 161</td>
<td>25</td>
<td>6360</td>
</tr>
<tr>
<td>E12 Rosewell, 1986b. New South Wales, Australia</td>
<td>(28.1[1 - 0.6 \exp(-0.04I)])</td>
<td>M=AD V=NA SI= 1 min</td>
<td>1 - 146</td>
<td>305</td>
<td>13438</td>
</tr>
<tr>
<td>E13 Sánchez-Moreno et al., 2012. Santiago Island, Cape Verde</td>
<td>(35 {1 - 0.79 \exp (-0.03I)})</td>
<td>M=OD V=Atl SI= 3 min.</td>
<td>0 - 157</td>
<td>321</td>
<td>82080</td>
</tr>
<tr>
<td>E14 Van Dijk et al., 2002. Universal</td>
<td>(28.3[1 - 0.52 \exp(-0.042I)])</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

2.3 Effect of time aggregation on KE-I relationships

The original formulations for estimating KE used hyetograph data (Ellison, 1944; Bisal, 1960; Wischmeier and Smith, 1958; Williams and Sheridan, 1991), and the relationship of I with KE was usually estimated using non-automated methods, such as the flour pellet or stained paper methods. The *USDA Agricultural Handbook No. 703* (Renard et al., 1997) specifies a minimum of 15 min of time-aggregated pluviograph data to compute the index proposed by Brown and Foster (E1; 1987), although linear correction can be used for coarser time resolutions (Renard et al., 1996). The sample interval influences the coefficients used in the KE–I equations (Petan et al., 2010). Thus, to obtain the best KE estimate from each equation, the input data should be time-aggregated at the same temporal resolution as the original equation (Williams and Sheridan, 1991). If this is not possible, then a conversion factor should be used (Yin et al., 2007; Panagos et al., 2015). In the present article, we evaluated the effect of time aggregation of rainfall intensity data on KE estimates. Thus, 1 min rainfall intensity was aggregated every 5, 10, 15, 30, and 60 min by calculating the means of 5, 10,
15, 30, and 60 consecutive records. Only complete consecutive records were considered. Events whose durations were less than the time aggregation interval were excluded. KE was then estimated using the 14 different KE–I equations (Table 1) and the aggregated rainfall intensities, and compared with observed KE₀ (1 min resolution).

2.4. Validation criteria

The observed (KE₀) and estimated (KE₁, KE₂, KE₃, E₁...E₁₄) KE values were compared using standard descriptive measurements of centrality and dispersion, and using several error and goodness-of-fit statistics. Among the former, we used the mean absolute error (MAE), a measure of the magnitude of errors in the same scale as the variable, and percent bias (PBIAS), a measure of percent deviation from the mean observed value. We used R² to assess goodness–of–fit, because there were no substantial differences of R² with other more refined, goodness–of–fit statistics such as the NSE coefficient of efficiency (Nash and Sutcliffe, 1970), or the agreement index D (Willmott, 1981) (data not shown). The validity of the empirical models was also evaluated graphically by goodness-of-fit plots.

3. Results

3.1 General description

We identified 821 rainfall episodes (107,623 1 min minute observations) during the 4 year study period. We lost approximately 1% of the data due to technical issues (power supply failures, data communication problems, etc.). Some precipitation (I > 0 mm h⁻¹) was registered in 80% of the data (85,993 min) during these events.
The 1-min rainfall intensity ranged from 0.0005 mm h\(^{-1}\) to 214 mm h\(^{-1}\), but almost 80% of these measurements were less than 6 mm h\(^{-1}\), and only 0.1% of measurements exceeded 50 mm h\(^{-1}\) (Figure 1). The KE ranged between 0.01 and 54.2 J m\(^{-2}\) mm\(^{-1}\), and about 80% of the observations were below 10 J m\(^{-2}\) mm\(^{-1}\). The relationship of KE and I was asymptotic, and KE became nearly constant at an I value of about 65 mm h\(^{-1}\). Table 2 provides descriptive statistics of all observed and calculated variables.

