Extreme amplitude pulse pairs in a laser model described by the Ginzburg-Landau equation

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We have found new dissipative solitons of the complex cubic-quintic Ginzburg-Landau equation with extreme amplitudes and short duration. At certain range of the equation parameters, these extreme spikes appear in pairs of slightly unequal amplitude. The bifurcation diagram of pulse amplitude versus dispersion parameter is constructed. © 2015 Optical Society of America

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The concept of dissipative soliton [1] helps to understand and to model passively mode-locked lasers [2]. It serves as a useful tool for the description of various modes of laser operation [2] and even for prediction of new effects [3] in simulating pulse generation by such lasers. The most common mathematical implementation of a dissipative system is the complex cubic-quintic Ginzburg-Landau equation (CCQGLE) [2]. Despite its deceptive simplicity, this equation admits highly complicated solutions. The complexity of its solutions literally created the world of the Ginzburg-Landau equation [4]. One of the reasons for such complexity is that this equation has several free parameters. Changing just one of them by a small amount may dramatically change the nature of the solution. Here, we are interested only in localized solutions or dissipative solitons [1]. A plethora of different types of dissipative solitons have been studied so far: stationary solitons [5,6], pulsating solitons [7,8], exploding solitons [9], creeping solitons [3,10], compos- ite solitons [11,12] and multisolitons [13,14]. Our present work shows that the list is still far from being complete.

The analysis of the whole set of parameters where soliton solutions can be found is a tedious task. Sophisticated searching numerical schemes such as the one described in the recent work [15] may help to some extent, but still there is always the possibility of missing some interesting regimes. Here, we have found, numerically, a regime of pulse generation which is characterized by an extreme growth of the pulse amplitude. The most impressive feature of this regime is that the process happens in a self-organized manner while the parameters of the system remain constant. Any localized initial profile reasonable close to the solution converges to it.

Up to now, there have been two models predicting the existence of waves with extreme amplitudes in dissipative systems [16,17]. Both are lumped models. The one developed in [16] relates the generation of very high amplitude pulses to the multiple collision of several dissipative solitons coexisting in the cavity. This model has been confirmed experimentally in [18,19]. The model developed in [17] operates with pulses that evolve chaotically in the cavity and, occasionally, may reach much higher amplitudes than the average amplitude of the evolving pulse. This last model roughly corresponds to the experimental observations of [20]. Presently, there are no studies of distributed models predicting the existence of a single pulse that can reach extremely high amplitudes. The present work fills this gap.

To be specific, we deal with the cubic-quintic complex Ginzburg-Landau equation, which in its standard form reads [2]:

\[
i\psi_z + \frac{D}{2} \psi_{tt} + |\psi|^2 \psi + \nu |\psi|^4 \psi = i\delta \psi + i\epsilon |\psi|^2 \psi + i\beta \psi_{tt} + i\mu |\psi|^2 \psi, \tag{1}\]

where for passively mode-locked lasers, \(t\) is the normalized time in a frame of reference moving with the group velocity, \(\psi\) is the complex envelope of the optical field and \(z\) is the propagation distance along the unfolded cavity. The meaning of the equation parameters on the left hand side is the following: \(D\) denotes the cavity dispersion, being anomalous when \(D > 0\) and normal if \(D < 0\), and \(\nu\) is the quintic refractive index coefficient. Dissipative terms are written on the right-hand-side of (1), and the meaning of the corresponding parameters is the following: \(\delta\) denotes linear gain/loss, \(\beta\) is the gain bandwidth coefficient, and \(\epsilon\) and \(\mu\) are the cubic and quintic gain/loss coefficients, respectively.

To make sure that the results we are presenting here are correct, for numerical simulations we used two different programs and three different computing platforms. We obtained the same result in all cases. Namely, we used a split-step technique with a fourth-order Runge-Kutta algorithm for solving the nonlinear part of the equation, while the linear part was solved in Fourier space. Modifications of this technique [21] have also been used. We also used step sizes, small enough to be sure that the parts of the solution with sharply growing amplitudes
are finely discretised along both $z$ and $t$ axes. We used up to 524288 mesh points in transverse direction that is sufficient to have practically zero values of the envelope function at the edges of the numerical grid. With this choice, the spectrum of the localized solutions are also well sampled in Fourier domain.

By solving Eq.(1) numerically for the set of parameters given in the caption of Fig.1 we have found quite unusual pulsations of a single pulse. The unusual feature of these pulsations is that they lead to a sharp growth of the amplitude of the pulse followed by an even sharper drop. The duration of the extra-high amplitude part of the pulse is significantly shorter than the period of pulsations. The amplitude of the pulse in this part is also much higher than the amplitude of the pedestal which is around 15. A second extreme growth, slightly different, follows immediately the first one, while the rest of the period the situation remains basically stationary. So for the above equation parameters the spikes do appear by asymmetric pairs, as the one shown in Fig.1.

