Affirmative Action through Minority Reserves: An Experimental Study on School Choice

Flip Klijn, Joana Pais, and Marc Vorsatz

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Background and Motivation

School choice: model of many–to–one matching markets where only one side is strategic.
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Concern for diversity has led to attempts to implement affirmative action in school choice programs.
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Abdulkadiroğlu and Sönmez (2003) and Abdulkadiroğlu (2005) propose a cap or **maximum quota** on the number of students from the same group that a school can admit.
Kojima (2012) shows that affirmative action policies based on **maximum quotas** can be detrimental to the very minorities they are supposed to help.
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Intuition: if there is a school that is mostly wanted by majority students, it may end up with unfilled seats and unassigned majority students may create competition for seats at other schools, thus hurting minority students.
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Possible solution: **minority reserves** (Hafalir et al., 2013).
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Possible solution: minority reserves (Hafalir et al., 2013).

Minority reserves: school gives higher priority to minority students up to the point when minorities fill their “reserved seats.” However, a school may assign some of its reserved seats to majority students provided that no minority student is interested.
Hafalir et al. (2013) adapt GS and TTC to minority reserves and show that strategy-proofness is preserved (i.e., no student can ever benefit by misrepresenting her preferences).
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For instance, Hafalir et al. (2013) show that

- GS with minority reserves (weakly) Pareto dominates GS with majority quotas and, considering minority students only, is not strictly Pareto dominated by the standard GS.
- for minority students, TTC with minority reserves is not strictly Pareto dominated by the standard TTC.
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**But:** experimental literature on school choice shows that despite strategy-proofness, students very often do not submit their true preferences.
If two strategy-proof mechanisms are perceived differently, then they may give rise to very different types and levels of non-truthful behavior.
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Therefore, whether or not affirmative action policies actually benefit minority students may well depend on how agents perceive the different mechanisms...

In our experimental study: four mechanisms, namely
- standard GS (GSs) and its counterpart with minority reserves (GSm);
- standard TTC (TTCs) and its counterpart with minority reserves (TTCm).
The school choice problem

Three schools: $s_1$, $s_2$, and $s_3$. Each school offers exactly two seats.

Six students look for a seat at one of three schools.

**Majority group (students):** $M_1$, $M_2$, $M_3$, and $M_4$.

**Minority group (students):** $m_1$ and $m_2$.

<table>
<thead>
<tr>
<th>Preferences</th>
<th>Priorities</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_1$</td>
<td>$M_2$</td>
</tr>
<tr>
<td>$s_1$</td>
<td>$s_1$</td>
</tr>
<tr>
<td>$m_1$</td>
<td>$M_3$</td>
</tr>
<tr>
<td>$s_2$</td>
<td>$s_2$</td>
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<tr>
<td>$m_2$</td>
<td>$M_4$</td>
</tr>
<tr>
<td>$s_3$</td>
<td>$s_3$</td>
</tr>
<tr>
<td>$M_4$</td>
<td>$M_2$</td>
</tr>
<tr>
<td>$M_3$</td>
<td>$M_1$</td>
</tr>
<tr>
<td>$M_1$</td>
<td>$m_1$</td>
</tr>
<tr>
<td>$M_2$</td>
<td>$m_2$</td>
</tr>
</tbody>
</table>

Centralized market: 1. students submit rank order lists, 2. a clearinghouse that uses one of the four mechanisms to assign students to schools.

For the mechanisms *with minority reserves*: each school *reserves one seat* for the minority group.
Gale-Shapley mechanisms (GSs and GSm)

Step 1. Each student sends an application to the school she ranked first.
Gale-Shapley mechanisms \((\text{GS}_s \text{ and } \text{GS}_m)\)

**Step 1.** Each student sends an application to the school she ranked first.

**Step 2.** Each school that receives at least 1 application acts as follows.

\((\text{GS}_s)\) It temporarily accepts the applicant with the highest priority. It also temporarily accepts the applicant with the highest priority among all remaining applicants (if any). The rest of the applicants (if any) are rejected.
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   (GSm) If the school receives no application from minority students, proceed as in Step 2 of GSs. If the school receives at least one application from minority students, then it temporarily accepts the minority applicant with the highest priority; it also temporarily accepts the applicant with the highest priority among all remaining (majority or minority) applicants (if any); the rest of the applicants (if any) are rejected.
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**Step 3.** Whenever a student is rejected by a school, she applies to the next highest ranked school.
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**Step 4.** Each school that receives at least 1 new application acts as follows.