**Table 2.** Descriptive statistics (1 min resolution) of observed and calculated variables.

Rainfall intensity (I, mm h\(^{-1}\)), rainfall amount (P, mm), and observed and estimated KE (J m\(^{-2}\) mm\(^{-1}\)) over the entire study period. Means, maxima, and standard deviations of KE and E are in units of J m\(^{-2}\) mm\(^{-1}\), and total cumulative kinetic energy is in units of J m\(^{-2}\).

<table>
<thead>
<tr>
<th>One-minute time resolution</th>
<th>Mean</th>
<th>Max</th>
<th>Sd</th>
<th>Cum. energy</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>0.78</td>
<td>213.2</td>
<td>4.00</td>
<td>-</td>
</tr>
<tr>
<td>P</td>
<td>0.01</td>
<td>3.55</td>
<td>0.07</td>
<td>-</td>
</tr>
<tr>
<td>KE(_0)</td>
<td>7.16</td>
<td>54.22</td>
<td>5.04</td>
<td>16458.3</td>
</tr>
<tr>
<td>KE(_1)</td>
<td>8.04</td>
<td>44.18</td>
<td>6.04</td>
<td>21151.5</td>
</tr>
<tr>
<td>KE(_2)</td>
<td>8.20</td>
<td>41.02</td>
<td>5.97</td>
<td>21298.1</td>
</tr>
<tr>
<td>KE(_3)</td>
<td>7.92</td>
<td>36.61</td>
<td>5.88</td>
<td>20340.5</td>
</tr>
<tr>
<td>E1</td>
<td>8.75</td>
<td>29.00</td>
<td>1.56</td>
<td>16986.3</td>
</tr>
<tr>
<td>E2</td>
<td>18.05</td>
<td>38.36</td>
<td>1.14</td>
<td>25953.1</td>
</tr>
<tr>
<td>E3</td>
<td>16.23</td>
<td>35.89</td>
<td>1.21</td>
<td>24131.9</td>
</tr>
<tr>
<td>E4</td>
<td>14.19</td>
<td>30.77</td>
<td>0.95</td>
<td>20645.3</td>
</tr>
<tr>
<td>E5</td>
<td>12.01</td>
<td>36.79</td>
<td>1.63</td>
<td>21331.0</td>
</tr>
<tr>
<td>E6</td>
<td>21.20</td>
<td>29.12</td>
<td>0.38</td>
<td>25520.7</td>
</tr>
<tr>
<td>E7</td>
<td>3.97</td>
<td>29.20</td>
<td>1.90</td>
<td>13265.3</td>
</tr>
<tr>
<td>E8</td>
<td>12.27</td>
<td>25.75</td>
<td>1.04</td>
<td>18516.0</td>
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<tr>
<td>E9</td>
<td>12.64</td>
<td>29.80</td>
<td>1.61</td>
<td>21043.1</td>
</tr>
<tr>
<td>E10</td>
<td>13.39</td>
<td>31.90</td>
<td>1.51</td>
<td>21800.2</td>
</tr>
<tr>
<td>E11</td>
<td>9.11</td>
<td>26.39</td>
<td>1.08</td>
<td>15503.1</td>
</tr>
<tr>
<td>E12</td>
<td>11.66</td>
<td>28.10</td>
<td>1.11</td>
<td>18392.5</td>
</tr>
<tr>
<td>E13</td>
<td>7.89</td>
<td>34.95</td>
<td>1.55</td>
<td>16606.8</td>
</tr>
<tr>
<td>E14</td>
<td>13.97</td>
<td>28.30</td>
<td>1.00</td>
<td>20382.7</td>
</tr>
</tbody>
</table>

