

Lorenzo Peña

Review of:  
Nicola Grana

*Sulla teoria delle valutazioni di N.C.A. da Costa  
and  
Contraddizione e incompletezza*

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Nicola Grana, *Sulla teoria delle valutazioni di N.C.A. da Costa*. Naples: Liguri Editore, 1990. Pp. 75. (ISBN 88-207-1942-8.)

Nicola Grana, *Contraddizione e incompletezza*. Naples: Liguri Editore, 1990. Pp. 70. (ISBN 88-207-1943-6.)

These two new little books by the Italian logician N. Grana provide us with new insights into some technical and philosophical aspects of paraconsistent logics, esp. of logics belonging to the «Brazilian» current initiated by N. da Costa. Both booklets are very well written, very clear, quite accessible to the layman and yet full of interesting content which emerges as truly novel to most logicians, let alone to such as don't devote their time to searching into logical subjects.

The former booklet goes into the semantics of da Costa's calculi  $C$ , which constitute the first important kind of paraconsistent copulative sentential logics to have been proposed. (Paraconsistent logic in general drop the Cornubia rule, namely:  $p, \text{not-}p \vdash q$ . Copulative logics are such as have the adjunction rule, namely:  $p, q \vdash p \wedge q$ .) The outstanding characteristic of the  $C$  calculi — as against, e.g., some relevant paraconsistent systems, such as Sylvan's, or transitive logic, which is a tensorial infinite-valued nonArchimedean system — lies in both rejecting the law of noncontradiction and yet possessing a strong negation defined as the thus negated formula's entailing some higher order contradiction. (The two other mentioned trends do instead contain the principle of noncontradiction and either lack any strong negation (relevant logics) or else introduce strong negation through a quite different procedure, by means of a primitive over-assertive functor read as '[it is] wholly [the case that]'.) Within da Costa's [meta]logical framework contradiction orders are characterized recursively:  $\langle p \wedge \neg p \rangle$  is of first order;  $\langle (p \wedge \neg p) \wedge \neg (p \wedge \neg p) \rangle$  is of second order, etc. The limit of those calculi is system  $C\omega$  with no strong negation. As semantics is concerned, the most useful, natural modelisation proposed for those systems is one wherein [simple] negation is not truth-functional: if  $v(p)=0$ , then  $v(\neg p)=1$ , but when  $v(p)=1$ , the value of  $\langle p \rangle$  may be either 0 or 1. Such is the kind of non-truth-functional 2-valued valuations, which N. Grana studies carefully in his booklet.

The author also goes into applications of da Costa's valuations method to other logical systems such as proving decidibility or reaching new decision procedures as regards intuitionistic logic, several deontic and temporal logics, finite-valued logics and some relevant logics. This little book contains useful information on those results. Its only defect is that it is too sketchy on higher level issues and developments and silent on how da Costa's calculi relate to other paraconsistent systems. The bibliography ought to have been enlarged with some omitted reviews.

The latter booklet goes into the relationship between logical systems which either lack noncontradiction or contain negations of [instances of] that principle and systems which do the same as regards the principle of excluded middle (the latter being the *paracomplete* logics, such as, most prominently, Heyting's intuitionistic logic.) It also puts forward some considerations linking the use of certain non-alethic systems (systems both paraconsistent and paracomplete) to a broad range of issues, such as: some views in physics and philosophy of science (Schrödinger, H. Weyl, Bachelard); some accounts of AI (artificial intelli-

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gence); treatment of lacunae and dilemmas in deontic and normative systems; coping with paradoxes; and so on. The author's leaning towards relativism clearly emerges when he brands what he calls «privileging the position of strong logos» (p. 58) and when he denounces any absolute truth whether for us or in itself. But of course the reader can profit from studying the book even if he keeps clear of such truth-relativism. Again some comparisons with other paraconsistent approaches would have been welcome.