(GSs) Among all new and retained applications, the school temporarily accepts the applicant with the highest priority. Among the remaining (new and retained) applications (if any), it also temporarily accepts the applicant with the highest priority (if any). The rest of the applicants (if any) are rejected.

( GSm ) The school considers the new and the retained applications. If none of these applications is from a minority student, proceed as in Step 4 of GSs. If there is at least one application from a minority student, the school temporarily accepts the minority applicant with the highest priority; it also temporarily accepts the applicant with the highest priority among all remaining (majority or minority) applicants (if any); the rest of the applicants (if any) are rejected.

Step 5. Steps 3 and 4 are repeated until there are no more rejections.

Matching becomes final.
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Step 1. Each student points to the school she ranked first.

\((TTCs)\) Each school points to the student with the highest priority.
\((TTCm)\) Each school points to the minority student with the highest priority.

There is at least one cycle of students and schools.

Each student in any of the cycles is matched to the school she is pointing to and the school’s number of available seats is reduced by one.
Step 2. Each unmatched student points to the school she ranks highest among all schools that still have available seats.
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(TTCm) A school that was not matched to a minority student before points to
- the unmatched minority student if there is one; otherwise it points to
- the majority student with the highest priority among all remaining students.

A school that was already matched to a minority student points to the student (minority or majority) with the highest priority among all remaining students.
Top Trading Cycles mechanisms (TTCs and TTCm)

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Step 3. Repeat Step 2 until all students are matched.
Procedures

- Eight sessions.
- Each session: two phases.
- In the two phases: different mechanisms. (So, 1 session = 2 mech’s.)
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- Subjects knew that
  - decisions in 1st phase would not affect their payoffs in 2nd phase;
  - they would not receive feedback nor information regarding the decisions of any other player.
Motivation

Experimental Design and Procedures

Hypotheses

Results

Conclusion

Procedures

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- Subjects knew that
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- First mechanism is explained.
- Each subject plays two games under the first mechanism: once in her ‘true’ and once in a ‘fictitious’ role. Subjects do not know when.
- Second mechanism is explained.
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Either the 1st or 2nd phase & true roles is payoff relevant (3 euros fee + 6/9/12 euros). Randomly determined by computer at the end.
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Null Hypothesis:

In all four mechanisms, preferences are revealed truthfully. Hence, all four mechanisms generate the student-optimal stable matching and are Pareto-efficient.
Alternative Hypothesis 1 (on truth-telling)

<table>
<thead>
<tr>
<th>Preferences</th>
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<tbody>
<tr>
<td>$M_1$</td>
<td>$M_2$</td>
</tr>
<tr>
<td>$s_1$</td>
<td>$s_1$</td>
</tr>
<tr>
<td>Second best match</td>
<td>$m_2$</td>
</tr>
<tr>
<td>Third best match</td>
<td>$s_3$</td>
</tr>
<tr>
<td>Fourth best match</td>
<td>$M_3$</td>
</tr>
<tr>
<td>Fifth best match</td>
<td>$M_2$</td>
</tr>
<tr>
<td>Sixth best match</td>
<td>$M_2$</td>
</tr>
</tbody>
</table>

Alternative Hypothesis 1:

*In the absence of minority reserves (i.e., GSs and TTCs),*
- $M_1$, $M_3$, and $m_1$ have same levels of truth-telling.
- $M_2$, $M_4$, and $m_2$ have same levels of truth-telling.
Alternative Hypothesis 2 (on truth-telling)

In case of minority reserves:

- Minority student $m_1$ will be assigned to her most preferred school in case she puts that school on the top of her reported preferences, independently of the behavior of the other students.
- As a consequence, minority student $m_2$ also experiences less strategic uncertainty. Moreover, still an advantage over majority students.