**Figure 1.** Kernel density of observed one-minute rainfall intensity (I, mm h\(^{-1}\)) and kinetic energy (KE\(_0\), J m\(^{-2}\) mm\(^{-1}\)).
Figure 2 shows time series of rainfall intensity, observed ($KE_0$), and estimated (E1) kinetic energy for 4 rainfall episodes. Initially, we identified long events that had relatively low rainfall rates, but that were quite erosive if the whole event was considered (A: June $6^{th}$, 2013), 6 h and 20 min, $P = 18$ mm and C: November $16^{th}$, 2013, 24 h, $P = 34$ mm); and short but very intense events (B: July $13^{th}$, 2013, 1 h and 30 min, $P = 44$ mm and D: April $21^{st}$, 2014, 30 minutes, $P = 8$ mm). Thus, while the time series of E1 and I appear similar, the time series of $KE_0$ appears very different. Unlike what is assumed in fixed KE-I equations, changes in the KE-I relationship occur within an event and between events.
Figure 2. Examples of 4 rainfall events: one-minute resolution time series of rainfall intensity (I, mm h⁻¹, red), observed kinetic energy (KE₀, J m⁻² mm⁻¹, green) and estimated kinetic energy (E₁, blue).

Figure 3 shows the number of drops of different diameters and velocities in the same 4 events. The smooth lines show the D-V theoretical relationships according to Atlas and Ulbrich.
The largest number of raindrops occurred when they had the smallest diameters (0.1 mm < D < 0.7 mm). The most intense events had more raindrops and larger raindrops (1.8 mm < D < 4 mm; Event B). Overall, there was good agreement between the observed D-V scatter and the theoretical relationships. Nevertheless, for the most intense rainfall events (e.g. event B), many large raindrops fell slower than expected. The Van Dijk et al. (2002) D-V equation provided a better fit for smaller drops (0.1 mm < D < 0.7 mm), whereas the Atlas and Ulbrich (1977) D-V equation provided a better fit for mid-size drops (1 mm < D < 3 mm). The Uplinger (1981) equation performed similarly to the Van Dijk equation, but it overestimated the D-V relationship for the smallest drops.
**Figure 3.** Number of raindrops per diameter class (x-axis) and velocity class (y-axis) in 4 rainfall events. The smooth lines show the raindrop diameter-velocity relationship as modeled by Atlas and Ulbrich (1977), in blue; by Uplinger (1981), in red; and by Van Dijk et al. (2002), in black.

3.2 Effect of raindrop terminal velocity on KE estimates: differences between measured and modelled values

Raindrop size and velocity have high variability, based on comparison of 4 selected rainfall events (Figure 3). Although the theoretical relationships provide a good fit to the average values, many small drops had higher measured velocities than expected from theory, and many large drops had lower measured velocities than expected from theory.
When we compared measured kinetic energy ($KE_0$) with estimates, assuming theoretical terminal raindrop fall velocity ($KE_1$, $KE_2$, and $KE_3$; Fig. 4), the 3 expressions yielded overestimates that were proportional to rainfall intensity. Despite an overall good fit (Table 3), with values of $R^2$ greater than 0.80 and 0.90 for the 1 min and the event scale respectively, there was a significant positive bias that was greater than 10% at both scales. A closer examination at different intensity intervals revealed greater differences at high intensities (above 60 mm h$^{-1}$), at which the over-estimates exceeded 50%.

Table 3. Goodness of fit and validation statistics of rainfall kinetic energy estimates $KE_1$ to $KE_3$ (J m$^{-2}$ mm$^{-1}$) with respect to the observed values ($KE_0$). Mean absolute error (MAE), percent bias (PBIAS), and coefficient of determination ($R^2$) are given at 1 min and event scales, and for rainfall intensities lower and higher than 60 mm h$^{-1}$.