This figure allows us to appreciate the dramatic increase of the pulse amplitude at two points of evolution. The sharp growth can be considered as unexpected, as during most of the period, the pulse amplitude varies slightly remaining near 15. We can refer to this part of the pulse as “pedestal”. The width of the extreme pulse is much narrower than the width of the pedestal. The same data are presented in a false color plot in Fig.2. The two red spots in this figure represent the two extreme pulses forming the pair. After the sharp growth up to the maximal value, the central amplitude sharply drops. The violet areas after each red spot indicate the drop in amplitude to a value $\approx 2$ which is much lower than the average amplitude of the pedestal. It is even more surprising that the extreme pulse generation is repeated after a short interval following the first peak.

The whole evolution is periodic with two extreme maxima in each period. Figure 3 shows the evolution of the maximal amplitude of the soliton at its center, i.e. at the point $t = 0$. The period of pulsations is much longer than the duration of each peak. It is also longer than the delay between the two peaks. The pair generation is repeated indefinitely. Once the parameters of the equation are fixed, the evolution pattern becomes well defined. Figure 3 also shows that the amplitudes of the two extreme peaks are comparable but not exactly the same. The amplitude of the first peak is slightly higher than the amplitude of the second one.

Another observation from Figure 3 is that the amplitude of the extreme peaks in the solution are approximately 5 times higher than the amplitude of the pedestal. This means that the intensity amplification of the peak is around 25. This creates interesting possibilities for potential practical applications of this effect. A single laser oscillator can be used for generation of high amplitude ultra-short pulses without additional compressors provided that the correct parameters of the system are
We have also calculated the total pulse energy $Q(z)$ which is $\int_{-\infty}^{\infty} |\psi(z, t)|^2 dt$. The corresponding plot is shown in Fig.4. The two local maxima in this plot correspond to the two amplitude peaks in the previous figures. The total energy grows due to the appearance of each extreme peak. However, the energy of the second peak is lower than the energy of the first one. This is in accordance with the fact that the amplitude of the second peak is lower than the amplitude of the first one. The major fraction of energy is contained within the pedestal. The increase of energy when the first extreme pulse appears is around 30% of the total energy of the pulse. Still, this is a significant amount which allows us to conclude that the central peak is highly energetic.

The evolution of the pulse spectrum is shown in Fig.5 in 3D-format. The $z$-interval of this plot is the same as in Fig.1. It includes the two extreme peaks of the pair. The extreme peaks obviously cause the widening of the spectrum. Two side-bands are clearly seen on each side of the main spectral peak. They vary in size from the first peak to the second one. These sidebands are not related to periodicity of the overall evolution. They quickly vanish when the high peaks disappear.

The presented regime of pulse generation is periodic with high accuracy. Pair of bound pulses within each period is an unusual feature of this periodic solution. As in other dissipative systems, periodic motion here should be considered as relaxation oscillations. These are usually sensitive to the parameters of the system. Changing any of them leads to drastic changes of the oscillation dynamics. Fig.6 shows the bifurcation diagram of the maximum amplitude of the pulses when the dispersion parameter $D$ is changed from $-2.68$ to $-2.49$. Let us recall that the previous figures correspond to the value $D = -2.5$. At this point, we have two maximal amplitudes that are shown by two dots above the other on this diagram. Reducing $D$ down to the value $\approx -2.535$ we observe the smooth increase of the two amplitudes.

The two peaks with maximal amplitudes still can be observed up to the point $D \approx -2.64$. At this point, another bifurcation leads to merging of two peaks into one.

An example of the pulse amplitude evolution for this case is shown in Fig.7. Thus, we have a single curve on the bifurcation diagram. The point $D \approx -2.664$ signifies the bifurcation to a chaotic state where the amplitude from one period to another changes chaotically. This would be the region where we can look for optical rogue waves [17,20,22,23].
of the pair of two peaks within a period into a single one. Still, the motion remains periodic in z although the period is changing from roughly 0.27 to 0.045, i.e. it is reduced approximately 6 times. Further increase of $\epsilon$ up to $\epsilon = 0.044$ leads to a chaotic pulse evolution while a small decrease of $\epsilon$ from 0.04 to $\epsilon = 0.038$, results in pulse decay and its subsequent disappearance.

![Fig. 7. Soliton amplitude evolution for $D = -2.65$. There is only one high peak in each period.](image)

In conclusion, we have found the region of parameters of the complex cubic-quintic Ginzburg-Landau equation that produces extremely high and short pulses. Although we considered a continuous model of a passively mode-locked laser, it may have an analog in the corresponding lumped model. Experimental confirmation of such effect would be highly desirable as it may provide a source of extremely high amplitude and short duration pulses from a single laser oscillator.

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