Alternative Hypothesis 2:

*Minority students report preferences truthfully more often in the presence than in the absence of minority reserves.*

*In both GS$m$ and TTC$m$, the level of truth-telling of student $m_1$ ($m_2$) is higher than the level of truth-telling of $M_1$ and $M_3$ ($M_2$ and $M_4$).*
Alternative Hypothesis 3: 
*The mechanisms with minority reserves generate more truth-telling among all students than the corresponding mechanisms without minority reserves.*
Alternative Hypothesis 4 (on stability)

<table>
<thead>
<tr>
<th>Student</th>
<th>Stable matching</th>
<th>( \mu_1 )</th>
<th>( \mu_2 )</th>
<th>( \mu_3 )</th>
<th>( \mu_4 )</th>
<th>( \mu_5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( M_1 )</td>
<td>( s_3 )</td>
<td>( s_3 )</td>
<td>( s_2 )</td>
<td>( s_1 )</td>
<td>( s_1 )</td>
<td></td>
</tr>
<tr>
<td>( M_2 )</td>
<td>( s_3 )</td>
<td>( s_2 )</td>
<td>( s_2 )</td>
<td>( s_2 )</td>
<td>( s_1 )</td>
<td></td>
</tr>
<tr>
<td>( M_3 )</td>
<td>( s_2 )</td>
<td>( s_2 )</td>
<td>( s_1 )</td>
<td>( s_3 )</td>
<td>( s_3 )</td>
<td></td>
</tr>
<tr>
<td>( M_4 )</td>
<td>( s_2 )</td>
<td>( s_1 )</td>
<td>( s_1 )</td>
<td>( s_1 )</td>
<td>( s_3 )</td>
<td></td>
</tr>
<tr>
<td>( m_1 )</td>
<td>( s_1 )</td>
<td>( s_1 )</td>
<td>( s_3 )</td>
<td>( s_2 )</td>
<td>( s_2 )</td>
<td></td>
</tr>
<tr>
<td>( m_2 )</td>
<td>( s_1 )</td>
<td>( s_3 )</td>
<td>( s_3 )</td>
<td>( s_3 )</td>
<td>( s_2 )</td>
<td></td>
</tr>
</tbody>
</table>

Expected payoff (in ECU) | 9 | 10.5 | 12 | 13.5 | 15

**Table:** Stable matchings.
Alternative Hypothesis 4 (on stability)

<table>
<thead>
<tr>
<th>Student</th>
<th>Stable under minority reserves matching</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\mu_1$</td>
</tr>
<tr>
<td>$M_1$</td>
<td>$s_3$</td>
</tr>
<tr>
<td>$M_2$</td>
<td>$s_3$</td>
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<tr>
<td>$M_3$</td>
<td>$s_2$</td>
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<tr>
<td>$M_4$</td>
<td>$s_2$</td>
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<tr>
<td>$m_1$</td>
<td>$s_1$</td>
</tr>
<tr>
<td>$m_2$</td>
<td>$s_1$</td>
</tr>
<tr>
<td>Expected payoff (in ECU)</td>
<td>9</td>
</tr>
</tbody>
</table>

**Table:** Stable under minority reserves matchings.

**Alternative Hypothesis 4:**

The probability of obtaining $\mu_4$ or $\mu_5$ ($\mu_4$, $\mu_5$, or $\mu_7$) relative to the other stable (stable under minority reserves) matchings is higher in the presence than in the absence of minority reserves.
In order to evaluate the success of the discriminatory policy, one definitively demands that the payoff to the minority students as a group is higher when minority reserves are present.

In our particular school choice problem, the effect of the minority reserves on the expected payoff of the majority group could be positive as well.