<table>
<thead>
<tr>
<th>One-minute time resolution</th>
<th>Event scale</th>
</tr>
</thead>
<tbody>
<tr>
<td>$KE_1$</td>
<td>$KE_2$</td>
</tr>
<tr>
<td>MAE $^{1}$</td>
<td>MAE $^{1}$</td>
</tr>
<tr>
<td>1.21</td>
<td>1.27</td>
</tr>
<tr>
<td>PBIAS $^{2}$</td>
<td>PBIAS $^{2}$</td>
</tr>
<tr>
<td>12.3</td>
<td>14.6</td>
</tr>
<tr>
<td>$R^2$</td>
<td>$R^2$</td>
</tr>
<tr>
<td>0.82</td>
<td>0.83</td>
</tr>
<tr>
<td>$^{1}$One minute time resolution at intensities $&lt;60$ mm h$^{-1}$</td>
<td>$^{1}$One minute time resolution at intensities $&gt;60$ mm h$^{-1}$</td>
</tr>
<tr>
<td>$KE_1$</td>
<td>$KE_2$</td>
</tr>
<tr>
<td>MAE $^{1}$</td>
<td>MAE $^{1}$</td>
</tr>
<tr>
<td>3.01</td>
<td>3.11</td>
</tr>
<tr>
<td>PBIAS $^{2}$</td>
<td>PBIAS $^{2}$</td>
</tr>
<tr>
<td>21</td>
<td>21.9</td>
</tr>
<tr>
<td>$R^2$</td>
<td>$R^2$</td>
</tr>
<tr>
<td>0.4</td>
<td>0.45</td>
</tr>
<tr>
<td>$^{1}$One minute time resolution at intensities $&lt;60$ mm h$^{-1}$</td>
<td>$^{1}$One minute time resolution at intensities $&gt;60$ mm h$^{-1}$</td>
</tr>
<tr>
<td>$KE_1$</td>
<td>$KE_2$</td>
</tr>
<tr>
<td>MAE $^{1}$</td>
<td>MAE $^{1}$</td>
</tr>
<tr>
<td>2.79</td>
<td>2.79</td>
</tr>
<tr>
<td>PBIAS $^{2}$</td>
<td>PBIAS $^{2}$</td>
</tr>
<tr>
<td>18.9</td>
<td>18.9</td>
</tr>
<tr>
<td>$R^2$</td>
<td>$R^2$</td>
</tr>
<tr>
<td>0.51</td>
<td>0.51</td>
</tr>
</tbody>
</table>

Among the 3 methods, $KE_2$ had the lowest MAE and PBIAS, and the greatest $R^2$, but the magnitude of the differences of $KE_3$ from $KE_1$ and $KE_2$ were marginal. $KE_2$ produced the worst results. Overestimation of KE, especially at high rainfall intensities, can be explained by the presence of slower velocities than expected for large raindrops during very intense events (Figure 3).

Figure 4. Relationship of rainfall intensity (I, mm h$^{-1}$) and energy ($KE_0$, J m$^{-2}$ mm$^{-1}$) based on measurements at 1 min intervals. The smooth lines are from a locally weighted scatterplot
3.3 Evaluation of rainfall kinetic energy – intensity (KE₀–I) relationships

Most of the rainfall events had relatively low I and KE, with occasional peaks. Proper evaluation of rainfall KE and its erosive potential should consider the total KE at different time scales. Thus, we must consider KE from minute to minute (which allows prediction of within-event peaks) and at the event scale (which accounts for the cumulative KE associated with a rainfall event). Figure 5 shows the density plot of KE₀–I at 1 min resolution, including a locally weighted scatterplot smoothing (LOESS) model for KE₀–I and theoretical terminal velocity models for KE₁ to KE₃. The theoretical models over-predicted KE, especially at the highest rainfall intensities (I > 10 mm h⁻¹). This was expected due to the greater than expected...
number of raindrops with lower velocities than the theoretical model (Figure 3). This overestimate was propagated to the event-scale, although the degree of over-estimation was reduced, most notably in the lower range of I. Tables 3 and 4 show the goodness-of-fit statistics. Despite the high $R^2$ at the 1 min (~0.85) and event (~0.95) time scales, the three models had quite large MAEs and the positive bias was greater than 10% in all cases.

Table 4. Goodness of fit and validation statistics of rainfall kinetic energy estimates (J m$^{-2}$ mm$^{-1}$) at 1 min resolution, at the event scale, and by intensity ranges. Analysis shows 1min kinetic energy records where I less than and greater than 60 mm h$^{-1}$. Observed KE values: $KE_0$, Simulated KE values: $E1...E14$.