Alternative Hypothesis 5:

*No student is harmed by the presence of minority reserves.*
Truth-telling

Notation (2,3,1): ranking where a student lists her second most preferred school first, her least preferred school second, and her most preferred school last. (The other five strategies have similar interpretations.)

<table>
<thead>
<tr>
<th>Mechanism</th>
<th>Submitted ranking</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1,2,3)</td>
</tr>
<tr>
<td><strong>GSs</strong></td>
<td>0.39</td>
</tr>
<tr>
<td><strong>GSm</strong></td>
<td>0.39</td>
</tr>
<tr>
<td><strong>TTCs</strong></td>
<td>0.48</td>
</tr>
<tr>
<td><strong>TTCm</strong></td>
<td>0.34</td>
</tr>
</tbody>
</table>

Truth-telling is salient, but much lower than predicted by theory (Null Hypothesis).
Truth-telling

<table>
<thead>
<tr>
<th>Mechanism</th>
<th>Student</th>
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<tbody>
<tr>
<td></td>
<td>$M_1$</td>
</tr>
<tr>
<td>GSs</td>
<td>0.47</td>
</tr>
<tr>
<td>GSm</td>
<td>0.25</td>
</tr>
<tr>
<td>TTCs</td>
<td>0.53</td>
</tr>
<tr>
<td>TTCm</td>
<td>0.50</td>
</tr>
</tbody>
</table>

**Result 1.a.** The proportion of truth-telling of the minority students is the same as that of their majority counterparts in the absence of minority reserves.
### Truth-telling

<table>
<thead>
<tr>
<th>Mechanism</th>
<th>Student</th>
<th>( M_1 )</th>
<th>( M_2 )</th>
<th>( M_3 )</th>
<th>( M_4 )</th>
<th>( m_1 )</th>
<th>( m_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>GSs</td>
<td>0.47</td>
<td>0.37</td>
<td>0.37</td>
<td>0.27</td>
<td>0.44</td>
<td>0.37</td>
<td></td>
</tr>
<tr>
<td>GSm</td>
<td>0.25</td>
<td>0.30</td>
<td>0.43</td>
<td>0.14</td>
<td>0.76</td>
<td>0.44</td>
<td></td>
</tr>
<tr>
<td>TTCs</td>
<td>0.53</td>
<td>0.55</td>
<td>0.50</td>
<td>0.33</td>
<td>0.58</td>
<td>0.39</td>
<td></td>
</tr>
<tr>
<td>TTCm</td>
<td>0.50</td>
<td>0.42</td>
<td>0.21</td>
<td>0.07</td>
<td>0.55</td>
<td>0.30</td>
<td></td>
</tr>
</tbody>
</table>

**Result 1.a.** The proportion of truth-telling of the minority students is the same as that of their majority counterparts in the absence of minority reserves.

But:

**Result 1.b.** There is a positive effect of minority reserves on the level of truth-telling of \( m_1 \) only under the Gale-Shapley mechanism. There are no spillover effects on other students.
Stability

<table>
<thead>
<tr>
<th>Mechanism</th>
<th>Overall</th>
<th>Breakdown by stable matching ($\mu_1$ worst, ... , $\mu_5$ best)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$\mu_1$</td>
</tr>
<tr>
<td>GSs</td>
<td>0.3452</td>
<td>0.0125</td>
</tr>
<tr>
<td>GSm</td>
<td>0.1318</td>
<td>0.0099</td>
</tr>
<tr>
<td>TTCs</td>
<td>0.0460</td>
<td>0.0152</td>
</tr>
<tr>
<td>TTCm</td>
<td>0.1157</td>
<td>0.0320</td>
</tr>
</tbody>
</table>

**Result 2.a.**

The probability distribution over stable matchings obtained under GSm first-order stochastically dominates the one obtained under GSs. Conversely, the probability distribution over stable matchings under TTCs first-order stochastically dominates the one obtained under TTCm!
Stability under minority reserves