<table>
<thead>
<tr>
<th>One-minute scale</th>
<th>Event scale</th>
<th>One-min $I&lt;60$ mm h$^{-1}$</th>
<th>One-min $I&gt;60$ mm h$^{-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>MAE</td>
<td>PBIAS</td>
<td>$R^2$</td>
</tr>
<tr>
<td>$E1$</td>
<td>3.86</td>
<td>22.3</td>
<td>0.27</td>
</tr>
<tr>
<td>$E2$</td>
<td>11.08</td>
<td>152.2</td>
<td>0.21</td>
</tr>
<tr>
<td>$E3$</td>
<td>9.36</td>
<td>126.8</td>
<td>0.23</td>
</tr>
<tr>
<td>$E4$</td>
<td>7.57</td>
<td>98.2</td>
<td>0.22</td>
</tr>
<tr>
<td>$E5$</td>
<td>5.75</td>
<td>67.8</td>
<td>0.24</td>
</tr>
<tr>
<td>$E6$</td>
<td>14.15</td>
<td>196.1</td>
<td>0.17</td>
</tr>
<tr>
<td>$E7$</td>
<td>3.98</td>
<td>-44.5</td>
<td>0.26</td>
</tr>
<tr>
<td>$E8$</td>
<td>6.00</td>
<td>71.4</td>
<td>0.27</td>
</tr>
<tr>
<td>$E9$</td>
<td>6.17</td>
<td>76.6</td>
<td>0.30</td>
</tr>
<tr>
<td>$E10$</td>
<td>6.81</td>
<td>87.0</td>
<td>0.28</td>
</tr>
<tr>
<td>$E11$</td>
<td>14.14</td>
<td>27.2</td>
<td>0.23</td>
</tr>
<tr>
<td>$E12$</td>
<td>5.57</td>
<td>62.9</td>
<td>0.24</td>
</tr>
<tr>
<td>$E13$</td>
<td>3.65</td>
<td>10.2</td>
<td>0.22</td>
</tr>
<tr>
<td>$E14$</td>
<td>7.37</td>
<td>95.1</td>
<td>0.25</td>
</tr>
</tbody>
</table>

Figure 5. Relationship of rainfall intensity (I, mm h$^{-1}$) and energy ($KE_0$, J m$^{-2}$ mm$^{-1}$) based on measurements at 1 min intervals. The smooth lines indicate locally weighted scatterplot smoothing (LOESS) model results for $KE_0$ vs. $I$. KE estimates assume theoretical terminal fall.
velocity ($KE_i$, Atlas and Ulbrich, 1977) and different empirical models ($E1$ to $E14$). Color legend is the same as in Figure 4.
We found similar results for the empirical models E1 to E14, all of which led to over-
estimates (Figure 5). The coefficient of determination ($R^2$) was low at 1 min resolution
($\sim 0.25$) but quite good at the event scale ($\sim 0.95$), and the different equations produced similar
results (Table 4). However, in all cases except E7, the empirical equations over-estimated the
observed kinetic energy ($KE_0$), as shown by positive MAE and PBIAS values. The bias
ranged from 10.2% (E13) to 196.1% (E6) at 1 min resolution, and from 0.9% (E13) to 57.7%
(E2) at the event scale. When the estimations were compared to the KE obtained by
calculation of theoretical terminal velocity instead of measured velocity ($KE_i$), the over-
estimation persisted for both scales. When 1 min observations were segregated by rainfall I,
we found much larger over-estimations of observed KE ($KE_0$) at high I ($I > 60$ mm h$^{-1}$),
although this did not occur for the KE estimates from theoretical terminal velocity ($KE_i$).

Our conclusions were similar for a comparison of observed and estimated mean KE per event
(Figure 6 and 7). In general, mean event KE estimations that assumed a theoretical terminal
velocity ($KE_i$ to $KE_3$) yielded the best approximations to the observed cumulative KE, despite
some over-estimation, although the empirical models led to greater over-estimation and a lack
of inter-event variability. Total event KE estimations by empirical models therefore depended
mostly on the duration of the event (Figure 7). The exponential KE–I parameterization
provides KE minima and maxima, with little variation between these limits.
Figure 6. Mean event kinetic energy ($KE_0$, X-axis) versus KE estimates (Y-axis) at the event scale.
Figure 7. Cumulative rainfall kinetic energy at the event scale ($KE_0$, X-axis) versus KE estimates (Y-axis).

3.3 Influence of time aggregation

Most of the KE–I equations were calibrated with data obtained at 1 min sampling intervals (Table 1), but data for some of these equations were collected at coarser time resolutions. Our analysis showed that using data at a different time aggregation than that used in calibrating an equation had a large influence on KE estimations. Figure 8 shows the results of comparing observed total KE ($KE_0$) by event with KE estimations using E1 (Brown and Foster, 1987, KE–I equation) at different time aggregations (from 1 to 60 min). As the aggregation interval
increased, there was an increasing underestimation of $KE_0$. Time aggregation did not allow for comparisons between equations E1…E14. The resulting KE estimates varied slightly among these equations as data aggregation time increased. This result indicates that there is little difference in which KE–I equation is used, since time aggregation had a much stronger effect.

Table 5 shows the results of using correction factors for the regression analysis between E1 and estimated rainfall intensity data with different time resolutions and observed $KE_0$ at a resolution of 1 min. Linear correction improved the fits, with $R^2$ values ranging from 0.87 to 0.90. The cumulative KE over the whole period was also well estimated, although there was a slight overestimation that ranged from 1.4% (1 min interval) to 7% (60 min interval).

Figure 8. Estimated kinetic energy (E1) at different time-aggregations vs. observed kinetic energy from 1 min sampling.
Table 5. Regression functions used to convert between E1 (Brown and Foster, 1987) at 1, 5, 10, 15, 30, and 60-min resolution and 1 min observed rainfall kinetic energy (KE0). The regression statistics were $R^2$ and SE, and the overall statistics were cumulative KE0 and E1, and PBIAS.

<table>
<thead>
<tr>
<th>Data resolution</th>
<th>N events</th>
<th>Regression function</th>
<th>$R^2$</th>
<th>se</th>
<th>Cum KE0</th>
<th>Cum E1</th>
<th>Cum PBIAS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-min</td>
<td>821</td>
<td>E1_m = 0.83*E1</td>
<td>0.90</td>
<td>523.9</td>
<td>615516.7</td>
<td>624607.9</td>
<td>1.4%</td>
</tr>
<tr>
<td>5-min</td>
<td>821</td>
<td>E1_m = 3.82*E1_5</td>
<td>0.88</td>
<td>507.0</td>
<td>612913.0</td>
<td>646521.0</td>
<td>5.4%</td>
</tr>
<tr>
<td>10-min</td>
<td>821</td>
<td>E1_m = 7.45*E1_10</td>
<td>0.87</td>
<td>519.8</td>
<td>605821.4</td>
<td>644601.8</td>
<td>6.4%</td>
</tr>
<tr>
<td>15-min</td>
<td>799</td>
<td>E1_m = 10.68*E1_15</td>
<td>0.85</td>
<td>561.8</td>
<td>597875.8</td>
<td>628340.2</td>
<td>5.1%</td>
</tr>
<tr>
<td>30-min</td>
<td>674</td>
<td>E1_m = 22.00*E1_30</td>
<td>0.87</td>
<td>572.9</td>
<td>566942.9</td>
<td>607320.7</td>
<td>7.1%</td>
</tr>
<tr>
<td>60-min</td>
<td>482</td>
<td>E1_m = 44.28*E1_60</td>
<td>0.87</td>
<td>651.2</td>
<td>499988.0</td>
<td>535742.3</td>
<td>7.1%</td>
</tr>
</tbody>
</table>

4. Discussion

Modern optical disdrometers are becoming more affordable, and are calibrated and ready for deployment in the field upon purchase, thereby simplifying use and facilitating comparisons. These instruments provide measurements of raindrop size and velocity, from which kinetic energy (KE) can be determined. The results of this study, similar to previous studies (e.g. Van Dijk et al., 2002), highlight the large variability in the size, velocity, and KE of raindrops among rainfall events that have similar intensity. This means that whenever possible, KE should be measured rather than estimated from empirical KE-I relationships.