<table>
<thead>
<tr>
<th>Mechanism</th>
<th>Overall</th>
<th>Breakdown by stable under minority reserves matching</th>
<th>$\mu_4$</th>
<th>$\mu_5$</th>
<th>$\mu_7$</th>
<th>$\mu_4 + \mu_5 + \mu_7$</th>
</tr>
</thead>
<tbody>
<tr>
<td>GSs</td>
<td>0.3928</td>
<td>0.0109</td>
<td>0.1533</td>
<td>0.5249</td>
<td>0.1833</td>
<td>0.0109</td>
</tr>
<tr>
<td>GSm</td>
<td>0.3553</td>
<td>0.0037</td>
<td>0.0507</td>
<td>0.0070</td>
<td>0.2989</td>
<td>0.0107</td>
</tr>
<tr>
<td>TTCs</td>
<td>0.0530</td>
<td>0.0141</td>
<td>0.0009</td>
<td>0.0450</td>
<td>0.4075</td>
<td>0.4000</td>
</tr>
<tr>
<td>TTCm</td>
<td>0.2403</td>
<td>0.0154</td>
<td>0.1319</td>
<td>0.0587</td>
<td>0.2185</td>
<td>0.0570</td>
</tr>
</tbody>
</table>

**Result 2.b.**
The probability of obtaining $\mu_4$, $\mu_5$, or $\mu_7$ relative to the other four stable under minority reserves matchings is higher in GS$m$ than in GS$s$. Also, the probability distribution over stable under minority reserves matchings under TTC$s$ first-order stochastically dominates the one obtained under TTC$m$. 
Stability (2)

<table>
<thead>
<tr>
<th>Mechanism</th>
<th>Students</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$M_1$</td>
</tr>
<tr>
<td>GSs</td>
<td>0.1938</td>
</tr>
<tr>
<td>GSs$m$</td>
<td>0.5882</td>
</tr>
<tr>
<td>TTCs</td>
<td>0.3471</td>
</tr>
<tr>
<td>TTCs$m$</td>
<td>0.4859</td>
</tr>
</tbody>
</table>

Table: Average probability of belonging to a blocking pair.

Result 2.c.
Introducing minority reserves increases (decreases) the probability that the average majority (minority) student forms part of a blocking pair. Minority students are less likely to form part of a blocking pair when the GS mechanism is employed.
### Expected payoffs

<table>
<thead>
<tr>
<th>Mechanism</th>
<th>Overall</th>
<th>Students</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$M_1$</td>
<td>$M_2$</td>
</tr>
<tr>
<td><strong>GSs</strong></td>
<td>12.06</td>
<td>12.33</td>
</tr>
<tr>
<td><strong>GSm</strong></td>
<td>12.27</td>
<td>11.97</td>
</tr>
<tr>
<td><strong>TTCs</strong></td>
<td>12.70</td>
<td>13.44</td>
</tr>
<tr>
<td><strong>TTCm</strong></td>
<td>12.25</td>
<td>12.29</td>
</tr>
</tbody>
</table>

**Result 3.** Minority reserves harm majority students, but benefit minority students inasmuch as the average payoff of this group increases. The distribution for $m_1$ under GSm first-order stochastic dominates the one under GSs.
Summary

- TTC: adding minority reserves decreases levels of truth telling of all types of students. It seems minority reserves increase the difficulty to understand the induced game.

- GS: adding minority reserves only enhances truth-telling of some minority students. While there is almost no difference in the overall efficiency between the two mechanisms with minority reserves, GS mechanism tends to be more stable than TTC independently of whether minority reserves are employed or not. Moreover, when minority reserves are introduced in GS, the better stable matchings are reached more often.

- Price to pay for introducing minority reserves in GS: (1) stability decreases and (2) majority students are hurt (which should not happen in our particular setting).
Summary

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Summary

- TTC: adding minority reserves decreases levels of truth-telling of all types of students. It seems minority reserves increase the difficulty to understand the induced game.
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