When we compared the observed raindrop velocities to their theoretical values from 3 different models, there was a large dispersion and relatively high density of raindrops that deviated significantly from the models. Thus, a larger than expected number of small drops fell at velocities greater than expected by theory, and a significant number of large drops fell at lower velocities than expected by theory. Overall, this led to over-estimation of KE by the
theoretical models. Similarly, Cerro et al. (1998) and Petan et al. (2010) reported an under-
estimation of raindrop velocity by up to 25% from optical disdrometers relative to predictions
from theoretical terminal velocities.

Measurement errors and meteorological factors may have contributed to the discrepancies
between the observed and expected size and velocity distributions of raindrops. Standard
optical disdrometers, such as those used in this study, are prone to errors from various
sources. Raindrop breakup may occur when drops impact the device, resulting in several
smaller drops that bounce and cross the laser beam at abnormally high velocities. Also,
especially at high rainfall intensities, two or more raindrops may cross the laser beam
simultaneously and be recorded as a single large drop travelling at sub-terminal speed. High
wind causing non-vertical raindrop trajectories may also bias the estimations of velocity. Niu
et al. (2010) reported that instrumental bias led to an overestimation of velocity for small
raindrops (diameter of ~0.3 mm), but this decreased sharply with increasing drop size. These
measurement errors could be possibly circumvented by use of more refined optical devices,
such as 2D-video disdrometers, which measure raindrop velocity from two different angles
and provide better discrimination of raindrops (Schönhuber et al., 2008).

Precipitation microphysics processes, such as break-up and coalescence, may have also
contributed to the discrepancy of observed and theoretical raindrop velocities. Small super-
terminal drops may actually be secondary drops that result from the natural fragmentation of
larger drops due to air drag forces; these would initially have a high velocity and then achieve
a new terminal velocity due to aerodynamic constrains. On the other hand, coalescence of
small drops into larger drops, which maintain smaller-than-theoretical velocities until
acceleration to a new terminal velocity, may explain our observations of sub-terminal large
drops. Raindrop break-up and coalescence are natural processes that occur during rainfall
events (Montero-Martínez et al. 2009; Niu et al., 2010), especially very intense events. We found that more intense events had more sub-terminal raindrops (Figures 2 and 3). Also differences in air density, wind shear from turbulent flows, or the inertial acceleration of particles in the presence of atmospheric turbulence can cause raindrop velocities to deviate from that expected by theory (Pinsky and Khain, 1996).

The KE–I relationship depends on geographical and meteorological factors. Thus, 8 of the 14 KE–I relationships that we evaluated were calibrated by or near the coast (i.e. E2, E3, E5, E13). Previous research suggested that the presence of lower KE in coastal regions was a consequence of temperature attenuation due to the nearby water (Van Dijk et al., 2002). Rosewell (1986) proposed two alternative equations (E11 and E12) for coastal and inland locations in Australia, with larger values of \( e_{\text{max}} \) (eq. 6) for the latter. Nevertheless, more recent studies of coastal regions (E2, Cerro et al., 1998; E3, Coutinho and Tomás, 1995; E13, Sánchez-Moreno et al, 2012) provided higher \( e_{\text{max}} \) values. In fact, the parameter \( e_{\text{max}} \) for equations E1, E7, and E12 was calibrated at interior locations and had lower values than those calibrated near the coast.

Altitude may also affect the distribution of raindrop size and velocity. Blanchard (1953) and McIsaac (1990) reported that lower KE is expected at higher elevations. Blanchard (1953) suggested that small drops evaporate as they fall over progressively longer distances, so that only larger drops reach the soil at lower elevations and leading to higher observed KE at lower rain rates. This may also help explain the larger values of \( e_{\text{max}} \) for equations calibrated near the coast. However, this hypothesis is not straightforward. A study in Slovenia (Petan et al., 2010, E9 and E10) showed higher KE values at more elevated sites for the same rain rate, although the authors did not explain the reason for this difference. Differences in rainfall generating regimes due to complex orography and other geographical characteristics together
with differences in instrumental sensitivity might also contribute to the differences in these parameters (Angulo-Martínez and Barros, 2015).

Van Dijk et al. (2002) proposed a universal KE–I relationship (E14) based on the re-analysis of many KE–I relationships determined throughout the world. In general, they found that the KE–I parameters had low values therefore proposed an universal equation with lower values of the parameters, especially in $e_{\text{max}}$, than the ones evaluated here.

Apart from spatial variations, there are also temporal variations in the KE-I relationship. Thus, the KE-I relationship changes within events as well as between events, because of changes in rainfall microphysics, including genetic rainfall processes and raindrop interactions in the path between the cloud and ground. Assuming a fixed KE-I relationship is therefore an oversimplification, and should be avoided in studies that assess the ability of rainfall to erode soil. For instance, recent work by Todisco (2014) and Wang et al. (2016) highlighted the importance of temporal intra-storm patterns of rainfall intensity in modulating erosion.

At the event scale, however, several KE-I relationships performed reasonably well in estimating total KE values. This is because the total amount of rain is much more important than within-event variations of rainfall intensity for event scale analysis. Hence, even though KE-I relationships performed poorly in predicting the energy expenditure per mm of rain (Figure 6), the cumulative KE over events was accurately predicted (Figure 7).

Evaluation of the effect of rainfall intensity time-aggregation when estimating KE by the KE–I relationships showed that the estimations were best when rainfall intensity data was aggregated as close as possible to the sampling interval used for calibrating the equation. Panagos et al. (2015) corrected for this underestimation by using a multiplier in their analysis.
of rainfall erosivity over Europe when using data from different climate and hydrographic agencies that had different time resolutions.

5. Summary and conclusions

We compared rainfall kinetic energy (KE) measurements obtained by an optical disdrometer over 4 years with estimations from 14 KE-I (rainfall intensity) relationships. KE measurements are difficult to obtain, and are usually restricted to research studies that are limited by time and space. Thus, most researchers use theoretical or empirical relationships, depending on data availability. However, none of these empirical expressions can be used universally, and their use far from the geographic and meteorological conditions where they were calibrated is very limited.

In our case, although we obtained reasonable KE estimates from some of the KE-I relationships when the total event KE was considered, a closer evaluation of KE estimates using a more stringent time resolution showed large deviations from measured KE in terms of magnitude and intra-storm temporal variability. Estimation of KE at the event scale is sufficient for most studies, such as soil erosion studies that need to calculate mean rainfall erosivity coefficients. However, even in such cases, it is advisable to check which KE-I relationship performs best for the local conditions, meaning that disdrometric measurements are required. In our study of the inner Ebro Basin, the KE-I equation of Brown and Foster (1987) performed quite well for predicting total KE, but this result should not be generalized to other regions.

The use of a fine time resolution (e.g. 1 min sampling) for determination of KE and KE-I variability is needed for experimental rainfall erosivity studies. For that purpose, empirical KE-I relationships perform poorly, and are not recommended. In fact, the KE-I relationship
usually changes during a rainfall event, depending on rainfall microphysics. Consequently, only direct observation of raindrops size and velocity, as obtained by an optical disdrometer, can be recommended.

Finally, the need for careful performance of data time-aggregation when using empirical KE-I relationships must be stressed. A simple linear correction can be easily used to correct for data aggregation effects provided that data at different time scales are available.

Acknowledgements

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